CP-conserving two-Higgs-doublet model: The approach to the decoupling limit

John F. Gunion
Davis Institute for High Energy Physics, University of California, Davis, California 95616

Howard E. Haber
Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064

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A *CP*-even neutral Higgs boson with standard-model-like couplings may be the lightest scalar of a two-Higgs-doublet model. We study the decoupling limit of the most general *CP*-conserving two-Higgs-doublet model, where the mass of the lightest Higgs scalar is significantly smaller than the masses of the other Higgs bosons of the model. In this case, the properties of the lightest Higgs boson are nearly indistinguishable from those of the standard model Higgs boson. The first nontrivial corrections to Higgs boson couplings in the approach to the decoupling limit are also evaluated. The importance of detecting such deviations in precision Higgs boson measurements at future colliders is emphasized. We also clarify the case in which a neutral Higgs boson can possess standard-model-like couplings in a regime where the decoupling limit does not apply. The two-Higgs-doublet sector of the minimal supersymmetric model illustrates many of the above features.

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I. INTRODUCTION

The minimal version of the standard model (SM) contains one complex Higgs doublet, resulting in one physical neutral *CP*-even Higgs boson $h_{\rm SM}$ after electroweak symmetry breaking (EWSB). However, the standard model is not likely to be the ultimate theoretical structure responsible for electroweak symmetry breaking. Moreover, the standard model must be viewed as an effective field theory that is embedded in a more fundamental structure, characterized by an energy scale Λ, which is larger than the scale of EWSB, $v=246$ GeV. Although Λ may be as large as the Planck scale, there are strong theoretical arguments that suggest that Λ is significantly lower, perhaps of order 1 TeV [1]. For example, Λ could be the scale of supersymmetry breaking [2–4], the compositeness scale of new strong dynamics [5], or associated with the inverse size of extra dimensions [6]. In many of these approaches, there exists an effective low-energy theory with elementary scalars that comprise a nonminimal Higgs sector [7]. For example, the minimal supersymmetric extension of the standard model (MSSM) contains a scalar Higgs sector corresponding to that of a two-Higgs-doublet model (2HDM) [8,9]. Models with Higgs doublets (and singlets) possess the important phenomenological property that $\rho=m_W/(m_Z\cos\theta_W)=1$ up to *finite* radiative corrections.

In this paper we focus on a general 2HDM. There are two possible cases. In the first case, there is never an energy range in which the effective low-energy theory contains only one light Higgs boson. In the second case, one *CP*-even neutral Higgs boson h is significantly lighter than a new scale $\Lambda_{\rm 2HDM}$, which characterizes the masses of all the remaining 2HDM Higgs states. In this latter case, the scalar sector of the effective field theory below $\Lambda_{\rm 2HDM}$ is that of the SM Higgs sector. In particular, if $\Lambda_{\rm 2HDM}\gg v$, and all dimensionless Higgs self-coupling parameters $\lambda_i\lesssim\mathcal{O}(1)$ [see Eq. (1)], then the couplings of h to gauge bosons and fermions and the h self-couplings approach the corresponding couplings of the $h_{\rm SM}$, with the deviations vanishing as some power of $v^2/\Lambda^2_{\rm 2HDM}$ [10]. This limit is called the decoupling limit [11] and is one of the main subjects of this paper.

The purpose of this paper is to fully define and explore the decoupling limit of the 2HDM.[1] We will explain the (often confusing) relations between different parameter sets (e.g., Higgs boson masses and mixing angles vs Lagrangian tree-level couplings) and give a complete translation table in Appendix A. We then make one simplifying assumption, namely, that the Higgs sector is *CP* conserving. (The conditions that guarantee that there is no explicit or spontaneous breaking of *CP* in the 2HDM are given in Appendix B. The more general *CP*-violating 2HDM is treated elsewhere [13,14].) In the *CP*-conserving 2HDM, there is still some freedom in the choice of Higgs-boson–fermion couplings. A number of different choices have been studied in the literature [7,15]: type I, in which only one Higgs doublet couples to the fermions; and type II, in which the neutral member of one Higgs doublet couples only to up-type quarks and the neutral member of the other Higgs doublet couples only to down-type quarks and leptons. For Higgs-boson–fermion couplings of type I or type II, tree-level flavor-changing neutral currents (FCNCs) mediated by Higgs bosons are automatically absent [16]. Type-I and type-II models can be implemented with an appropriately chosen discrete symmetry (which may be softly broken without dire phenomenologically consequences). The type-II model Higgs sector also arises in the MSSM. In this paper, we allow for the most general Higgs-boson–fermion Yukawa couplings (the so-called type-III model [17]). For type-III Higgs-boson–fermion Yukawa couplings, tree-level Higgs-boson-mediated FCNCs are present, and one must be careful to choose Higgs boson parameters that ensure that these FCNC effects are numerically small. We will demonstrate in this paper that in the approach to the decoupling limit, FCNC effects generated by tree-level Higgs boson exchanges are suppressed by a factor of $\mathcal{O}(v^2/\Lambda^2_{\rm 2HDM})$.

[1]Some of the topics of this paper have also been addressed recently in Ref. [12].

In Sec. II, we define the most general *CP*-conserving 2HDM and provide a number of useful relations among the parameters of the scalar Higgs potential and the Higgs boson masses in Appendixes C and D. In Appendix E, we note that certain combinations of the scalar potential parameters are invariant with respect to the choice of basis for the two scalar doublets. In particular, the Higgs boson masses and the physical Higgs boson interaction vertices can be written in terms of these invariant coupling parameters. The decoupling limit of the 2HDM is defined in Sec. III and its main properties are examined. In this limit, the properties of the lightest *CP*-even Higgs boson h precisely coincide with those of the SM Higgs boson. This is shown in Sec. IV, where we exhibit the tree-level Higgs boson couplings to vector bosons, fermions, and Higgs bosons, and evaluate them in the decoupling limit (cubic and quartic Higgs boson self-couplings are written out explicitly in Appendixes F and G, respectively). The first nontrivial corrections to the Higgs boson couplings as one moves away from the decoupling limit are also given. In Sec. V, we note that certain parameter regimes exist outside the decoupling regime in which one of the *CP*-even Higgs bosons exhibits tree-level couplings that approximately coincide with those of the SM Higgs boson. We discuss the origin of this behavior and show how one can distinguish this region of parameter space from that of true decoupling. In Sec. VI, the two-Higgs-doublet sector of the MSSM is used to illustrate the features of the decoupling limit when $m_A \gg m_Z$. In addition, we briefly describe the impact of radiative corrections and show how these corrections satisfy the requirements of the decoupling limit. We emphasize that the rate of approach to decoupling can be delayed at large $\tan\beta$, and we discuss the possibility of a SM-like Higgs boson in a parameter regime in which all Higgs boson masses are in the range $\lesssim \mathcal{O}(v)$. Finally, our conclusions are give in Sec. VII.

II. THE *CP*-CONSERVING TWO-HIGGS-DOUBLET MODEL

We first review the general (nonsupersymmetric) two-Higgs-doublet extension of the standard model [7]. Let Φ_1 and Φ_2 denote two complex $Y=1$, $\mathrm{SU}(2)_L$ doublet scalar fields. The most general gauge invariant scalar potential is given by[2]

$$\begin{aligned}\mathcal{V}=&\, m_{11}^2\Phi_1^\dagger\Phi_1+m_{22}^2\Phi_2^\dagger\Phi_2-[m_{12}^2\Phi_1^\dagger\Phi_2+\text{H.c.}]\\&+\tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2+\tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\\&+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2+[\lambda_6(\Phi_1^\dagger\Phi_1)\\&+\lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2+\text{H.c.}\}.\end{aligned} \tag{1}$$

In general, m_{12}^2, λ_5, λ_6, and λ_7 can be complex. In many discussions of two-Higgs-doublet models, the terms proportional to λ_6 and λ_7 are absent. This can be achieved by imposing a discrete symmetry $\Phi_1\to-\Phi_1$ on the model. Such a symmetry would also require $m_{12}^2=0$ unless we allow a soft violation of this discrete symmetry by dimension-two terms.[3] In this paper, we refrain in general from setting any of the coefficients in Eq. (1) to zero.

We next derive the constraints on the parameters λ_i such that the scalar potential $\mathcal{V}$ is bounded from below. It is sufficient to examine the quartic terms of the scalar potential (which we denote by $\mathcal{V}_4$). We define $a\equiv\Phi_1^\dagger\Phi_1$, $b\equiv\Phi_2^\dagger\Phi_2$, $c\equiv\mathrm{Re}\,\Phi_1^\dagger\Phi_2$, $d\equiv\mathrm{Im}\,\Phi_1^\dagger\Phi_2$, and note that $ab\geqslant c^2+d^2$. Then, one can rewrite the quartic terms of the scalar potential as follows:

$$\begin{aligned}\mathcal{V}_4=&\,\tfrac{1}{2}[\lambda_1^{1/2}a-\lambda_2^{1/2}b]^2+[\lambda_3+(\lambda_1\lambda_2)^{1/2}](ab-c^2-d^2)\\&+2[\lambda_3+\lambda_4+(\lambda_1\lambda_2)^{1/2}]c^2\\&+[\mathrm{Re}\,\lambda_5-\lambda_3-\lambda_4-(\lambda_1\lambda_2)^{1/2}](c^2-d^2)-2cd\,\mathrm{Im}\,\lambda_5\\&+2a[c\,\mathrm{Re}\,\lambda_6-d\,\mathrm{Im}\,\lambda_6]+2b[c\,\mathrm{Re}\,\lambda_7-d\,\mathrm{Im}\,\lambda_7].\end{aligned} \tag{2}$$

We demand that no directions exist in field space in which $\mathcal{V}\to-\infty$. (We also require that no flat directions exist for $\mathcal{V}_4$.) Three conditions on the λ_i are easily obtained by examining asymptotically large values of a and/or b with $c=d=0$:

$$\lambda_1>0,\quad \lambda_2>0,\quad \lambda_3>-(\lambda_1\lambda_2)^{1/2}. \tag{3}$$

A fourth condition arises by examining the direction in field space where $\lambda_1^{1/2}a=\lambda_2^{1/2}b$ and $ab=c^2+d^2$. Setting $c=\xi d$, and requiring that the potential is bounded from below for all ξ leads to a condition on a quartic polynomial in ξ, which must be satisfied for all ξ. There is no simple analytical constraint on the λ_i that can be derived from this condition. If $\lambda_6=\lambda_7=0$, the resulting polynomial is quadratic in ξ, and a constraint on the remaining nonzero λ_i is easily derived [18]:

$$\lambda_3+\lambda_4-|\lambda_5|>-(\lambda_1\lambda_2)^{1/2}\quad (\text{assuming }\ \lambda_6=\lambda_7=0). \tag{4}$$

In this paper, we shall ignore the possibility of explicit *CP*-violating effects in the Higgs potential by choosing all coefficients in Eq. (1) to be real (see Appendix B).[4] The scalar fields will develop nonzero vacuum expectation values

[2]In Refs. [7] and [9], the scalar potential is parametrized in terms of a different set of couplings, which are less useful for the decoupling analysis. In Appendix A, we relate this alternative set of couplings to the parameters appearing in Eq. (1).

[3]This discrete symmetry is also employed to restrict the Higgs-boson–fermion couplings so that no tree-level Higgs-boson-mediated FCNCs are present. If $\lambda_6=\lambda_7=0$ but $m_{12}^2\neq0$, the soft breaking of the discrete symmetry generates *finite* Higgs-boson-mediated FCNCs at one loop.

[4]The most general *CP*-violating 2HDM will be examined in Ref. [14].

if the mass matrix m_{ij}^2 has at least one negative eigenvalue. We assume that the parameters of the scalar potential are chosen such that the minimum of the scalar potential respects the $\mathrm{U}(1)_{\mathrm{e.m.}}$ gauge symmetry. Then, the scalar field vacuum expectation values are of the form

$$\langle\Phi_1\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix},\quad \langle\Phi_2\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix}, \tag{5}$$

where the v_i are taken to be real, i.e., we assume that spontaneous *CP* violation does not occur.[5] The corresponding potential minimum conditions are

$$m_{11}^2=m_{12}^2t_\beta-\tfrac{1}{2}v^2[\lambda_1c_\beta^2+\lambda_{345}s_\beta^2+3\lambda_6s_\beta c_\beta+\lambda_7s_\beta^2t_\beta], \tag{6}$$

$$m_{22}^2=m_{12}^2t_\beta^{-1}-\tfrac{1}{2}v^2[\lambda_2s_\beta^2+\lambda_{345}c_\beta^2+\lambda_6c_\beta^2t_\beta^{-1}+3\lambda_7s_\beta c_\beta], \tag{7}$$

where we have defined

$$\lambda_{345}\equiv\lambda_3+\lambda_4+\lambda_5,\quad t_\beta\equiv\tan\beta\equiv\frac{v_2}{v_1}, \tag{8}$$

and

$$v^2\equiv v_1^2+v_2^2=\frac{4m_W^2}{g^2}=(246\ \ \mathrm{GeV})^2. \tag{9}$$

It is always possible to choose the phases of the scalar doublet Higgs fields such that both v_1 and v_2 are positive; henceforth we take $0\leq\beta\leq\pi/2$.

Of the original eight scalar degrees of freedom, three Goldstone bosons ($G^\pm$ and G) are absorbed by the $W^\pm$ and Z. The remaining five physical Higgs particles are two *CP*-even scalars (h and H, with $m_h\leq m_H$), one *CP*-odd scalar (A), and a charged Higgs pair ($H^\pm$). The squared-mass parameters m_{11}^2 and m_{22}^2 can be eliminated by minimizing the scalar potential. The resulting squared masses for the *CP*-odd and charged Higgs states are[6]

$$m_A^2=\frac{m_{12}^2}{s_\beta c_\beta}-\tfrac{1}{2}v^2(2\lambda_5+\lambda_6t_\beta^{-1}+\lambda_7t_\beta), \tag{10}$$

$$m_{H^\pm}^2=m_{A^0}^2+\tfrac{1}{2}v^2(\lambda_5-\lambda_4). \tag{11}$$

The two *CP*-even Higgs states mix according to the following squared-mass matrix:

$$\mathcal{M}^2\equiv m_{A^0}^2\begin{pmatrix}s_\beta^2 & -s_\beta c_\beta\\ -s_\beta c_\beta & c_\beta^2\end{pmatrix}+\mathcal{B}^2, \tag{12}$$

where

$$\mathcal{B}^2\equiv v^2\begin{pmatrix}\lambda_1c_\beta^2+2\lambda_6s_\beta c_\beta+\lambda_5s_\beta^2 & (\lambda_3+\lambda_4)s_\beta c_\beta+\lambda_6c_\beta^2+\lambda_7s_\beta^2\\ (\lambda_3+\lambda_4)s_\beta c_\beta+\lambda_6c_\beta^2+\lambda_7s_\beta^2 & \lambda_2s_\beta^2+2\lambda_7s_\beta c_\beta+\lambda_5c_\beta^2\end{pmatrix}. \tag{13}$$

Defining the physical mass eigenstates

$$H=(\sqrt{2}\,\mathrm{Re}\,\Phi_1^0-v_1)c_\alpha+(\sqrt{2}\,\mathrm{Re}\,\Phi_2^0-v_2)s_\alpha,$$

$$h=-(\sqrt{2}\,\mathrm{Re}\,\Phi_1^0-v_1)s_\alpha+(\sqrt{2}\,\mathrm{Re}\,\Phi_2^0-v_2)c_\alpha, \tag{14}$$

the masses and mixing angle α are found from the diagonalization process

$$\begin{pmatrix}m_H^2 & 0\\ 0 & m_h^2\end{pmatrix}=\begin{pmatrix}c_\alpha & s_\alpha\\ -s_\alpha & c_\alpha\end{pmatrix}\begin{pmatrix}\mathcal{M}_{11}^2 & \mathcal{M}_{12}^2\\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2\end{pmatrix}\begin{pmatrix}c_\alpha & -s_\alpha\\ s_\alpha & c_\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\mathcal{M}_{11}^2c_\alpha^2+2\mathcal{M}_{12}^2c_\alpha s_\alpha+\mathcal{M}_{22}^2s_\alpha^2 & \mathcal{M}_{12}^2(c_\alpha^2-s_\alpha^2)+(\mathcal{M}_{22}^2-\mathcal{M}_{11}^2)s_\alpha c_\alpha\\ \mathcal{M}_{12}^2(c_\alpha^2-s_\alpha^2)+(\mathcal{M}_{22}^2-\mathcal{M}_{11}^2)s_\alpha c_\alpha & \mathcal{M}_{11}^2s_\alpha^2-2\mathcal{M}_{12}^2c_\alpha s_\alpha+\mathcal{M}_{22}^2c_\alpha^2\end{pmatrix}. \tag{15}$$

[5]The conditions required for the absence of explicit and spontaneous *CP* violation in the Higgs sector are elucidated in Appendix B.

[6]Here and in the following, we use the shorthand notation $c_\beta\equiv\cos\beta$, $s_\beta\equiv\sin\beta$, $c_\alpha\equiv\cos\alpha$, $s_\alpha\equiv\sin\alpha$, $c_{2\alpha}\equiv\cos2\alpha$, $s_{2\alpha}\equiv\cos2\alpha$, $c_{\beta-\alpha}\equiv\cos(\beta-\alpha)$, $s_{\beta-\alpha}\equiv\sin(\beta-\alpha)$, etc.

The mixing angle α is evaluated by setting the off-diagonal elements of the CP-even scalar squared-mass matrix [Eq. (15)] to zero, and demanding that $m_H \geqslant m_h$. The end result is

$$m_{H,h}^2 = \tfrac{1}{2}\left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \pm \sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}\right], \tag{16}$$

and the corresponding CP-even scalar mixing angle is fixed by

$$s_{2\alpha} = \frac{2\mathcal{M}_{12}^2}{\sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}},$$

$$c_{2\alpha} = \frac{\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2}{\sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}}. \tag{17}$$

We shall take $-\pi/2 \leqslant \alpha \leqslant \pi/2$.

It is convenient to define the following four combinations of parameters:

$$m_D^4 \equiv \mathcal{B}_{11}^2 \mathcal{B}_{22}^2 - [\mathcal{B}_{12}^2]^2,$$

$$m_L^2 \equiv \mathcal{B}_{11}^2 \cos^2\beta + \mathcal{B}_{22}^2 \sin^2\beta + \mathcal{B}_{12}^2 \sin 2\beta,$$

$$m_T^2 \equiv \mathcal{B}_{11}^2 + \mathcal{B}_{22}^2,$$

$$m_S^2 \equiv m_A^2 + m_T^2, \tag{18}$$

where the $\mathcal{B}_{ij}^2$ are the elements of the matrix defined in Eq. (13). In terms of these quantities we have the exact relations

$$m_{H,h}^2 = \tfrac{1}{2}\left[m_S^2 \pm \sqrt{m_S^4 - 4m_A^2 m_L^2 - 4m_D^4}\right]. \tag{19}$$

and

$$c_{\beta-\alpha}^2 = \frac{m_L^2 - m_h^2}{m_H^2 - m_h^2}. \tag{20}$$

Equation (20) is most easily derived by using the identity $c_{\beta-\alpha}^2 = \frac{1}{2}(1 + c_{2\beta}c_{2\alpha} + s_{2\beta}s_{2\alpha})$ and the results of Eq. (17). Note that the case of $m_h = m_H$ is special and must be treated carefully. We do this in Appendix C, where we explicitly verify that $0 \leqslant c_{\beta-\alpha}^2 \leqslant 1$.

