Supersymmetric standard model of inflation with extra dimensions

M. Bastero-Gil*
Centre of Theoretical Physics, University of Sussex, Falmer, Brighton, BN1 9QJ, United Kingdom

V. Di Clemente† and S. F. King‡
Department of Physics and Astronomy, University of Southampton, Highfield, Southampton, SO17 1BJ, United Kingdom

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We embed the supersymmetric standard model of hybrid inflation based on the next-to-minimal superpotential term $\lambda N H_u H_d$ supplemented by an inflaton term $\kappa \phi N^2$ into an extra-dimensional framework, in which all the Higgs fields and singlets live in the bulk, while all the matter fields live on the brane. All the parameters of the effective 4D model can then be naturally understood in terms of a fundamental (''string'') scale $M_* \sim 10^{13}$ GeV and a brane supersymmetry breaking scale 10^8 GeV, of the same order as the height of the inflaton potential during inflation. In particular, the very small Yukawa couplings $\lambda \sim \kappa \sim 10^{-10}$, necessary for the model to solve the strong *CP* problem and to generate the correct effective μ term after inflation, can be naturally understood in terms of volume suppression factors. The brane scalar masses are naturally of order a TeV while the bulk inflaton mass is naturally in the MeV range sufficient to satisfy the slow roll constraints. Curvature perturbations are generated after inflation from the isocurvature perturbations of the supersymmetric Higgs field as discussed in a companion paper.

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I. INTRODUCTION

Although inflation provides a solution to the flatness and horizon problems, and is also supported by mounting evidence from detailed studies of the cosmic microwave background (CMB) spectrum [1], its relation to particle physics remains obscure. Some time ago two of us [Bastero-Gil and King (BGK)] proposed a supersymmetric hybrid inflation model based on the next-to-minimal superpotential term $\lambda N H_u H_d$ supplemented by an inflaton term $\kappa \phi N^2$ [2]. The motivation for this model was to construct a realistic model of inflation which was motivated by particle physics considerations. The idea was that, at the end of inflation, the singlet N would develop a vacuum expectation value (VEV) of the order of 10^{13} GeV, breaking a Peccei-Quinn symmetry in the process and providing a solution to the strong *CP* problem, as well as providing an effective origin for the Higgs boson mass term (μ term) at the TeV scale. Unfortunately the BGK model appears to suffer from a number of naturalness problems. The first problem is that in order to generate a TeV scale effective μ term, and satisfy other requirements of inflation, the dimensionless couplings must be very small $\lambda \sim \kappa \sim 10^{-10}$. The second problem is that in order for the inflaton to provide curvature perturbations of the correct order of magnitude the inflaton ϕ mass also has to be extremely small being in the eV range. Finally the height of the inflaton potential of order 10^8 GeV is much smaller than the generic 10^{11} GeV which is typical of supergravity explanations for the generation of TeV scale soft masses.[1]

Recently in [3] it was pointed out that the second problem of the BGK model, namely that of the eV inflaton mass, could be alleviated by relaxing the requirement that the inflaton be responsible for generating the observed curvature perturbations [1]. The basic idea is that the inflaton is only required to satisfy the slow roll conditions for inflation, and the curvature perturbations may be generated after inflation from the isocurvature perturbations of some late decaying scalar field called the ''curvaton'' [4–6]. In the BGK model it was pointed out [3] that this means that the inflaton need only have a mass of order MeV and not eV as in the original version of the model, thereby alleviating extreme fine tuning in this model. However, no candidate was proposed for the curvaton, and the remaining naturalness problems of the smallness of the couplings λ, κ, the small height of the inflaton potential and the less extreme but still unnatural requirement of an MeV inflaton mass was not addressed in [3]. In a companion paper [7] we show that the Higgs scalars H_u, H_d of the BGK model could be responsible for generating the curvature perturbations responsible for large scale structure. This represents an alternative to the late decaying scalar mechanism in which the curvature perturbations are generated during the reheating stage. As in the curvaton approach [3] this allows the inflaton mass to be in the MeV range, but does not solve any of the remaining naturalness problems of the model.

The purpose of the present paper is to show how, by embedding the BGK model into an extra dimensional framework, all the remaining naturalness problems of the model may be resolved. The extra dimensional setup has all the Higgs fields and singlets in the bulk, and all the matter fields live on the branes. All the parameters of the effective 4D model can then be naturally understood in terms of a funda-

*Email address: mbg20@pact.cpes.susx.ac.uk
†Email address: vicente@hep.phys.soton.ac.uk
‡Email address: sfk@hep.phys.soton.ac.uk

[1]In supergravity the soft masses are given by $m \approx F_s / m_p$, where m_p is the Planck scale and $\sqrt{F_s} \approx 10^{11}$ GeV is the supersymmetric breaking scale. On the other hand, F_s^2 is the natural order of magnitude for the vacuum energy $V(0)$ (the height of the inflaton potential).

mental ("string") scale $M_* \sim 10^{13}$ GeV and a brane supersymmetry (SUSY) breaking F term of order 10^8 GeV. In particular, the very small Yukawa couplings $\lambda \sim \kappa \sim 10^{-10}$, necessary for the model to solve the strong *CP* problem and to generate the correct effective μ term after inflation, can be naturally understood in terms of volume suppression factors [8,9]. Also MeV inflaton masses for scalars in the bulk, and TeV scale soft masses for scalars on the branes are naturally generated.

