Braneworld reheating in the bulk inflaton model

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(Received 5 January 2003; published 21 April 2003)

In the context of the braneworld inflation driven by a bulk scalar field, we study the energy dissipation from the bulk scalar field into the matter on the brane in order to understand the reheating after inflation. Deriving the late-time behavior of the bulk field with dissipation by using the Green's function method, we give a rigorous justification of the statement that the standard reheating process is reproduced in this bulk inflaton model as long as the Hubble parameter on the brane and the mass of the bulk scalar field are much smaller than the five-dimensional inverse curvature scale. Our result supports the idea that the brane inflation model caused by a bulk scalar field is expected to be a viable alternative scenario of the early universe.

DOI: 10.1103/PhysRevD.67.084014 PACS number(s): 04.50.+h, 98.80.Cq

I. INTRODUCTION

The braneworld scenario has opened up a new perspective on higher dimensional theory [1]. In particular, a model proposed by Randall and Sundrum (RS2) [2] has ignited active research on braneworld cosmology because of its attractive features [3–6].

An alternative scenario of the inflationary universe in the context of the RS2 model was proposed by Himemoto and Sasaki [7]. In this scenario, inspired by higher dimensional gravitational theory including a dilatonic scalar field or higher curvature terms, a five-dimensional Einstein-scalar system was studied. Considering a minimally coupled massive scalar field ϕ for a five-dimensional scalar field as a toy model, a solution of the field equation in the anti–de Sitter background with an inflating brane was found under the assumption of a separable form of solution. It was shown that a five-dimensional scalar field, which we call a bulk scalar field, can realize slow-roll inflation as long as $|m^2|/H^2 \ll 1$ is satisfied. Here, $|m^2|$ is the mass squared of the bulk scalar field and H is the Hubble parameter on the brane.

As a next step, in order to show the generality of this inflation model, we investigated the dynamics of a bulk scalar field without assuming a separable form of solution, focusing on the cases with $|m^2|\ell^2 \ll 1$ and $H^2\ell^2 \ll 1$. There, by analyzing the Green's function for the bulk scalar field, the late-time behavior of the bulk scalar field was clarified, and it was shown that the bulk scalar field effectively behaves as a four-dimensional scalar field Φ with mass $m_{\rm eff}=m/\sqrt{2}$ during both the slow-rolling phase $|m^2|/H^2 \ll 1$ and the rapidly oscillating phase $|m^2|/H^2 \gg 1$, irrespective of the initial field configuration [8]. Moreover, the lowest order back reaction to the geometry starting with the second order in the amplitude of ϕ was found to be consistently represented by a four-dimensional effective theory with the same four-dimensional scalar field Φ mentioned above. The overall normalization of Φ is related to ϕ by a simple scaling $\Phi = \sqrt{\ell}\,\phi$, where ℓ is the curvature radius of AdS_5. We showed that a bulk scalar field in the braneworld can mimic four-dimensional inflaton dynamics, at least as long as the Hubble parameter and the mass of the bulk scalar field are sufficiently small compared with ℓ^{-1}. We also studied the quantum fluctuations in this inflation model in order to see whether this model is observationally acceptable or not. Then it was shown that the correction due to the five-dimensional nature of the inflaton field is small, as long as $H^2\ell^2 \ll 1$ and $|m^2|\ell^2 \ll 1$ [9]. Kobayashi, Koyama, and Soda also considered the quantum fluctuations of a massless bulk scalar field in the context of this inflation model and obtained a consistent result [10].

In [11,12], assuming the dominance of a single oscillation mode, it was shown that braneworld reheating also mimics the four-dimensional standard process. In this short paper, in order to give a rigorous justification of the result obtained in [11,12], we investigate the late-time behavior of the evolution of a bulk scalar field with arbitrary, regular initial data using the Green's function method. We find that the argument of the correspondence between the five-dimensional and four-dimensional systems mentioned in the previous paragraph can be extended to include the reheating process.