Finally, for completeness we record the expressions for the original hypercharge-1 scalar fields Φ_i in terms of the physical Higgs states and the Goldstone bosons:

$$\Phi_1^{\pm} = c_\beta G^{\pm} - s_\beta H^{\pm},$$

$$\Phi_2^{\pm} = s_\beta G^{\pm} + c_\beta H^{\pm},$$

$$\Phi_1^0 = \frac{1}{\sqrt{2}}[v_1 + c_\alpha H - s_\alpha h + i c_\beta G - i s_\beta A],$$

$$\Phi_2^0 = \frac{1}{\sqrt{2}}[v_2 + s_\alpha H + c_\alpha h + i s_\beta G + i c_\beta A]. \tag{21}$$

III. THE DECOUPLING LIMIT

In an effective field theory, we may examine the behavior of the theory characterized by two disparate mass scales, $m_L \ll m_S$, by integrating out all particles with masses of order m_S, assuming that all the couplings of the "low-mass" effective theory comprising particles with masses of order m_L can be kept fixed. In the 2HDM, the low-mass effective theory, if it exists, must correspond to the case where one of the Higgs doublets is integrated out. That is, the resulting effective low-mass theory is precisely equivalent to the one-scalar-doublet SM Higgs sector. These conclusions follow from electroweak gauge invariance; namely, there are two relevant scales—the electroweak scale characterized by the scale $v = 246$ GeV and a second scale $m_S \gg v$. The underlying electroweak symmetry requires that scalar mass splittings within doublets cannot be larger than $\mathcal{O}(v)$ [assuming that dimensionless couplings of the theory are no larger than $\mathcal{O}(1)$]. It follows that the $H^{\pm}$, A, and H masses must be of $\mathcal{O}(m_S)$, while $m_h \sim \mathcal{O}(v)$. Moreover, since the effective low-mass theory consists of a one-doublet Higgs sector, the properties of h must be indistinguishable from those of the SM Higgs boson.

We can illustrate these results more explicitly as follows. Suppose that all the Higgs boson self-coupling parameters λ_i are held fixed such that $|\lambda_i| \lesssim \mathcal{O}(1)$, while taking $m_A^2 \gg |\lambda_i| v^2$. In particular, we constrain the $\alpha_i \equiv \lambda_i/(4\pi)$ so that the Higgs sector does not become strongly coupled, implying no violations of tree unitarity [19–23]. Then the $\mathcal{B}_{ij}^2 \sim \mathcal{O}(v^2)$, and it follows that

$$m_h \simeq m_L = \mathcal{O}(v), \tag{22}$$

$$m_H, m_A, m_{H^{\pm}} = m_S + \mathcal{O}(v^2/m_S), \tag{23}$$

and

$$\begin{aligned}\cos^2(\beta-\alpha) &\simeq \frac{m_L^2(m_T^2 - m_L^2) - m_D^4}{m_A^4} \\ &= \frac{\left[\frac{1}{2}(\mathcal{B}_{11}^2 - \mathcal{B}_{22}^2)s_{2\beta} - \mathcal{B}_{12}^2 c_{2\beta}{}^2\right]}{m_A^4} \\ &= \mathcal{O}\left(\frac{v^4}{m_S^4}\right).\end{aligned} \tag{24}$$

We shall establish the above results in more detail below.

The limit $m_A^2 \gg |\lambda_i| v^2$ (subject to $|\alpha_i| \lesssim 1$) is called the *decoupling limit* of the model.[7] Note that Eq. (24) implies that in the decoupling limit $c_{\beta-\alpha} = \mathcal{O}(v^2/m_A^2)$. We will demonstrate that this implies that the couplings of h in the decoupling limit approach values that correspond precisely to

[7]In Sec. IV [see Eq. (51) and surrounding discussion], we refine this definition slightly, and also require that $m_A^2 \gg |\lambda_6| v^2 \cot\beta$ and $m_A^2 \gg |\lambda_7| v^2 \tan\beta$, in order to guarantee that at large $\cot\beta$ ($\tan\beta$) the couplings of h to up-type (down-type) fermions approach the corresponding SM Higgs-boson–fermion couplings.

those of the SM Higgs boson. We will also obtain explicit expressions for the squared-mass differences between the heavy Higgs bosons (as a function of the λ_i couplings in the Higgs potential) in the decoupling limit.

One can give an alternative condition for the decoupling limit. As above, we assume that all $|\alpha_i|\lesssim 1$. First consider the following special cases. If neither $\tan\beta$ nor $\cot\beta$ is close to 0, then $m_{12}^2 \gg |\lambda_i| v^2$ [see Eq. (10)] in the decoupling limit. On the other hand, if $m_{12}^2 \sim \mathcal{O}(v^2)$ and $\tan\beta \gg 1$ ($\cot\beta \gg 1$), then it follows from Eqs. (6) and (7) that $m_{11}^2 \gg \mathcal{O}(v^2)$ if $\lambda_7<0$ [$m_{22}^2 \gg \mathcal{O}(v^2)$ if $\lambda_6<0$] in the decoupling limit. All such conditions depend on the original choice of the scalar field basis Φ_1 and Φ_2. For example, we can diagonalize the squared-mass terms of the scalar potential [Eq. (1)] thereby setting $m_{12}=0$. In the decoupling limit in the new basis, one is simply driven to the second case above. A basis-independent characterization of the decoupling limit is simple to formulate. Starting from the scalar potential in an arbitrary basis, form the matrix m_{ij}^2 [made up of the coefficients of the quadratic terms in the potential; see Eq. (1)]. Denote the eigenvalues of this matrix by m_a^2 and m_b^2, respectively; note that the eigenvalues are real but can be of either sign. By convention, we can take $|m_a^2| \leqslant |m_b^2|$. Then the decoupling limit corresponds to $m_a^2<0$, $m_b^2>0$ such that $m_b^2 \gg |m_a^2|, v^2$ (with $|\alpha_i|\lesssim 1$).

For some choices of the scalar potential, no decoupling limit exists. Consider the case of $m_{12}^2=\lambda_6=\lambda_7=0$ (and all other $|\alpha_i|\lesssim 1$). Then the potential minimum conditions [Eqs. (6) and (7)] do not permit either m_{11}^2 or m_{22}^2 to become large; m_{11}^2, $m_{22}^2 \sim \mathcal{O}(v^2)$, and clearly all Higgs boson masses are of $\mathcal{O}(v)$. Thus, in this case no decoupling limit exists.[8] The case of $m_{12}^2=\lambda_6=\lambda_7=0$ corresponds to the existence of a discrete symmetry in which the potential is invariant under the change of sign of one of the Higgs doublet fields. Although the latter statement is basis dependent, one can check that the following stronger condition holds: no decoupling limit exists if and only if $\lambda_6=\lambda_7=0$ in the basis where $m_{12}^2=0$. Thus, the absence of a decoupling limit implies the existence of some discrete symmetry under which the scalar potential is invariant (although the precise form of this symmetry is most evident for the special choice of basis).

We now return to the results for the Higgs boson masses and the *CP*-even Higgs boson mixing angle in the decoupling limit. For fixed values of λ_6, λ_7, α, and β, there are two equivalent parameter sets: (i) λ_1, λ_2, λ_3, λ_4, and λ_5; (ii) m_h^2, m_H^2, m_{12}^2, $m_{H^\pm}^2$, and m_A^2. The relations between these two parameter sets are given in Appendix D. Using the results Eqs. (D3)–(D7) we can give explicit expressions in the decoupling limit for the Higgs boson masses in terms of the potential parameters and the mixing angles. First, it is convenient to define the following four linear combinations of the λ_i:[9]

$$\lambda \equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}(\lambda_6 c_\beta^2 + \lambda_7 s_\beta^2), \tag{25}$$

$$\hat{\lambda} \equiv \tfrac{1}{2} s_{2\beta}[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}] - \lambda_6 c_\beta c_{3\beta} - \lambda_7 s_\beta s_{3\beta}, \tag{26}$$

$$\lambda_A \equiv c_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \lambda_{345} s_{2\beta}^2 - \lambda_5 + 2\lambda_6 c_\beta s_{3\beta} - 2\lambda_7 s_\beta c_{3\beta}, \tag{27}$$

$$\lambda_F \equiv \lambda_5 - \lambda_4, \tag{28}$$

where λ_{345} is defined in Eq. (8). The significance of these coupling combinations is discussed in Appendix E. We consider the limit $c_{\beta-\alpha}\to 0$, corresponding to the decoupling limit $m_A^2 \gg |\lambda_i| v^2$. In nearly all of the parameter space, $\mathcal{M}_{12}^2<0$ [see Eq. (12)], and it follows from Eq. (17) that $-\pi/2 \leqslant \alpha \leqslant 0$ (which implies that $c_{\beta-\alpha}\to 0$ is equivalent to $\beta-\alpha\to\pi/2$ given that $0\leqslant\beta\leqslant\pi/2$). However, in the small regions of parameter space in which β is near zero (or $\pi/2$), roughly corresponding to $m_A^2 \tan\beta < \lambda_6 v^2$ (or $m_A^2\cot\beta < \lambda_7 v^2$), one finds $\mathcal{M}_{12}^2>0$ (and consequently $0<\alpha<\pi/2$). In these last two cases, the decoupling limit is achieved for $\alpha=\pi/2-\beta$ and $\cot\beta\gg 1$ ($\tan\beta \gg 1$). That is, $\cos(\beta-\alpha) = \sin 2\beta \ll 1$ and $\sin(\beta-\alpha)\simeq -1\ (+1)$.[10] In practice, since $\tan\beta$ is fixed and cannot be arbitrarily large (or arbitrarily close to zero), one can always find a value of m_A large enough such that $\mathcal{M}_{12}^2<0$. This is equivalent to employing the refined version of the decoupling limit mentioned in footnote 7. In this case, the decoupling limit simply corresponds to $\beta-\alpha\to\pi/2$ [i.e., $\sin(\beta-\alpha)=1$] independently of the value of β.

In the approach to the decoupling limit where $\alpha\simeq\beta-\pi/2$ (that is, $|c_{\beta-\alpha}|\ll 1$ and $s_{\beta-\alpha}\simeq 1-\frac{1}{2}c_{\beta-\alpha}^2$), we may use Eqs. (D9)–(D12) and Eq. (11) to obtain[11]

$$m_A^2 \simeq v^2\left[\frac{\hat{\lambda}}{c_{\beta-\alpha}} + \lambda_A - \frac{3}{2}\hat{\lambda} c_{\beta-\alpha}\right], \tag{29}$$

$$m_h^2 \simeq v^2(\lambda - \hat{\lambda} c_{\beta-\alpha}), \tag{30}$$

[8]However, it may be difficult to distinguish between the nondecoupling effects of the SM with a heavy Higgs boson and those of the 2HDM where all Higgs bosons are heavy [24].

[9]We make use of the triple-angle identities $c_{3\beta}=c_\beta(c_\beta^2-3s_\beta^2)$ and $s_{3\beta}=s_\beta(3c_\beta^2-s_\beta^2)$.

[10]We have chosen a convention in which $-\pi/2\leqslant\alpha\leqslant\pi/2$. An equally good alternative is to choose $\sin(\beta-\alpha)\geqslant 0$. If negative, one may simply change the sign of $\sin(\beta-\alpha)$ by taking $\alpha\to\alpha\pm\pi$, which is equivalent to the field redefinitions $h\to -h$, $H\to -H$.

[11]In obtaining Eqs. (29), (31), and (32) we divided both sides of each equation by $c_{\beta-\alpha}$, so these equations need to be treated with care if $c_{\beta-\alpha}=0$ exactly. In this latter case, it suffices to note that $\hat{\lambda}/c_{\beta-\alpha}$ has a finite limit whose value depends on m_A and λ_A [see Eq. (36)].

$$m_H^2 \simeq v^2\left[\frac{\hat{\lambda}}{c_{\beta-\alpha}}+\lambda-\tfrac{1}{2}\,\hat{\lambda}c_{\beta-\alpha}\right]$$
$$\simeq m_A^2+(\lambda-\lambda_A+\hat{\lambda}c_{\beta-\alpha})v^2, \tag{31}$$

$$m_{H^\pm}^2 \simeq v^2\left[\frac{\hat{\lambda}}{c_{\beta-\alpha}}+\lambda_A+\tfrac{1}{2}\lambda_F-\frac{3}{2}\hat{\lambda}c_{\beta-\alpha}\right]$$
$$=m_A^2+\tfrac{1}{2}\lambda_F v^2. \tag{32}$$

The condition $m_H > m_h$ implies the inequality (valid to first order in $c_{\beta-\alpha}$)

$$m_A^2 > v^2(\lambda_A - 2\hat{\lambda}c_{\beta-\alpha}) \tag{33}$$

[cf. Eq. (D32)]. The positivity of m_h^2 also imposes a useful constraint on the Higgs potential parameters. For example, $m_h^2>0$ requires that $\lambda>0$.

In the decoupling limit (where $m_A^2 \gg |\lambda_i| v^2$), Eqs. (29)–(32) provide the first nontrivial corrections to Eqs. (22) and (23). Finally, we employ Eq. (10) to obtain

$$m_{12}^2 \simeq v^2 s_\beta c_\beta\left[\frac{\hat{\lambda}}{c_{\beta-\alpha}}+\lambda_A+\lambda_5+\tfrac{1}{2}\lambda_6 t_\beta^{-1}+\tfrac{1}{2}\lambda_7 t_\beta \right.$$
$$\left. -\tfrac{3}{2}\hat{\lambda}c_{\beta-\alpha}\right]. \tag{34}$$

This result confirms our previous observation that $m_{12}^2 \gg |\lambda_i| v^2$ in the decoupling limit as long as β is not close to 0 or $\pi/2$. However, m_{12}^2 can be of $\mathcal{O}(v^2)$ in the decoupling limit ($c_{\beta-\alpha}\to 0$) if either $t_\beta \gg 1$ [and $c_\beta/c_{\beta-\alpha}\sim\mathcal{O}(1)$] or $t_\beta^{-1}\gg 1$ [and $s_\beta/c_{\beta-\alpha}\sim\mathcal{O}(1)$].

The significance of Eq. (30) is easily understood by noting that the decoupling limit corresponds to integrating out the second heavy Higgs doublet. The resulting low-mass effective theory is the one-Higgs-doublet model with corresponding scalar potential $V=m^2(\Phi^\dagger\Phi)+(\lambda/2)(\Phi^\dagger\Phi)^2$, where λ is given by Eq. (25) and

$$m^2 \equiv m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - 2m_{12}^2 s_\beta c_\beta . \tag{35}$$

Imposing the potential minimum conditions [Eqs. (6) and (7)], we see that $v^2=-2m^2/\lambda$ (where $\langle\Phi^0\rangle\equiv v/\sqrt{2}$) as expected. Moreover, the Higgs boson mass is given by $m_h^2=\lambda v^2$, in agreement with the $c_{\beta-\alpha}\to 0$ limit of Eq. (30).

We can rewrite Eq. (29) in another form [or equivalently use Eqs. (D30) and (D31) to obtain]

$$\cos(\beta-\alpha)\simeq\frac{\hat{\lambda}v^2}{m_A^2-\lambda_A v^2}\simeq\frac{\hat{\lambda}v^2}{m_H^2-m_h^2}. \tag{36}$$

This yields an $\mathcal{O}(v^2/m_A^2)$ correction to Eq. (24). Note that Eq. (36) also implies that in the approach to the decoupling limit, the sign of $\cos(\beta-\alpha)$ is given by the sign of $\hat{\lambda}$.

IV. TWO-HIGGS-DOUBLET MODEL COUPLINGS IN THE DECOUPLING LIMIT

The phenomenology of the two-Higgs-doublet model depends in detail on the various couplings of the Higgs bosons to gauge bosons, Higgs bosons, and fermions [7]. The Higgs boson couplings to gauge bosons follow from gauge invariance and are thus model independent:

$$g_{hVV}=g_V m_V s_{\beta-\alpha}, \quad g_{HVV}=g_V m_V c_{\beta-\alpha}, \tag{37}$$

where $g_V\equiv 2m_V/v$ for $V=W$ or Z. There are no tree-level couplings of A or $H^\pm$ to VV. In the decoupling limit where $c_{\beta-\alpha}=0$, we see that $g_{hVV}=g_{h_{\rm SM}VV}$, whereas the HVV coupling vanishes. Gauge invariance also determines the strength of the trilinear couplings of one gauge boson to two Higgs bosons:

$$g_{hAZ}=\frac{g c_{\beta-\alpha}}{2\cos\theta_W}, \quad g_{HAZ}=\frac{-g s_{\beta-\alpha}}{2\cos\theta_W}. \tag{38}$$

In the decoupling limit, the *hAZ* coupling vanishes, while the *HAZ* coupling attains its maximal value. This pattern is repeated in all the three-point and four-point couplings of h and H to VV, $V\phi$, and $VV\phi$ final states (where V is a vector boson and ϕ is one of the Higgs scalars). These results can be summarized as follows: the coupling of h and H to vector boson pairs or vector-scalar boson final states is proportional to either $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$ as indicated below [7,9]:

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$
HW^+W^-	hW^+W^-
HZZ	hZZ
ZAh	ZAH
$W^\pm H^\mp h$	$W^\pm H^\mp H$
$ZW^\pm H^\mp h$	$ZW^\pm H^\mp H$
$\gamma W^\pm H^\mp h$	$\gamma W^\pm H^\mp H$

(39)

Note in particular that *all* vertices in the theory that contain at least one vector boson and *exactly one* of the nonminimal Higgs boson states (H, A, or $H^\pm$) are proportional to the factor $\cos(\beta-\alpha)$ and hence vanish in the decoupling limit.

The Higgs boson couplings to fermions are model dependent. The most general structure for the Higgs-boson–fermion Yukawa couplings, often referred to as the type-III model [17], is given by

$$-\mathcal{L}_Y=\overline{Q}_L^0\tilde{\Phi}_1\eta_1^{U,0}U_R^0+\overline{Q}_L^0\Phi_1\eta_1^{D,0}D_R^0+\overline{Q}_L^0\tilde{\Phi}_2\eta_2^{U,0}U_R^0$$
$$+\overline{Q}_L^0\Phi_2\eta_2^{D,0}D_R^0+\text{H.c.}, \tag{40}$$

where $\Phi_{1,2}$ are the Higgs doublets, $\tilde{\Phi}_i\equiv i\sigma_2\Phi_i^*$, Q_L^0 is the weak isospin quark doublet, and U_R^0,D_R^0 are weak isospin quark singlets. [The right- and left-handed fermion fields are defined as usual: $\psi_{R,L}\equiv P_{R,L}\psi$, $P_{R,L}\equiv\frac{1}{2}(1\pm\gamma_5)$.] Here, Q_L^0,U_R^0,D_R^0 denote the interaction basis states, which are vectors in flavor space, whereas $\eta_1^{U,0},\eta_2^{U,0},\eta_1^{D,0},\eta_2^{D,0}$ are matrices in flavor space. We have omitted the leptonic couplings in Eq. (40); these follow the same pattern as the down-type quark couplings.

