

Heisenberg's universal $\ln^2 s$ increase of total cross sections

H. G. Dosch

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

P. Gauron and B. Nicolescu

LPNHE—Theory Group, Université Pierre et Marie Curie, Tour 12 E3, 4 Place Jussieu, 75252 Paris Cedex 05, France*

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The $\ln^2 s$ behavior of total cross sections, first obtained by Heisenberg 50 years ago, has received increased interest both on phenomenological and theoretical levels. In this paper we present a modification of Heisenberg's model in connection with the presence of glueballs and we show that it leads to a realistic description of all existing hadron total cross-sections data.

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I. INTRODUCTION

In a remarkable paper of 1952, Heisenberg investigated the production of mesons as a problem of shock waves [1]. One of his results was that the total cross section increases like the square of the logarithm of the center-of-mass energy. It is noteworthy that this result coincides with very recent calculations based on AdS conformal field theory (CFT) dual string-gravity theory [2] or on the color glass condensate approach [3] and, of course, saturates the Froissart-Martin bound [4]. In contradistinction to the latter case however the coefficient of the \ln^2 term is an estimate at finite energies and not an asymptotic bound as the one obtained by Lukaszuk and Martin [5].

We show in this Brief Report that by modifications of the original model of Heisenberg motivated by the enormous progress of knowledge in the 50 years that have since passed, the model yields some general and even some quantitative results which describe the data very well.

Our article is organized as follows. In Sec. II we briefly discuss the original model of Heisenberg; in Sec. III we modify it and compare it with the data and in Sec. IV we discuss the results, their merits and their shortcomings and we present our conclusions.

II. THE HEISENBERG MODEL FOR THE TOTAL CROSS SECTION

The considerations of Heisenberg concerning the total cross section are essentially geometrical ones, but the crucial ingredient is that the energy density and not the hadronic density is the essential quantity to be taken into account. The major part of Heisenberg's paper is related to dynamical questions of meson production and is for our present investigation only of interest as dynamical background.

Proton-proton collisions are considered in the center-of-mass system and the energy \sqrt{s} is supposed to be high enough that Lorentz contraction allows us to view the nucleons as disks (see Fig. 1).

Interaction takes place only in the overlap region (shaded area in Fig. 1) and the crucial assumption is made that a reaction can only occur if the energy density is high enough in order to create at least a meson pair.

Let $\gamma\sqrt{s}/V$ be the energy per unit volume disposable for meson production, where γ is some positive constant smaller than 1. According to the assumption stated above, a reaction can only take place if $\gamma\sqrt{s}$ is sufficient to create two mesons. If we denote the energy of the two mesons by k_0 we thus have the condition

$$\gamma \cdot \sqrt{s} \geq k_0. \quad (1)$$

Heisenberg took k_0 as the average energy of two produced mesons. In his shock wave approach with a nonrenormalizable meson interaction, k_0 was only increasing very slowly (logarithmically) with energy. Next Heisenberg assumed that, at least for large impact factors b (see Fig. 1), γ is proportional to the overlap of the meson clouds, that is

$$\gamma = \alpha \cdot e^{-m \cdot b}, \quad (2)$$

where α is some constant smaller than 1 and m is the mass of the mesons forming the cloud. From

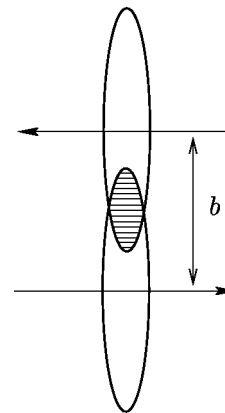


FIG. 1. Scattering of two Lorentz contracted hadrons in the center of mass system; the interaction region is shaded.