The layout of the remainder of the paper is the following. In Sec. II we review the BGK model in more detail. In Sec. III we embed the model in an extra dimensional framework, and show how this leads to volume suppressed 4D effective Yukawa couplings λ, κ of order 10^{-10}. In Sec. IV we describe the SUSY breaking mechanism due to a brane singlet F-term F_S, and show how this leads to both TeV scale soft masses for brane scalars and trilinears and MeV scale soft masses for bulk scalars such as the inflaton. A summary and conclusions are presented in Sec. V.

II. BRIEF REVIEW OF THE BGK MODEL

In this section we first reexamine the main features of the four-dimensional supersymmetric hybrid model, based on the superpotential[2] [2]

$$W = \lambda N H_u H_d - \kappa \phi N^2, \tag{1}$$

where N and ϕ are singlet fields, and $H_{u,d}$ the Higgs fields of the minimal supersymmetric standard model (MSSM). The first term is familiar from the nonminimal SSM (NMSSM), and the second terms includes another singlet ϕ (the inflaton). As in the NMSSM, the combination $\lambda \langle N \rangle$ gives rise to an effective μ term in the Higgs superpotential. The usual cubic term of the NMSSM N^3 has been replaced here by an interaction term between N and ϕ. In order to keep the superpotential linear in the inflaton field ϕ, other cubic terms in the superpotential are forbidden by imposing a global $U(1)_{PQ}$ Peccei-Quinn symmetry. The global symmetry is broken by the VEVs of the singlets, leading to a very light axion and solving the strong *CP* problem [12]. The axion scale f_a is then set by the VEVs of the singlets, and is constrained by astrophysical and cosmological observations to be roughly in the window 10^{10} GeV$\leqslant f_a \leqslant 10^{13}$ GeV [13,14].

[2]In this paper the superpotential in Eq. (1) is regarded only as the *effective* four-dimensional superpotential obtained after dimensional reduction. It has been pointed out in Ref. [10] that in the brane world setup the Hubble parameter H is proportional to the energy density on the brane, ρ, instead of the usual $H \sim \sqrt{\rho}$ of the standard big bang cosmology. However, by putting fields in the bulk whose density dominates over the brane density and requiring stability of the extra dimensions during the inflationary period it is possible to show that the standard cosmology is recovered [11]. Therefore the 4D results in this section remain effectively valid when the theory is embedded in extra dimensions as is done in the next section.

Inflation takes place below the SUSY breaking scale. Including the soft SUSY breaking terms, trilinears A_κ and masses for the singlets m_ϕ, m_N, the inflationary potential is given by [2]

$$V(\phi, N) = V(0) + \frac{\kappa^2}{4} N^4 + \left(\kappa^2 \phi^2 - \frac{1}{\sqrt{2}} \kappa A_\kappa \phi + \frac{1}{2} m_N^2 \right) N^2 + \frac{1}{2} m_\phi^2 \phi^2, \tag{2}$$

where ϕ and N represent the real part of the complex fields, and we have set the axionic part and the Higgs fields to zero for simplicity. In addition, we have introduced a constant term $V(0)$, whose origin will be discussed later. The inflationary trajectory is obtained when the inflaton field ϕ takes values larger than the critical one:

$$\phi_c \simeq \frac{A_\kappa}{\sqrt{2}\kappa}. \tag{3}$$

As long as $\phi > \phi_c$, the N field dependent squared mass is positive and then N is trapped at the origin; the potential energy in Eq. (2) is then dominated by the vacuum energy $V(0)$. When ϕ reaches the critical value ϕ_c, the squared mass of the N field changes sign, and both fields roll down towards the global minimum at $\phi_0 = \phi_c/2$, $N_0 = \phi_c/\sqrt{2}$, ending inflation.

The required values of the couplings and masses are derived by combining cosmological and particle physics constraints. In order to have slow-roll inflation in the first place, the inflaton mass m_ϕ needs to be small enough compared to the Hubble rate of expansion H, as given by the η parameter

$$\eta_\phi = \frac{|m_\phi^2|}{3H^2} = m_P^2 \frac{|m_\phi^2|}{V(0)} < 1, \tag{4}$$

where $m_P = M_P/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and η_ϕ is evaluated some N e-folds before the end of inflation. Assuming that m_ϕ satisfies the above condition, the other physical scales in the problem are the soft breaking term $A_\kappa \simeq 1$ TeV, and the axion scale $f_a \sim \phi_c \sim \phi_0 \sim N_0 \sim 10^{13}$ GeV. From Eq. (3), this unavoidably leads to a tiny coupling constant of the order $\kappa \sim 10^{-10}$. The same applies to λ, with $\mu = \lambda N_0 \sim 1$ TeV. Thus, demanding a zero vacuum energy at the global minimum, $V(\phi_0, N_0) = 0$, the height of the potential during inflation is given by

$$V(0)^{1/4} \simeq \sqrt{\frac{\kappa}{2}} N_0 \simeq (10^8 \text{ GeV}). \tag{5}$$

The Hubble parameter during inflation is then of the order of $O(10 \text{ MeV})$. And from Eq. (4), this means that inflaton soft mass m_ϕ can be at most of the order of some MeVs in order to satisfy the slow roll conditions.

In order to meet the Cosmic Background Explorer (COBE) value $\delta_H = 1.95 \times 10^{-5}$ [15], we would require having $\kappa m_\phi \sim 10^{-18}$ GeV, i.e., a tiny inflaton mass of the order

of a few[3] eV. This is a much stronger constraint on the mass of the inflaton than just requiring the inflaton to be a "light" field during inflation and satisfying the slow-roll condition Eq. (4).

