Model of the universe including dark energy accounted for by both a quintessence field and a (negative) cosmological constant

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In this work we present a model of the universe in which dark energy is modeled explicitly with both a dynamical quintessence field and a cosmological constant. Our results confirm the possibility of a future collapsing universe (for a given region of the parameter space), which is necessary for a consistent formulation of both string and quantum field theories. The predictions of this model for distance modulus of supernovae are similar to those of the standard ΛCDM model.

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I. INTRODUCTION

From 1998 to date several important discoveries in the astrophysical sciences have been made, which have given rise to the so-called new cosmology [1,2]. Among its more important facts we may cite that the universe expands in an accelerated way [3,4], and the first Doppler peak in the cosmic microwave background is strongly consistent with a flat universe whose density is the critical one [5], while several independent observations indicate that matter energy density is about one-third of the aforementioned critical density [6,7]. The last two facts imply that some unknown component of the universe ''was missing''; it is called dark energy, and it represents nearly two-thirds of the energy density of the universe. The leading candidates to be identified with dark energy involve fundamental physics and include a cosmological constant (vacuum energy), a rolling scalar field (quintessence), and a network of light, frustrated topological defects [8].

On the other hand, an eternally accelerating universe seems to be at odds with string theory, because of the impossibility of formulating the *S* matrix. In de Sitter space the presence of an event horizon, signifying causally disconnected regions of space, implies the absence of asymptotic particle states which are needed to define transition amplitudes [9,10]. This objection against accelerated expansion also applies to quantum field theory [11].

Because of the above there is renewed interest in exponential quintessence, because in several scenarios exponential potentials can reproduce the present acceleration and predict future deceleration, so again string theory has well defined asymptotic states [10,12]. It is worthwhile noticing that exponential quintessence had been so far overlooked because of fine-tuning arguments, but several authors have recently pointed out that the degree of fine-tuning needed in these scenarios is no more than in others usually accepted [10,12–14].

The cosmological constant can be incorporated into the quintessence potential as a constant which shifts the potential value, especially the value of the minimum of the potential, where the quintessence field rolls towards. Conversely, the height of the minimum of the potential can also be regarded as a part of the cosmological constant. Usually, for separating them, the possible nonzero height of the minimum of the potential is incorporated into the cosmological constant and then set to be zero. The cosmological constant can be provided by various kinds of matter, such as the vacuum energy of quantum fields and the potential energy of classical fields, and may also be originated in the intrinsic geometry. So far there is no sufficient reason to set the cosmological constant (or the height of the minimum of the quintessence potential) to zero [15]. In particular, some mechanisms to generate a negative cosmological constant have been pointed out [16,17].

The goal of this paper is to present a model of the universe in which the dark energy component is accounted for by both a quintessence field and a negative cosmological constant. The quintessence field accounts for the present stage of accelerated expansion of the universe. Meanwhile, the inclusion of a negative cosmological constant warrants that the present stage of accelerated expansion will be, eventually, followed by a period of collapse into a final cosmological singularity (AdS universe).

II. MODEL

Our scenario is a further generalization of that originally proposed by Rubano and Scudellaro [13] (also studied by us in Ref. [18]). We consider a model consisting of a three-component cosmological fluid: matter, scalar field (quintessence with an exponential potential), and a negative cosmological constant. We point out that we model dark energy with both the quintessence field and the negative cosmological constant, resulting in a positive effective cosmological constant, in agreement with experimental data [19]. ''Matter'' means barionic + cold dark matter, with no pressure, and the scalar field is minimally coupled and noninteracting with matter, so the action is

$$S=\int d^4x \sqrt{-g}\left(\frac{c^2}{16\pi G}(R-2\Lambda)+\mathcal{L}_\phi+\mathcal{L}_m\right), \quad (2.1)$$