We next shift the scalar fields according to their vacuum expectation values, and then reexpress the scalars in terms of the physical Higgs states and Goldstone bosons [see Eq. (21)]. In addition, we diagonalize the quark mass matrices and define the quark mass eigenstates. The resulting Higgs-boson–fermion Lagrangian can be written in several ways [25]. We choose to display the form that makes the type-II model limit of the general type-III couplings apparent. The type-II model (where $\eta_1^{U,0}=\eta_2^{D,0}=0$) automatically has no tree-level flavor-changing neutral Higgs boson couplings, whereas these are generally present for type-III couplings. The fermion mass eigenstates are related to the interaction eigenstates by biunitary transformations:

$$P_L U = V_L^U P_L U^0, \quad P_R U = V_R^U P_R U^0,$$
$$P_L D = V_L^D P_L D^0, \quad P_R D = V_R^D P_R D^0, \tag{41}$$

and the Cabibbo-Kobayashi-Maskawa matrix is defined as $K\equiv V_L^U V_L^{D\dagger}$. It is also convenient to define ''rotated'' coupling matrices

$$\eta_i^U \equiv V_L^U \eta_i^{U,0} V_R^{U\dagger}, \quad \eta_i^D \equiv V_L^D \eta_i^{D,0} V_R^{D\dagger}. \tag{42}$$

The diagonal quark mass matrices are obtained by replacing the scalar fields with their vacuum expectation values

$$M_D = \frac{1}{\sqrt{2}}(v_1\eta_1^D + v_2\eta_2^D), \quad M_U = \frac{1}{\sqrt{2}}(v_1\eta_1^U + v_2\eta_2^U). \tag{43}$$

After eliminating η_2^U and η_1^D, the resulting Yukawa couplings are

$$\begin{aligned}\mathcal{L}_Y = {}& \frac{1}{v}\overline{D}M_D D\left(\frac{s_\alpha}{c_\beta}h - \frac{c_\alpha}{c_\beta}H\right) + \frac{i}{v}\overline{D}M_D\gamma_5 D(t_\beta A - G) - \frac{1}{\sqrt{2}c_\beta}\overline{D}(\eta_2^D P_R + \eta_2^{D\dagger}P_L)D(c_{\beta-\alpha}h - s_{\beta-\alpha}H) - \frac{i}{\sqrt{2}c_\beta}\overline{D}(\eta_2^D P_R \\ & - \eta_2^{D\dagger}P_L)DA - \frac{1}{v}\overline{U}M_U U\left(\frac{c_\alpha}{s_\beta}h + \frac{s_\alpha}{s_\beta}H\right) + \frac{i}{v}\overline{U}M_U\gamma_5 U(t_\beta^{-1}A + G) + \frac{1}{\sqrt{2}s_\beta}\overline{U}(\eta_1^U P_R + \eta_1^{U\dagger}P_L)U(c_{\beta-\alpha}h - s_{\beta-\alpha}H) \\ & - \frac{i}{\sqrt{2}s_\beta}\overline{U}(\eta_1^U P_R - \eta_1^{U\dagger}P_L)UA + \frac{\sqrt{2}}{v}[\overline{U}KM_D P_R D(t_\beta H^+ - G^+) + \overline{U}M_U K P_L D(6 t_\beta^{-1}H^+ + G^+) + \text{H.c.}] \\ & - \left[\frac{1}{s_\beta}\overline{U}\eta_1^{U\dagger}KP_L DH^+ + \frac{1}{c_\beta}\overline{U}K\eta_2^D P_R DH^+ + \text{H.c.}\right].\end{aligned} \tag{44}$$

In general, η_1^U and η_2^D are complex nondiagonal matrices. Thus, the Yukawa Lagrangian displayed in Eq. (44) exhibits both flavor-nondiagonal and CP-violating couplings between the neutral Higgs bosons and the quarks.

In the decoupling limit (where $c_{\beta-\alpha}\to 0$), the Yukawa Lagrangian displays a number of interesting features. First, the flavor nondiagonal and the CP-violating couplings of h vanish (although the corresponding couplings to H and A persist). Moreover, in this limit, the h coupling to fermions reduces precisely to its standard model value $\mathcal{L}_Y^{\rm SM} = -(m_f/v)\overline{f}fh$. To better see the behavior of couplings in the decoupling limit, the following trigonometric identities are particularly useful:

$$h\overline{D}D: \quad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha), \tag{45}$$

$$h\overline{U}U: \quad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha), \tag{46}$$

$$H\overline{D}D: \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha), \tag{47}$$

$$H\overline{U}U: \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha), \tag{48}$$

where we have indicated the type of Higgs-boson–fermion coupling with which a particular trigonometric expression arises. It is now easy to read off the corresponding Higgs-boson–fermion couplings in the decoupling limit and one verifies that the h-fermion couplings reduce to their standard model values. Working to $\mathcal{O}(c_{\beta-\alpha})$, the Yukawa couplings of h are given by

$$\begin{aligned}\mathcal{L}_{hQQ} = {}& -\overline{D}\left[\frac{1}{v}M_D - \tan\beta\left(\frac{1}{v}M_D - \frac{1}{\sqrt{2}s_\beta}(S_D \right.\right. \\ & \left.\left. + iP_D\gamma_5)\right)c_{\beta-\alpha}\right]Dh - \overline{U}\left[\frac{1}{v}M_U + \cot\beta\left(\frac{1}{v}M_U \right.\right. \\ & \left.\left. - \frac{1}{\sqrt{2}c_\beta}(S_U + iP_U\gamma_5)\right)c_{\beta-\alpha}\right]Uh,\end{aligned} \tag{49}$$

where

$$S_D \equiv \tfrac{1}{2}(\eta_2^D + \eta_2^{D\dagger}), \quad P_D \equiv -\frac{i}{2}(\eta_2^D - \eta_2^{D\dagger}) \tag{50}$$

are 3×3 Hermitian matrices and S_U and P_U are defined similarly by making the replacements $D\to U$ and $2\to 1$. Note that both h-mediated FCNC interactions (implicit in the off-diagonal matrix elements of S and P) and CP-violating interactions proportional to P are suppressed by a factor of $c_{\beta-\alpha}$ in the decoupling limit. Moreover, FCNCs and CP-violating effects mediated by A and H are suppressed by the square of the heavy Higgs boson masses (relative to v), due to the propagator suppression. Since $m_h \ll m_H, m_A$ and $c_{\beta-\alpha} \simeq \mathcal{O}(v^2/m_A^2)$ near the decoupling limit, we see that the flavor- and CP-violating processes mediated by h, H, and A are all suppressed by the same factor. Thus, for $m_A \gtrsim \mathcal{O}(1\text{ TeV})$, the decoupling limit provides a viable mechanism for suppressed Higgs-boson-mediated FCNCs and suppressed Higgs-boson-mediated CP-violating effects in the most general 2HDM.

Note that the approach to decoupling can be delayed if either $\tan\beta \gg 1$ or $\cot\beta \gg 1$, as is evident from Eq. (49). For example, decoupling at large $\tan\beta$ or $\cot\beta$ occurs when $|c_{\beta-\alpha}\tan\beta| \ll 1$ or $|c_{\beta-\alpha}\cot\beta| \ll 1$, respectively. Using Eqs. (36) and (26), these conditions are respectively equivalent to

$$m_A^2 \gg |\lambda_6| v^2 \cot\beta \quad \text{and} \quad m_A^2 \gg |\lambda_7| v^2 \tan\beta, \tag{51}$$

which supplement the usual requirement of $m_A^2 \gg \lambda_i v^2$. That is, there are two possible ranges of the CP-odd Higgs boson squared mass, $\lambda_i v^2 \ll m_A^2 \lesssim |\lambda_7| v^2 \tan\beta$ (or $\lambda_i v^2 \ll m_A^2 \lesssim |\lambda_6| v^2 \cot\beta$) when $\tan\beta \gg 1$ (or $\cot\beta \gg 1$), where the h couplings to VV, hh, and hhh are nearly indistinguishable from the corresponding $h_{\rm SM}$ couplings, whereas one of the $hf\bar{f}$ couplings can deviate significantly from the corresponding $h_{\rm SM}f\bar{f}$ couplings.

The cubic and quartic Higgs boson self-couplings depend on the parameters of the 2HDM potential [Eq. (1)], and are listed in Appendixes F and G, respectively. In the decoupling limit (DL) of $\alpha\to\beta-\pi/2$, we denote the terms of the scalar potential corresponding to the cubic Higgs boson couplings by $\mathcal{V}_{\rm DL}^{(3)}$ and the terms corresponding to the quartic Higgs boson couplings by $\mathcal{V}_{\rm DL}^{(4)}$. The coefficients of the quartic terms in the scalar Higgs potential can be written more simply in terms of the linear combinations of couplings defined earlier [Eqs. (25)–(28)] and three additional combinations (see Appendix E for a discussion of the significance of these combinations):

$$\begin{aligned}\lambda_T \equiv {}& \tfrac{1}{4}s_{2\beta}^2(\lambda_1+\lambda_2)+\lambda_{345}(s_\beta^4+c_\beta^4)-2\lambda_5\\ &-s_{2\beta}c_{2\beta}(\lambda_6-\lambda_7),\end{aligned} \tag{52}$$

$$\lambda_U \equiv \tfrac{1}{2}s_{2\beta}(s_\beta^2\lambda_1-c_\beta^2\lambda_2+c_{2\beta}\lambda_{345})-\lambda_6 s_\beta s_{3\beta}-\lambda_7 c_\beta c_{3\beta}. \tag{53}$$

$$\lambda_V \equiv \lambda_1 s_\beta^4+\lambda_2 c_\beta^4+\tfrac{1}{2}\lambda_{345}s_{2\beta}^2-2s_{2\beta}(\lambda_6 s_\beta^2+\lambda_7 c_\beta^2). \tag{54}$$

The resulting expressions for $\mathcal{V}_{\rm DL}^{(3)}$ and $\mathcal{V}_{\rm DL}^{(4)}$ are

$$\begin{aligned}\mathcal{V}_{\rm DL}^{(3)} = {}& \tfrac{1}{2}\lambda v(h^3+hG^2+2hG^+G^-)+(\lambda_T+\lambda_F)vhH^+H^-+\tfrac{1}{2}\hat{\lambda}v[3Hh^2+HG^2+2HG^+G^--2h(AG+H^+G^-+H^-G^+)]\\ &+\tfrac{1}{2}\lambda_U v(H^3+HA^2+2HH^+H^-)+[\lambda_A-\lambda+\tfrac{1}{2}\lambda_F]vH(H^+G^-+H^-G^+)+(\lambda_A-\lambda)vHAG+\tfrac{1}{2}\lambda_T vhA^2\\ &+(\lambda-\lambda_A+\tfrac{1}{2}\lambda_T)vhH^2+\frac{i}{2}\lambda_F vA(H^+G^--H^-G^+)\end{aligned} \tag{55}$$

and

$$\begin{aligned}\mathcal{V}_{\rm DL}^{(4)} = {}& \tfrac{1}{8}\lambda(G^2+2G^+G^-+h^2)^2+\hat{\lambda}(h^3H-h^2AG-h^2H^+G^--h^2H^-G^++hHG^2+2hHG^+G^--AG^3-2AGG^+G^-\\ &-G^2H^-G^+-G^2H^+G^--2H^+G^-G^+G^--2H^-G^-G^+)+\tfrac{1}{2}(\lambda_T+\lambda_F)(h^2H^+H^-+H^2G^+G^-+A^2G^+G^-\\ &+G^2H^+H^-)+\lambda_U(hH^3+hHA^2+2hHH^+H^--H^2AG-H^2H^+G^--H^2H^-G^+-A^3G-A^2H^+G^--A^2H^-G^+\\ &-2AGH^+H^--2H^+H^-H^+G^--2H^-H^+H^-G^+)+[2(\lambda_A-\lambda)+\lambda_F](hHH^+G^-+hHH^-G^+\\ &-AGH^+G^-AGH^-G^+)+\tfrac{1}{4}\lambda_V(H^4+2H^2A^2+A^4+4H^2H^+H^-+4A^2H^+H^-+4H^+H^-H^+H^-)+\tfrac{1}{2}(\lambda-\lambda_A)\\ &\times(H^+H^+G^-G^-+H^-H^-G^+G^+-2hHAG)+\tfrac{1}{4}\lambda_T(h^2A^2+H^2G^2)+\tfrac{1}{4}[2(\lambda-\lambda_A)+\lambda_T](h^2H^2+A^2G^2)\\ &+(\lambda-\lambda_A+\lambda_T)H^+H^-G^+G^-+\frac{i}{2}\lambda_F(hAH^+G^-hAH^-G^++HGH^+G^--HGH^-G^+),\end{aligned} \tag{56}$$

where G and $G^{\pm}$ are the Goldstone bosons (absorbed by the Z and $W^{\pm}$, respectively). Moreover, for $c_{\beta-\alpha}=0$, we have $m_h^2=\lambda v^2$ and $m_H^2-m_A^2=(\lambda-\lambda_A)v^2$, whereas $m_{H^\pm}^2-m_A^2=\frac{1}{2}\lambda_F v^2$ is exact at the tree level. As expected, in the decoupling limit, the low-energy effective scalar theory (which includes h and the three Goldstone bosons) is precisely the same as the corresponding SM Higgs theory, with λ proportional to the Higgs boson quartic coupling.

One can use the results of Appendixes F and G to compute the first nontrivial $\mathcal{O}(c_{\beta-\alpha})$ corrections to Eqs. (55) and (56) as one moves away from the decoupling limit. These results are given in Tables I and II in the Appendixes. For example, the *hhh* and *hhhh* couplings in the decoupling limit are given by

$$g_{hhh}\simeq -3v(\lambda-3\,\hat{\lambda}c_{\beta-\alpha})\simeq\frac{-3m_h^2}{v}+6\,\hat{\lambda}c_{\beta-\alpha}v, \quad (57)$$

$$g_{hhhh}\simeq -3(\lambda-4\,\hat{\lambda}c_{\beta-\alpha})\simeq\frac{-3m_h^2}{v^2}+9\,\hat{\lambda}c_{\beta-\alpha}, \quad (58)$$

where we have used Eq. (30). Precision measurements of these couplings could in principle (modulo radiative corrections, which are known within the SM [26]) provide evidence for a departure from the corresponding SM relations.

Using the explicit forms for the quartic Higgs boson couplings given in Appendix G, it follows that all quartic couplings are $\lesssim\mathcal{O}(1)$ if we require that the $\lambda_i\lesssim\mathcal{O}(1)$. Unitarity constraints on Goldstone and Higgs boson scattering processes can be used to impose numerical limitations on the contributing quartic couplings [19–23]. If we apply tree-level unitarity constraints for $\sqrt{s}$ larger than all Higgs boson masses, then $\lambda_i/4\pi\lesssim\mathcal{O}(1)$ (the precise analytic upper bounds are given in Ref. [22]). One can also investigate a less stringent requirement if the Higgs sector is close to the decoupling limit; namely, assuming $m_h\ll m_H, m_A, m_{H^\pm}$, one can simply impose unitarity constraints on the low-energy effective scalar theory. One must check, for example, that all $2\to 2$ scattering processes involving the $W^{\pm}$, Z, and h satisfy partial-wave unitarity [20,22,23]. At the tree level, one simply obtains the well known SM result $\lambda\leqslant 8\pi/3$, where λ is given by Eq. (25).[12] At one loop, the heavier Higgs scalars can contribute via virtual exchanges, and the restrictions on the self-couplings now involve both the light and the heavier Higgs scalars. For example, in order to avoid large one-loop corrections to the four-point interaction $W^+W^-\to hh$ via an intermediate loop of a heavy Higgs pair, the quartic interactions among h^2H^2, h^2A^2, and $h^2H^+H^-$ must be perturbative. In this case, Eq. (56) implies that $|\lambda-\lambda_A|,|\lambda_F|\lesssim 1$. It follows that there is a bound on the squared-mass splittings among the heavy Higgs boson states of $\mathcal{O}(v^2)$. Thus, to maintain unitarity and perturbativity, the decoupling limit demands rather degenerate heavy Higgs bosons.

[12]Using $m_h^2=\lambda v^2$, this bound is a factor of 2 more stringent than that of Ref. [20] based on the requirement $|\mathrm{Re}\, a_0|\leqslant 1/2$ for the s-wave partial-wave amplitude [27].

Using the explicit forms for the cubic Higgs boson couplings given in Appendix F, it follows that all cubic couplings are $\lesssim\mathcal{O}(v)$ if we require that the $\lambda_i\lesssim\mathcal{O}(1)$. The cubic couplings can also be rewritten in terms of the Higgs boson masses. For example, one possible form for the *hhh* coupling is given in Eq. (F6). Here, we shall consider two equivalent expressions for the hH^+H^- coupling:

$$\begin{aligned} g_{hH^+H^-} &= -\frac{1}{v}\Bigg[\left(m_h^2-\frac{m_{12}^2}{s_\beta c_\beta}\right)\frac{c_{\beta+\alpha}}{s_\beta c_\beta}+(2m_{H^\pm}^2-m_h^2)s_{\beta-\alpha} \\ &\qquad +\tfrac{1}{2}v^2\left(\frac{\lambda_6}{s_\beta^2}-\frac{\lambda_7}{c_\beta^2}\right)c_{\beta-\alpha}\Bigg] \\ &= \frac{1}{v}\Bigg[(2m_A^2-2m_{H^\pm}^2-m_h^2)s_{\beta-\alpha}+2(m_A^2-m_h^2) \\ &\qquad \times\frac{c_{2\beta}c_{\beta-\alpha}}{s_{2\beta}}+v^2\left(\frac{\lambda_5 c_{\beta+\alpha}}{s_\beta c_\beta}-\frac{\lambda_6 s_\alpha}{s_\beta}+\frac{\lambda_7 c_\alpha}{c_\beta}\right)\Bigg]. \end{aligned} \quad (59)$$

From the first equality of Eq. (59), it appears that $g_{hH^+H^-}$ grows quadratically with the heavy charged Higgs boson mass. However, this is an illusion, as can be seen in the subsequent expression for $g_{hH^+H^-}$. In particular, $m_A^2-m_{H^\pm}^2\sim\mathcal{O}(v^2)$ follows from Eq. (11), while in the decoupling limit $m_A^2 c_{\beta-\alpha}\sim\mathcal{O}(v^2)$ follows from Eq. (D3). Hence, $g_{hH^+H^-}\sim\mathcal{O}(v)$ as expected. One can also check that the apparent singular behavior as $s_\beta\to 0$ or $c_\beta\to 0$ is in fact absent, since the original form of $g_{hH^+H^-}$ was well behaved in this limit. Clearly, the most elegant form for $g_{hH^+H^-}$ is given in Eq. (F1). No matter which form is used, it is straightforward to perform an expansion for small $c_{\beta-\alpha}$ to obtain

$$g_{hH^+H^-}=-v(\lambda_F+\lambda_T)+\mathcal{O}(c_{\beta-\alpha}), \quad (60)$$

which agrees with the corresponding result given in Table I of Appendix F.

One can also be misled by writing the cubic couplings in terms of Λ_i, which are employed in an alternate parametrization of the 2HDM scalar potential given in Appendix A. In particular, in the CP-conserving case, $m_{12}^2=1/2v^2 s_\beta c_\beta\Lambda_5$, which becomes large in the approach to the decoupling limit. Consequently, all the $\Lambda_i(i=1,...,6)$ are large in the decoupling limit [see Eq. (A3)], even though the magnitudes of the λ_i are all $\lesssim\mathcal{O}(1)$.