As we discuss in a companion paper [7] primordial curvature perturbations can be originated by the Higgs perturbations instead of the inflaton perturbations. The inflaton mass is then only restricted by the slow-roll condition, and therefore no extremely tiny values of the masses are required. However, the spectral index of the spectrum of curvature perturbations is now controlled by the Higgs parameters:

$$n-1\simeq 2m_h^2/(3H^2), \tag{6}$$

which is constrained by observations [1] to be $n<1.06$. And this is a slightly stronger constraint that just demanding slow roll, i.e., for the Higgs boson mass we will have $m_h<0.3H \simeq 3$ MeV.

III. EMBEDDING THE BGK MODEL IN EXTRA DIMENSIONS

We now embed the BGK model into an extra-dimensional framework, in which all the Higgs fields and singlets live in the bulk, while all the matter fields live on the brane. In this section we shall show how the very small Yukawa couplings $\lambda\sim\kappa\sim 10^{-10}$ necessary for the model to solve the strong *CP* problem and generate the correct effective μ term after inflation will be naturally understood in terms of a volume suppression factor $(M_*/m_p)^2$ where M_* is a fundamental ("string") scale $M_*\sim 10^{13}$ GeV and m_p is the effective reduced Planck scale whose value will also be explained. In the next section we shall consider SUSY breaking and show how the required MeV soft masses for the inflaton in the bulk and the TeV soft masses on the brane may result from a brane supersymmetry breaking F term of order 10^8 GeV, which also naturally sets the scale for the height of the inflaton potential $V(0)$.

Let us consider two 3-branes spatially separated along d extra dimensions with a common radius R. These extra dimensions are compactified on some orbifold that leads at least to two fixed points at $y_j=0,\pi R$ $(j=1,\ldots,d)$, where the two D3-branes are located. All the quarks/squarks fields $(Q_i,U_i,D_i$, where $i=1,2,3)$ live on the "Yukawa" brane at[4] $y_j=0$, while SUSY is broken by the F term of a gauge singlet field S on the SUSY breaking brane at $y_j=\pi R$. The gauge fields $\hat{G}_A^{(1)}$, the inflaton field $\hat{\phi}$, the singlet field $\hat{N}$ and both Higgs fields $\hat{H}_u$ and $\hat{H}_d$ feel all the dimensions of the theory $(4+d$ dimensions). Also we include an additional gauge group $\hat{G}_B^{(2)}$ in such a way that at some scale M the total gauge group[5] $G_A^{(1)}\times G_B^{(2)}$ breaks down to the standard model gauge group $G_{SM}=SU(3)\times SU(2)\times U(1)$. This is depicted in Fig. 1.

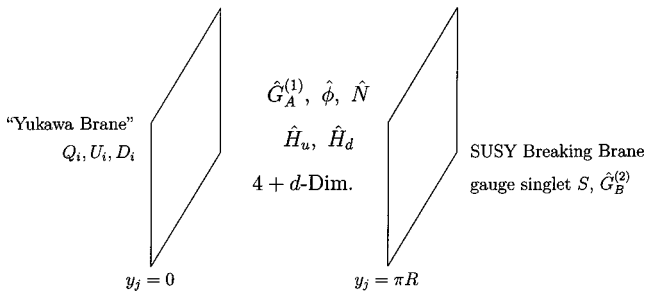

FIG. 1. The model showing the parallel 3-branes spatially separated along d extra dimensions with coordinates $\mathbf{y}=(y_1,\ldots,y_d)$ and a common radius R.

In each of the fixed points of the full manifold we have $N=1$ supersymmetry. Nevertheless, in order to get supermultiplets associated with the Kaluza-Klein (KK) tower after compactifying the $4+d$ dimensions down to 4 dimensions, we need the infinite degrees of freedom to fall down to some extended supersymmetry. Strictly speaking, the extra Kaluza-Klein tower of states will effectively be $N=2$ supersymmetric only for one or two extra dimensions. For higher values of d, the situation is a bit more complicated. For example, for $d=6$ we naively expect the Kaluza-Klein tower of states to be $N=4$ supersymmetric [16]. In general, the enhanced supersymmetry for the excited Kaluza-Klein arises because the minimum number of supersymmetries in higher dimensions (as counted in terms of four-dimensional gaugino spinors) grows with the space-time dimensions. However, by making suitable choices of orbifolds, it is always possible to project the relevant Kaluza-Klein towers down to representations of $N=2$ supersymmetry, even if $d>2$ [17]. Hence, without loss of generality, we shall consider $N=2$ supersymmetric Kaluza-Klein towers for arbitrary values of d.

In a $N=2$ supersymmetric theory there is a global $SU(2)_R$ automorphism group defined in the supersymmetric algebra. The off-shell hypermultiplet Φ_a is given by[6] $\Phi_a=(\phi_a^l,\Psi_a,F_a^l)$, where $l=1,2$ is the $SU(2)_R$ index and $\Psi_a=(\psi_{a,L},\psi_{a,R})$ is a Dirac spinor. On the other hand, it is well known that in $N=2$ there is no $SU(2)_R$ invariant cubic interactions involving hypermultiplets Φ_a [19]. One possible way to define the supersymmetric Yukawa couplings is sticking the superpotential in one of the fixed points ($D3$-branes) of the orbifold, where only one of the supersymmetry N

[3]We notice that this value can be entirely due to one-loop radiative corrections $\delta m_\phi^2\sim\kappa^2(\kappa\phi_c)^2$, once the tree-level value is set to zero.

[4]In this brane, for reasons we shall discuss later, we define the quark's Yukawa couplings. In Sec. IV D we shall address the issue regarding the localization of leptons (so far they can live either in the "Yukawa" brane or in the SUSY breaking brane).

