Dependence of hadronic properties on quark masses and constraints on their cosmological variation

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We follow our previous paper on the possible cosmological variation of the weak scale (quark masses) and the strong scale, inspired by data on the cosmological variation of the electromagnetic fine structure constant from distant quasar absorption spectra. In this work we identify the *strange quark mass* m_s as the most important quantity, and the *sigma meson mass* as the ingredient of the nuclear forces most sensitive to it. As a result, we claim significantly stronger limits on the ratio of weak/strong scale ($W = m_s/\Lambda_{QCD}$) variation following from our previous discussion on primordial big-bang nucleosynthesis ($|\delta W/W| < 0.006$) and the Oklo natural nuclear reactor [$|\delta W/W| < 1.2\times 10^{-10}$; there is also a nonzero solution $\delta W/W = (-0.56 \pm 0.05)\times 10^{-9}$].

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I. INTRODUCTION

The understanding of how exactly the fundamental parameters of the standard model enter any observable is certainly one of the most important aims of hadronic or nuclear physics. Two old profound questions drive its discussion: (i) Can there be ‘‘alternative universes’’ with different sets of parameters, and what are the boundaries of the world we know in the parameter space? (ii) How do we observe the cosmological variations of the weak and strong scales?

The discussion of both issues has been significantly revived recently. We will not go into question (i) (see, e.g., [1]) and only mention the latter (ii). The issue of the cosmological time variation of major constants of physics has been recently revived by astronomical data which seem to suggest a variation of electromagnetic α at the 10^{-5} level for the time scale 10×10^{12} yr, see [2]. The statistical significance of the effect at the moment obviously excludes any random fluctuations, so the effect definitely exists. Whether it has or not a conventional explanation is not yet clear: more experimental work is clearly needed to reach any conclusions.

Nevertheless, it is quite timely to have another look at the existing limits on time variation of all the fundamental constants. In particular, since the electromagnetic and weak forces are mixed together in the standard model, one may expect a similar modification of the weak couplings, the weak scale in general and quark masses in particular. In fact, one can measure only variation of dimensionless parameters. Therefore, we obtain limits on variation of m_s/Λ_{QCD}, where m_s is the strange quark mass and Λ_{QCD} is the QCD scale defined as a position of Landau pole in logarithm for running coupling constant. It is convenient to put $\Lambda_{QCD} = \text{const}$.

A generic further argument goes as follows. The masses of three heavy quarks—c,b,t—are *too large* to be important in hadronic and nuclear physics. The masses of two light ones—u,d—are important, in particular, via the pion contribution to nuclear forces we studied in our previous paper [3] and a subsequent one [4]. Much more extensive discussion on this issue in context of chiral perturbation theory can be found in literature, see, e.g., [5], and references therein. The conclusion reached in those studies is that the m_u, m_d are *too small* to be really important.

Thus, we focus on the dependence on the *strange* quark mass m_s, the only one which has the right magnitude to generate a maximal sensitivity of hadronic or nuclear physics to the weak scale. Indeed, as follows from QCD phenomenology, its variation from 0 to experimental value influences vacuum parameters such as the quark condensates $\overline{q}q$ at the factor-2 level. It also affects the masses of even *nonstrange* hadrons such as the nucleon at about 20% level, etc. The fundamental explanation of why such unexpected ‘‘strangeness’’ is in fact present is related with the important role of the instanton-induced effects in QCD. As it follows from the chiral anomaly relation and is explained in details, e.g., in a review [6], the multifermion ’t Hooft interaction necessarily involves all three flavors at all steps, even in the interactions of light u,d quarks.

The main new element of the present work is the identification of the specific hadronic state (or set of $\pi\pi$ states), the σ or $f_0(600)$ meson by the Particle Data Group(PDG) Properties [10], as the important ingredient of the nuclear forces most sensitive to m_s. Strong variation of the phase shift at 400–600 MeV is known for a very long time in $\pi\pi$ scattering, and was also identified in the attractive part of the nuclear forces, see e.g. a review on applications of Walecka model [7]. Whether one would like to call it a resonance or not, the fact remains that it dominates the attractive part of NN interaction, and is thus responsible for nuclear binding and for our very existence.