The organization of this paper is as follows. In Sec. II, we review the basic picture of the reheating scenario after braneworld inflation driven by a bulk scalar field as proposed in [11]. In Sec. III, we consider the late-time behavior of the bulk scalar field on the brane by using the properties of the retarded Green's function. Then we explain how to construct the Green's function for a scalar field with dissipation. Subsequently, we discuss an effective description of the dynamics from the four-dimensional point of view and show that the conventional reheating process is effectively reproduced in the braneworld. Section IV is devoted to a summary and discussion.

II. MODEL OF REHEATING AFTER BRANEWORLD INFLATION

We briefly review the model for the reheating process after braneworld inflation driven by a bulk scalar field dis-

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cussed in Ref. [11]. We consider a five-dimensional Einstein-scalar system in a braneworld scenario of the RS2 type. The bulk geometry satisfies the five-dimensional Einstein equations

$$R_{ab}-\frac{1}{2}g_{ab}R+\Lambda_5 g_{ab}=\kappa_5^2[T_{ab}+S_{ab}\delta(r-r_0)], \quad (2.1)$$

where r is the coordinate normal to the brane, and the brane is assumed to be located at $r=r_0$ at the fixed point of Z_2 symmetry. S_{ab} represents the energy-momentum tensor composed of the brane tension σ and the contribution of matter fields confined on the brane τ_{ab}:

$$S_{ab}=-\sigma q_{ab}+\tau_{ab}, \quad (2.2)$$

where q_{ab} is the induced metric on the brane. T_{ab} is the energy-momentum tensor of the bulk scalar field. For simplicity, we assume a minimally coupled massive scalar field with an interaction on the brane. We set $\Lambda_5=-\kappa_5^4\sigma^2/6$ so as to recover the Randall-Sundrum flat braneworld when T_{ab} and τ_{ab} vanish.

In order to represent the energy dissipation from the bulk scalar field ϕ to the matter fields on the brane, the equation for ϕ is modified to

$$(\Box_5-m^2)\phi=\Gamma_d\ell\,\delta(r-r_0)\dot{\phi}, \quad (2.3)$$

where $\ell=6/(\kappa_5^2\sigma)$ is the AdS_5 curvature radius, and m is the mass of the bulk scalar field. An overdot indicates differentiation with respect to the cosmological proper time. Γ_d represents the decay constant. Integrating Eq. (2.3) together with Z_2 symmetry, we obtain the boundary condition

$$\partial_r\phi|_{r=r_0}=\frac{\Gamma_d\ell}{2}\dot{\phi}(t,r_0). \quad (2.4)$$

In the analyses below, we neglect the terms quadratic in Γ_d. Here, we do not consider the energy release to the bulk space, although such a dissipation process in the bulk may be worthy of investigation.

III. BULK SCALAR DYNAMICS WITH DISSIPATION

We consider solutions of Eq. (2.3) under the conditions

$$m\ell,\ H\ell,\ \text{and}\ \Gamma_d\ell\ll 1, \quad (3.1)$$

where m and H are, respectively, the mass of the bulk scalar field and the expansion rate of the four-dimensional metric induced on the brane. Then we investigate the late-time behavior of solutions by extending the Green's function method used in Ref. [8].

A. Initial value problem

We consider the field equation

$$\mathcal{L}_X\Psi(X)\equiv[-\partial_t^2+\partial_y^2-W(y)-U(y)\partial_t]\Psi(X)=0, \quad (3.2)$$

where X denotes $\{t,y\}$. We assume that $W(y)$ and $U(y)$ are functions whose asymptotic value at $|y|\to\infty$ is supposed to be a constant. We solve this field equation by imposing Z_2 symmetry at $y=0$.

Let us consider the retarded Green's function that satisfies

$$\mathcal{L}_X G(X,X')=-\delta(t-t')\delta(y-y'), \quad (3.3)$$

with the causal condition that $G(X,X')=0$ for X' not in the causal past of X. Note that this Green's function satisfies reciprocity for the time reversal operation $t\to -t$, namely, $G(t,y,t',y')=G(-t',y',-t,y)$. Using this reciprocity, the evolution of Ψ for any given initial data can be described as

$$\Psi=\int_{t'=t_i}\left[G(X,X')\frac{\partial\Psi(X')}{\partial t'}-\frac{\partial G(X,X')}{\partial t'}\Psi(X')+U(y')G(X,X')\Psi(X')\right]dy', \quad (3.4)$$

where t_i is the initial time. This representation of Ψ implies that the asymptotic behavior of Ψ can be understood by the late-time behavior of $G(X,X')$.