*Unité de Recherche des Universités Paris 6 et Paris 7, Associée au CNRS.

$$\gamma \geq \gamma_{min} = \frac{k_0}{\sqrt{s}}, \quad (3)$$

we deduce the maximal impact parameter for which interaction takes place

$$b_{max} = \frac{-1}{m} \ln \frac{k_0}{\sqrt{s} \alpha} \quad (4)$$

and therefore

$$\sigma = 2\pi \int_0^{b_{max}} b db = \frac{\pi}{m^2} \ln^2 \frac{\sqrt{s} \alpha}{k_0} \quad (5)$$

which, apart from the factor α , is the result obtained by Heisenberg [1]. We see that implicitly the assumption has been made that if a meson production is energetically possible, it will happen (black disk). Of course, Heisenberg was taking the pion mass for the meson mass. For the energy of the produced mesons he deduced, in his dynamical considerations, assuming interactions of maximal strength, that the energy k_0 increases only slowly with energy, at any rate not by a power of s . Therefore the asymptotically leading term in the cross section is $(\pi/4m_\pi^2) \ln^2 s$, the coefficient $\pi/4m_\pi^2$ being 1/4 of the Lukaszuk-Martin bound [5]. The argument can be extended easily to hadron-hadron scattering in general, and therefore we have the result that the coefficient of the $\ln^2 s$ term is *universal* for all hadron reactions.

The assumption that the interaction is of maximal strength, that is highly divergent in the UV region, is essential in Heisenberg's argument. Only in that way enough energy is dissipated from the shock-wave front into lower energies to ensure copious particle production and to obtain a powerlike decrease of γ_{min} in Eq. (3). Also in our present understanding multiparticle production is essential for the increase of cross sections with energy. This has been studied in great detail in perturbative QCD by Lipatov and co-workers [6,7]. The increase in phase space with $\log s$ for each additional particle leads to a power series in $\alpha_s \log s$ and demands in the high energy limit for resummation. This has been done by solving the famous Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation. In leading order this leads to a singularity in the complex J plane at $J = 1 + 4 \log 2(\alpha_s N_c / \pi) \approx 1.5$ and hence to a tremendous increase with energy. Though next to leading order corrections are huge [8,9] partial resummations seem to lead to a stable result [10] at $J \approx 1.3$. But this can be only applied if at least one hard scale is present, that is, in practice, only to γ^* hadron or better to $\gamma^* \gamma^*$ scattering with high photon virtualities. In this case the Froissart bound cannot be derived. Nevertheless it seems plausible that also for scattering of small objects the asymptotic theorems hold. In a recent paper [3] it was shown that gluon saturation effects together with confinement lead indeed to a moderation of the BFKL-type behavior to a $\log^2 s$ behavior at high energies. Quite generally speaking it is very plausible that a strong increase of the cross section eventually violating unitarity bounds is eventu-

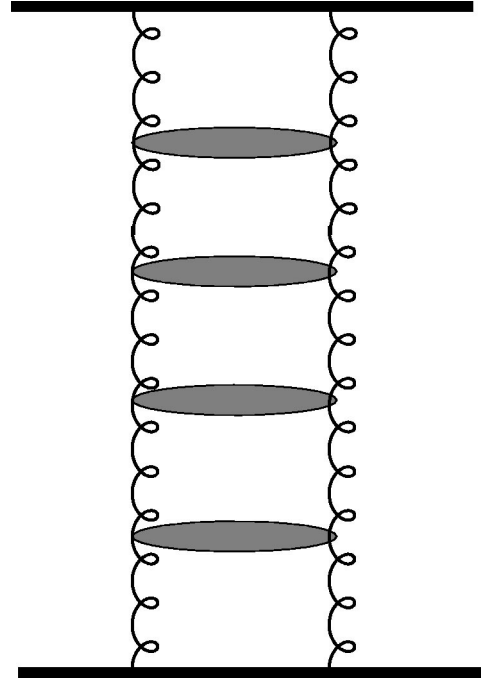


FIG. 2. Multiparticle production leading to an increasing cross section according to [15], the gray ellipses represent hadrons (e.g., two pions) or may indicate instanton induced reactions.

ally damped to satisfy the Froissart bound (see for instance [11]). The connection between the Heisenberg model and the considerations of [3] has been discussed in [12]. Here it is suggested to get a more quantitative connection in a U(1) gauge theory with massive gauge bosons and fermions. This would in some way be a return to the model of Cheng and Wu [13].