The reader may take ‘‘sigma’’ in what follows as a short-hand for "a part of two-pion continuum with invariant mass 400–600 MeV contributing to NN scattering." However for completeness, we mention here some points in the ongoing debate about its acceptance as a real hadronic state. Nonrelativistic quark models have scalar as a $l=1$ or p state, and tend to predict its mass to be around 1.4 GeV or so, in striking contradiction to $m_\sigma \sim 500$ MeV. Also large width of σ makes this state to be easily deformed by all kind of effects, see e.g. recent work by one of us predicting drastic modification on the sigma shape in pp and heavy ion collisions [9]. On the other hand, recent data have elucidated sigma production in a set of much simpler situations from heavy quark hadrons, Υ transitions and D decays. Those consistently point toward the smaller width $\Gamma_\sigma \sim 250$ MeV.

In the next section, we will argue that the sigma mass has stronger sensitivity to m_s than that of ordinary nonstrange hadrons such as N or ω. This happens because of a valence strange part plus the repulsion from the nearby $\bar{K}K, \eta\eta$ continuum thresholds. We then estimate the derivative of the deuteron binding and neutron resonance energies to m_s.

II. SENSITIVITY OF HADRONIC MASSES TO VARIATION OF QUARK MASSES

A. Why sigma?

The very first appearance of the sigma mesons was as a two-pion scalar-isoscalar resonance. It has been gradually learned that the corresponding channel for $\bar{q}q$ interaction is the "maximally attractive channel," with attraction so strong that it breaks spontaneously chiral symmetry and produces the nonzero quark condensate. The mechanism of that attraction is attributed mostly to instanton-induced 't Hooft interaction, for a review, see [6].

Sigma meson is an excitation on top of the scalar condensate, a kind of a Higgs boson of strong interactions. If indeed one naively assumes that it underlines all hadronic masses, e.g., that of the nucleon, the corresponding coupling can be estimated as

$$g_s = M_n / f_\pi \approx 10. \tag{1}$$

This large value in turn implies that the perturbation theory can only be used as a qualitative guide at best.

In passing, let us mention that arguments about development of the most optimum effective description of hadronic or nuclear physics in terms of mesonic degrees of freedom, known also as quantum hadrodynamics, are still going on. Using some field variables can be better than others: in this respect let us mention the paper [8] which emphasized that instead of the traditional σ of the linear sigma model, a chiral partner of the pion, one can better use the radial field $\sqrt{\sigma^2+\vec{\pi}^2}$ which has normal derivative coupling to pions. There is extensive literature on loop corrections and related observables, such as resonance mass and shape modification in nuclear and excited hadronic matters, see, e.g., [9] as a recent example.

For the purpose of this work, it would be sufficient to use simple and widely used Walecka model, which keeps only the sigma and omega exchanges in the effective nuclear forces,

$$V = -\frac{g_s^2}{4\pi}\frac{e^{-rm_\sigma}}{r} + \frac{g_v^2}{4\pi}\frac{e^{-rm_\omega}}{r}. \tag{2}$$

The very important lesson about nuclear forces this model emphasizes is that the nuclear potential is in fact a highly tuned small difference of two large terms.

We will argue in the next section that there are reasons to think that the sensitivity of these two terms to the fundamental weak scale is quite different. Sigma (scalars) involve all quark flavors while omega (vectors) do not, forbidden by the Zweig rule. As a result, *there is no fine tuning in the derivative over* m_s, which significantly enhances the effect to be derived.

The values of the two coupling constants used in the nuclear matter applications of this model [7] are

$$g_s^2 = 357.4 m_\sigma^2 / m_N^2 \approx 100,$$

$$g_v^2 = 273.8 m_\omega^2 / m_N^2 \approx 190 \tag{3}$$

for $m_\sigma = 500$ MeV. Note that the effective scalar coupling is close to naive value (1) mentioned above.