Let us now construct the Green's function that satisfies Eq. (3.3). Because of time translation invariance, we can express the Green's function in terms of Fourier transformation as

$$G(X,X')=\frac{1}{2\pi}\int_{\mathcal{C}}dp\,G_p(y,y')e^{-ip(t-t')}, \quad (3.5)$$

where we choose the path $\mathcal{C}$ so that the retarded condition is satisfied; namely, $\mathcal{C}$ runs from $p=-\infty$ to $p=+\infty$ on the complex plane for the integrand G_p to contain neither pole nor the branch cut above $\mathcal{C}$.

The equation for G_p follows from Eqs. (3.3) and (3.5) as

$$\mathcal{L}_y^{(p)}G_p(y,y')\equiv[\partial_y^2+p^2-W(y)+ipU(y)]G_p(y,y')=-\delta(y-y'). \quad (3.6)$$

The homogeneous equation $\mathcal{L}_y^{(p)}u(y)=0$ has two independent solutions, which respectively behave asymptotically as $u_p^{(\mathrm{out})}(y)\sim e^{ik|y|}$ and $u_p^{(\mathrm{in})}(y)\sim e^{-ik|y|}$ at $|y|\to\infty$, where we defined $k^2=\lim_{|y|\to\infty}p^2-W(y)+ipU(y)$. We can describe the solution satisfying the condition of Z_2 symmetry as a linear combination of these two independent solutions, $u_p^{(Z_2)}(y)=u_p^{(\mathrm{out})}(y)-\gamma_p u_p^{(\mathrm{in})}(y)$, where γ_p is determined by the Z_2 symmetry condition

$$\mathcal{D}[u_p^{(Z_2)}(z)]\equiv[\partial_y-\hat{W}+ip\hat{U}]u_p^{(Z_2)}(y)|_{y=0+}=0 \quad (3.7)$$

where $\hat{W}$ and $\hat{U}$ are the coefficients in front of $\delta(y)$ contained in $W(y)/2$ and $U(y)/2$, respectively. With these mode functions, we can express $G_p(y,y')$ satisfying the boundary condition at infinity and on the brane as

$$G_p(y,y')=\frac{1}{-2ip\gamma_p}[u_p^{(\rm out)}(y)u_p^{(Z_2)}(y')\theta(|y|-|y'|)+u_p^{(Z_2)}(y)u_p^{(\rm out)}(y')\theta(|y'|-|y|)] \tag{3.8}$$

with $\gamma_p=\mathcal{D}[u_p^{(\rm out)}(y)]/\mathcal{D}[u_p^{(\rm in)}(y)]$.

The late-time behavior of Ψ is understood by investigating the structure of singularities such as poles and branch cuts in $G_p(y,y')$. The singularity on the complex p plane with the largest imaginary part dominates the late-time behavior. In particular, poles exist at the values of p for which

$$\mathcal{D}[u_p^{(\rm out)}(y)]=0 \tag{3.9}$$

holds.

B. Application to braneworld

Using the formulas developed in the previous section, we consider the evolution of a massive bulk scalar field with dissipation, which well approximates the situation where ϕ oscillates at the bottom of the potential. We assume that the background spacetime is five-dimensional anti–de Sitter space with a boundary de Sitter brane given by $ds^2=dr^2+\{H\ell\sinh(r/\ell)\}^2(-dt^2+H^{-2}e^{2Ht}d\boldsymbol{x}_{(3)}^2)$ [4]. In order to use the formulas presented in the preceding subsection, we introduce the conformal coordinates $\{\tau,y\}$ defined by $\tau\equiv Ht$ and $R(y)\equiv\ell\sinh^{-1}(|y|+y_0)=\ell\sinh(r/\ell)$, where y_0 is specified by $\sinh(y_0)=\sinh^{-1}(r_0/\ell)=H\ell$. Then the metric becomes