[5]In this paper we are not going to consider or specify a particular gauge group for $G_A^{(1)}\times G_B^{(2)}$, it can be either some string motivated gauge group (i.e. Pati-Salam group, E_8, etc.) or some grand unified theory (GUT) group [$SU(5)$,$SO(10)$, etc].

[6]For more detail about the off-shell formulation of the vector and hypermultiplets in $N=2$ supersymmetry see, for example, Ref. [18].

$=1$ survives after the orbifolding. This is what we have called the Yukawa brane.

The Lagrangian associated with the superpotential can be written as[7]

$$\mathcal{L}^{W}_{4+d} = -\int d^2\theta \;\; \delta^d(\mathbf{y})\left[\frac{\hat{\lambda}}{M_*^{3d/2}}\hat{N}\hat{H}_u\hat{H}_d - \frac{\hat{\kappa}}{M_*^{3d/2}}\hat{\phi}\hat{N}^2 + \frac{\hat{y}_t}{M_*^{d/2}}Q_3\hat{H}_u U_3 + \frac{\hat{y}_b}{M_*^{d/2}}Q_3\hat{H}_d D_3\right], \tag{7}$$

with hat mean couplings and fields in extra dimensions, where λ and κ are the couplings defined in Eq. (1) and y_t (y_b) is the top (bottom) Yukawa coupling. $\delta^d(\mathbf{y})$ is the d-dimensional generalization of the delta function. M_* is the string scale in $4+d$ dimensions and it is related to the Planck scale in four dimensions through the very well known formula

$$m_p^2 = M_*^{2+d} V_d\,, \tag{8}$$

with $V_d \approx R^d$ the volume factor of the compact manifold defined in the extra dimensional bulk. The bulk fields have a mass dimensions $1+d/2$, while the brane fields have the standard mass dimension 1. The higher dimensional fields lead to non-renormalizable interaction terms, where the suppression is now given by the fundamental scale in $4+d$ dimensions (M_*) instead of the four dimensional Planck mass (m_p).

As we have explained above, the full theory should be written in terms of hypermultiplets and vector multiplets in $N=2$ supersymmetry. However, the Lagrangian (7) is a function of the $N=1$ supermultiplets (ϕ^1, ψ_L, F^1) only, because we are assuming that the orbifolding casts the two $N=1$ multiplets in the $N=2$ hypermultiplet in such a way that just one of them is even under some orbifold discrete group (for example, Z_2) and then at the fixed point only this multiplet is different from zero.[8]

Upon dimensional reduction a plethora of particles [Kaluza-Klein (KK)-modes] comes out in the effective four dimensional Lagrangian. In fact, there are mixing among different KK numbers since the interaction (7) does not preserve the translational invariance along the extra dimensions due to the presence of the delta function [$\delta^d(\mathbf{y})$] which breaks explicitly the Poincaré invariance of the theory. However, making some assumption about the number of extra dimensions d, we can neglect the contribution from the infinite tower of KK and write down the effective Lagrangian as a function only of the zero mode of each bulk fields. Indeed, using Eq. (8) we have that the compactification scale $1/R$ is given by

$$\frac{1}{R} = M_*\left(\frac{M_*}{m_p}\right)^{2/d}. \tag{9}$$

On the other hand, we will see that consistency of the model demands the string scale of the theory be present at some intermediate scale; in particular, it has to be of the same order of the axion scale $M_* \sim f_a \sim 10^{13}$ GeV. This implies that if the number of extra dimensions is larger than two, $d>2$, the compactification scale [using Eq. (5)] is then $1/R > V(0) \sim 10^8$ GeV.

This means that the energy scale for inflation (governed mainly by the vacuum energy) is below the first excitation of the KK propagating in the bulk. Therefore, the KK modes are not produced during the early stage of inflation and then the decoupling of these particles is a good approximation for our purpose. From now on, we only consider a number of extra dimensions larger than two, even if the result we will show here will be independent of the number of extra dimensions.

We are going now to study in details the effective four dimensional Yukawa couplings and the gauge coupling.

A. Yukawa couplings

After integrating out the d extra dimensions and considering only the zero mode of the bulk fields, from Eq. (7) we get

$$\mathcal{L}^{W}_{4} = -\int d^2\theta \left[\hat{\lambda}\left(\frac{M_*}{m_p}\right)^3 N H_u H_d - \hat{\kappa}\left(\frac{M_*}{m_p}\right)^3 \phi N^2 + \hat{y}_t\left(\frac{M_*}{m_p}\right) Q_3 H_u U_3 + \hat{y}_b\left(\frac{M_*}{m_p}\right) Q_3 H_d D_3\right], \tag{10}$$

where now all the fields are four dimensional ones. From the last equation we found that the four dimensional couplings are naturally suppressed if $M_* < m_p$,

$$\lambda = \left(\frac{M_*}{m_p}\right)^3 \hat{\lambda}, \quad \kappa = \left(\frac{M_*}{m_p}\right)^3 \hat{\kappa}, \qquad Y_t = (y_t) = \left(\frac{M_*}{m_p}\right)\hat{y}_t, \quad Y_b = (y_b) = \left(\frac{M_*}{m_p}\right)\hat{y}_b\,. \tag{11}$$

A natural assumption is to consider all the multidimensional couplings to be of the same order,

$$\hat{\lambda} \sim \hat{\kappa} \sim \hat{y}_t \sim \hat{y}_b\,, \tag{12}$$

in which case we get the following relationship between the Yukawa couplings:

$$\frac{\lambda}{Y_{t(b)}} \sim \frac{\kappa}{Y_{t(b)}} \sim \left(\frac{M_*}{m_p}\right)^2. \tag{13}$$

[7]For simplicity, we set all the Yukawa couplings except the third generation ones to zero.