B. Strange valence and strange sea of the σ meson

Scalar and pseudoscalar mesons are different from more familiar vectors and axial ones in terms of their flavor composition. In the latter case, the so called Zweig rule applies, forbidding flavor mixing: so, for example, the ω meson we will discuss in this work has a truly negligible mixing with a strange counterpart, ϕ meson. Scalar and pseudoscalar mesons are on the contrary nearly ideally $SU(3)$ octets and singlets, so different flavors are very strongly mixed together. The pseudoscalar channel has been studied extensively, and we know that in this case η' is much heavier than (twice more strange) η meson. This is the famous Weinberg $U(1)$ problem, resolved by existence of the instanton-induced repulsive interaction pushing the singlet state upward.

The same instanton-induced interaction has *opposite sign* in the $SU(3)$ singlet scalar channel, pushing m_σ downward, see [6] for details. The magnitude of flavor mixing matrix element in the scalar channel has been also evaluated on the lattice, and the results agree with the instanton-based predictions in sign and magnitude.

Apart from theoretical motivation, there is a simple phenomenological fact that no purely strange $\bar{s}s$ counterpart to sigma resonance $f_0(600)$ is seen. There are strong evidences that the pair of states $f_0(980), a_0(980)$ are the $\bar{K}K$ molecule, so the next f_0's are at 1300 and 1500 MeV.

For these reasons, we think that a description of σ as a $SU(3)$ singlet state

$$\sigma = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s) \tag{4}$$

strongly split from the $SU(3)$ octet one is a reasonable approximation. This means that the valence contribution to the derivative is

$$\left.\frac{\partial m_\sigma}{\partial m_s}\right|_{val} = \langle\sigma|\bar{s}s|\sigma\rangle = 2/3, \tag{5}$$

since with probability 1/3 there is a strange *pair*. A mixing between different scalar mesons f_0 ($\sigma \equiv f_0(600), f_0(980), f_0(1370)$) would further change the valence contributions downward. However, based on large gap between sigma and other states, and also based on better studied $\eta\eta'$ mixing, one might think that the relative change due to mixing with next f_0 states is not significant.

Let us now consider the contribution of the so called strange sea, virtual $\bar{s}s$ pairs, which are always present even in a completely nonstrange hadron such as nucleon. The relatively well studied case is the nucleon mass sensitivity to m_s, already discussed in [3]. Let us briefly remind that KN scattering data imply that

$$\frac{\partial m_N}{\partial m_s} = \langle N|\bar{s}s|N\rangle \approx 1.5 \tag{6}$$

and thus about 1/5 of the nucleon mass comes from the strange sea ($m_s \approx 120$ MeV).

Similar matrix elements for σ, ω mesons are not possible to obtain experimentally, although it can be done on the lattice. To estimate it, we will adopt a simple constituent quark picture, assuming *additivity* of the strange sea. If so, the derivatives analogous to Eq. (6) for all mesons should be 2/3 of it, or

$$\left.\frac{\partial m_{mesons}}{\partial m_s}\right|_{sea} \approx 1. \tag{7}$$

As we will see later, the exact value of the common sea contribution is not actually important, since its contributions to σ and ω mesons tend to cancel each other nearly exactly when we calculate variation of N-N interaction. What matters is the *difference* between their strange seas, to be discussed in the next section.

It is convenient to present the effect of the possible quark mass variation on the σ mass in the following form:

$$\frac{\delta m_\sigma}{m_\sigma} = 0.4\left(\frac{\delta m_s}{m_s} + \frac{m_u + m_d}{m_s}\frac{\delta m_q}{m_q}\right) = 0.4\frac{\delta m_s}{m_s} + 0.04\frac{\delta m_q}{m_q}, \tag{8}$$

where we have used $m_s = 120$ MeV. We see that the relative change of the strange quark mass produces much larger effect than the relative change of the light quark mass. This is similar to the case of the nucleon mass variation

$$\frac{\delta m_N}{m_N} = 0.19\frac{\delta m_s}{m_s} + 0.045\frac{\delta m_q}{m_q}. \tag{9}$$

C. $\bar{K}K, \eta\eta$ mixing with σ

As the second approximation, we will discuss loop effects or mixing with two-meson states. These are also completely different for σ and ω mesons. As we mentioned earlier, the latter practically does not mix with $\bar{K}K$ states, while σ does mix with them strongly. An admixture of virtual $\bar{K}K, \eta\eta$ pairs can be viewed as an additional contribution to the strange sea, on top of the strange content of the nonstrange constituent quarks (7).