One important consequence of $g_{hH^+H^-}\sim\mathcal{O}(v)$ is that the one-loop amplitude for $h\to\gamma\gamma$ reduces to the corresponding SM result in the decoupling limit (where $m_{H^\pm}\gg v$). To prove this, we observe that in the decoupling limit all h couplings to SM particles that enter the one-loop Feynman diagrams for $h\to\gamma\gamma$ are given by the corresponding SM values. However, there is a new contribution to the one-loop amplitude that arises from a charged Higgs boson loop. But this contribution is suppressed by $\mathcal{O}(v^2/m_{H^\pm}^2)$ because $g_{hH^+H^-}\sim\mathcal{O}(v)$, and our assertion is proved. In addition, the first nontrivial corrections to decoupling, of $\mathcal{O}(v^2/m_A^2)$, can easily be computed and arise from two sources. First, the contribution of the charged Higgs boson loop yields a contribution to the $h\rightarrow\gamma\gamma$ amplitude proportional to $g_{hH^+H^-}v/m_{H^\pm}^2$. Second, the contributions of the fermion loops are altered due to the modified $hf\bar{f}$ couplings [see Eq. (49)], which yield corrections of $\mathcal{O}(c_{\beta-\alpha})\sim\mathcal{O}(v^2/m_A^2)$. Both corrections enter at the same order. Note that the contribution of the W loop is also modified, but the corresponding first order correction is of $\mathcal{O}(c_{\beta-\alpha}^2)$ (since the hW^+W^- coupling is proportional to $s_{\beta-\alpha}$) and thus can be neglected.

The above considerations can be generalized to all loop-induced processes which involve the h and SM particles as external states. As long as $\lambda_i\lesssim\mathcal{O}(1)$, the Appelquist-Carazzone decoupling theorem [28] guarantees that for $m_A\rightarrow\infty$ the amplitudes for such processes approach the corresponding SM values. The same result also applies to radiatively corrected h decay rates and cross sections.

V. A SM-LIKE HIGGS BOSON WITHOUT DECOUPLING

We have demonstrated above that the decoupling limit (where $m_A^2\gg|\lambda_i|v^2$) implies that $|c_{\beta-\alpha}|\ll 1$. However, the $|c_{\beta-\alpha}|\ll 1$ limit is more general than the decoupling limit. From Eq. (36), one learns that $|c_{\beta-\alpha}|\ll 1$ implies that either (i) $m_A^2\gg\lambda_A v^2$, and/or (ii) $|\hat{\lambda}|\ll 1$ subject to the condition specified by Eq. (33). Case (i) is the decoupling limit described in Sec. III. Although case (ii) is compatible with $m_A^2\gg\lambda_i v^2$, which is the true decoupling limit, there is no requirement *a priori* that m_A be particularly large [as long as Eq. (33) is satisfied]. It is even possible to have $m_A<m_h$, implying that all Higgs boson masses are $\lesssim\mathcal{O}(v)$, in contrast to the true decoupling limit. In this latter case, there does not exist an effective low-energy scalar theory consisting of a single Higgs boson.

Although the tree-level couplings of h to vector bosons may appear to be SM-like, a significant deviation of either the $h\bar{D}D$ or $h\bar{U}U$ coupling from the corresponding SM value is possible. For example, for $|c_{\beta-\alpha}|\ll 1$, the h couplings to quark pairs normalized to their SM values [see Eqs. (36), (45), and (46)] are given by

$$h\bar{D}D:\quad 1-\frac{\hat{\lambda}v^2\tan\beta}{m_A^2-\lambda_A v^2},\quad h\bar{U}U:\quad 1+\frac{\hat{\lambda}v^2\cot\beta}{m_A^2-\lambda_A v^2}. \tag{61}$$

If $m_A\lesssim\mathcal{O}(v)$ and $\tan\beta\gg 1$ ($\cot\beta\gg 1$), then the deviation of the $h\bar{D}D$ ($h\bar{U}U$) coupling from the corresponding SM value can be significant even though $|\hat{\lambda}|\ll 1$. A particularly nasty case is one where the $h\bar{D}D$ ($h\bar{U}U$) coupling is equal in magnitude but opposite in sign to the corresponding SM value [29].[13] For example, the $h\bar{D}D$ coupling of Eq. (61) is equal to -1 when $\tan\beta\simeq 2[(m_A^2/v^2)-\lambda_A]/\hat{\lambda}\gg 1$. Of course, the latter corresponds to an isolated point of the parameter space; it is far more likely that the $h\bar{D}D$ coupling will exhibit a discernible deviation in magnitude from its SM value.

Even if the tree-level couplings of h to both vector bosons and fermions appear to be SM-like, radiative corrections can introduce deviations from SM expectations [29] if m_A is not significantly larger than v.[14] For example, consider the amplitude for $h\rightarrow\gamma\gamma$ (which corresponds to a dimension-5 effective operator). If $m_A\lesssim\mathcal{O}(v)$ [implying that $m_{H^\pm}\sim\mathcal{O}(v)$] *and* $|\hat{\lambda}|\ll 1$ (implying that tree-level couplings of h approach their SM values), then the charged Higgs boson loop contribution to the $h\rightarrow\gamma\gamma$ amplitude will *not* be suppressed. Hence the resulting amplitude will be shifted from the SM result, thus revealing that true decoupling has not been achieved, and the h is not the SM Higgs boson [29].

Radiative corrections can also introduce deviations from SM expectations if the Higgs boson self-coupling parameters are large [30]. We can illustrate this in a model in which h is SM-like and all other Higgs bosons are very heavy, and yet the decoupling limit does not apply. Consider a model in which $m_{12}^2=\lambda_6=\lambda_7=0$ and the Higgs potential parameters are chosen to yield $m_H=m_A=m_{H^\pm}$ and $c_{\beta-\alpha}=0$. This can be achieved by taking $m_{11}^2=m_{22}^2$ and[15]

$$\lambda_1=\lambda_3+\frac{\lambda_5 c_{2\beta}}{c_\beta^2},\quad \lambda_2=\lambda_3-\frac{\lambda_5 c_{2\beta}}{s_\beta^2},\quad \lambda_4=\lambda_5, \tag{62}$$

with $\lambda_5<0$ and $-(\lambda_1\lambda_2)^{1/2}<\lambda_{345}<0$ [thereby ensuring that $m_A^2>0$, $m_h<m_H$, and Eq. (4) are satisfied]. These results are most easily obtained by using Eqs. (D20)–(D23). One immediately finds that $m_h^2=(\lambda_3+\lambda_5)v^2$ and $m_H^2=m_A^2=m_{H^\pm}^2=-\lambda_5 v^2$. It is easy to check that $\hat{\lambda}=0$ is exact, which yields $c_{\beta-\alpha}=0$ (since $\lambda_{345}<0$ implies that $m_A^2>m_L^2$ and $m_h^2=m_L^2$ [cf. Eqs. (19) and (20)]), and $\lambda=\lambda_A=\lambda_3+\lambda_5$. Note that, although $\hat{\lambda}=c_{\beta-\alpha}=0$, Eq. (36) implies that the ratio $\hat{\lambda}/c_{\beta-\alpha}=-\lambda_{345}=(m_A^2-m_h^2)/v^2$ can be taken to be an arbitrary positive parameter. This example exhibits a model in which the properties of h are indistinguishable from those of the SM Higgs boson, but the decoupling limit can never be achieved (since $m_{12}^2=0$). One cannot take the masses of the mass-degenerate H, A, and $H^\pm$ arbitrarily large with $m_h\sim\mathcal{O}(m_Z)$ without taking all the $|\lambda_i|(i=1,...,5)$ arbitrarily large (thereby violating unitarity). Nevertheless, if one takes the $|\lambda_i|$ close to their unitarity limits, one can find a region of parameter space in which $m_H=m_A=m_{H^\pm}\gg m_h\sim\mathcal{O}(m_Z)$. If only h were observed, it would appear to be difficult to dis-

[13] Note that for $|\hat{\lambda}|\ll 1$ (i.e., for $|c_{\beta-\alpha}|\ll 1$ with m_A arbitrary, where the hVV couplings are SM-like), there is no choice of parameters for which *both* the $h\bar{D}D$ and $h\bar{U}U$ couplings are equal in magnitude but opposite in sign relative to the corresponding SM couplings.

[14] Radiative corrections that contribute to shifts in the coefficients of operators of dimension $\leqslant 4$ will simply renormalize the parameters of the scalar potential. Hence the deviation from the SM of the properties of h associated with dimension $\leqslant 4$ operators will continue to be suppressed in the limit of the renormalized parameter $|\hat{\lambda}|\ll 1$.

[15] In this case, Eqs. (6) and (7) imply that $\tan^2\beta=(\lambda_{345}-\lambda_1)/(\lambda_{345}-\lambda_2)$.

tinguish this case from a Higgs sector close to the decoupling limit. However, when the $|\lambda_i|$ are large one expects large radiative corrections due to loops that depend on the Higgs boson self-couplings. For example, the one-loop corrections to the *hhh* coupling (which at the tree level is given by $g_{hhh}=-3m_h^2/v^2$ when $c_{\beta-\alpha}=0$) can deviate by as much as 100% or more from the corresponding corrections in the standard model in the above model where $c_{\beta-\alpha}=0$ and $m_H=m_A=m_{H^\pm}\gg m_h\sim\mathcal{O}(m_Z)$ [30]. More generally, a model with a light SM-like Higgs boson and all other Higgs bosons heavy could be distinguished from a Higgs sector near the decoupling limit only by observing the effects of one-loop corrections proportional to the (large) Higgs boson self-coupling parameters. Such radiative corrections could deviate significantly from the corresponding loop corrections in the standard model.

Two additional examples in which the $|\hat{\lambda}|\ll 1$ limit is realized are given by (1) $\tan\beta\gg 1$, $\lambda_6=\lambda_7=0$, and $m_A^2>(\lambda_2-\lambda_5)v^2$, and (2) $\lambda_1=\lambda_2=\lambda_{345}$, $\lambda_6=\lambda_7=0$, and $m_A^2>(\lambda_2-\lambda_5)v^2$ [31].

The condition on m_A^2 in the two cases is required by Eq. (33). In case 1, $\hat{\lambda}=0$ when $\beta=\pi/2$, whereas in case 2, $\hat{\lambda}=0$ independently of $\tan\beta$. In both these cases, it is straightforward to use Eqs. (12) and (16) to obtain

$$m_{h,H}^2=\begin{cases}\lambda_2 v^2,\\ m_A^2+\lambda_5 v^2.\end{cases} \tag{63}$$

Since $m_L^2=\lambda_2 v^2$, Eq. (20) yields $\cos(\beta-\alpha)=0$ as expected.

Two special limits of case 2 above are treated in Ref. [31], where scalar potentials with $\lambda_1=\lambda_2=\lambda_3=\mp\lambda_4=\pm\lambda_5>0$ (and $\lambda_6=\lambda_7=0$) are considered. Assuming that $m_A^2>(\lambda_2-\lambda_5)v^2$, the resulting Higgs spectrum is given by $m_{H^\pm}^2=m_H^2=m_A^2\pm m_h^2$ and $m_h^2=\lambda_1 v^2$ (m_A^2 is a free parameter that depends on m_{12}^2). In the case of $\lambda_5>0$, one has $m_A^2>0$ and it is possible to have a Higgs spectrum in which *A* is very light, while the other Higgs bosons (including *h*) are heavy and approximately degenerate in mass. In the case of $\lambda_5<0$, one has $m_A^2>2m_h^2$, and a light *A* would imply that all the Higgs bosons of the model are light. In both cases $c_{\beta-\alpha}=0$, and the tree-level couplings of *h* correspond precisely to those of the SM Higgs boson (see Sec. IV). These are clearly very special cases, corresponding to a distinctive form of the quartic terms of the Higgs potential:

$$\mathcal{V}_4=\tfrac{1}{2}\lambda_1[(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2)^2\pm(\Phi_1^\dagger\Phi_2-\Phi_2^\dagger\Phi_1)^2], \tag{64}$$

where the choice of sign corresponds to the sign of λ_5. Note that $\mathcal{V}_4$ above exhibits a flat direction if $\lambda_5>0$, whereas the scalar potential possesses a globally stable minimum if $\lambda_5<0$ [see Eq. (4)].

Next, we examine a region of Higgs parameter space where $|\sin(\beta-\alpha)|\ll 1$, in which the heavier *CP*-even Higgs boson *H* is SM-like (also considered in Refs. [29] and [31]). In this case, the *h* couplings to vector boson pairs are highly suppressed. This is far from the decoupling regime. Nevertheless, this region does merit a closer examination, which we now perform.

When $s_{\beta-\alpha}\to 0$, we have $\beta-\alpha=0$ or π. We shall work to first nontrivial order in an $s_{\beta-\alpha}$ expansion, with $c_{\beta-\alpha}\simeq\pm(1-\frac{1}{2}s_{\beta-\alpha}^2)$. Using the results of Eqs. (D9)–(D11) and Eq. (11), we obtain[16]

$$m_A^2\simeq v^2\left[\mp\frac{\hat{\lambda}}{s_{\beta-\alpha}}+\lambda_A\pm\tfrac{3}{2}\hat{\lambda}s_{\beta-\alpha}\right], \tag{65}$$

$$\begin{aligned}m_h^2&\simeq v^2\left[\mp\frac{\hat{\lambda}}{s_{\beta-\alpha}}+\lambda\pm\tfrac{1}{2}\hat{\lambda}s_{\beta-\alpha}\right]\\ &\simeq m_A^2+(\lambda-\lambda_A\mp\hat{\lambda}s_{\beta-\alpha})v^2,\end{aligned} \tag{66}$$

$$m_H^2\simeq v^2(\lambda\pm\hat{\lambda}s_{\beta-\alpha}), \tag{67}$$

$$\begin{aligned}m_{H^\pm}^2&\simeq v^2\left[\mp\frac{\hat{\lambda}}{s_{\beta-\alpha}}+\lambda_A+\tfrac{1}{2}\lambda_F\pm\tfrac{3}{2}\hat{\lambda}s_{\beta-\alpha}\right]\\ &=m_A^2+\tfrac{1}{2}\lambda_F v^2.\end{aligned} \tag{68}$$

The condition $m_H>m_h$ imposes the inequality (valid to first order in $s_{\beta-\alpha}$)

$$m_A^2<v^2(\lambda_A\pm 2\hat{\lambda}s_{\beta-\alpha}), \tag{69}$$

[cf. Eq. (D32)]. Note that Eq. (69) implies that all Higgs boson squared masses are of $\mathcal{O}(v^2)$. We may also use Eq. (10) to obtain

$$\begin{aligned}m_{12}^2\simeq v^2 s_\beta c_\beta\biggl[&\mp\frac{\hat{\lambda}}{s_{\beta-\alpha}}+\lambda_A+\lambda_5+\tfrac{1}{2}\lambda_6 t_\beta^{-1}+\tfrac{1}{2}\lambda_7 t_\beta\\ &\pm\tfrac{3}{2}\hat{\lambda}s_{\beta-\alpha}\biggr].\end{aligned} \tag{70}$$

We can rewrite Eq. (65) in another form [or equivalently use Eqs. (D30) and (D31) to obtain]

$$\sin(\beta-\alpha)\simeq\frac{\mp\hat{\lambda}v^2}{m_A^2-\lambda_A v^2}\simeq\frac{\pm\hat{\lambda}v^2}{m_H^2-m_h^2}. \tag{71}$$

The $|s_{\beta-\alpha}|\ll 1$ limit is achieved when $|\hat{\lambda}|\ll 1$, subject to the condition given in Eq. (69). Clearly, *H* is SM-like, since if

[16]Note that Eqs. (D4) and (D5) are interchanged under the transformation $m_h^2\leftrightarrow m_H^2$ and $c_{\beta-\alpha}\leftrightarrow -s_{\beta-\alpha}$. Thus, applying these transformations to Eqs. (29)–(32) yields the results given in Eqs. (65)–(68) with $c_{\beta-\alpha}=+1$.

$s_{\beta-\alpha}\simeq 0$, then the couplings of H to VV, HH, and HHH coincide with the corresponding SM Higgs boson couplings.[17]

The couplings of H to fermion pairs are obtained from Eq. (44) by expanding the Yukawa couplings of H to $\mathcal{O}(s_{\beta-\alpha})$:

$$\mathcal{L}_{HQQ}=-\bar{D}\left[\pm\frac{1}{v}M_D+\tan\beta\left(\frac{1}{v}M_D-\frac{1}{\sqrt{2}s_\beta}(S_D+iP_D\gamma_5)\right)s_{\beta-\alpha}\right]DH-\bar{U}\left(\pm\frac{1}{v}M_U-\cot\beta\left[\frac{1}{v}M_U-\frac{1}{\sqrt{2}c_\beta}(S_U+iP_U\gamma_5)\right]s_{\beta-\alpha}\right)UH, \tag{72}$$

where $\pm$ corresponds to $c_{\beta-\alpha}=\pm 1$ and S_D and P_D are given by Eq. (50). If $|\hat{\lambda}|\tan\beta\ll 1$ or $|\hat{\lambda}|\cot\beta\ll 1$, then the $Hf\bar{f}$ couplings reduce to the corresponding $h_{\rm SM}f\bar{f}$ couplings. However, if $|\hat{\lambda}|\ll 1\lesssim|\hat{\lambda}|\tan\beta$ (or $|\hat{\lambda}|\ll 1\lesssim|\hat{\lambda}|\cot\beta$) when $\tan\beta\gg 1$ (or $\cot\beta\gg 1$), then the $Hf\bar{f}$ couplings can deviate significantly from the corresponding $h_{\rm SM}f\bar{f}$ couplings. This behavior is qualitatively different from the decoupling limit, where for fixed λ_i and large $\tan\beta$ (or large $\cot\beta$), one can always choose m_A large enough such that the $hf\bar{f}$ couplings approach the corresponding SM values. In contrast, when $|s_{\beta-\alpha}|\ll 1$, the size of m_A is restricted by Eq. (69), and so there is no guarantee of SM-like $Hf\bar{f}$ couplings when either $\tan\beta$ or $\cot\beta$ is large.

Although the tree-level properties of H are SM-like when $|\hat{\lambda}|\ll 1$, deviations can occur for loop-induced processes as noted earlier. Again, the $H\to\gamma\gamma$ amplitude will deviate from the corresponding SM amplitude due to the contribution of the charged Higgs boson loop which is not suppressed since $m_{H^\pm}\sim\mathcal{O}(v)$. Thus, departures from true decoupling can in principle be detected for $|s_{\beta-\alpha}|\ll 1$.

We now briefly examine some model examples in which $|s_{\beta-\alpha}|\ll 1$ is realized. These examples are closely related to the ones previously considered in the case of $c_{\beta-\alpha}=0$. First, consider the model in which $m_{12}^2=\lambda_6=\lambda_7=0$ and the Higgs potential parameters are chosen to yield $m_h=m_A=m_{H^\pm}$ and $s_{\beta-\alpha}=0$. This can be achieved by taking $m_{11}^2=m_{22}^2$ and the nonzero λ_i given by Eq. (62) with $\lambda_5<0$ and $\lambda_{345}>0$. In this case, $m_H^2=(\lambda_3+\lambda_5)v^2$ and $m_h^2=m_A^2=m_{H^\pm}^2=-\lambda_5 v^2$. It is easy to check that $\hat{\lambda}=0$ is exact and yields $s_{\beta-\alpha}=0$ (since $\lambda_{345}>0$ implies that $m_A^2<m_L^2$ and $m_H^2=m_L^2$ [cf. Eqs. (19) and (20)]). Thus, the properties of H are indistinguishable from those of the SM Higgs boson. However, all the other mass-degenerate Higgs bosons are lighter than the SM-like Higgs boson H. Thus, one expects that all Higgs bosons can be observed (once the SM-like Higgs boson is discovered). That is, there is little chance of confusing H with the Higgs boson of the standard model.