$$ds^2=R(y)^2(dy^2-d\tau^2+e^{2\tau}\,d\boldsymbol{x}_{(3)}^2). \tag{3.10}$$

Setting $\phi=R^{-3/2}e^{-3\tau/2}\Psi(y,\tau)$, Eq. (2.3) reduces to Eq. (3.2) with

$$W(y)=\frac{15+4m^2\ell^2}{4\sinh^2(|y|+y_0)}-\left(\frac{3\sqrt{1+H^2\ell^2}}{H\ell}+\frac{3}{2}\Gamma_d\ell\right)\delta(y), \tag{3.11}$$

$$U(y)=\Gamma_d\ell\quad\delta(y). \tag{3.12}$$

We note that both $W(y)$ and $U(y)$ vanish for $|y|\to\infty$, and hence the conditions that they asymptotically become constant are satisfied. Although $U(y)$ is localized on the brane in the present model, dissipation in the bulk can also be discussed by using the same technique as long as $U(y)$ satisfies this property.

The mode solution satisfying the outgoing asymptotic behavior is

$$u_p^{(\rm out)}(y)=\Gamma(1-ip)P_{\nu-1/2}^{ip}[\coth(|y|+y_0)], \tag{3.13}$$

where $\nu=\sqrt{m^2\ell^2+4}$ [13]. In the present case, the operator defined in Eq. (3.7) reduces to

$$\mathcal{D}=\partial_y+\frac{3}{2}\frac{\sqrt{1+H^2\ell^2}}{H\ell}+\left(ip+\frac{3}{2}\right)\frac{\Gamma_d\ell}{2}. \tag{3.14}$$

Then, from Eqs. (3.9) and (3.13) under the condition (3.1), we obtain the location of the poles with the largest imaginary parts in the complex p plane as

$$p_\pm\approx\frac{1}{H}\left[-i\frac{\Gamma_d}{2}\pm\sqrt{m_{\rm eff}^2-\frac{(3H+\Gamma_d)^2}{4}}\right], \tag{3.15}$$

where

$$m_{\rm eff}^2=\frac{m^2}{2}. \tag{3.16}$$

Here the terms neglected are quadratic or higher order in ℓ. In addition to two poles, there is an infinite sequence of poles. However, they do not give a dominant contribution to the late-time behavior since they have smaller imaginary parts. (See Ref. [8].) Therefore the asymptotic behavior of the field ϕ is dominated by the poles $p_\pm$, and hence we have

$$\phi\propto e^{-3\tau/2}G(X,X')\propto e^{-(3/2+ip_\pm)Ht}. \tag{3.17}$$

C. Effective four-dimensional description

In the preceding section, we confirmed that the late-time behavior of a bulk scalar field on the brane can be effectively described by a four-dimensional scalar field Φ which satisfies

$$\ddot\Phi+(3H+\Gamma_d)\dot\Phi+m_{\rm eff}^2\Phi=0. \tag{3.18}$$

Our main interest here is to compare the law of cosmic expansion on the brane to that expected in the usual four-dimensional model with the scalar field Φ. First, we present a rather general framework to discuss the cosmic expansion of a homogeneous universe realized on the brane without specifying the explicit form of the energy-momentum tensor in the bulk.

We introduce a unit vector in the time direction u^a parallel to the brane. Toward the outside of the brane u^a is extended so as to satisfy $u^a{}_{;b}n^b=0$, where n_b is the outgoing unit normal vector of the brane. Then, considering integration of the five-dimensional conservation law

$$0=\int\sqrt{-g}\quad d^5x[S_a{}^b\delta(r-r_0)+T_a{}^b]_{;b}u^a \tag{3.19}$$

for a thin region surrounding the brane, we obtain

$$\dot\rho+4H\rho+H\tau^\mu{}_\mu=2T_{ab}u^an^b, \tag{3.20}$$

where $\rho:=u^\mu u^\nu\tau_{\mu\nu}$, and latin indices run over four-dimensional coordinates on the brane.