The σ mixing with continuum of pseudoscalars is described by the standard mass operator given by the usual loop diagram

$$\Sigma(Q^2) = \int \frac{d^4k}{(2\pi)^4} \times \frac{\lambda^2(k,Q)}{[(k+Q/2)^2 - m^2 + i\epsilon][(k-Q/2)^2 - m^2 + i\epsilon]}, \tag{10}$$

where λ is the $KK\sigma, \eta\eta\sigma$, or $\pi\pi\sigma$ couplings. Its real part describes the shift of m_σ due to repulsion from these states: the imaginary part (for pions only) gives the width. The sign of the shift due to $\bar{K}K$ is obviously negative, since $m_\sigma < 2m_K$. The effect of $\pi\pi$ has contributions of both signs. For constant coupling the shift is logarithmically divergent, in reality it has to be regulated by form factors in the vertexes. The total shift is negative and large and is of the order of very large width $\Gamma_\sigma \sim 300$ MeV. Note that this large negative shift is partly the reason as why the σ mass is so small. However, in this paper we focus on the dependence on quark masses. Assuming that the main dependence comes from masses of Goldstone bosons, m_π, m_K, we differentiate the mass operator over these masses and obtain the convergent result. Thus one can ignore form factors and extract the effective coupling constant out of the integral.

For the derivative at $Q = m_\sigma = 0.5$ GeV, we get the following numerical value for the shifts:

$$\frac{\partial\Sigma}{\partial m_K^2} = 0.0229\ \text{GeV}^{-2}\lambda^2_{\sigma KK}, \tag{11}$$

$$\frac{\partial\Sigma}{\partial m_\eta^2} = 0.019\ \text{GeV}^{-2}\lambda^2_{\sigma\eta\eta}. \tag{12}$$

The couplings are not experimentally known, so we rely on the $SU(3)$ symmetry and relate them to $\lambda_{\sigma\pi\pi}$, which is in turn related to sigma meson width

$$\Gamma_\sigma = \frac{3}{2}\frac{\lambda^2_{\sigma\pi\pi}}{16\pi m_\sigma}\sqrt{1 - \frac{4m_\pi^2}{m_\sigma^2}}. \tag{13}$$

Taking $\Gamma_\sigma = 250$ MeV we obtain $\lambda^2_{\sigma\pi\pi} = 5\ \text{GeV}^2$. The factor 3/2 account for $\pi^+\pi^-, \pi^0\pi^0$ modes. However, in the mass shift there are contribution of $K^+K^-, \bar{K}^0K^0, \eta\eta$ channels which we would count as 5/2. Substituting numbers and using standard Gell-Mann–Oaks-Renner expressions for

m_η, m_K ($m^2 \propto m_s \Lambda_{QCD}$), we obtain the additional sensitivity of the sigma mass shift arising from the mixing effects,

$$\frac{\delta m_\sigma}{m_\sigma} = \frac{\delta m_s}{m_s}\left[2\frac{m_K^2}{2m_\sigma^2}\frac{\partial\Sigma}{\partial m_K^2} + 0.5\frac{m_\eta^2}{2m_\sigma^2}\frac{\partial\Sigma}{\partial m_\eta^2}\right] \approx \frac{\delta m_s}{m_s}\,0.14. \tag{14}$$

We included $K^+K^-, \overline{K}^0K^0$ modes with the coefficient 1 and $\eta\eta$ with 1/2: the latter only contribute about 1/5 of the final answer. We ignored even smaller contribution of the $\eta'\eta'$ loop.