[8]Notice that terms like $\hat{\phi}(\partial_y \hat{N}')^2$, where N' is the supermultiplet which belongs to the *other* $N=1$ supersymmetry, can be allowed by the orbifold symmetry in the Lagrangian (7), but are heavily suppressed by higher powers of the string scale M_*, and can therefore be neglected.

Therefore, if $M_* \approx 10^{13}$ GeV we naturally get $\lambda \sim \kappa \sim \mathcal{O}(10^{-10})$ where the Yukawa couplings for the third generation are of the order one, $Y_u \sim Y_d \sim \mathcal{O}(1)$. Notice that the $4+d$-dimensional couplings in Eq. (11) are extremely large. This only indicates that our D-dimensional model is non-perturbative. The ''duality'' through dimensional reduction between a non-perturbative theory in extra dimensions and a perturbative theory in the effective four dimensions has been pointed out some time ago in Ref. [20].

B. Gauge coupling

The Lagrangian in $4+d$ dimensions associated with the gauge coupling for the Higgs (bulk fields) and the quarks (brane fields) has the following form:

$$\mathcal{L}^g_{4+d} = \left[\frac{\hat{g}}{M_*^{d/2}} \gamma_M \hat{A}^M \overline{Q}_i Q_i + (Q_i \leftrightarrow U_i, D_i) \right] \delta^d(\mathbf{y}) + \left[\frac{\hat{g}^2}{M_*^d} \hat{A}_M \hat{A}^M \hat{H}_u^\dagger \hat{H}_u + (\hat{H}_u \leftrightarrow \hat{H}_d) \right], \tag{14}$$

where $\hat{A}_M$ are the gauge boson in higher dimensions, being $M = \mu, 5 = 0,1,2,3,5$. The bulk gauge fields have mass dimensions $1 + d/2$ (like the Higges fields).[9] After integrating out the d extra dimensions from the above Lagrangian we get

$$\mathcal{L}^g_4 = \left[\frac{\hat{g}}{M_*^{d/2}} \gamma_\mu \left(\frac{A^\mu}{\sqrt{V_d}} \right) \overline{Q}_i Q_i + (Q_i \leftrightarrow U_i, D_i) \right] + \left[\frac{\hat{g}^2}{M_*^d} \left(\frac{A_\mu A^\mu}{V_d} \right) \left(\frac{H_u^\dagger H_u}{V_d} \right) + (\hat{H}_u \leftrightarrow \hat{H}_d) \right] \times V_d . \tag{15}$$

Notice that the fifth component of the gauge fields, A_5, has been removed from the Lagrangian since it does not have zero mode. From the previous Lagrangian and using Eq. (8) we can read the effective four dimensional gauge coupling as

$$g = \left(\frac{M_*}{m_p} \right) \hat{g}. \tag{16}$$

Comparing Eqs. (11) and (16) we observe that both the gauge coupling and the Yukawa couplings in four dimensions have the same suppression factor. Thus $g/Y_{t(b)} \sim 1$ assuming that their higher dimensional couplings are of the same order.

IV. SUPERSYMMETRY BREAKING

We shall suppose that SUSY is broken by the F term of a $4D$ gauge-singlet field S on the source brane localized at the fixed point $y_p = \{y_i\} = \pi R$, and mediated across the extra dimensional space by bulk fields propagating in a loop correction like gaugino mediations [21,22]. Moreover, S is also neutral under the $U(1)_{PQ}$ symmetry. Because of that, no μ term is generated by the Giudice-Masiero mechanism in this model [23]. The solution to the μ problem relies on the coupling of the singlet N to the Higgs bosons. Like in the NMSSM the μ term is given by $\mu \sim \lambda N_0$, once the singlet N gets a non-zero VEV N_0 after inflation. All the bulk fields (gaugino, higgsino, Higgs, inflaton, singlet N) get a tree-level SUSY mass term through direct coupling with the SUSY breaking brane. The rest of the particles which live in the Yukawa brane only get one-loop SUSY mass terms and then they will be neglected in this paper.

In the next sections we are going to discuss the origin of all the SUSY breaking terms necessary to generate the inflaton potential Eq. (2): the vacuum energy $V(0)$, the trilinear A_k term and the quadratic mass terms, m_N^2 and m_ϕ^2. For completeness we will discuss the other soft terms as well, i.e. the soft mass term for the Higgs bosons, the $B\mu$ terms and the gaugino masses. In the SUSY breaking sector there are two free parameters, the F term of the singlet S (F_S) and the cutoff M_*. However, demanding the solution of the CP problem, and imposing that the F_S^2 term explains the origin of the vacuum energy, we will see that all the parameters (both dimensionful and dimensionless) of the potential Eq. (2) are fully determined.

A. Vacuum energy

The SUSY breaking brane will typically introduce a vacuum energy of the order of F_S^2 providing a vacuum energy $V(0)$ in the potential (2). We simply set this constant from the Lagrangian in $4+d$ dimensions:

$$\Delta \mathcal{L}^{soft}_{4+d} = -\int d^4\theta \ \delta^d(\mathbf{y} - y_p) S^\dagger S. \tag{17}$$

In the effective four dimensions and when SUSY is broken, we get a vacuum energy

$$V(0) = F_S^2. \tag{18}$$

From Eq. (5) we see that the F term has to be $\sqrt{F_S} \sim 10^8$ GeV. This result is indeed interesting because it states that the vacuum energy and the SUSY breaking scale are of the same order of magnitude, avoiding any fine tuning regarding the Kähler potential [24].