The conservation law for the matter field localized on the brane $\tau^{\mu\nu}{}_{;\nu}=0$ is violated because there is energy transfer between the brane and bulk fields. However, from the Bianchi identity the four-dimensional conservation law is guaranteed to be preserved in total. The four-dimensional effective Einstein equation is derived from Eq. (2.1) as [3]

$$ {}^{(4)}G_{\mu\nu}=\kappa_4^2(\tau_{\mu\nu}+\tau^{(\pi)}_{\mu\nu}+\tau^{(s)}_{\mu\nu}+\tau^{(E)}_{\mu\nu}), \tag{3.21} $$

where $\kappa_4^2=\kappa_5^2/\ell$ and

$$ \tau^{(\pi)}_{\mu\nu}=-\frac{\kappa_5^2\ell}{24}[6\tau_{\mu\alpha}\tau_\nu{}^\alpha-2\tau\tau_{\mu\nu}-q_{\mu\nu}(3\tau_{\alpha\beta}\tau^{\alpha\beta}-\tau^2)], $$

$$ \tau^{(s)}_{\mu\nu}=\frac{2\ell}{3}\left[T_{ab}q^a{}_\mu q^b{}_\nu+q_{\mu\nu}\left(T_{ab}n^an^b-\frac{1}{4}T^a{}_a\right)\right], $$

$$ \tau^{(E)}_{\mu\nu}=-\frac{\ell}{\kappa_5^2}{}^{(5)}C_{rbrd}\ \ q^b_\mu\ \ q^d_\nu. \tag{3.22} $$

Here ${}^{(5)}C_{rbrd}$ is the five-dimensional Weyl tensor with its two indices projected in the normal direction. Then we have

$$ \tau^{(\mathrm{tot})\mu\nu}{}_{|\nu}=0 \tag{3.23} $$

with

$$ \tau^{(\mathrm{tot})}_{\mu\nu}:=\tau_{\mu\nu}+\tau^{(\pi)}_{\mu\nu}+\tau^{(s)}_{\mu\nu}+\tau^{(E)}_{\mu\nu}. \tag{3.24} $$

Here the vertical bar represents covariant differentiation with respect to the four-dimensional induced metric $q_{\mu\nu}$. In a homogeneous universe, the temporal component of the conservation law reduces to

$$ \dot\rho^{(\mathrm{tot})}+4H\rho^{(\mathrm{tot})}+H\tau^{(\mathrm{tot})\mu}{}_\mu=0, \tag{3.25} $$

where $\rho^{(\mathrm{tot})}:=u^\mu u^\nu\tau^{(\mathrm{tot})}_{\mu\nu}$. Similarly, we define the energy density for each component by $\rho^{(i)}:=u^\mu u^\nu\tau^{(i)}_{\mu\nu}$. Then, using the facts that $\rho^{(\pi)}=\rho^2/2\sigma$, $\tau^{(\pi)\mu}{}_\mu=(\rho/\sigma)(\tau^\mu{}_\mu+2\rho)$ [3], and $\tau^{(s)\mu}{}_\mu=2\ell T_{ab}n^an^b$, we find that the effective energy density of the bulk field $\rho_{\mathrm{eff}}:=\rho^{(E)}+\rho^{(s)}$ satisfies

$$ \dot\rho_{\mathrm{eff}}+4H\rho_{\mathrm{eff}}=-2\left(1+\frac{\rho}{\sigma}\right)T_{ab}u^an^b-2H\ell T_{ab}n^an^b. \tag{3.26} $$

Now let us return to our specific model, in which the bulk energy-momentum tensor is given by

$$ T_{ab}=\phi_{,a}\phi_{,b}-g_{ab}\left(\frac{1}{2}g^{cd}\phi_{,c}\phi_{,d}+\frac{1}{2}m^2\phi^2\right). \tag{3.27} $$