D. The total sensitivity of m_σ as compared to m_N

Together with the one estimated in the previous section, it leads to total

$$\frac{\delta m_\sigma}{m_\sigma} \approx (0.24 + 0.16 + 0.14)\frac{\delta m_s}{m_s} = 0.54\frac{\delta m_s}{m_s}, \tag{15}$$

where three terms are the contributions of the common strange sea (7), valence strangeness (5), and the loop mixing (14), respectively.

In the same units the sensitivity of the nucleon mass is

$$\frac{\delta m_N}{m_N} \approx 0.19\frac{\delta m_s}{m_s}. \tag{16}$$

We conclude that the sigma mass is about three times more sensitive to the variation of the strange quark mass than the nucleon mass.

We also need the sensitivity of ω meson to the strange mass variation. This meson does not have valence strange quarks and practically does not mix with ϕ, K, and η mesons. Therefore, only the strange see contributes,

$$\frac{\delta m_\omega}{m_\omega} \approx 0.15\frac{\delta m_s}{m_s}. \tag{17}$$

III. THE MODIFICATION OF THE DEUTERON BINDING

A. Preliminary analytic estimates

Simple analytic estimate for sensitivity of the deuteron binding to sigma and omega mass modification is obtained by the differentiation of the potential over the mass and averaging the resulting expression $\sim\exp(-mr)$ over the radial wave function.

The simplest short range approximation leads to the free motion wave function,

$$\psi(r) = \frac{\sqrt{2\kappa}}{r}\exp(-\kappa r). \tag{18}$$

This wave function tends to infinity at small distances. However, the real deuteron wave function should be small there because of the repulsive core in the potential $V(r)$. Therefore, we introduced a small cutoff radius b in the integration over r. We estimated b from the condition $V(b)=0$ which gives $b=0.45$ fm. We get the following shift of the deuteron binding:

$$\frac{\delta Q_d}{Q_d} = -\frac{m_\sigma}{Q_d}\frac{\delta m_\sigma}{m_\sigma}\frac{g_s^2}{4\pi}\frac{2\kappa}{2\kappa+m_\sigma}e^{-(2\kappa+m_\sigma)b} \approx -75\frac{\delta m_\sigma}{m_\sigma}. \tag{19}$$

A variation of m_ω gives

$$\frac{\delta Q_d}{Q_d} = \frac{m_\omega}{Q_d}\frac{\delta m_\omega}{m_\omega}\frac{g_v^2}{4\pi}\frac{2\kappa}{2\kappa+m_\omega}e^{-(2\kappa+m_\omega)b} \approx 80\frac{\delta m_\omega}{m_\omega}. \tag{20}$$

The sign difference between these two derivatives and very large derivative value is due to fine tuning between omega and sigma terms, which are separately much larger than the sum.

The next step one can do analytically is to add a simplest square potential to the hard core. The energy of a shallow level in such potential is equal to [11]

$$E_d = -Q_d = -\frac{\pi^2}{16}\frac{(U-U_0)^2}{U_0}, \tag{21}$$

$$U_0 = \frac{\pi^2}{8ma^2}. \tag{22}$$

Here U and a are the depth and width of the potential well ($a=c-b$, where c and b are outer and inner radii), $m=m_N/2$ is the reduced mass. Selecting the width and depth of the well to be $a=1.6$ fm, $U_0=40.2$ MeV, $U=52.6$ MeV, we get

$$\frac{\delta Q_d}{Q_d} \approx -81.6e^{-m_\sigma b}\frac{\delta m_\sigma}{m_\sigma}, \tag{23}$$

$$\frac{\delta Q_d}{Q_d} \approx 87.4e^{-m_\omega b}\frac{\delta m_\omega}{m_\omega}. \tag{24}$$

By changing the core radius from $b=0$ to $b=0.4$ fm, one can vary the answer by about factor 3. Simple exponential dependence on the core radius b appears because of translational invariance of $1D$ Schrödinger equation for $r\psi(r)$.