Two additional examples in which the $|s_{\beta-\alpha}|\ll 1$ limit is realized are given by (1) $\tan\beta\gg 1$, $\lambda_6=\lambda_7=0$, and $m_A^2<(\lambda_2-\lambda_5)v^2$, and (2) $\lambda_1=\lambda_2=\lambda_{345}$, $\lambda_6=\lambda_7=0$, and $m_A^2<(\lambda_2-\lambda_5)v^2$ [31]. The condition on m_A^2 in the two cases is required by Eq. (69). In case 1, $\hat{\lambda}=0$ when $\beta=\pi/2$, whereas in case 2, $\hat{\lambda}=0$ independently of $\tan\beta$. In both these cases, it is straightforward to use Eqs. (12) and (16) to obtain

$$m_{h,H}^2=\begin{cases} m_A^2+\lambda_5 v^2,\\ \lambda_2 v^2.\end{cases} \tag{73}$$

Since $m_L^2=\lambda_2 v^2$, it follows from Eq. (20) that $c_{\beta-\alpha}^2=1$. Hence, $\sin(\beta-\alpha)=0$, which implies that H is SM-like.[18]

Finally, we note that the SM-like Higgs bosons resulting from the limiting cases above where $\hat{\lambda}=0$ can be easily understood in terms of the squared-mass matrix entries of Eqs. (12) and (13). In order to achieve $c_{\beta-\alpha}=0$ or $s_{\beta-\alpha}=0$, we demand that $\tan 2\beta=\tan 2\alpha$. This implies [see Eq. (17)] that the entries in the $\mathcal{B}^2$ matrix be in the same ratio as the entries in the term proportional to m_A^2 in Eq. (12):

$$\frac{2\mathcal{M}_{12}^2}{\mathcal{M}_{11}^2-\mathcal{M}_{22}^2}=\tan 2\beta. \tag{74}$$

It is easy to check that

$$\hat{\lambda}v^2=\tfrac{1}{2}(\mathcal{B}_{11}^2-\mathcal{B}_{22}^2)\sin 2\beta-\mathcal{B}_{12}^2\cos 2\beta. \tag{75}$$

Equations (12) and (74) immediately imply that $\hat{\lambda}=0$ is equivalent to $\tan 2\beta=\tan 2\alpha$. Moreover, to determine whether $c_{\beta-\alpha}=0$ or $s_{\beta-\alpha}=0$, simply note that if the sign of $\sin 2\alpha/\sin 2\beta$ is negative (positive), then $c_{\beta-\alpha}=0$ ($s_{\beta-\alpha}=0$). In the convention where $\tan\beta$ is positive, it follows that $\sin 2\beta>0$. Using Eqs. (12) and (13), if the sign of

$$\mathcal{M}_{12}^2=s_\beta c_\beta[(\lambda_{345}-\lambda_5)v^2-m_A^2]+v^2(\lambda_6 c_\beta^2+\lambda_7 s_\beta^2) \tag{76}$$

is negative (positive), then $c_{\beta-\alpha}=0$ ($s_{\beta-\alpha}=0$). One can check that the conditions given by Eqs. (33) and (69) correspond precisely to the negative (positive) sign of $\mathcal{M}_{12}^2$ [Eq. (76)], after imposing $\hat{\lambda}=0$.[19] The condition $\hat{\lambda}=0$ can be achieved not only for appropriate choices of the λ_i and $\tan\beta$ in the general 2HDM, but also can be satisfied in the MSSM when radiative corrections are incorporated (see Sec. VI).

VI. DECOUPLING EFFECTS IN THE MSSM HIGGS SECTOR

The Higgs sector of the MSSM is a *CP*-conserving two-Higgs-doublet model, with a Higgs potential whose dimension-4 terms respect supersymmetry and with type-II Higgs-boson–fermion couplings. The quartic couplings λ_i are given by [9]

[17] When $\hat{\lambda}=0$, the H couplings to VV, HH, and $f\bar{f}$ [see Eq. (72)] all differ by an overall sign from the corresponding $h_{\rm SM}$ couplings if $c_{\beta-\alpha}=-1$. However, this sign is unphysical, since one can eliminate it with a redefinition $h\to -h$ and $H\to -H$, which is equivalent to replacing α with $\alpha\pm\pi$.

$$\lambda_1=\lambda_2=-\lambda_{345}=\tfrac{1}{4}(g^2+g'^2),\quad \lambda_4=-\tfrac{1}{2}g^2,$$

$$\lambda_5=\lambda_6=\lambda_7=0. \tag{77}$$

The squared-mass parameters defined in Eq. (18) simplify to $m_L^2=m_Z^2\cos^2 2\beta$, $m_D^2=0$, $m_T^2=m_Z^2$, and $m_S^2=m_A^2+m_Z^2$. Using Eq. (77), the invariant coupling parameters defined in Eqs. (25)–(28) and Eqs. (52)–(54) reduce to

$$\lambda=-\lambda_T=\lambda_V=\tfrac{1}{4}(g^2+g'^2)\cos^2 2\beta,$$

$$\hat{\lambda}=-\lambda_U=\tfrac{1}{4}(g^2+g'^2)\sin 2\beta\cos 2\beta,$$

$$\lambda_A=\tfrac{1}{4}(g^2+g'^2)\cos 4\beta,$$

$$\lambda_F=\tfrac{1}{2}g^2. \tag{78}$$

The results of Sec. II can then be used to obtain the well-known tree-level results

$$m_A^2=m_{12}^2(\tan\beta+\cot\beta),\quad m_{H^\pm}^2=m_A^2+m_W^2, \tag{79}$$

and a neutral *CP*-even squared-mass matrix given by

$$\mathcal{M}_0^2=\begin{pmatrix} m_A^2\sin^2\beta+m_Z^2\cos^2\beta & -(m_A^2+m_Z^2)\sin\beta\cos\beta \\ -(m_A^2+m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta+m_Z^2\sin^2\beta \end{pmatrix}, \tag{80}$$

with eigenvalues

$$m_{H^0,h^0}^2=\tfrac{1}{2}\left(m_A^2+m_Z^2\pm\sqrt{(m_A^2+m_Z^2)^2-4m_Z^2m_A^2\cos^2 2\beta}\right), \tag{81}$$

and the diagonalizing angle α given by

$$\cos 2\alpha=-\cos 2\beta\left(\frac{m_A^2-m_Z^2}{m_{H^0}^2-m_{h^0}^2}\right),$$

$$\sin 2\alpha=-\sin 2\beta\left(\frac{m_{H^0}^2+m_{h^0}^2}{m_{H^0}^2-m_{h^0}^2}\right). \tag{82}$$

One can also write

$$\cos^2(\beta-\alpha)=\frac{m_h^2(m_Z^2-m_h^2)}{m_A^2(m_H^2-m_h^2)}. \tag{83}$$

In the decoupling limit where $m_A\gg m_Z$, the above formulas yield

$$m_h^2\simeq m_Z^2\cos^2 2\beta,\quad m_H^2\simeq m_A^2+m_Z^2\sin^2 2\beta,$$

$$m_{H^\pm}^2=m_A^2+m_W^2,\quad \cos^2(\beta-\alpha)\simeq\frac{m_Z^4\sin^2 4\beta}{4m_A^4}. \tag{84}$$

That is, $m_A\simeq m_H\simeq m_{H^\pm}$ up to corrections of $\mathcal{O}(m_Z^2/m_A)$, and $\cos(\beta-\alpha)=0$ up to corrections of $\mathcal{O}(m_Z^2/m_A^2)$.

It is straightforward to work out all the tree-level Higgs couplings, both in general and in the decoupling limit. Since the Higgs-boson–fermion couplings follow the type-II pattern, the Higgs-boson–fermion Yukawa couplings are given by Eq. (44) with $\eta_1^U=\eta_2^U=0$. However, one-loop radiative corrections can lead in some cases to significant shifts from the tree-level couplings. It is of interest to examine how the approach to the decoupling limit is affected by the inclusion of radiative corrections.

First, we note that in some cases, one-loop effects mediated by loops of supersymmetric particles can generate a deviation from standard model expectations, even if $m_A\gg m_Z$ where the corrections to the decoupling limit are negligible. As a simple example, if squarks are relatively light, then squark loop contributions to the $h\to gg$ and $h\to\gamma\gamma$ amplitudes can be significant [32]. Of course, in the limit of large squark masses, the contributions of the supersymmetric loops decouple as well [33]. Thus, in the MSSM, there are two separate decoupling limits that must be analyzed. For simplicity, we assume henceforth that supersymmetric particle masses are large (say of order 1 TeV), so that supersymmetric loop effects of the type just mentioned are negligible.

The leading contributions to the radiatively corrected Higgs boson couplings arise in two ways. First, the radiative corrections to the *CP*-even Higgs boson squared-mass matrix results in a shift of the *CP*-even Higgs boson mixing angle α from its tree-level value. That is, the dominant Higgs propagator corrections can to a good approximation be absorbed into an effective (‘‘radiatively corrected’’) mixing angle α [34]. In this approximation, we can write

$$\mathcal{M}^2\equiv\begin{pmatrix}\mathcal{M}_{11}^2 & \mathcal{M}_{12}^2\\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2\end{pmatrix}=\mathcal{M}_0^2+\delta\mathcal{M}^2, \tag{85}$$

where the tree-level contribution $\mathcal{M}_0^2$ was given in Eq. (80) and $\delta\mathcal{M}^2$ is the contribution from the radiative corrections. Then, $\cos(\beta-\alpha)$ is given by

[18]Since $\lambda_6=\lambda_7=0$, if we additionally set $m_{12}^2=0$, then we recover the discrete symmetry of the Higgs potential previously noted in Sec. III. Thus, there is no true decoupling limit in this model. Moreover, since $m_A^2=-\lambda_5 v^2$ (which implies that $\lambda_5<0$), Eq. (73) yields $m_h=0$, although this result would be modified once radiative corrections are included.

[19]It is simplest to use $\hat{\lambda}=0$ to eliminate the quantity $\lambda_1 c_\beta^2-\lambda_2 s_\beta^2$ from λ_A in Eqs. (33) and (69).

$$\cos(\beta-\alpha)=\frac{(\mathcal{M}_{11}^2-\mathcal{M}_{22}^2)\sin 2\beta-2\mathcal{M}_{12}^2\cos 2\beta}{2(m_H^2-m_h^2)\sin(\beta-\alpha)}$$
$$=\frac{m_Z^2\sin 4\beta+(\delta\mathcal{M}_{11}^2-\delta\mathcal{M}_{22}^2)\sin 2\beta-2\,\delta\mathcal{M}_{12}^2\cos 2\beta}{2(m_H^2-m_h^2)\sin(\beta-\alpha)}. \tag{86}$$

Using tree-level Higgs boson couplings with α replaced by its renormalized value provides a useful first approximation to the radiatively corrected Higgs boson couplings.

Second, contributions from the one-loop vertex corrections to tree-level Higgs-boson–fermion couplings can modify these couplings in a significant way, especially in the limit of large $\tan\beta$. In particular, although the tree-level Higgs boson–fermion coupling follow the type-II pattern, when radiative corrections are included, all possible dimension-4 Higgs-boson–fermion couplings are generated. These results can be summarized by an effective Lagrangian that describes the coupling of the neutral Higgs bosons to the third generation quarks:

$$-\mathcal{L}_{\rm eff}=[(h_b+\delta h_b)\overline{b}_R b_L\Phi_1^{0*}+(h_t+\delta h_t)\overline{t}_R t_L\Phi_2^0]$$
$$+\Delta h_t\overline{t}_R t_L\Phi_1^0+\Delta h_b\overline{b}_R b_L\Phi_2^{0*}+\text{H.c.}, \tag{87}$$

resulting in a modification of the tree-level relation between h_t (h_b) and m_t (m_b) as follows [35–38]:

$$m_b=\frac{h_b v}{\sqrt{2}}\cos\beta\left(1+\frac{\delta h_b}{h_b}+\frac{\Delta h_b\tan\beta}{h_b}\right)\equiv\frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b), \tag{88}$$

$$m_t=\frac{h_t v}{\sqrt{2}}\sin\beta\left(1+\frac{\delta h_t}{h_t}+\frac{\Delta h_t\cot\beta}{h_t}\right)\equiv\frac{h_t v}{\sqrt{2}}\sin\beta(1+\Delta_t). \tag{89}$$

The dominant contributions to Δ_b are $\tan\beta$ enhanced, with $\Delta_b\simeq(\Delta h_b/h_b)\tan\beta$; for $\tan\beta\gg 1$, $\delta h_b/h_b$ provides a small correction to Δ_b. [In the same limit, $\Delta_t\simeq\delta h_t/h_t$, with the additional contribution of $(\Delta h_t/h_t)\cot\beta$ providing a small correction.]

From Eq. (87) we can obtain the couplings of the physical neutral Higgs bosons to third generation quarks. The resulting interaction Lagrangian is of the form

$$\mathcal{L}_{\rm int}=-\sum_{q=t,b}[g_{hq\overline{q}}hq\overline{q}+g_{Hq\overline{q}}Hq\overline{q}-ig_{Aq\overline{q}}A\overline{q}\gamma_5 q]. \tag{90}$$

Using Eqs. (88) and (89), one obtains [39,40]

$$g_{hb\overline{b}}=-\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1+\cot\alpha\cot\beta)\right], \tag{91}$$

$$g_{Hb\overline{b}}=\frac{m_b}{v}\frac{\cos\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1-\tan\alpha\cot\beta)\right], \tag{92}$$

$$g_{Ab\overline{b}}=\frac{m_b}{v}\tan\beta\left[1+\frac{1}{(1+\Delta_b)\sin^2\beta}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)\right], \tag{93}$$

$$g_{ht\overline{t}}=\frac{m_t}{v}\frac{\cos\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\alpha)\right], \tag{94}$$

$$g_{Ht\overline{t}}=\frac{m_t}{v}\frac{\sin\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta-\cot\alpha)\right], \tag{95}$$

$$g_{At\overline{t}}=\frac{m_t}{v}\cot\beta\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\beta)\right]. \tag{96}$$

We now turn to the decoupling limit. First consider the implications for the radiatively corrected value of $\cos(\beta-\alpha)$. Since $\delta\mathcal{M}_{ij}^2\sim\mathcal{O}(m_Z^2)$, and $m_H^2-m_h^2=m_A^2+\mathcal{O}(m_Z^2)$, one finds [39]

$$\cos(\beta-\alpha)=c\left[\frac{m_Z^2\sin 4\beta}{2m_A^2}+\mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right)\right] \tag{97}$$

in the limit of $m_A\gg m_Z$, where

$$c\equiv 1+\frac{\delta\mathcal{M}_{11}^2-\delta\mathcal{M}_{22}^2}{2m_Z^2\cos 2\beta}-\frac{\delta\mathcal{M}_{12}^2}{m_Z^2\sin 2\beta}. \tag{98}$$

The effect of the radiative corrections has been to modify the tree-level definition of $\hat{\lambda}$:

$$\hat{\lambda}v^2=cm_Z^2\sin 2\beta\cos 2\beta. \tag{99}$$

Equation (97) exhibits the expected decoupling behavior for $m_A\gg m_Z$. However, Eqs. (86) and (97) exhibit another way in which $\cos(\beta-\alpha)=0$ can be achieved—simply choose the MSSM parameters (that govern the Higgs boson mass radiative corrections) such that the numerator of Eq. (86) vanishes. That is,

$$2m_Z^2\sin 2\beta=2\,\delta\mathcal{M}_{12}^2-\tan 2\beta(\delta\mathcal{M}_{11}^2-\delta\mathcal{M}_{22}^2). \tag{100}$$

This condition is equivalent to $c=0$, and thus corresponds precisely to the case of $\hat{\lambda}=0$ discussed at the beginning of Sec. V. Although $\hat{\lambda}\neq 0$ at the tree level, the above analysis shows that $|\hat{\lambda}|\ll 1$ can arise due to the effects of one-loop radiative corrections that approximately cancel the tree-level result.[20] In particular, Eq. (100) is independent of the value of m_A. Typically, Eq. (100) yields a solution at large $\tan\beta$. That is, by approximating $\tan 2\beta \simeq -\sin 2\beta \simeq -2/\tan\beta$, one can determine the value of β at which $\hat{\lambda}\simeq 0$ [39]:

$$\tan\beta \simeq \frac{2m_Z^2 - \delta\mathcal{M}_{11}^2 + \delta\mathcal{M}_{22}^2}{\delta\mathcal{M}_{12}^2}. \qquad (101)$$

Hence, there exists a value of $\tan\beta$ (which depends on the choice of MSSM parameters) where $\cos(\beta-\alpha)\simeq 0$ independently of the value of m_A. If m_A is not much larger than m_Z, then h is a SM-like Higgs boson outside the decoupling regime.[21] Of course, as explained in Sec. V, this SM-like Higgs boson can be distinguished in principle from the SM Higgs boson by measuring its decay rate to two photons and looking for a deviation from SM predictions.

Finally, we analyze the radiatively corrected Higgs-boson–fermion couplings [Eqs. (91)–(96)] in the decoupling limit. Here it is useful to note that, for $m_A \gg m_Z$,

$$\cot\alpha = -\tan\beta - \frac{2m_Z^2}{m_A^2}\tan\beta\cos 2\beta + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right). \qquad (102)$$

Applying this result to Eqs. (91) and (94), it follows that in the decoupling limit $g_{hq\bar{q}} = g_{h_{\rm SM}q\bar{q}} = m_q/v$. Away from the decoupling limit, the Higgs boson couplings to down-type fermions can deviate significantly from their tree-level values due to enhanced radiative corrections at large $\tan\beta$ [where $\Delta_b \simeq \mathcal{O}(1)$]. In particular, because $\Delta_b \propto \tan\beta$, the leading one-loop radiative correction to $g_{hb\bar{b}}$ is of $\mathcal{O}(m_Z^2\tan\beta/m_A^2)$, which formally decouples only when $m_A^2 \gg m_Z^2\tan\beta$. This behavior is called *delayed decoupling* in Ref. [41], although this phenomenon can also occur in a more general 2HDM (with tree-level couplings), as noted previously in Sec. IV [below Eq. (50)].

VII. DISCUSSION AND CONCLUSIONS

In this paper, we have studied the decoupling limit of a general *CP*-conserving two-Higgs-doublet model. In this limit, the lightest Higgs boson of the model is a *CP*-even neutral Higgs scalar (h) with couplings identical to those of the SM Higgs boson. Near the decoupling limit, the first order corrections for the Higgs boson couplings to gauge and Higgs bosons, the Higgs-boson–fermion Yukawa couplings, and the Higgs boson cubic and quartic self-couplings have also been obtained. These results exhibit a definite pattern for the deviations of the h couplings from those of the SM Higgs boson. In particular, the rate of the approach to decoupling depends on the particular Higgs boson coupling as follows:

$$\frac{g_{hVV}^2}{g_{h_{\rm SM}VV}^2} \simeq 1 - \frac{\hat{\lambda}^2 v^4}{m_A^4}, \qquad (103)$$

$$\frac{g_{hhh}^2}{g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}}^2} \simeq 1 - \frac{6\,\hat{\lambda}^2 v^2}{\lambda m_A^2}, \qquad (104)$$

$$\frac{g_{htt}^2}{g_{h_{\rm SM}tt}^2} \simeq 1 + \frac{2\,\hat{\lambda} v^2\cot\beta}{m_A^2}(1-\xi_t), \qquad (105)$$

$$\frac{g_{hbb}^2}{g_{h_{\rm SM}bb}^2} \simeq 1 - \frac{2\,\hat{\lambda} v^2\tan\beta}{m_A^2}(1-\xi_b), \qquad (106)$$

where ξ_t and ξ_b reflect the terms proportional to S and P in Eq. (49). Thus, the approach to decoupling is fastest for the h couplings to vector bosons and slowest for the couplings to down-type (or up-type) quarks if $\tan\beta > 1$ (or $\tan\beta < 1$). We may apply the above results to the MSSM (see Sec. VI). Including the leading ($\tan\beta$)-enhanced radiative corrections, $\xi_b = v\,\Delta h_b/(\sqrt{2}s_\beta m_b) = \Delta_b/[s_\beta^2(1+\Delta_b)]$ (whereas $\xi_t \ll 1$ can be neglected) and $\hat{\lambda}$ is given by Eq. (99). Plugging into Eqs. (103)–(106), one reproduces the results obtained in Ref. [39].