Then we find

$$ T_{ab}u^an^b=\frac{\Gamma_d\ell}{2}\dot\phi^2, $$

$$ T_{ab}n^an^b=\frac{1}{2}\dot\phi^2-\frac{1}{2}m^2\phi^2, \tag{3.28} $$

where the terms quadratic in $\Gamma_d\ell$ were neglected. If we set the identification [8]

$$ \Phi=\sqrt{\ell}\ \phi, \tag{3.29} $$

from Eqs. (3.20) and (3.26), we obtain the formulas for the cosmic expansion valid in the low energy regime (3.1) as

$$ H^2=\frac{\kappa_4^2}{3}(\rho_{\mathrm{eff}}+\rho), $$

$$ \dot\rho_{\mathrm{eff}}+4H\rho_{\mathrm{eff}}=-H(\dot\Phi^2-4V_{\mathrm{eff}})-\Gamma_d\dot\Phi^2, $$

$$ \dot\rho+4H\rho+H\tau^\mu{}_\mu=\Gamma_d\dot\Phi^2, \tag{3.30} $$

where $V_{\mathrm{eff}}(\Phi)=\frac{1}{2}m_{\mathrm{eff}}^2\Phi^2=(\ell/4)m^2\phi^2$. This set of equations is the same as that satisfied by a four-dimensional model having a scalar field Φ with mass squared $m_{\mathrm{eff}}^2=m^2/2$ and decay width $\Gamma_d/2$. To conclude, provided that the conditions (3.1) are satisfied, the effective dynamics of the Einstein-scalar system on the brane is indistinguishable from a four-dimensional theory even if we consider energy dissipation from the bulk scalar field to the matter fields on the brane.

IV. SUMMARY AND DISCUSSION

We have investigated the late-time behavior of a bulk scalar field with dissipation to the matter fields on the brane. We have shown that a bulk scalar field observed on the brane effectively behaves as a four-dimensional scalar field. Furthermore, we have shown that the set of equations to determine the cosmic expansion is also indistinguishable from that of the corresponding standard four-dimensional model. This result reinforces the speculation previously presented in Refs. [11,12]. Although we have analyzed perturbatively only the dynamics of a bulk scalar field on a fixed five-dimensional anti–de Sitter background with a boundary de Sitter brane, our result suggests that four-dimensional inflaton dynamics including the reheating era is effectively reproduced by the dynamics of a five-dimensional scalar field. Thus the brane inflation model caused by a bulk scalar field is expected to be a viable alternative scenario of the early universe.

Genuine braneworld effects that can be used to test the scenario are, however, in the deviations from the standard model. Our analysis shows that the corrections are suppressed by a factor $H^2\ell^2$ or $|m^2|\ell^2$. We did not go into detail about this correction in this paper. Here we just mention that we have to consider the dynamics of a bulk scalar field with a general Friedman-Robertson-Walker (FRW) brane boundary when we discuss corrections of this order, because the expected suppression of the correction due to the variation of the expansion rate is of the same order: $\dot H\ell^2\sim H^2\ell^2$.

Another regime where $H^2\ell^2\gg1$ and $|m^2|\ell^2\gg1$ in this bulk inflaton model is also interesting. Even in this regime, slow-roll inflation seems to be realized on the brane as long as $|m^2|/H^2\ll1$ is satisfied [7], although many unsolved issues remain there.

Last, we should mention dissipation in the bulk. When we consider a specific well-motivated model, the bulk inflaton field may naturally have interaction with the other fields in the bulk. Once a model is specified, we will be able to estimate the strength of the coupling. Thus, investigating dissipation in the bulk might give an important constraint on the construction of a realistic model. The formulas developed in this paper might also be useful for this purpose. Investigations in this direction are in progress.

ACKNOWLEDGMENTS

We are grateful to M. Sasaki and J. Yokoyama for useful communications. T.T. would like to thank L. Sorbo and D. Langlois for useful discussions during his stay at IAP. To complete this work, the discussion during the YITP workshop YITP-W-01-15 on ''Braneworld—Dynamics of Spacetime Boundary'' was useful. This work was supported in part by the Monbukagakusho Grant-in-Aid No. 14740165 (T.T.).

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