For hadron-hadron scattering the perturbative treatment of BFKL is not possible, but there has been a promising development [14,15] to explain the increase of the hard cross section by multiple particle production in a way analogous to BFKL. But, in contrast to the perturbative treatment in QCD, the produced “particles” (rungs in the BFKL ladder) are not gluons but colorless objects, pions for instance (see Fig. 2). In this way a singularity in the complex J plane at $J \approx 1.1$ has been obtained [15]. It is interesting to note that in the more recent publications [16–18] emphasis is given to instanton induced interactions as source of many-particle production. All these attempts have in common that the rise of energy in hadron-hadron cross sections is a genuine nonperturbative effect which brings them closer to Heisenberg's original idea.

III. MODIFICATIONS OF THE MODEL

There are two obvious necessary modifications of the Heisenberg model: (1) If we want to apply it to all kind of hadrons, we have to take care of the different hadron sizes, since in the above treatment all sizes are equal to $1/m$; (2) we have to take into account that *direct* pion exchange, though being the exchange with the lightest particle, is not relevant at high energies. This is due to the fact that exchanged gluons have spin 1 and pions spin 0. Therefore already in Born

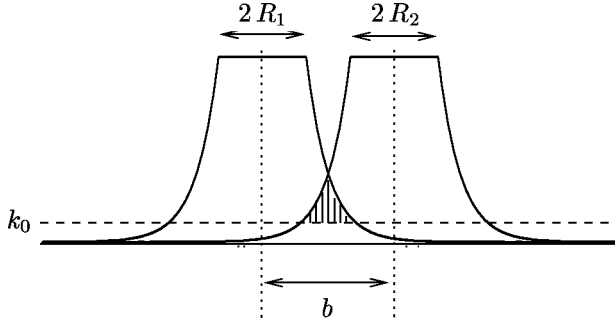


FIG. 3. Interaction region (shaded) of two hadrons in the modified Heisenberg model of high energy scattering.

approximation gluon exchange dominates at high energies. In Regge theory this is manifested by the fact that intercept of the pion is much lower than that of the Pomeron.

We therefore modify the model in two respects.

(1) We make a rough approximation for the overlap (see Fig. 3):

$$\gamma = \begin{cases} \alpha & \text{for } b \leq R_1 + R_2, \\ \alpha e^{-m(b-R_1-R_2)} & \text{for } b \geq R_1 + R_2. \end{cases} \quad (6)$$

(2) For the mass we rather insert a mass M in the range of the glueball mass instead of the pion mass m , since we believe that the high-energy behavior is dominated by gluon exchange.

We then obtain for b_{max}

$$b_{max} = R_1 + R_2 + \frac{1}{M} \log \frac{\sqrt{s}\alpha}{k_0}, \quad (7)$$

assuming \sqrt{s} large enough that $\ln(\sqrt{s}\alpha/k_0) > 0$.

For the total cross section we thus obtain

$$\begin{aligned} \sigma &= e \frac{\pi}{M^2} \ln^2 \frac{\sqrt{s}\alpha}{k_0} + 2\pi(R_1 + R_2) \ln \frac{\sqrt{s}\alpha}{k_0} \frac{1}{M} + \pi(R_1 + R_2)^2, \\ &= \frac{\pi}{4M^2} \ln^2 s + \frac{\pi}{M} \ln s \left\{ (R_1 + R_2) + \frac{1}{M} \ln \frac{\alpha}{k_0} \right\} \\ &\quad + \pi(R_1 + R_2)^2 + \frac{\pi}{M^2} \ln^2 \frac{\alpha}{k_0} + \frac{2\pi}{M} (R_1 + R_2) \ln \frac{\alpha}{k_0}, \end{aligned} \quad (8)$$

where, as usual, implicit scale factors of 1 GeV^2 and 1 GeV , respectively, are obviously assumed in writing σ as a quadratic form in $\ln s$. We see that the leading $\ln^2 s$ term is still universal, but now dominated rather by a glueball than by the pion mass. Since R_1 and R_2 are supposed to be of the size of the electromagnetic radii, the second term in Eq. (8) will dominate over the π/M^2 term except at high energies, $s \gg k_0^2/\alpha^2$.