B. Trilinear soft terms for scalars

The trilinear soft terms allowed for the PQ charges are

$$\Delta \mathcal{L}^{soft}_{4+d} = -\int d^2\theta \ \delta^d(\mathbf{y} - y_p) \left(\frac{\hat{\lambda}}{M_*^{3d/2}} \hat{N} \hat{H}_u \hat{H}_d - \frac{\hat{\kappa}}{M_*^{3d/2}} \hat{\phi} \hat{N}^2 \right) \frac{S}{M_*}. \tag{19}$$

[9] The gauge coupling remains dimensionless.

From the second term in Eq. (19) we get the A_k term defined in the potential (19), while the $B\mu$ term arises from the first term of Eq. (19) when the N field develops a VEV at the end of inflation.

Integrating out the extra dimensions coordinates we get

$$\Delta\mathcal{L}_4^{soft}=-\int d^2\theta\ (\lambda N H_u H_d-\kappa\phi N^2)\frac{S}{M_*}, \quad (20)$$

where we have used Eq. (11) to redefine the effective Yukawa couplings λ and κ. When the F term of the singlet S gets VEVs (F_S) from Eq. (20) we obtain an A_k term during inflation and a B term at the end of inflation,

$$A_k\sim B\sim\frac{F_S}{M_*}. \quad (21)$$

We have seen that $\sqrt{F_S}\sim 10^8$ GeV, but M_* seems to be a free parameter. From the minimization of the potential (2) we get the VEV of the field N given by $N_0=A_k/(\sqrt{2}k)$. Using Eqs. (13) and (21) we get

$$N_0=\frac{m_p^2}{\sqrt{2}}\frac{F_S}{M_*^3}. \quad (22)$$

On the other hand, from Eqs. (5), (13), (18) we have

$$F_S^{1/2}=\left(\frac{M_*}{m_p}\right)\frac{N_0}{\sqrt{2}}. \quad (23)$$

Casting the last two equations together we can relate M_* with N_0 as[10]

$$N_0=\sqrt{8}M_*. \quad (24)$$

This means that if we want to solve the CP problem in our model, we immediately need a string scale defined at $M_*\sim 10^{13}$ GeV. Using also that $\sqrt{F_S}\sim 10^8$ GeV, from Eq. (21) we get $A_k\sim B\sim 1$ TeV.

C. Quadratic soft terms for bulk scalars

The quadratic soft masses for the scalars are given by

$$\Delta\mathcal{L}_{4+d}^{soft}=-\int d^4\theta\ \delta^d(\mathbf{y}-y_p)\frac{c_X}{M_*^d}\hat{X}^\dagger\hat{X}\frac{S^\dagger S}{M_*^2}, \quad (25)$$

where X runs over all the bulk fields, ϕ, N, H_u, H_d and c_X are constants of the order one. After dimensional reduction, we have the $4D$ Lagrangian,

$$\Delta\mathcal{L}_4^{soft}=-\int d^4\theta\ \frac{c_X}{(M_*^d V_d)}X^\dagger X\frac{S^\dagger S}{M_*^2}. \quad (26)$$

Using Eq. (8) we get the mass term for the scalars when the F term of the field S gets a VEV:

$$m_X^2=c_X\left(\frac{F_S}{m_p}\right)^2. \quad (27)$$

From Eqs. (21) and (27) we see that the values for the trilinear and the mass terms are non-equal (non-universality) as long as the Planck scale in four dimensions, m_p, and the Planck scale in $4+d$ dimensions, M_*, are different. In fact, their ratio is given by

$$\frac{m_X}{A_\kappa}=\sqrt{c_X}\left(\frac{M_*}{m_p}\right). \quad (28)$$

Therefore, we have that the quadratic soft term for bulk fields are small, in particular $m_X/\sqrt{c_X}\ll A_\kappa$. Below we will see that this is no longer true for particles defined in one of the branes. For $M_*\sim 10^{13}$ GeV, $\sqrt{F_S}\sim 10^8$ GeV and imposing[11] $c_X\sim 1/(4\pi)^2\sim\mathcal{O}(10^{-2})$, from the last equation we get very tiny soft masses for the bulk fields

$$m_\phi\sim m_N\sim m_{h_u}\sim m_{h_d}\sim\mathcal{O}(1\ \text{MeV}). \quad (29)$$

The quadratic soft mass for the inflaton, m_ϕ^2, generated in the SUSY breaking brane, is the same that appears in the inflaton potential (2). As we have already said following Eq. (4), this mass has to be $m_\phi<\mathcal{O}(1\ \text{MeV})$ in order to satisfy the slow-roll condition for the potential. However, this mass is quite large to satisfy the COBE constraint. Nevertheless, in our model the inflaton does not play an important role to generate the density perturbation; instead a new mechanism is proposed in a companion paper [7] in which the Higgs field can generate the large-scale curvature perturbation from an efficient conversion of isocurvature perturbation to curvature one during the reheating era.

D. Quadratic soft terms for brane scalars

So far we have discussed how to generate the soft terms for the fields living in the bulk. The situation is slightly different for the scalars living on one of the branes. The scalars living on the SUSY breaking brane get a soft mass term from the following operator:

[10]Note that N_0 is the effective field in four dimensions and this field can be larger than the string scale M_*. The point is that the higher dimensional field Φ_d carries a volume suppression factor, $\Phi_d=\Phi_4/\sqrt{V_d}$, where Φ_4 is a four dimensional field and V_d is the extra dimension volume. When we use the above relationship between Φ_d and Φ_4, integrate out the extra dimensions, and use relation (8), the natural cutoff for the effective four dimensional field is seen to be m_P and not M_*.