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where Λ is the cosmological constant, $\mathcal{L}_m$ is the Lagrangian for the matter degrees of freedom, and the Lagrangian for the quintessence field is given by

$$\mathcal{L}_\phi = -\frac{1}{2}\phi_{,n}\phi^{,n} - V(\phi). \tag{2.2}$$

This model cannot be used from the very beginning of the universe, but only since decoupling of radiation and dust. Thus we do not take into account inflation, creation of matter, nucleosynthesis, etc. We apply the same technique of adimensional variables we used in Ref. [18] (this allows us to determine the integration constants without additional assumptions). We use the dimensionless time variable $\tau = H_0 t$, where t is the cosmological time and H_0 is the present value of the Hubble parameter. In this case $a(\tau) = a(t)/a(0)$ is the scale factor. Then we have that, at present $(\tau = 0)$,

$$\begin{aligned} a(0) &= 1, \\ \dot{a}(0) &= 1, \\ H(0) &= 1. \end{aligned} \tag{2.3}$$

Considering a homogeneous and isotropic universe, and using the experimental fact of a spatially flat universe [20], the field equations derivable from Eq. (2.1) are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2}{9}\sigma^2\left(\frac{\overline{D}}{a^3} + \frac{1}{2}\dot{\phi}^2 + \overline{V}(\phi) + \frac{3}{2}\frac{\overline{\Lambda}}{\sigma^2}\right), \tag{2.4}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{2}{3}\sigma^2\left(\frac{1}{2}\dot{\phi}^2 - \overline{V}(\phi) - \frac{3}{2}\frac{\overline{\Lambda}}{\sigma^2}\right), \tag{2.5}$$

and

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \overline{V}'(\phi) = 0, \tag{2.6}$$

where the dot means derivative in respect to τ and

$$\overline{V}(\phi) = \overline{B}^2 e^{-\sigma\phi}, \tag{2.7}$$

$$\overline{X} = \frac{X}{H_0^2}$$

$$\left(\text{except for } \overline{D} = \frac{D}{a_0^3 H_0^2} = \frac{\rho_{m_0}}{H_0^2}\right), \tag{2.8}$$

$$\sigma^2 = \frac{12\pi G}{c^2}, \tag{2.9}$$

with ρ_{m_0}—the present density of matter and B^2—a generic constant. We stress that the particular choice of Eq. (2.9) for σ allows for general exact integrations of equations. Indeed, this choice has been used in the context of inflationary theory [21–24], and in the Rubano-Scudellaro model [13,18]. Applying the Noether symmetry approach [25,21,22,26], it can be shown that the new variables we should introduce to simplify the field equations are the same used in Ref. [13]:

$$a^3 = uv, \tag{2.10}$$

and

$$\phi = -\frac{1}{\sigma}\ln\left(\frac{u}{v}\right). \tag{2.11}$$

In these variables the field Eqs. (2.4)–(2.6) may be written as the following pair of equations (from now on we use the energy density Ω_Λ instead of the cosmological constant $\overline{\Lambda}$):

$$\ddot{u} = \frac{9\Omega_\Lambda}{4}u, \tag{2.12}$$

and

$$\ddot{v} = \frac{9\Omega_\Lambda}{4}v + \sigma^2\overline{B}^2 u. \tag{2.13}$$

The solutions of Eqs. (2.12) and (2.13) are found to be

$$u(\tau) = u_1\sin(1.5\sqrt{-\Omega_\Lambda}\tau) + u_2\cos(1.5\sqrt{-\Omega_\Lambda}\tau), \tag{2.14}$$

and

$$\begin{aligned} v(\tau) = &\left(v_2 - \frac{\sigma^2\overline{B}^2}{9\Omega_\Lambda}u_2 - \frac{\sigma^2\overline{B}^2}{3\sqrt{-\Omega_\Lambda}}u_1\tau\right)\cos(1.5\sqrt{-\Omega_\Lambda}\tau) \\ &+ \left(v_1 - \frac{\sigma^2\overline{B}^2}{9\Omega_\Lambda}u_1 + \frac{\sigma^2\overline{B}^2}{3\sqrt{-\Omega_\Lambda}}u_2\tau\right)\sin(1.5\sqrt{-\Omega_\Lambda}\tau), \end{aligned} \tag{2.15}$$

where u_1, u_2, v_1, and v_2 are the integration constants.