Another effect one should consider is the modification of the nucleon mass: its contribution to modification of the deuteron binding is

$$\delta Q_d = \frac{\delta M_N}{M_N}\left\langle d\left|\frac{p^2}{2M_N}\right|d\right\rangle \tag{25}$$

which leads to

$$\frac{\delta Q}{Q} = \frac{U+U_0}{U-U_0}\frac{\delta m_N}{m_N} \simeq 7.7\frac{\delta m_N}{m_N}. \tag{26}$$

Although the sensitivity to the nucleon mass is much weaker than that for mesons, it is still quite strong: we attribute it to the fact that the small deuteron binding energy is in turn the delicate balance between larger kinetic and potential energies.

B. Using the Walecka potential

The estimates of the preceding section are given for orientation only, and in fact one of course has to solve numerically the radial Schrödinger equation and obtain the correct wave function. Then one can either average the potential derivative over it or simply vary all masses involved explicitly. We did the latter and determined the sensitivity of Q_d to the sigma, omega, and nucleon masses.

Strictly speaking, at this point it is no longer possible to limit ourselves to Walecka model with the coupling constants (3), since it does not describe correctly the deuteron binding. In fact, by ignoring all spin-dependent forces one cannot even separate the spin-singlet and the spin-triplet states. The tensor forces, attributed to pion and rho exchanges, are needed for this task. Instead of doing so, we have chosen to modify a bit the strength of the omega term, reducing g_v^2 by a factor 0.953 as compared to Eq. (3) and obtaining the correct deuteron binding.

Our results are[1]

$$\frac{\delta Q_d}{Q_d} \approx -48\frac{\delta m_\sigma}{m_\sigma} \approx -26\frac{\delta m_s}{m_s}, \tag{27}$$

$$\frac{\delta Q_d}{Q_d} \approx 50\frac{\delta m_\omega}{m_\omega} \approx 7.5\frac{\delta m_s}{m_s}, \tag{28}$$

$$\frac{\delta Q_d}{Q_d} \approx 6\frac{\delta m_N}{m_N} \approx 1.1\frac{\delta m_s}{m_s}. \tag{29}$$

One can see that the first two derivatives are more sensitive to the exact shape of the wave function: they agree qualitatively but not quantitatively with the analytic estimates above. We will not show here such details as exact and approximate wave functions, but just comment that the difference between them explain the difference in the integrals.

Summing all the contributions we find

$$\frac{\delta Q_d}{Q_d} \approx -17\frac{\delta m_s}{m_s}. \tag{30}$$

Using limits on big-bang nucleosynthesis from [3]

$$\left|\frac{\delta Q_d}{Q_d}\right| < 0.1, \tag{31}$$

one gets the final limit on the m_s variation to be

$$\left|\frac{\delta(m_s/\Lambda_{QCD})}{(m_s/\Lambda_{QCD})}\right| < 0.006. \tag{32}$$

[1]In the derivative over the omega mass we have divided back by the factor 0.953, restoring the original strength of the vector term.

IV. OKLO

In this section, we extract limits on δm_s following from data on natural nuclear reactor in Oklo active about 2×10^9 yr ago. The most sensitive phenomenon (used previously for limits on the variations of the electromagnetic α) is disappearance of certain isotopes (especially Sm^{149}) possessing a neutron resonance close to zero [12]. To date the lowest resonance energy $E_0 = 0.0973 \pm 0.0002$ eV is large compared to its width, so the neutron capture cross section $\sigma \sim 1/E_0^2$. The data constrain the ratio of this cross section to a nonresonance one (which was used to measure number of neutrons emitted by the reactor). It therefore implies[2] that these data constrain the variation of the following ratio $\delta(E_0/E_1)$ where $E_1 \sim 1$ MeV is a typical single-particle energy scale, which may be viewed as the energy of some one-body "doorway" state.