In order to perform a rough numerical estimate, we may insert for the glueball mass a value between 1.4 and 1.7 GeV, yielding

$$\frac{\pi}{4M^2} = 0.11 - 0.16 \text{ mb}. \quad (9)$$

For R_1 and R_2 we may insert the electromagnetic radii. In contrast to Heisenberg, we insert for k_0 the minimal energy of two produced particles. Since production seems to occur in clusters with mass around 1.3 GeV [19], we can put $k_0 = 2.6 \text{ GeV}$. The value of α ($0 \leq \alpha \leq 1$) might be process dependent. For very small objects (“onia”) it might be very small.

In the past, application of the Heisenberg model to the global analyses of the forward hadronic data were performed in [20], but the universality of the leading term was not discussed there. This universality was treated by Gershtein and Logunov [21], who made the assumption, as in the present paper, that the growth of the hadron-hadron total cross sections is related to resonance production of glueballs.

A very good fit for all forward data has been recently obtained by assuming a universal $\ln^2 s$ dependence and a constant contribution dependent on the process [22,23]. Apart from Reggeon exchanges, which are of no concern here, the cross section was fitted [23] to

$$\sigma^{HH} = B \ln^2 \left(\frac{s}{s_0} \right) + Z^{HH}, \quad (10)$$

with $B \approx 0.32 \text{ mb}$, $Z^{pp} \approx 36 \text{ mb}$, $Z^{\pi p} \approx 21 \text{ mb}$, $Z^{Kp} \approx 18 \text{ mb}$, and $s_0 \approx 34 \text{ GeV}^2$.

Such a value of B would correspond to a mass M of 1 GeV, a bit small for a glueball, but not unreasonable given the crude approximations. Z^{HH} are in the right order of magnitude of R^2 .

The scale s_0 is related to the quantities α and k_0 in Eqs. (2) and (3). Using for k_0 the cluster mass [19] we obtain for the scattering of normal hadrons a value $\alpha \approx 0.28$, that is about a fourth of the energy goes into particle production.

The term Z^{HH} in Eq. (10) has been derived here from the purely geometrical model (8). The *transverse* hadronic radii following from Eq. (8) and the fitted value for Z^{HH} are $R_p = 0.53 \text{ fm}$, $R_\pi = 0.28 \text{ fm}$, and $R_K = 0.23 \text{ fm}$. They are smaller than the corresponding electromagnetic radii, but of the same order of magnitude. An energy independent term has been also obtained in a nonperturbative model for high energy scattering. It is based on a functional-integral approach to high energy scattering [24], where the functional integrals are evaluated in a specific model for nonperturbative QCD [25], based on the gauge invariant gluon-gluon correlator [26,27]. The hadron dependent values of Z for pp , πp , and Kp scattering in this model [28] are 35 mb, 23 mb and 19 mb, respectively.

IV. CONCLUSIONS

We have seen that a geometrical model, where the energy density available for particle production is the relevant quantity, leads to the quite general result that the leading term in energy is of $\ln^2 s$ type and universal for all hadronic processes. Such a type of energy behavior has been obtained as

the best choice testing many models and even the parameters found in the fit to all forward data [23] are in qualitative agreement with a reasonable choice of the parameters of the present model.

A consequence of the universal $\ln^2 s$ term is that at asymptotic energies all hadron cross sections become equal. At finite but high energies the pion and kaon proton cross sections are therefore expected to rise somewhat faster than the nucleon-nucleon cross sections. This seems indeed to be indicated by the data.

A crucial modification of the original model of Heisenberg consisted in replacing the pion mass determining the falloff of the energy density available for high-energy reactions by a mass close to the expected glueball mass. This is necessitated by the fact that *direct* pion exchange does not contribute to high-energy scattering. This may explain why the Lukaszuk-Martin bound could not be essentially lowered. If one insists on rigor, one has to use the lightest particles to take into account the nearest singularities, but it is quite certain that these nearest singularities are irrelevant for high-energy scattering.

We are fully aware that the model proposed here is very sketchy but we think it interesting enough to investigate further the energy dependence of high energy reactions suggested by that paper. It leaves however many questions open, some of them are quite obvious: (1) How does one generalize to differential cross sections? (2) How does one incorporate this s -channel picture into the Regge approach? (3) How does one include possible effects, such as an asymptotically leading $C = -1$ exchange (odderon) [29]? (4) How does one unify this approach with a treatment of deep inelastic scattering (DIS) and especially how does one explain the sharp rise in $1/x$ for small x and large Q^2 ?

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