Although the results of this paper were derived from a tree-level analysis of couplings, these results can also be applied to the radiatively corrected couplings that multiply operators of dimension 4 or less. An example of this was given in Sec. VI, where we showed how the decoupling limit applies to the radiatively corrected Higgs-boson–fermion Yukawa couplings. In particular, near the decoupling limit one can neglect radiative corrections that are generated by the exchange of heavy Higgs bosons. These contributions are suppressed by a loop factor in addition to the suppression factor of $\mathcal{O}(v^2/m_A^2)$ and thus are smaller than corrections to tree-level Higgs boson couplings that enter at first order in $c_{\beta-\alpha}$. This should be contrasted with loop-induced Higgs boson couplings (e.g., $h\to\gamma\gamma$, which is generated by a dimension-5 effective operator), where the corrections of $\mathcal{O}(c_{\beta-\alpha})$ to tree-level Higgs boson couplings that appear in the one-loop amplitude and the effects of a heavy Higgs boson loop are both of $\mathcal{O}(v^2/m_A^2)$ (in addition to the overall one-loop factor). Consequently, both contributions are equally important in determining the overall correction to the loop-induced Higgs couplings due to the departure from the decoupling limit.

If a neutral Higgs boson h is discovered at a future collider, it may turn out that its couplings are close to those expected of the SM Higgs boson. The challenge for future experiments is then to determine whether the observed state is the SM Higgs boson, or whether it is the lowest-lying scalar state of a nonminimal Higgs sector [42]. If the latter, then it is likely that the additional scalar states of the model are heavy, and the decoupling limit applies. In this case, it is possible that the heavier scalars cannot be detected at the CERN Large Hadron Collider (LHC) or at an e^+e^- linear collider (LC) with a center-of-mass energy in the range of 350–800 GeV. Moreover, it may not be possible to distin-

[20]The one-loop corrections arise from the exchange of supersymmetric particles, whose contributions can be enhanced for certain MSSM parameter choices. One can show that the two-loop corrections are subdominant, so that the approximation scheme is under control.

[21]For large $\tan\beta$ and $m_A \lesssim \mathcal{O}(m_Z)$, one finds that $\sin(\beta-\alpha)\simeq 0$, implying that H is the SM-like Higgs boson, as discussed in Sec. V.

guish between the h and the SM Higgs boson at the LHC. However, the measurements of Higgs boson observables at the LC can provide sufficient precision to observe deviations from SM Higgs boson properties at the few percent level. In this case, one can begin to probe deep into the decoupling regime [12].

In this paper, we also clarified a Higgs boson parameter regime in which h possesses SM-like couplings to vector bosons but where $m_A^2 \lesssim \mathcal{O}(v^2)$ and the decoupling limit does not apply (see Sec. V). In this case, the couplings of h to fermion pairs can deviate significantly from the corresponding SM Higgs-boson–fermion couplings if either $\tan\beta$ or $\cot\beta$ is large. Moreover, the masses of H, A, and $H^\pm$ are not particularly large, and all scalars would be accessible at the LHC and/or the LC.

The discovery of the Higgs boson will be a remarkable achievement. Nevertheless, the lesson of the decoupling limit is that a SM-like Higgs boson provides very little information about the nature of the underlying electroweak symmetry-breaking dynamics. It is essential to find evidence for departures from SM Higgs boson predictions. Such departures can reveal crucial information about the existence of a nonminimal Higgs sector. Precision Higgs boson measurements can also provide critical tests of possible new physics beyond the standard model. As an example, in the MSSM, deviations in Higgs boson couplings from the decoupling limit can yield indirect information about the MSSM parameters. In particular, at large $\tan\beta$ the sensitivity to MSSM parameters may be increased due to enhanced radiative corrections. The decoupling limit is both a curse and an opportunity. If nature chooses the Higgs sector parameters to lie deep in the decoupling regime, then it may not be possible to distinguish the observed h from the SM Higgs boson. On the other hand, given sufficient precision of the measurements of h branching ratios and cross sections [40], it may be possible to observe a small but statistically significant deviation from SM expectations and provide a first glimpse of the physics responsible for electroweak symmetry breaking.

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APPENDIX A: AN ALTERNATIVE PARAMETRIZATION OF THE 2HDM SCALAR POTENTIAL

In this Appendix, we give the translation of the parameters of Eq. (1) employed in this paper to the parameters employed in the *Higgs Hunter's Guide* (HHG) [7]. While the HHG parametrization was useful for some purposes (e.g., the scalar potential minimum is explicitly exhibited), it obscures the decoupling limit.

In the HHG parametrization, the most general 2HDM scalar potential, subject to a discrete symmetry $\Phi_1 \to -\Phi_1$ that is only softly violated by dimension-2 terms, is given by[22]

$$\begin{aligned}\mathcal{V} = {} & \Lambda_1(\Phi_1^\dagger\Phi_1 - V_1^2)^2 + \Lambda_2(\Phi_2^\dagger\Phi_2 - V_2^2)^2 + \Lambda_3[(\Phi_1^\dagger\Phi_1 - V_1^2) + (\Phi_2^\dagger\Phi_2 - V_2^2)]^2 + \Lambda_4[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)] \\ & + \Lambda_5[\mathrm{Re}(\Phi_1^\dagger\Phi_2) - V_1V_2\cos\xi]^2 + \Lambda_6[\mathrm{Im}(\Phi_1^\dagger\Phi_2) - V_1V_2\sin\xi]^2 \\ & + \Lambda_7[\mathrm{Re}(\Phi_1^\dagger\Phi_2) - V_1V_2\cos\xi][\mathrm{Im}(\Phi_1^\dagger\Phi_2) - V_1V_2\sin\xi], \end{aligned} \tag{A1}$$

where the Λ_i are real parameters.[23] The $V_{1,2}$ are related to the $v_{1,2}$ of Eq. (5) by $V_{1,2} = v_{1,2}/\sqrt{2}$. The conversion from these Λ_i to the λ_i and m_{ij}^2 of Eq. (1) is

$$\begin{aligned} \lambda_1 &= 2(\Lambda_1 + \Lambda_3) \\ \lambda_2 &= 2(\Lambda_2 + \Lambda_3), \\ \lambda_3 &= 2\Lambda_3 + \Lambda_4, \\ \lambda_4 &= -\Lambda_4 + \tfrac{1}{2}(\Lambda_5 + \Lambda_6), \\ \lambda_5 &= \tfrac{1}{2}(\Lambda_5 - \Lambda_6 - i\Lambda_7), \\ \lambda_6 &= \lambda_7 = 0 \\ m_{11}^2 &= -2V_1^2\Lambda_1 - 2(V_1^2 + V_2^2)\Lambda_3, \\ m_{22}^2 &= -2V_2^2\Lambda_2 - 2(V_1^2 + V_2^2)\Lambda_3, \\ m_{12}^2 &= V_1V_2\left(\Lambda_5\cos\xi - i\Lambda_6\sin\xi - \frac{i}{2}e^{i\xi}\Lambda_7\right). \end{aligned} \tag{A2}$$

[22] In the HHG, V_i and Λ_i are denoted by v_i and λ_i, respectively. In Eq. (A1), we employ the former notation in order to distinguish between the HHG parametrization and the notation of Eqs. (1) and (5).

[23] In Eq. (A1) we include the Λ_7 term that was left out in the hardcover edition of the HHG. See the erratum that has been included in the paperback edition of the HHG (Perseus Publishing, Cambridge, MA, 2000).

Excluding λ_6 and λ_7, the scalar potentials [Eqs. (1) and (A1)] are fixed by ten real parameters. The *CP*-conserving limit of Eq. (A1) is most easily obtained by setting $\xi=0$ and $\Lambda_7=0$. In the *CP*-conserving limit, it is easy to invert Eq. (A2) and solve for the Λ_i $(i=1,...,6)$. The result is

$$\Lambda_1=\tfrac{1}{2}[\lambda_1-\lambda_{345}+2m_{12}^2/(v^2 s_\beta c_\beta)],$$

$$\Lambda_2=\tfrac{1}{2}[\lambda_2-\lambda_{345}+2m_{12}^2/(v^2 s_\beta c_\beta)],$$

$$\Lambda_3=\tfrac{1}{2}[\lambda_{345}-2m_{12}^2/(v^2 s_\beta c_\beta)],$$

$$\Lambda_4=2m_{12}^2/(v^2 s_\beta c_\beta)-\lambda_4-\lambda_5,$$

$$\Lambda_5=2m_{12}^2/(v^2 s_\beta c_\beta),$$

$$\Lambda_6=2m_{12}^2/(v^2 s_\beta c_\beta)-2\lambda_5, \tag{A3}$$

where $\lambda_{345}\equiv\lambda_3+\lambda_4+\lambda_5$ and $v^2 s_\beta c_\beta=2V_1V_2$.

APPENDIX B: CONDITIONS FOR *CP* CONSERVATION IN THE TWO-HIGGS-DOUBLET MODEL

First, we derive the conditions such that the Higgs sector does not exhibit explicit *CP* violation.[24] It is convenient to adopt a convention in which one of the vacuum expectation values, say v_1, is real and positive.[25] This still leaves one additional phase redefinition for the Higgs doublet fields. If there is no explicit *CP* violation, it should be possible to choose the phases of the Higgs fields so that there are no explicit phases in the Higgs potential parameters of Eq. (1). If we consider $\Phi_1^\dagger\Phi_2\to e^{-i\eta}\Phi_1^\dagger\Phi_2$, then the η-dependent terms in $\mathcal{V}$ are given by

$$\mathcal{V}\ni-m_{12}^2e^{-i\eta}\Phi_1^\dagger\Phi_2+\tfrac{1}{2}\lambda_5e^{-2i\eta}(\Phi_1^\dagger\Phi_2)^2+\lambda_6e^{-i\eta}(\Phi_1^\dagger\Phi_1)\times(\Phi_1^\dagger\Phi_2)+\lambda_7e^{-i\eta}(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2)+\text{H.c.} \tag{B1}$$

Let us write

$$m_{12}^2=|m_{12}^2|e^{i\theta_m},\quad \lambda_{5,6,7}=|\lambda_{5,6,7}|e^{i\theta_{5,6,7}}. \tag{B2}$$

Then, all explicit parameter phases are removed if

$$\theta_m-\eta=n_m\pi,\quad \theta_5-2\eta=n_5\pi,\quad \theta_{6,7}-\eta=n_{6,7}\pi, \tag{B3}$$

where $n_{m,5,6,7}$ are integers. Writing $\eta=\theta_m-n_m\pi$ from the first condition of Eq. (B3) and substituting into the other conditions, gives

$$\theta_5-2\theta_m=(n_5-2n_m)\pi\Rightarrow\text{Im}[(m_{12}^2)^2\lambda_5^*]=0, \tag{B4}$$

$$\theta_6-\theta_m=(n_6-n_m)\pi\Rightarrow\text{Im}[m_{12}^2\lambda_6^*]=0, \tag{B5}$$

$$\theta_7-\theta_m=(n_7-n_m)\pi\Rightarrow\text{Im}[m_{12}^2\lambda_7^*]=0. \tag{B6}$$

Equations (B4)–(B6) constitute the conditions for the absence of explicit *CP* violation in the (tree-level) Higgs sector. A useful convention is one in which m_{12}^2 is real (by a suitable choice of the phase η). It then follows that λ_5, λ_6, and λ_7 are also real. Henceforth, we shall assume that all parameters in the scalar potential are real.

Let us consider now the conditions for the absence of spontaneous *CP* violation.[26] Let us write $\langle\Phi_1^\dagger\Phi_2\rangle=\frac{1}{2}v_1v_2e^{i\xi}$ with v_1 and v_2 real and positive and $0\leqslant\xi\leqslant\pi$. The ξ-dependent terms in $\mathcal{V}$ are given by

$$\mathcal{V}\ni-m_{12}^2v_1v_2\cos\xi+\tfrac{1}{4}\lambda_5v_1^2v_2^2\cos2\xi+\tfrac{1}{2}\lambda_6v_1^3v_2\cos\xi+\tfrac{1}{2}\lambda_7v_2^3v_1\cos\xi, \tag{B7}$$

which yields

$$\frac{\partial\mathcal{V}}{\partial\cos\xi}=-m_{12}^2v_1v_2+\lambda_5v_1^2v_2^2\cos\xi+\tfrac{1}{2}\lambda_6v_1^3v_2+\tfrac{1}{2}\lambda_7v_2^3v_1 \tag{B8}$$

and

$$\frac{\partial^2\mathcal{V}}{\partial(\cos\xi)^2}=\lambda_5v_1^2v_2^2. \tag{B9}$$

Spontaneous *CP* violation occurs when $\xi\neq0$, $\pi/2$, or π at the potential minimum. That is, $\lambda_5>0$ and there exists a *CP*-violating solution to

$$\cos\xi=\frac{m_{12}^2-\frac{1}{2}\lambda_6v_1^2-\frac{1}{2}\lambda_7v_2^2}{\lambda_5v_1v_2}. \tag{B10}$$

Thus, we conclude that the criterion for spontaneous *CP* violation (in a convention where all parameters of the scalar potential are real) is

$$0\neq|m_{12}^2-\tfrac{1}{2}\lambda_6v_1^2-\tfrac{1}{2}\lambda_7v_2^2|<\lambda_5v_1v_2\quad\text{and}\quad\lambda_5>0. \tag{B11}$$

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[24]For another approach, in which invariants are employed to identify basis-independent conditions for *CP* violation in the Higgs sector, see Refs. [43] and [44].

[25]Due to the $U(1)$-hypercharge symmetry of the theory, it is always possible to make a phase rotation on the scalar fields such that $v_1>0$.

[26]Similar considerations can be found in Refs. [44–46] and [13].

Otherwise, the minimum of the potential occurs either at $\xi=0$, $\pi/2$ or π and *CP* is conserved.[27] The case of $\xi=\pi/2$ is singular and arises when $m_{12}^2=1/2\lambda_6 v_1^2+1/2\lambda_7 v_2^2$ and $\lambda_5>0$.[28] It is convenient to choose a convention where $\langle\Phi_1^0\rangle$ is real and $\langle\Phi_2^0\rangle$ is pure imaginary. One must then reevaluate the Higgs boson mass eigenstates. As shown in Ref. [47], the neutral Goldstone boson is now a linear combination of $\mathrm{Im}\,\Phi_1^0$ and $\mathrm{Re}\,\Phi_2^0$, while the physical *CP*-odd scalar *A* corresponds to the orthogonal combination. The two *CP*-even Higgs scalars are orthogonal linear combinations of $\mathrm{Re}\,\Phi_1^0$ and $\mathrm{Im}\,\Phi_2^0$. Most of the results of this paper do not apply for this case without substantial revision. Nevertheless, it is clear that the decoupling limit ($m_A^2\gg\lambda_i v^2$) does not exist due to the condition on m_{12}^2.

We shall not consider the $\xi=\pi/2$ model further in this paper. Then, if the parameters of the scalar potential are real and if there is no spontaneous *CP* violation, then it is always possible to choose the phase η in Eq. (B1) so that the potential minimum corresponds to $\xi=0$.[29] In this convention,

$$m_{12}^2-\tfrac{1}{2}\lambda_6 v_1^2-\tfrac{1}{2}\lambda_7 v_2^2\geqslant\lambda_5 v_1 v_2 \quad \text{for } \lambda_5>0, \tag{B12}$$

$$m_{12}^2-\tfrac{1}{2}\lambda_6 v_1^2-\tfrac{1}{2}\lambda_7 v_2^2\geqslant 0 \quad \text{for } \lambda_5\leqslant 0, \tag{B13}$$

where Eq. (B12) follows from Eq. (B11), and Eq. (B13) is a consequence of the requirement that $\mathcal{V}(\xi=0)\leqslant\mathcal{V}(\xi=\pi)$. Since $\xi=0$ and both v_1 and v_2 are real and positive, this convention corresponds to the one chosen below Eq. (9). Note that if we rewrite Eq. (10) as[30]

$$m_A^2=\frac{v^2}{v_1 v_2}[m_{12}^2-\lambda_5 v_1 v_2-\tfrac{1}{2}\lambda_6 v_1^2-\tfrac{1}{2}\lambda_7 v_2^2], \tag{B14}$$

it follows that if $\lambda_5>0$ then the condition $m_A^2\geqslant 0$ is equivalent to Eq. (B12). However, if $\lambda_5\leqslant 0$, then Eq. (B13) implies that $m_A^2\geqslant|\lambda_5|v^2$.

APPENDIX C: A SINGULAR LIMIT: $m_h=m_H$

By definition, $m_h\leqslant m_H$. The limiting case of $m_h=m_H$ is special and requires careful treatment in some cases. For example, despite the appearance of $m_H^2-m_h^2$ in the denominator of Eq. (20), one can show that $0\leqslant c_{\beta-\alpha}^2\leqslant 1$. To prove this, we first write

$$c_{\beta-\alpha}^2=\tfrac{1}{2}\left[1-\frac{m_S^2-2m_L^2}{\sqrt{m_S^4-4m_A^2 m_L^2-4m_D^4}}\right]. \tag{C1}$$

Next, we use Eq. (18) to explicitly compute

$$m_S^4-4m_A^2m_L^2-4m_D^4=m_A^4-2m_A^2[(\mathcal{B}_{22}^2-\mathcal{B}_{11}^2)c_{2\beta}+2\mathcal{B}_{12}^2 s_{2\beta}] +(\mathcal{B}_{11}^2-\mathcal{B}_{22}^2)^2+4[\mathcal{B}_{12}^2]^2 \tag{C2}$$

and

$$(m_S^2-2m_L^2)^2=m_S^4-4m_A^2m_L^2-4m_D^4 -[(\mathcal{B}_{11}^2-\mathcal{B}_{22}^2)s_{2\beta}-2\mathcal{B}_{12}^2c_{2\beta}]^2. \tag{C3}$$

Note that Eq. (C2), viewed as a quadratic function of m_A^2 (of the form $Am_A^4+Bm_A^2+C$), is non-negative if $B^2-4AC=[(\mathcal{B}_{11}^2-\mathcal{B}_{22}^2)s_{2\beta}-2\mathcal{B}_{12}^2c_{2\beta}]^2\geqslant 0$. It then follows from Eq. (C1) that $0\leqslant c_{\beta-\alpha}^2\leqslant 1$ if

$$(m_S^2-2m_L^2)^2\leqslant m_S^4-4m_A^2m_L^2-4m_D^4, \tag{C4}$$

a result which is manifestly true [see Eq. (C3)].

We now turn to the case of $m_h=m_H$. This can arise if and only if the *CP*-even Higgs boson squared-mass matrix (in any basis) is proportional to the unit matrix. From Eq. (12), it then follows that

$$\mathcal{B}_{11}^2-\mathcal{B}_{22}^2=m_A^2c_{2\beta}, \quad 2\mathcal{B}_{12}^2=m_A^2 s_{2\beta}. \tag{C5}$$

where $m_h^2=m_H^2=\mathcal{B}_{11}^2+m_A^2s_\beta^2=\mathcal{B}_{22}^2+m_A^2c_\beta^2$. Alternatively, from Eq. (19), the condition for $m_h=m_H$ is given by $m_S^4-4m_A^2m_L^2-4m_D^4\equiv Am_A^4+Bm_A^2+C=0$. However, one must check that this quadratic equation possesses a positive (real) solution for m_A^2. Noting the discussion above Eq. (C4), such a solution can exist if and only if $B^2-4AC=0$, which is indeed consistent with Eq. (C5). Of course, the results of Eq. (C5) are not compatible with the decoupling limit, since it is not possible to have $m_h=m_H$ and $m_A^2\gg|\lambda_i|v^2$.