In finding the integration constants we use Eqs. (2.3) and field equations evaluated at $\tau = 0$, introducing the deceleration parameter of the universe q_0. Finally, using $\Omega_{m_0} + \Omega_{Q_0} + \Omega_\Lambda = 1$, the above integration constants can be written in the following way:

$$u_2^{(\pm)} = \pm\sqrt{\frac{3(2 - q_0 - 1.5\Omega_{m_0} - 3\Omega_\Lambda)}{2\sigma^2\overline{B}^2}}, \tag{2.16}$$

$$v_2^{(\pm)} = \frac{1 + \dfrac{\sigma^2\overline{B}^2}{9\ \Omega_\Lambda}\ u_2^2}{u_2^{(\pm)}}, \tag{2.17}$$

$$u_{1[\pm]}^{(\pm)} = \frac{\{\sqrt{3} - [\pm]\sqrt{1 + q_0 - 1.5\Omega m_0}\}}{\sqrt{-3\Omega_\Lambda}}u_2^{(\pm)}, \tag{2.18}$$

and

$$v_{1[\pm]}^{(\pm)}=\frac{2-\sqrt{-\Omega_\Lambda}v_2^{(\pm)}u_{1[\pm]}^{(\pm)}}{\sqrt{-\Omega_\Lambda}u_2^{(\pm)}}, \tag{2.19}$$

respectively. The subscript null indicates present values, and we recall that Ω_i are component densities (in units of the critical density; m stands for matter, Q for quintessence field, and Λ for the cosmological constant).

III. ANALYSIS OF RESULTS

We see from the expressions for the constants that our solution has four branches. We used the ''all-pluses'' branch, in which the upper plus signs are preferred over the lower minus ones.

Since $\sqrt{1+q_0-1.5\Omega_{m_0}}$ should be real, the following constrain on the present value of the deceleration parameter follows: $q_0 \geqslant -1+1.5\Omega_{m_0}$. It can be noticed that the constants (and, consequently, the solutions) depend on four parameters: Ω_{m_0}, Ω_Λ, q_0, and $\overline{B}^2$. σ^2 is fixed, and from now on we assign the value 3/2. In this normalization, today's value of the critical density of the universe $\rho_{c_0}=9H(0)^2/2\ \sigma^2=3$. In general, the experimental values for Ω_{m_0} and q_0 are model dependent, because these magnitudes are not directly measured (though Turner and Riess have developed a model-independent test for past deceleration [27]). We chose $\Omega_{m_0}=0.3$ and $q_0=-0.44$, perfectly acceptable for most available models. Though we made calculations for several values of Ω_Λ in the range -0.01–-0.30, for simplicity we present results for -0.15, bearing in mind that they change little for other values. Concerning $\overline{B}^2$, it was shown analytically that the relevant cosmological magnitudes we studied in this paper are independent of it. That is the case of the scale factor, the Hubble and deceleration parameters, and the energy density, pressure, and state parameter of the quintessence field. However, it can be easily shown that $\overline{B}^2$ can be of the order of the critical density of the universe [14]. From Eqs. (2.3), (2.10), (2.14), and (2.15) there can also be shown a relation between this parameter and today's value of the scalar field ϕ_0:

$$\phi_0=-\frac{1}{\sigma}\ln\frac{2-q_0-1.5\Omega_{m_0}-3\Omega_\Lambda}{\overline{B}^2}. \tag{3.1}$$

The above considerations lead us to choose for subsequent calculations $\overline{B}^2=1$, which just means a determined rescaling in ϕ_0. We postpone for further work the question of whether we need finely tuned initial conditions to get a determined value of ϕ_0.

Figure 1 shows the evolution of the scale factor for $\Omega_\Lambda=-0.15$. For the above values of the other parameters, we obtained a collapsing universe, no matter what the value of Ω_Λ. We also saw that with the decrease (modular increase) of Ω_Λ, the time of collapse diminishes.