[11]One might think that the operators (25) arise integrating out some massive string excitation propagating a one-loop. It turns out that the coefficient c_X has to contain the one-loop factor, $c_X\approx 1/(4\pi)^2$.

$$\Delta\mathcal{L}_{4+d}^{soft}=-\int d^4\theta\ \ \delta^d(\mathbf{y}-y_p)c_Y Y^\dagger Y\frac{S^\dagger S}{M_*^2}, \tag{30}$$

where Y runs over all the brane's superfields. This leads to a soft mass term,

$$m_Y=\sqrt{c_Y}\left(\frac{F_S}{M_*}\right). \tag{31}$$

Using again $\sqrt{F_S}\sim 10^8$ GeV and $M_*\sim 10^{13}$ GeV, we got a soft term, $m_Y\sim 1$ TeV.

On the other hand, the bulk scalar masses receive a suppression factor M_*/m_p with respect to the former case due to the finite extra dimension volume. In quantum field theory one is free to choose in which branes the particles live. It can be either the SUSY breaking brane or what we have called the Yukawa brane. A possible motivation to put fermions in the Yukawa brane is to solve the flavor changing neutral current (FCNC) problem since all the interactions which violate flavor will be suppressed by a factor $\exp(-M_*R)$ [21]. However, at tree level all their soft masses will be zero since there is no contact term with the singlet S. Therefore the only way to produce soft masses in this case will be through radiative corrections via renormalization group running. In the case of squarks, the main contribution at one-loop comes from the gauginos and the exact spectra will be quite similar to what happen with non-scale supergravity or gaugino mediation [21]. It is well known that there may be phenomenological difficulties with such models in the slepton sector [25]. One possibility is to leave the quarks and/or squarks on the Yukawa brane but localize the lepton sector in the SUSY breaking brane. In this way the sleptons will get TeV soft masses and we still have the FCNC problem for the quark sector resolved.

E. Gaugino mass

The gauge group in our model is given by the direct product of two groups, $G_A^{(1)}\times G_B^{(2)}$. This group will diagonally break down to the standard model gauge group. The gaugino mass for the gauge group which live in the bulk, $G_A^{(1)}$, is given by the operator

$$\Delta\mathcal{L}_{4+d}^{soft}=-\int d^2\theta\ \delta^d(\mathbf{y}-y_p)\frac{c_\lambda^{(1)}}{M_*^d}(W_\alpha^{(1)}W^{\alpha(1)})\frac{S}{M_*}, \tag{32}$$

where $c_\lambda^{(1)}$ is a constant of the order one and $W_\alpha^{(1)}$ is the field strength of the gauge group $G_A^{(1)}$. After dimensional reduction we get a soft gaugino mass for the group $G_A^{(1)}$:

$$m_{\lambda^{(1)}}=\frac{F_S}{M_*}\frac{1}{(M_*R)^d}=\frac{F_S}{M_*}\left(\frac{M_*}{m_p}\right)^2, \tag{33}$$

where we have used the relation Eq. (8) in the last equality. This mass is very tiny; in fact, using $M_*\sim 10^{13}$ GeV and $\sqrt{F_S}\sim 10^8$ GeV, we found $m_{\lambda^{(1)}}\sim\mathcal{O}(100\text{ eV})$. In the case where the only group in our model was the standard model one embedded in extra dimensions, this result would rule out this setup by a direct search of the gauginos. However, localizing the other gauge group, $G_B^{(2)}$, in the SUSY breaking brane it is possible to overcome this problem.[12]

The gauginos of the group $G_B^{(2)}$ get a mass through the operator

$$\Delta\mathcal{L}_{4+d}^{soft}=-\int d^2\theta\ \delta^d(\mathbf{y}-y_p)c_\lambda^{(2)}(W_\alpha^{(2)}W^{\alpha(2)})\frac{S}{M_*}. \tag{34}$$

Using dimensional reduction we obtain

$$m_{\lambda^{(2)}}=\frac{F_S}{M_*}. \tag{35}$$

This mass is exactly the same as that for the A_k term (21), and therefore by using $M_*\sim 10^{13}$ GeV and $\sqrt{F_S}\sim 10^8$ GeV we get $m_{\lambda^{(2)}}\approx 1$ TeV. Hence we have that $m_{\lambda^{(2)}}\gg m_{\lambda^{(1)}}$.

Once the total gauge group is diagonally broken to the standard model gauge group, $G_A^{(1)}\times G_B^{(2)}\to G^{SM}$, the eigenvalue mass for the lowest states are given by [26]

$$m_{\lambda^{SM}}=f(g_1,g_2)m_{\lambda^{(1)}}+h(g_1,g_2)m_{\lambda^{(2)}}\approx m_{\lambda^{(2)}}\approx 1\ \text{TeV}, \tag{36}$$

where $f(g_1,g_2)\sim h(g_1,g_2)\sim\mathcal{O}(1)$ are some functions of the large gauge group $G^{(1)}$ and $G^{(2)}$. It turns out that the standard model gauginos in our model have mass of the order of the SUSY breaking scale $\sim\mathcal{O}(\text{TeV})$.