If we take $B^2-4AC=0$ but keep m_A arbitrary, then Eq. (C1) yields

$$c_{\beta-\alpha}^2=\begin{cases}0 & \text{if } m_L^2<\tfrac{1}{2}m_S^2,\\ 1 & \text{if } m_L^2>\tfrac{1}{2}m_S^2.\end{cases} \tag{C6}$$

For $m_L^2=1/2m_S^2$, we have $m_h^2=m_H^2=1/2m_S^2$, and the angle α is not well defined. In this case, one cannot distinguish between *h* and *H* in either production or decays, and the corresponding squared amplitudes should be (incoherently) added

[27] The *CP*-conserving minimum corresponding to $\xi=0$ or $\xi=\pi$ does not in general correspond to an extremum in $V(\cos\xi)$. Specifically, for $\lambda_5<0$, the extremum corresponds to a maximum in $\mathcal{V}$, while for $\lambda_5>0$ the extremum corresponding to a minimum of $\mathcal{V}(\cos\xi)$ arises for $|\cos\xi|>1$. In both cases, when restricted to the physical region corresponding to $|\cos\xi|\leqslant 1$, the minimum of $\mathcal{V}(\cos\xi)$ is attained on the boundary $|\cos\xi|=1$.

[28] Note that the case of $\xi=\pi/2$ arises automatically in the case of the discrete symmetry discussed in Sec. III, $m_{12}^2=\lambda_6=\lambda_7=0$, when $\lambda_5>0$.

[29] In particular, if $\xi=\pi$, simply choose $\eta=\pi$, which corresponds to changing the overall sign of $\Phi_1^\dagger\Phi_2$. This is equivalent to redefining the parameters $m_{12}^2\to-m_{12}^2$, $\lambda_6\to-\lambda_6$, and $\lambda_7\to-\lambda_7$.

[30] Under the assumption that v_1 and v_2 are positive, Eq. (10) implicitly employs the convention in which $\xi=0$.

in all processes. It is easy to check that the undetermined angle α that appears in the relevant Higgs boson couplings would then drop out in any such sum of squared amplitudes. The singular point of parameter space corresponding to $m_h = m_H$ will not be considered further in this paper.

APPENDIX D: RELATIONS AMONG HIGGS BOSON POTENTIAL PARAMETERS AND MASSES

It is useful to express the physical Higgs boson masses in terms of the parameters of the scalar potential [Eq. (1)]. First, inserting Eqs. (12) and (13) into Eq. (15) and examining the diagonal elements yields the *CP*-even Higgs boson squared masses

$$\begin{aligned} m_h^2 = m_A^2 c_{\beta-\alpha}^2 + v^2[&\lambda_1 c_\beta^2 s_\alpha^2 + \lambda_2 s_\beta^2 c_\alpha^2 - 2\lambda_{345} c_\alpha c_\beta s_\alpha s_\beta \\ &+ \lambda_5 c_{\beta-\alpha}^2 - 2\lambda_6 c_\beta s_\alpha c_{\beta+\alpha} + 2\lambda_7 s_\beta c_\alpha c_{\beta+\alpha}], \end{aligned} \tag{D1}$$

$$\begin{aligned} m_H^2 = m_A^2 s_{\beta-\alpha}^2 + v^2[&\lambda_1 c_\beta^2 c_\alpha^2 + \lambda_2 s_\beta^2 s_\alpha^2 + 2\lambda_{345} c_\alpha c_\beta s_\alpha s_\beta \\ &+ \lambda_5 s_{\beta-\alpha}^2 + 2\lambda_6 c_\beta c_\alpha s_{\beta+\alpha} + 2\lambda_7 s_\beta s_\alpha s_{\beta+\alpha}], \end{aligned} \tag{D2}$$

while the requirement that the off-diagonal entries in Eq. (15) are zero yields

$$\begin{aligned} m_A^2 s_{\beta-\alpha} c_{\beta-\alpha} = \tfrac{1}{2} v^2[&s_{2\alpha}(-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2) + \lambda_{345} s_{2\beta} c_{2\alpha} \\ &- 2\lambda_5 s_{\beta-\alpha} c_{\beta-\alpha} + 2\lambda_6 c_\beta c_{\beta+2\alpha} \\ &+ 2\lambda_7 s_\beta s_{\beta+2\alpha}], \end{aligned} \tag{D3}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. We can now eliminate m_A^2 from Eqs. (D1) and (D2) and Eqs. (10) and (11) using the result of Eq. (D3). This yields equations for the other three physical Higgs boson squared masses and the scalar potential mass parameter m_{12}^2 in terms of the Higgs scalar quartic couplings:

$$\begin{aligned} \frac{m_h^2}{v^2} s_{\beta-\alpha} = &-\lambda_1 c_\beta^3 s_\alpha + \lambda_2 s_\beta^3 c_\alpha \\ &+ \tfrac{1}{2}\lambda_{345} c_{\beta+\alpha} s_{2\beta} + \lambda_6 c_\beta^2 (c_\beta c_\alpha - 3 s_\beta s_\alpha) \\ &+ \lambda_7 s_\beta^2 (3 c_\beta c_\alpha - s_\beta s_\alpha), \end{aligned} \tag{D4}$$

$$\begin{aligned} \frac{m_H^2}{v^2} c_{\beta-\alpha} = &\lambda_1 c_\beta^3 c_\alpha + \lambda_2 s_\beta^3 s_\alpha + \tfrac{1}{2}\lambda_{345} s_{\beta+\alpha} s_{2\beta} \\ &+ \lambda_6 c_\beta^2 (3 s_\beta c_\alpha + c_\beta s_\alpha) + \lambda_7 s_\beta^2 (s_\beta c_\alpha + 3 c_\beta s_\alpha), \end{aligned} \tag{D5}$$

$$\begin{aligned} \frac{2m_{H^\pm}^2}{v^2} s_{\beta-\alpha} c_{\beta-\alpha} = &-s_{2\alpha}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \lambda_{345} s_{2\beta} c_{2\alpha} \\ &- (\lambda_4 + \lambda_5) s_{\beta-\alpha} c_{\beta-\alpha} + 2\lambda_6 c_\beta c_{\beta+2\alpha} \\ &+ 2\lambda_7 s_\beta s_{\beta+2\alpha}, \end{aligned} \tag{D6}$$

$$\begin{aligned} \frac{2m_{12}^2}{v^2} s_{\beta-\alpha} c_{\beta-\alpha} = &-\tfrac{1}{2} s_{2\beta} s_{2\alpha}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 c_{2\alpha} \\ &+ \lambda_6 c_\beta^2 [3 c_\beta s_\beta c_{2\alpha} - c_\alpha s_\alpha (1 + 2 s_\beta^2)] \\ &+ \lambda_7 s_\beta^2 [3 s_\beta c_\beta c_{2\alpha} + c_\alpha s_\alpha (1 + 2 c_\beta^2)]. \end{aligned} \tag{D7}$$

Note that Eq. (D6) is easily derived by inserting Eq. (D3) into Eq. (11). A related useful result is easily derived from Eqs. (D3) and (D5):

$$\begin{aligned} \frac{(m_A^2 - m_H^2)}{v^2} s_{\beta-\alpha} = \tfrac{1}{2} s_{2\beta}(&-\lambda_1 c_\alpha c_\beta + \lambda_2 s_\alpha s_\beta + \lambda_{345} c_{\beta+\alpha}) \\ &- \lambda_5 s_{\beta-\alpha} + \lambda_6 c_\beta [c_\beta c_{\beta+\alpha} - 2 s_\beta^2 c_\alpha] \\ &+ \lambda_7 s_\beta [s_\beta c_{\beta+\alpha} + 2 c_\beta^2 s_\alpha]. \end{aligned} \tag{D8}$$

It is remarkable that the left-hand side of Eq. (D8) is proportional only to $s_{\beta-\alpha}$ (i.e., the factor of $c_{\beta-\alpha}$ has canceled). As a result, in the decoupling limit where $c_{\beta-\alpha} \to 0$, we see that $m_A^2 - m_H^2 = \mathcal{O}(v^2)$.

The expressions given in Eqs. (D3)–(D6) are quite complicated. These results simplify considerably when expressed in terms of λ, $\hat{\lambda}$, and λ_A [Eqs. (25)–(27)]:

$$m_A^2 = v^2\left[\lambda_A + \hat{\lambda}\left(\frac{s_{\beta-\alpha}}{c_{\beta-\alpha}} - \frac{c_{\beta-\alpha}}{s_{\beta-\alpha}}\right)\right], \tag{D9}$$

$$m_h^2 = v^2\left[\lambda - \frac{\hat{\lambda} c_{\beta-\alpha}}{s_{\beta-\alpha}}\right], \tag{D10}$$

$$m_H^2 = v^2\left[\lambda + \frac{\hat{\lambda} s_{\beta-\alpha}}{c_{\beta-\alpha}}\right]. \tag{D11}$$

One can then rewrite Eq. (D8) as

$$m_H^2 - m_A^2 = v^2\left[\lambda - \lambda_A + \frac{\hat{\lambda} c_{\beta-\alpha}}{s_{\beta-\alpha}}\right]. \tag{D12}$$

We can invert Eqs. (D3)–(D7) and solve for any five of the scalar potential parameters in terms of the physical Higgs boson masses and the remaining three undetermined variables [12,48,49]. It is convenient to solve for $\lambda_1,\ldots,\lambda_5$ in terms of λ_6, λ_7, m_{12}^2, and the Higgs boson masses. We obtain

$$\lambda_1 = \frac{m_H^2 c_\alpha^2 + m_h^2 s_\alpha^2 - m_{12}^2 t_\beta}{v^2 c_\beta^2} - \tfrac{3}{2}\lambda_6 t_\beta + \tfrac{1}{2}\lambda_7 t_\beta^3, \tag{D13}$$

$$\lambda_2 = \frac{m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_{12}^2 t_\beta^{-1}}{v^2 s_\beta^2} + \tfrac{1}{2}\lambda_6 t_\beta^{-3} - \tfrac{3}{2}\lambda_7 t_\beta^{-1}, \tag{D14}$$

$$\lambda_3=\frac{(m_H^2-m_h^2)c_\alpha s_\alpha+2m_{H^\pm}^2 s_\beta c_\beta-m_{12}^2}{v^2 s_\beta c_\beta}-\tfrac{1}{2}\lambda_6 t_\beta^{-1}-\tfrac{1}{2}\lambda_7 t_\beta \tag{D15}$$

$$\lambda_4=\frac{(m_A^2-2m_{H^\pm}^2)s_\beta c_\beta+m_{12}^2}{v^2 s_\beta c_\beta}-\tfrac{1}{2}\lambda_6 t_\beta^{-1}-\tfrac{1}{2}\lambda_7 t_\beta, \tag{D16}$$

$$\lambda_5=\frac{m_{12}^2-m_A^2 s_\beta c_\beta}{v^2 s_\beta c_\beta}-\tfrac{1}{2}\lambda_6 t_\beta^{-1}-\tfrac{1}{2}\lambda_7 t_\beta. \tag{D17}$$

In addition, the minimization conditions of Eqs. (6) and (7) reduce to

$$m_{11}^2=-\frac{1}{2c_\beta}(m_H^2 c_\alpha c_{\beta-\alpha}-m_h^2 s_\alpha s_{\beta-\alpha})+m_{12}^2 t_\beta, \tag{D18}$$

$$m_{22}^2=-\frac{1}{2s_\beta}(m_h^2 c_\alpha s_{\beta-\alpha}+m_H^2 s_\alpha c_{\beta-\alpha})+m_{12}^2 t_\beta^{-1}. \tag{D19}$$

Note that λ_6 and λ_7 do not appear when m_{11}^2 and m_{22}^2 are expressed entirely in terms of m_{12}^2 and physical Higgs boson masses.

In some cases, it proves more convenient to eliminate m_{12}^2 in favor of λ_5 using Eq. (D17). The end result is

$$\lambda_1=\frac{m_H^2 c_\alpha^2+m_h^2 s_\alpha^2-m_A^2 s_\beta^2}{v^2 c_\beta^2}-\lambda_5 t_\beta^2-2\lambda_6 t_\beta, \tag{D20}$$

$$\lambda_2=\frac{m_H^2 s_\alpha^2+m_h^2 c_\alpha^2-m_A^2 c_\beta^2}{v^2 s_\beta^2}-\lambda_5 t_\beta^{-2}-2\lambda_7 t_\beta^{-1}, \tag{D21}$$

$$\lambda_3=\frac{(m_H^2-m_h^2)s_\alpha c_\alpha+(2m_{H^\pm}^2-m_A^2)s_\beta c_\beta}{v^2 s_\beta c_\beta}-\lambda_5-\lambda_6 t_\beta^{-1}-\lambda_7 t_\beta, \tag{D22}$$

$$\lambda_4=\frac{2(m_A^2-m_{H^\pm}^2)}{v^2}+\lambda_5, \tag{D23}$$

and

$$m_{11}^2=-\frac{1}{2c_\beta}(m_H^2 c_\alpha c_{\beta-\alpha}-m_h^2 s_\alpha s_{\beta-\alpha})+(m_A^2+\lambda_5 v^2)s_\beta^2+\tfrac{1}{2}v^2(\lambda_6 s_\beta c_\beta+\lambda_7 s_\beta^2 t_\beta), \tag{D24}$$

$$m_{22}^2=-\frac{1}{2s_\beta}(m_h^2 c_\alpha s_{\beta-\alpha}+m_H^2 s_\alpha c_{\beta-\alpha})+(m_A^2+\lambda_5 v^2)c_\beta^2+\tfrac{1}{2}v^2(\lambda_6 c_\beta^2 t_\beta^{-1}+\lambda_7 s_\beta c_\beta). \tag{D25}$$

Using Eqs. (D9)–(D11), one may obtain simple expressions for λ, $\hat{\lambda}$, and λ_A [Eqs. (25)–(27)] in terms of the neutral Higgs boson squared masses:

$$\lambda v^2=m_h^2 s_{\beta-\alpha}^2+m_H^2 c_{\beta-\alpha}^2, \tag{D26}$$

$$\hat{\lambda} v^2=(m_H^2-m_h^2)s_{\beta-\alpha}c_{\beta-\alpha}, \tag{D27}$$

$$\lambda_A v^2=m_A^2+(m_H^2-m_h^2)(c_{\beta-\alpha}^2-s_{\beta-\alpha}^2), \tag{D28}$$

$$\lambda_F v^2=2(m_{H^\pm}^2-m_A^2), \tag{D29}$$

where we have also included an expression for $\lambda_F\equiv\lambda_5-\lambda_4$ in terms of the Higgs boson squared masses [see Eq. (11)]. Thus, four of the invariant coupling parameters can be expressed in terms of the physical Higgs boson masses and the basis-independent quantity $\beta-\alpha$ (see Appendix E).

Finally, we note that Eqs. (D27) and (D28) also yield a simple expression for $\beta-\alpha$, which plays such a central role in the decoupling limit. We find two forms that are noteworthy:

$$\tan[2(\beta-\alpha)]=\frac{-2\hat{\lambda}v^2}{m_A^2-\lambda_A v^2} \tag{D30}$$

and

$$\sin[2(\beta-\alpha)]=\frac{2\hat{\lambda}v^2}{m_H^2-m_h^2}. \tag{D31}$$

Indeed, if $\hat{\lambda}=0$ then either $c_{\beta-\alpha}=0$ or $s_{\beta-\alpha}=0$ as discussed in Sec. V. For $\hat{\lambda}\neq 0$, the condition $m_H>m_h$ implies that $\hat{\lambda}s_{\beta-\alpha}c_{\beta-\alpha}>0$. This inequality, when applied to Eq. (D9), imposes the following constraint on m_A:

$$v^2\left[\lambda_A-\frac{2\hat{\lambda}c_{\beta-\alpha}}{s_{\beta-\alpha}}\right]<m_A^2<v^2\left[\lambda_A+\frac{2\hat{\lambda}s_{\beta-\alpha}}{c_{\beta-\alpha}}\right]. \tag{D32}$$

In addition, we require that $m_A^2\geqslant 0$.

The expressions for the Higgs boson masses [Eqs. (D9)–(D11)] and $\beta-\alpha$ [Eq. (D30) or (D31)] are especially useful when considering the approach to the decoupling limit, where $|c_{\beta-\alpha}|\ll 1$. For example, Eqs. (D9)–(D11) reduce in this limit to the results of Eqs. (29)–(31). Moreover, $\sin[2(\beta-\alpha)]\simeq-\tan[2(\beta-\alpha)]\simeq 2c_{\beta-\alpha}$, and Eqs. (D30) and (D31) reduce to the results given by Eq. (36). The corresponding results in the limiting case of $|s_{\beta-\alpha}|\ll 1$ treated in Sec. V are also similarly obtained.

APPENDIX E: INVARIANT COMBINATIONS OF THE HIGGS SCALAR POTENTIAL PARAMETERS

In the most general 2HDM model, there is no distinction between the two $Y=1$ complex doublets Φ_1 and Φ_2. In principle, one could choose any two orthogonal linear combinations of Φ_1 and Φ_2 (i.e., choose a new basis for the scalar doublets) and construct the scalar sector Lagrangian with respect to the new basis. Clearly, the parameters of Eq. (1), m_{ij}^2 and the λ_i, would all be modified, along with α and β. However, there exist seven invariant combinations of the λ_i that are independent of basis choice [50]. These are λ, $\hat{\lambda}$, λ_A, λ_F defined in Eqs. (25)–(28), and λ_T, λ_U, and λ_V defined in Eqs. (52)–(54). In addition, the combination $\beta-\alpha$ is clearly basis independent. Thus, all physical Higgs boson masses and Higgs boson self-couplings can be expressed in terms of the above invariant coupling parameters and $\beta-\alpha$. In Appendix D, we have already shown how to express the Higgs boson masses in terms of the invariant parameters. In Appendixes F and G we also exhibit the three- and four-Higgs-boson couplings in terms of the invariant parameters.[31]

To obtain expressions for the Higgs boson self-couplings in terms of invariant parameters, one must invert the relations between the λ_i and the invariant coupling parameters. The end result is

$$\begin{aligned}
\lambda_1 &= c_\beta^2(1+3s_\beta^2)\lambda+2s_{2\beta}(c_\beta^2\hat{\lambda}+s_\beta^2\lambda_U)\\
&\quad -\tfrac{1}{2}s_{2\beta}^2(2\lambda_A-\lambda_T)+s_\beta^4\lambda_V,\\
\lambda_2 &= s_\beta^2(1+3c_\beta^2)\lambda-2s_{2\beta}(s_\beta^2\hat{\lambda}+c_\beta^2\lambda_U)\\
&\quad -\tfrac{1}{2}s_{2\beta}^2(2\lambda_A-\lambda_T)+c_\beta^4\lambda_V,\\
\lambda_{345} &= (2c_{2\beta}^2-c_\beta^2 s_\beta^2)\lambda-3s_{2\beta}c_{2\beta}(\hat{\lambda}-\lambda_U)\\
&\quad -(c_{2\beta}^2-2c_\beta^2 s_\beta^2)(2\lambda_A-\lambda_T)+\tfrac{3}{4}s_{2\beta}^2\lambda_V,\\
\lambda_5 &= (c_{2\beta}^2+c_\beta^2 s_\beta^2)\lambda-s_{2\beta}c_{2\beta}(\hat{\lambda}-\lambda_U)-c_{2\beta}^2\lambda_A\\
&\quad +\tfrac{1}{4}s_{2\beta}^2(\lambda_V-2\lambda_T),\\
\lambda_6 &= \tfrac{1}{2}s_{2\beta}(3s_\beta^2-1)\lambda-c_\beta c_{3\beta}\hat{\lambda}-s_\beta s_{3\beta}\lambda_U\\
&\quad +\tfrac{1}{2}s_{2\beta}c_{2\beta}(2\lambda_A-\lambda_T)-\tfrac{1}{2}s_\beta^2 s_{2\beta}\lambda_V,\\
\lambda_7 &= \tfrac{1}{2}s_{2\beta}(3c_\beta^2-1)\lambda-s_\beta s_{3\beta}\hat{\lambda}-c_\beta c_{3\beta}\lambda_U\\
&\quad -\tfrac{1}{2}s_{2\beta}c_{2\beta}(2\lambda_A-\lambda_T)-\tfrac{1}{2}c_\beta^2 s_{2\beta}\lambda_V,
\end{aligned} \tag{E1}$$

and $\lambda_4=\lambda_5-\lambda_F$.