A generic expression for the level energy in terms of fundamental parameters of QCD can be written as follows:

$$\delta E_0 = A\,\delta\Lambda_{QCD} + B_q\delta m_q + B_s\delta m_s + C\,\delta\alpha\Lambda_{QCD}, \tag{33}$$

where A,B,C are some coefficients. The first term is the basic QCD term, while others are corrections due to modification of the quark masses and the electromagnetic α.

In this section we provide a new estimate of the B_s. More specifically, we estimate the variation of the resonance energy resulting from a modification of the sigma mass. The energy of the resonance $E_0 = E_{excitation} - S_n$ consists of excitation energy of a compound nucleus minus the neutron separation energy S_n. This, in turn, is a depth of the potential well V minus the neutron Fermi energy ϵ_F, $S_n = V - \epsilon_F$. The latter scales like $1/R^2$ if the radius of the well is changed. The kinetic part of the excitation energy $E_{excitation}$ scales in the same way. So

$$E_0 = E_{excitation} - S_n = E_{excitation} + \epsilon_F - V = K\frac{\hbar^2}{MR} - V, \tag{34}$$

where K is a numerical constant which can be found from the present time condition $E_0 \approx 0$. The shift of the resonance then is

$$\delta E_0 = -K\frac{\hbar^2}{MR}\left(\frac{\delta M}{M} + \frac{2\,\delta R}{R}\right) - \delta V = -V\left(\frac{\delta M}{M} + \frac{2\,\delta R}{R} + \frac{\delta V}{V}\right). \tag{35}$$

Using Eq. (2) we can find the depth of the potential well[3]

[2]Of course, under assumption that the *same* resonance was the lowest one at the time of Oklo reactor.

[3]Note that the suppression of N-N wave function at a small separation due to the repulsive core reduces the depth of the effective potential V. However, this effect is not so important in the ratio $\delta V/V$.

$$V=\frac{3}{4\pi r_0^3}\left(\frac{g_s^2}{m_\sigma^2}-\frac{g_v^2}{m_\omega^2}\right). \tag{36}$$

Here $r_0=1.2$ fm is an internucleon distance. Numerical estimates show that the contribution of the variation of r_0 [and the variation of $R=A^{1/3}r_0$ in Eq. (35)] is not as important as the direct contribution of the m_σ variation in the equation above. This gives us

$$\frac{\delta V}{V}=-8.6\frac{\delta m_\sigma}{m_\sigma}+6.6\frac{\delta m_\omega}{m_\omega}=-3.5\frac{\delta m_s}{m_s}, \tag{37}$$

$$\delta E_0=1.7\times10^8\ \text{eV}\times\frac{\delta m_s}{m_s}. \tag{38}$$

We used $V=50$ MeV in Eq. (35). Comparison of this result with the the observational limits claimed in [12] $|\delta E_0|<0.02$ eV gives a very strong limit

$$\left|\frac{\delta(m_s/\Lambda_{QCD})}{(m_s\Lambda_{QCD})}\right|<1.2\times10^{-10} \tag{39}$$

at time $\approx1.8\times10^9$ yr ago.

Note that the authors of the last work in [12] found also the nonzero solution $\delta E_0=-0.097\pm0.008$ eV. This solution corresponds to the same resonance moved below thermal neutron energy. In this case

$$\frac{\delta(m_s/\Lambda_{QCD})}{(m_s\Lambda_{QCD})}=-(0.56\pm0.05)\times10^{-9}. \tag{40}$$

The error here does not include the theoretical uncertainty.

The production of nuclei with $A>5$ during big-bang nucleosynthesis (BBN) is strongly suppressed because of the absence of stable nuclei with $A=5$. ^{5}He is unstable nucleus which is seen as a resonance in n-α elastic scattering. The ground state lies at 0.89 MeV above neutron threshold. The variation of the strange quark mass may influence the position of the resonance making, for example, ^{5}He stable. Stable ^{5}He at the time of BBN would change strongly the primordial abundances of light elements. The estimate similar to that we made for Sm nucleus gives us a limit

$$\frac{\delta(m_s/\Lambda_{QCD})}{(m_s\Lambda_{QCD})}>-0.006. \tag{41}$$

This limit corresponds to $\delta E>-0.89$ MeV at the time of BBN.