Figure 2 shows the behavior of the deceleration parameter as function of the redshift z. In agreement with Turner and Riess [27] and other authors, this figure shows that the acceleration is a relatively recent phenomenon, being the transition from the decelerated phase to the accelerated one at redshift near 0.5. However, as follows from Fig. 1, acceleration is not eternal: in the future $q>0$ again, which gives rise to the collapse. Figure 3 shows the energy densities of matter and dark energy (quintessence field plus cosmological constant, i.e., effective quintessence). In the literature it is widely accepted that using an exponential potential leads to a dark energy density which scales like matter, which implies a constant ratio of quintessence to matter energy density, at least in the matter domination regime [28,29,13]. But this is a consequence of assuming the state parameter ω of dark energy almost perfectly constant, which in our case is far from being true, as seen in Fig. 3. We appreciate that matter dominates in a redshift interval by 0.4–1.6, which is roughly consistent with the decelerated universe shown in Fig. 2. For higher redshifts dark energy dominates, but Fig. 4 (state parameter of effective quintessence versus redshift) shows that in that epoch its state parameter is positive. This points at a past epoch in the evolution when gravity of the dark energy was attractive, which is consistent with the deceleration, and with the increase of the deceleration parameter at higher redshifts (given the fact that then both matter and dark energy have attractive gravity).

Now we proceed to make a first indirect check of our model with the supernovae observations, in the same way it

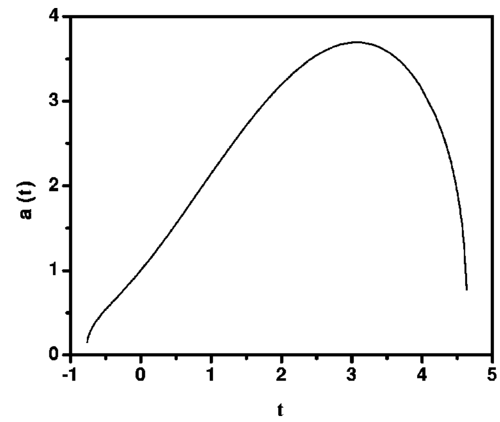

FIG. 1. This plot of scale factor versus time shows a collapsing universe.

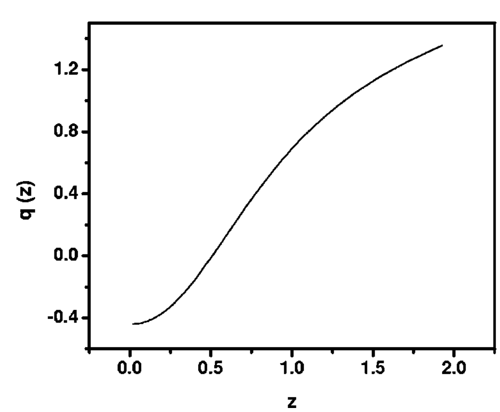

FIG. 2. The acceleration is a rather recent phenomenon, as can be seen in this graphic of deceleration parameter versus redshift.

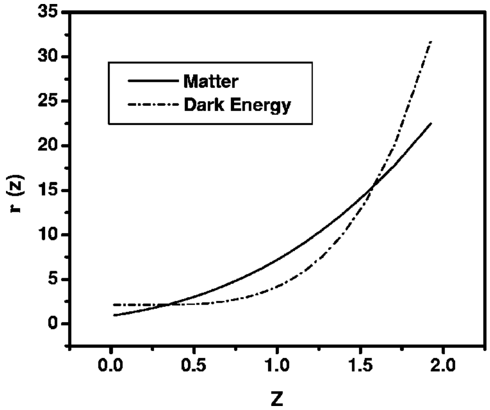

FIG. 3. Matter energy density dominates in a redshift interval from 0.4 to 1.6 (roughly).

is done in Ref. [13]. From these observations, we can have the distance modulus δ of each supernova, which is the difference between its apparent and absolute magnitudes (m and M, respectively):

$$\delta = m - M. \tag{3.2}$$

On the other hand, the distance modulus is related to the luminosity distance $d_L(z)$, through

$$\delta(z) = 5\,\log_{10} d_L(z) + 25. \tag{3.3}$$

The luminosity distance (in Mpc) can be calculated for each model, once the expression for the Hubble parameter $H(z)$ is known:

$$d_L(z) = 3000(1+z)\int_0^z \frac{dz'}{H(z')}. \tag{3.4}$$

Having the expressions for $H(z)$ of both the cold dark matter model with a cosmological constant (ΛCDM) and our model, we used Eqs. (3.4) and (3.3) to plot distance modulus $\delta(z)$ versus redshift z. So, in Fig. 5 we just compare the predictions of our model with those of the ΛCDM one. We observe that towards higher redshifts, the relative deviations grow up to near 2% at $z=1$. Possible causes are the