V. SUMMARY

We have shown how, by embedding the BGK model into an extra dimensional framework, all the naturalness problems of the model may be resolved. An underlying assumption of our approach is that the radii of the extra dimensions are stabilized, for example, by the mechanism proposed in [27]. The extra dimensional setup has all the Higgs fields and singlets in the bulk, and all the matter fields live on the branes. All the parameters of the effective 4D model, including the Planck scale, can then be naturally understood in terms of a fundamental "string" scale $M_*\sim 10^{13}$ GeV and a brane SUSY breaking term $\sqrt{F_S}\sim 10^8$ GeV. Once the number of extra dimensions d is specified, the reduced Planck scale m_p in Eq. (8) then fixes the size of the extra dimensions. From these scales everything else follows: the height of the inflaton potential during inflation is of order $\sqrt{F_S}$; the singlet VEVs after inflation associated with the axion solution to the strong CP problem are of order M_*; the small Yukawa couplings $\lambda\sim\kappa\sim 10^{-10}$ necessary for the model to solve the strong CP problem and generate the correct effective μ term after inflation are of order $(M_*/m_p)^2$; the TeV scale soft masses and trilinears for scalars on the branes are naturally understood as F_S/M_*; and the MeV inflaton

[12]This group can either be a replica of the same bulk gauge group $G_A^{(1)}$ or a different one.

masses for scalars in the bulk are suppressed relative to the TeV scale soft masses by a factor M_*/m_p. Although we do not address the question of neutrino masses in this paper, we note that the natural scale of our model $M_* \sim 10^{13}$ GeV is also typical of right-handed Majorana masses in see saw models.

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[1] X.m. Wang, M. Tegmark, and M. Zaldarriaga, Phys. Rev. D **65**, 123001 (2002).
[2] M. Bastero-Gil and S.F. King, Phys. Lett. B **423**, 27 (1998).
[3] K. Dimopoulos and D.H. Lyth, hep-ph/0209180.
[4] D.H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002); T. Moroi and T. Takahashi *ibid.* **522**, 215 (2001); **539**, 303(E) (2002); K. Enqvist and M.S. Sloth, Nucl. Phys. **B626**, 395 (2002).
[5] T. Moroi and T. Takahashi, Phys. Rev. D **66**, 063501 (2002); D.H. Lyth, C. Ungarelli, and D. Wands, *ibid.* **67**, 023503 (2003); M.S. Sloth, hep-ph/0208241.
[6] N. Bartolo and A. Liddle, Phys. Rev. D **65**, 121301 (2002); R. Durrer and F. Vernizzi, *ibid.* **66**, 083503 (2002); A. Notari and A. Riotto, Nucl. Phys. **B644**, 371 (2002); V. Bozza, M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Lett. B **543**, 14 (2002).
[7] M. Bastero-Gil, V. Di Clemente, and S.F. King, hep-ph/0211011.
[8] P. Kanti and K.A. Olive, Phys. Rev. D **60**, 043502 (1999); Phys. Lett. B **464**, 192 (1999).
[9] R.N. Mohapatra, A. Perez-Lorenzana, and C.A. de Sousa Pires, Phys. Rev. D **62**, 105030 (2000); A.M. Green and A. Mazumdar, *ibid.* **65**, 105022 (2002).
[10] P. Binetruy, C. Deffayet, and D. Langlois, Nucl. Phys. **B565**, 269 (2000).
[11] R.N. Mohapatra, A. Perez-Lorenzana, and C.A. de Sousa Pires, Int. J. Mod. Phys. A **16**, 1431 (2001).
[12] For a review see, for example, J.E. Kim, Phys. Rep. **150**, 1 (1987).
[13] G.G. Raffelt, Annu. Rev. Nucl. Part. Sci. **49**, 163 (1999).
[14] M.S. Turner, Phys. Rev. D **33**, 889 (1986); K. Choi, E.J. Chun, and J.E. Kim, Phys. Lett. B **403**, 209 (1997).
[15] G.F. Smoot *et al.*, Astrophys. J. Lett. **396**, L1 (1992); E.L. Wright *et al.*, *ibid.* **396**, L13 (1992); M.J. White and E.F. Bunn, *ibid.* **443**, L53 (1995); C.L. Bennett *et al.*, *ibid.* **464**, L1 (1996); E.F. Bunn and M. White, Astrophys. J. **480**, 6 (1997).
[16] J.D. Lykken, "Introduction to supersymmetry," hep-th/9612114.
[17] K.R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B537**, 47 (1999).
[18] E.A. Mirabelli and M.E. Peskin, Phys. Rev. D **58**, 065002 (1998).
[19] M.F. Sohnius, Phys. Rep. **128**, 39 (1985).
[20] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[21] D.E. Kaplan, G.D. Kribs, and M. Schmaltz, Phys. Rev. D **62**, 035010 (2000); Z. Chacko, M.A. Luty, A.E. Nelson, and E. Ponton, J. High Energy Phys. **01**, 003 (2000); S.F. King and D.A. Rayner, Nucl. Phys. **B607**, 77 (2001).
[22] V. Di Clemente, S.F. King, and D.A. Rayner, Nucl. Phys. **B617**, 71 (2001); **B646**, 24 (2002).
[23] G.F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).
[24] M. Bastero-Gil and S.F. King, Nucl. Phys. **B549**, 391 (1999).
[25] M. Schmaltz and W. Skiba, Phys. Rev. D **62**, 095004 (2000).
[26] S.F. King and M. Oliveira, Phys. Rev. D **60**, 035003 (1999).
[27] L. Amendola, E.W. Kolb, M. Litterio, and F. Occhionero, Phys. Rev. D **42**, 1944 (1990); E. Ponton and E. Poppitz, J. High Energy Phys. **06**, 019 (2001); P. Kanti, K.A. Olive, and M. Pospelov, Phys. Lett. B **538**, 146 (2002).