We obtained limits on variation of m_s/Λ_{QCD} during the interval between the big bang and present time and on shorter time scale from Oklo natural nuclear reactor which was active 1.8×10^9 yr ago. It is also possible to obtain limits on the intermediate time scale. One possibility is related to position of the resonance in ^{12}C during production of this element in stars. This famous resonance at $E=380$ kev is needed to produce enough carbon and create life. According to Ref. [13] the position of this resonance cannot shift by more than 60 kev (one can also find in Ref. [13] the limits on the strong interactions and other relevant references). We have made a very rough estimate of the limit on the strange quark mass variation which can be obtained via m_σ mechanism:

$$\left|\frac{\delta(m_s/\Lambda_{QCD})}{(m_s\Lambda_{QCD})}\right|<0.001. \tag{42}$$

This limit can be improved after an accurate calculation.

V. SUMMARY

In this work we focused at the effect of posible variation of *strange quark mass* m_s relative to Λ_{QCD}. We argued that attractive scalar part of the nuclear forces is more sensitive to it than vector repulsive one. As a result, we claim significantly stronger limits on ratio of weak/strong scale ($W=m_s/\Lambda_{QCD}$) variation following from our previous discussion of primordial big-bang nucleosynthesis ($|\delta W/W|<0.006$) and Oklo natural nuclear reactor $|\delta W/W|<1.2\times10^{-10}$.

We should stress that the errors given for the limits include the errors of the experimental inputs only. In view of the many assumptions made herein, it is very difficult to attempt to place theoretical error estimates on our conclusions. Optimistically, however, we do not believe that our estimates will be eventually changed by a factor of 2.

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[1] J.D. Bjorken, Phys. Rev. D **67**, 043508 (2003), and references therein.

[2] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Rev. Lett. **82**, 884 (1999); J.K. Webb, M.T. Murphy, V.V. Flambaum, V.A. Dzuba, J.D. Barrow, C.W. Churchill, J.X. Prochaska, and A.M. Wolfe, *ibid.* **87**, 091301 (2001); M.T. Murphy, J.K. Webb, V.V. Flambaum, V.A. Dzuba, C.W. Churchill, J.X. Prochaska, J.D. Barrow, and A.M. Wolfe, Mon. Not. R. Astron. Soc. **327**, 1208 (2001); M.T. Murphy, J.K. Webb, V.V. Flambaum, C.W. Churchill, and J.X. Prochaska, *ibid.* **327**, 1223 (2001); M.T. Murphy, J.K. Webb, V.V. Flambaum, C.W. Churchill, J.X. Prochaska, and A.M. Wolfe, *ibid.* **327**, 1237 (2001).

[3] V.V. Flambaum and E.V. Shuryak, Phys. Rev. D **65**, 103503

(2002).
[4] V.F. Dmitriev and V.V. Flambaum, Phys. Rev. D **67**, 063513 (2003).
[5] S.R. Beane and M.J. Savage, Nucl. Phys. **A717**, 91 (2003).
[6] T. Schafer and E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998); hep-lat/0005025.
[7] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
[8] G. Chanfray, M. Ericson, and P.A. Guichon, Phys. Rev. C **63**, 055202 (2001).
[9] E.V. Shuryak and G.E. Brown, Nucl. Phys. **A717**, 322 (2003).
[10] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[11] L.D. Landau and E.M. Lifshits, *Quantum Mechanics, Theoretical Physics* (Nauka, Moscow, 1974), Vol. 3.
[12] A.I. Shlyakhter, Nature (London) **264**, 340 (1976); T. Damour and F.J. Dyson, Nucl. Phys. **B480**, 37 (1996); Y. Fujii, A. Iwamoto, T. Fukahori, T. Ohnuki, M. Nakagawa, H. Hidaka, Y. Oura, and P. Moller *ibid.* **B573**, 377 (2000).
[13] H. Oberhummer, R. Pichler, and A. Csoto, nucl-th/9810057.