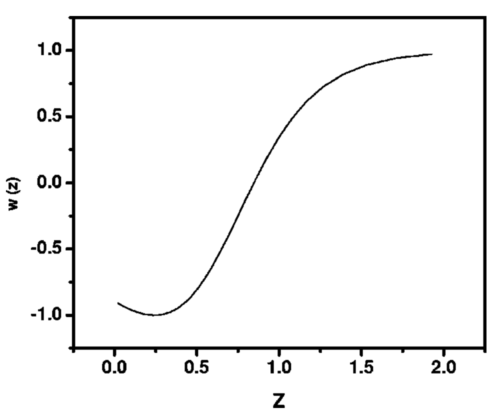

FIG. 4. The state parameter of dark energy (quintessence field plus cosmological constant) is far from being constant.

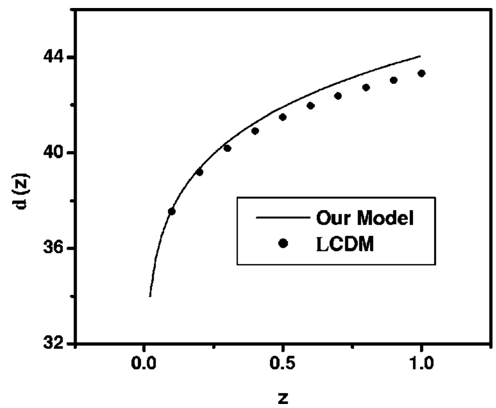

FIG. 5. The predictions of our model and ΛCDM model for distance moduli of supernovae are quite similar, the maximum relative deviations are of about 2%.

different dynamics in the energy densities budget in these two models (as seen in Fig. 3, in our model matter domination becomes greater as one runs from $z=0.4$ back to $z=1$), and the abrupt change in the state parameter of dark energy in this interval, as seen in Fig. 4 (in ΛCDM model ω is constant). In fact, supernovae observations are thought to be useful to determine the value of ω for dark energy, but there is considerable controversy concerning the possibility of doing this [30,31]. Indeed, it is known that luminosity distance has small sensitivity respect to the variation of the state parameter $\omega(z)$ of dark energy, because the complex integral relation between these magnitudes smears out the differences. For instance, in Ref. [30] it is shown how nine different models of dark energy give similar predictions for luminosity distance (but also see Ref. [31]). Anyway, it is known that, after proper fitting, the standard ΛCDM model makes satisfactory predictions for the *observed* distance moduli of the supernovae, which means that, in the context of this qualitative discussion, we have a first indication that our model will also do that. We made a rough preliminary calculation with one of the other branches and also got maximum differences with ΛCDM model predictions near 2% at $z=1$. Thus we guess that the integral relation (3.4) [and probably the logarithmic one (3.3)] could smooth the different "initial" expressions for $H(z)$ that every model supply into "final" similar predictions for luminosity distances and distance moduli. Anyway, we postpone this issue for further work, in the context of the comparison with the real data.

IV. CONCLUSIONS

In a recent paper [15] it was pointed out that the ultimate fate of the evolution of our universe is much more sensitive to the presence of the cosmological constant than any other matter content. In particular, the universe with a negative cosmological constant will always collapse eventually, even though the cosmological constant may be nearly zero and undetectable at all at the present time. Our results support the very general assertions of Ref. [15], we have shown that for a determined region of the parameter space, the universe collapses. This also favors the consistent formulation of string and quantum field theories, as explained in the Introduction. The predictions of our model for distance moduli from the supernovae are very similar to those made by the standard ΛCDM model. So far, we have investigated one of the several possible branches of the solution, leaving for the future the investigation of the others. We have also reserved for future work the careful examination of this universe near its beginning (i.e., just after the decoupling of matter and radiation).

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