The significance of the invariant coupling parameters is most evident in the so-called Higgs basis of Ref. [44], in which only the neutral component of one of the two Higgs doublets (say, the first one) possesses a vacuum expectation value. Let us denote the two Higgs doublets in this basis by Φ_a and Φ_b. Then, after a rotation from the Φ_1-Φ_2 basis by an angle β,

$$\begin{aligned}
\Phi_a &= \Phi_1\cos\beta+\Phi_2\sin\beta,\\
\Phi_b &= -\Phi_1\sin\beta+\Phi_2\cos\beta,
\end{aligned} \tag{E2}$$

one obtains

$$\Phi_a=\begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+\varphi_a^0+iG^0)\end{pmatrix},\quad \Phi_b=\begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_b^0+iA)\end{pmatrix}, \tag{E3}$$

where φ_a^0 and φ_b^0 are related in the CP-conserving model to the CP-even neutral Higgs bosons by

$$H=\varphi_n^0\cos(\beta-\alpha)-\varphi_b^0\sin(\beta-\alpha), \tag{E4}$$

$$h=\varphi_a^0\sin(\beta-\alpha)+\varphi_b^0\cos(\beta-\alpha). \tag{E5}$$

Here, we see that $\beta-\alpha$ is the invariant angle that characterizes the direction of the CP-even mass eigenstates (in the two-dimensional Higgs "flavor" space) relative to that of the vacuum expectation value.

In the Higgs basis, the corresponding values of $\lambda_1,\ldots,\lambda_7$ are easily evaluated by putting $\beta=0$ in Eq. (E1). Thus, the scalar potential takes the following form:

$$\begin{aligned}
\mathcal{V} &= m_{aa}^2\Phi_a^\dagger\Phi_a+m_{bb}^2\Phi_b^\dagger\Phi_b-[m_{ab}^2\Phi_a^\dagger\Phi_b+\text{H.c.}]\\
&\quad +\tfrac{1}{2}\lambda(\Phi_a^\dagger\Phi_a)^2+\tfrac{1}{2}\lambda_V(\Phi_b^\dagger\Phi_b)^2+(\lambda_T+\lambda_F)(\Phi_a^\dagger\Phi_a)\\
&\quad \times(\Phi_b^\dagger\Phi_b)+(\lambda-\lambda_A-\lambda_F)(\Phi_a^\dagger\Phi_b)(\Phi_b^\dagger\Phi_a)\\
&\quad +\{\tfrac{1}{2}(\lambda-\lambda_A)(\Phi_a^\dagger\Phi_b)^2-[\hat{\lambda}(\Phi_a^\dagger\Phi_a)\\
&\quad +\lambda_U(\Phi_b^\dagger\Phi_b)]\Phi_a^\dagger\Phi_b+\text{H.c.}\},
\end{aligned} \tag{E6}$$

where three new invariant quantities are revealed:

[31]The Higgs boson couplings to vector bosons depend only on $\beta-\alpha$ [see Eqs. (37)–(39)]. The Higgs boson couplings to fermions in the type-III model (in which both up-and down-type fermions couple to both Higgs doublets) can also be written in terms of invariant parameters. However, one would then have to identify the appropriate invariant combinations of the Higgs-boson–fermion Yukawa coupling parameters [50] η_i^U and η_i^D [see Eq. (42)].

$$m_{aa}^2=m_{11}^2c_\beta^2+m_{22}^2s_\beta^2-[m_{12}^2+(m_{12}^2)^*]s_\beta c_\beta, \tag{E7}$$

$$m_{bb}^2=m_{11}^2s_\beta^2+m_{22}^2c_\beta^2+[m_{12}^2+(m_{12}^2)^*]s_\beta c_\beta, \tag{E8}$$

$$m_{ab}^2=(m_{11}^2-m_{22}^2)s_\beta c_\beta+m_{12}^2c_\beta^2-(m_{12}^2)^*s_\beta^2. \tag{E9}$$

In the *CP*-conserving theory where m_{12}^2 is real, the corresponding potential minimum conditions [Eqs. (6),(7)] simplify to

$$m_{aa}^2=-\tfrac{1}{2}v^2\lambda, \quad m_{ab}^2=-\tfrac{1}{2}v^2\hat{\lambda}, \tag{E10}$$

with no constraint on m_{bb}^2. In fact, m_{bb}^2 is related to m_A^2:

$$m_A^2=\mathrm{Tr}\, m^2+\tfrac{1}{2}v^2(\lambda+\lambda_T)=m_{bb}^2+\tfrac{1}{2}v^2\lambda_T \tag{E11}$$

after imposing the potential minimum condition [Eq. (E10)]. It is convenient to trade the free parameter m_{bb}^2 for $\beta-\alpha$. Using the results of Eqs. (D30) and (D31), it follows that

$$\tan[2(\beta-\alpha)]=\frac{2\hat{\lambda}}{\lambda_A-\tfrac{1}{2}\lambda_T-m_{bb}^2/v^2}, \tag{E12}$$

where the sign of $\sin[2(\beta-\alpha)]$ is equal to the sign of $\hat{\lambda}$.

It is now straightforward to obtain the three- and four-Higgs-boson couplings in terms of the invariant coupling parameters and $\beta-\alpha$, by inserting Eqs. (E3)–(E5) into Eq. (E6).

APPENDIX F: THREE-HIGGS-BOSON VERTICES IN THE TWO-HIGGS-DOUBLET MODEL

In this appendix, we list the Feynman rules for the three-point Higgs boson interaction in the most general *CP*-conserving two-Higgs-doublet extension of the standard model. The Feynman rule for the *ABC* vertex is denoted by ig_{ABC}.[32] For completeness, *R*-gauge Feynman rules involving the Goldstone bosons ($G^\pm$ and G) are also listed.

The Feynman rules are obtained from the scalar potential by multiplying the corresponding coefficients of $\mathcal{V}$ by $-i$ times the appropriate symmetry factor. To obtain the three-Higgs-boson couplings in terms of $\beta-\alpha$ and the invariant coupling parameters, we insert Eqs. (E3)–(E5) into Eq. (E6), and identify the terms that are cubic in the Higgs boson fields. The resulting three-point Higgs boson couplings (which are proportional to $v\equiv 2m_W/g$) are given by

$$g_{hAA}=-v[\lambda_T s_{\beta-\alpha}-\lambda_U c_{\beta-\alpha}],$$

$$g_{HAA}=-v[\lambda_T c_{\beta-\alpha}+\lambda_U s_{\beta-\alpha}],$$

$$g_{hHH}=3v[\lambda s_{\beta-\alpha}(-\tfrac{2}{3}+c_{\beta-\alpha}^2)+\hat{\lambda}c_{\beta-\alpha}(1-3s_{\beta-\alpha}^2) +(2\lambda_A-\lambda_T)s_{\beta-\alpha}(\tfrac{1}{3}-c_{\beta-\alpha}^2)+\lambda_U s_{\beta-\alpha}^2 c_{\beta-\alpha}],$$

$$g_{Hhh}=3v[\lambda c_{\beta-\alpha}(-\tfrac{2}{3}+s_{\beta-\alpha}^2)-\hat{\lambda}s_{\beta-\alpha}(1-3c_{\beta-\alpha}^2) +(2\lambda_A-\lambda_T)c_{\beta-\alpha}(\tfrac{1}{3}-s_{\beta-\alpha}^2)-\lambda_U c_{\beta-\alpha}^2 s_{\beta-\alpha}],$$

$$g_{hhh}=-3v[\lambda s_{\beta-\alpha}(1+c_{\beta-\alpha}^2)-3\hat{\lambda}c_{\beta-\alpha}s_{\beta-\alpha}^2 -(2\lambda_A-\lambda_T)s_{\beta-\alpha}c_{\beta-\alpha}^2-\lambda_U c_{\beta-\alpha}^3],$$

$$g_{HHH}=-3v[\lambda c_{\beta-\alpha}(1+s_{\beta-\alpha}^2)+3\hat{\lambda}s_{\beta-\alpha}c_{\beta-\alpha}^2 -(2\lambda_A-\lambda_T)c_{\beta-\alpha}s_{\beta-\alpha}^2+\lambda_U s_{\beta-\alpha}^3],$$

$$g_{hH^+H^-}=-v[(\lambda_T+\lambda_F)s_{\beta-\alpha}-\lambda_U c_{\beta-\alpha}],$$

$$g_{HH^+H^-}=-v[(\lambda_T+\lambda_F)c_{\beta-\alpha}+\lambda_U s_{\beta-\alpha}]. \tag{F1}$$

TABLE I. Three-Higgs-boson vertex Feynman rules in the approach to the decoupling limit are given by $ig_{ABC}=iv(X_{ABC}+Y_{ABC}c_{\beta-\alpha})$, where the coefficients X and Y are listed below.

ABC	X_{ABC}	Y_{ABC}
hhh	-3λ	$9\hat{\lambda}$
hhH	$-3\hat{\lambda}$	$\lambda+2(\lambda_T-2\lambda_A)$
hHH	$2(\lambda_A-\lambda)-\lambda_T$	$3(\lambda_U-2\hat{\lambda})$
hAA	$-\lambda_T$	λ_U
hH^+H^-	$-\lambda_T-\lambda_F$	λ_U
HHH	$-3\lambda_U$	$6(\lambda_A-\lambda)-3\lambda_T$
HAA	$-\lambda_U$	$-\lambda_T$
HH^+H^-	$-\lambda_U$	$-\lambda_T-\lambda_F$

In the approach to the decoupling limit, the three-Higgs-boson vertices simplify considerably as exhibited in Table I. Here, we have listed all the cubic couplings in the form

$$g_{ABC}=v(X_{ABC}+Y_{ABC}c_{\beta-\alpha}), \tag{F2}$$

where the coefficients X and Y are given in terms of various linear combinations of the invariant coupling parameters. These results follow trivially from Eq. (F1).

The couplings involving the Goldstone bosons are given by

[32]To obtain g_{ABC}, multiply the coefficient of *ABC* that appears in the interaction Lagrangian by the appropriate symmetry factor $n!$, where n is the number of identical particles at the vertex. Note that H^+ and H^- are not considered identical.

$$
\begin{aligned}
g_{hGG} &= g_{hG^+G^-} = v[\hat{\lambda} c_{\beta-\alpha} - \lambda s_{\beta-\alpha}],\\
g_{HGG} &= g_{HG^+G^-} = -v[\hat{\lambda} s_{\beta-\alpha} + \lambda c_{\beta-\alpha}],\\
g_{hAG} &= v[\hat{\lambda} s_{\beta-\alpha} - (\lambda-\lambda_A) c_{\beta-\alpha}],\\
g_{HAG} &= v[\hat{\lambda} c_{\beta-\alpha} + (\lambda-\lambda_A) s_{\beta-\alpha}],\\
g_{hH^\pm G^\pm} &= v[\hat{\lambda} s_{\beta-\alpha} - (\lambda-\lambda_A-\tfrac{1}{2}\lambda_F) c_{\beta-\alpha}],\\
g_{HH^\pm G^\mp} &= v[\hat{\lambda} c_{\beta-\alpha} + (\lambda-\lambda_A-\tfrac{1}{2}\lambda_F) s_{\beta-\alpha}],\\
g_{AH^\pm G^\mp} &= \pm \frac{i}{2} v \lambda_F .
\end{aligned}
\tag{F3}
$$

In the rule for the $AH^\pm G^\mp$ vertex, the sign corresponds to $H^\pm$ entering the vertex and $G^\pm$ leaving the vertex.

One can also express the three-Higgs-boson vertices in terms of the Higgs boson masses by using Eqs. (D26)–(D29). The Feynman rules for the three-point Higgs boson vertices that involve Goldstone bosons then take on rather simple forms:

$$
\begin{aligned}
g_{hGG} &= g_{hG^+G^-} = \frac{-g}{2m_W} m_h^2 s_{\beta-\alpha},\\
g_{HGG} &= g_{HG^+G^-} = \frac{-g}{2m_W} m_H^2 c_{\beta-\alpha},\\
g_{hAG} &= \frac{-g}{2m_W}(m_h^2 - m_A^2) c_{\beta-\alpha},\\
g_{HAG} &= \frac{g}{2m_W}(m_H^2 - m_A^2) s_{\beta-\alpha},\\
g_{hH^\pm G^\mp} &= \frac{g}{2m_W}(m_{H^\pm}^2 - m_h^2) c_{\beta-\alpha},\\
g_{HH^\pm G^\mp} &= \frac{-g}{2m_W}(m_{H^\pm}^2 - m_H^2) s_{\beta-\alpha},\\
g_{AH^\pm G^\mp} &= \frac{\pm ig}{2m_W}(m_{H^\pm}^2 - m_A^2).
\end{aligned}
\tag{F4}
$$

The cubic couplings of the physical Higgs bosons, expressed in terms of the Higgs boson masses, are more complicated. For example, let us first compute g_{hhh} in terms of $\lambda_1,\ldots,\lambda_7$:

$$
\begin{aligned}
g_{hhh} = 3v[&\lambda_1 s_\alpha^3 c_\beta - \lambda_2 c_\alpha^3 s_\beta + \lambda_{345} s_\alpha c_\alpha c_{\alpha+\beta}\\
&-\lambda_6 s_\alpha^2(3c_\alpha c_\beta - s_\alpha s_\beta) + \lambda_7 c_\alpha^2(3 s_\alpha s_\beta - c_\alpha c_\beta)].
\end{aligned}
\tag{F5}
$$

This can then be reexpressed in terms of the Higgs boson masses using Eqs. (D20)–(D23). The end result is [12]

$$
\begin{aligned}
g_{hhh} = -3v\Bigg[&\frac{m_h^2 s_{\beta-\alpha}}{v^2} + \left(\frac{m_h^2 - m_A^2 - \lambda_5 v^2}{v^2 s_\beta c_\beta}\right) c_{\beta-\alpha}^2 c_{\beta+\alpha}\\
&+\left(\lambda_6 \frac{s_\alpha}{s_\beta} - \lambda_7 \frac{c_\alpha}{c_\beta}\right) c_{\beta-\alpha}^2\Bigg].
\end{aligned}
\tag{F6}
$$

Note that the decoupling limit result [Eq. (57)] follows easily after using Eq. (29) to obtain the $\mathcal{O}(c_{\beta-\alpha})$ correction. We have also exhibited $g_{hH^+H^-}$ in Eq. (59). Expressions for the other three-Higgs-boson couplings in terms of the Higgs boson masses can be found in Ref. [12] (see also Ref. [48] for the case of $\lambda_6=\lambda_7=0$ and Ref. [51] for other special cases). However, in the most general case, such expressions are less useful. Finally, using Eq. (D29) we note the relations

$$
\begin{aligned}
v[g_{hH^+H^-} - g_{hAA}] &= -2(m_{H^\pm}^2 - m_A^2) s_{\beta-\alpha},\\
v[g_{HH^+H^-} - g_{HAA}] &= -2(m_{H^\pm}^2 - m_A^2) c_{\beta-\alpha}.
\end{aligned}
\tag{F7}
$$

TABLE II. Four-Higgs-boson vertex Feynman rules in the approach to the decoupling limit are given by $ig_{ABCD} = i(X_{ABCD} + Y_{ABCD} c_{\beta-\alpha})$, where the coefficients X and Y are listed below. The rules for $AAAA$, AAH^+H^-, and $H^+H^-H^+H^-$ are exact (since they are independent of $\beta-\alpha$).

$ABCD$	X_{ABCD}	Y_{ABCD}
$hhhh$	-3λ	$12\hat{\lambda}$
$hhhH$	$-3\hat{\lambda}$	$3(\lambda+\lambda_T-2\lambda_A)$
$hhHH$	$2(\lambda_A-\lambda)-\lambda_T$	$6(\lambda_U-\hat{\lambda})$
$hhAA$	$-\lambda_T$	$2\lambda_U$
hhH^+H^-	$-\lambda_T-\lambda_F$	$2\lambda_U$
$hHHH$	$-3\lambda_U$	$3(\lambda_V-\lambda_T)+6(\lambda_A-\lambda)$
$hHAA$	$-\lambda_U$	$-\lambda_T+\lambda_V$
hHH^+H^-	$-\lambda_U$	$\lambda_V-\lambda_T-\lambda_F$
$HHHH$	$-3\lambda_V$	$-12\lambda_U$
$HHAA$	$-\lambda_V$	$-2\lambda_U$
HHH^+H^-	$-\lambda_V$	$-2\lambda_U$
$AAAA$	$-3\lambda_V$	0
AAH^+H^-	$-\lambda_V$	0
$H^+H^-H^+H^-$	$-2\lambda_V$	0

APPENDIX G: FOUR-HIGGS-BOSON VERTICES IN THE TWO-HIGGS-DOUBLET MODEL

In this appendix, we list the Feynman rules for the four-point Higgs boson interaction in the most general *CP*-conserving two-Higgs-doublet extension of the standard model. Recalling that $\mathcal{L}_{\rm int} \ni -\mathcal{V}$, the Feynman rules are obtained from the scalar potential[33] by multiplying the corresponding coefficients of $\mathcal{V}$ by $-i$ times the appropriate symmetry factor. We find it convenient to write the terms of the potential that are quartic in the Higgs fields as a sum of two pieces: $\mathcal{V} \ni \mathcal{V}_A + \mathcal{V}_B$, where $\mathcal{V}_A$ depends explicitly on $\beta-\alpha$ and $\mathcal{V}_B$ is independent of $\beta-\alpha$. To obtain the four-Higgs-boson couplings in terms of $\beta-\alpha$ and the invariant coupling parameters, we insert Eqs. (E3)–(E5) into Eq. (E6) and identify the terms that are quartic in the Higgs boson fields. For completeness, the quartic interaction terms involving the Goldstone bosons ($G^\pm$ and G) are also listed. The end result is

[33]Note, e.g., that the term proportional to hAH^+G^- in $\mathcal{V}$ corresponds to H^+ and G^- directed *into* the vertex, etc.

$$
\begin{aligned}
8\mathcal{V}_A = {} & h^4[\lambda s^2_{\beta-\alpha}(3c^2_{\beta-\alpha}+1)-4\hat{\lambda}c_{\beta-\alpha}s^3_{\beta-\alpha}-2(2\lambda_A-\lambda_T)c^2_{\beta-\alpha}s^2_{\beta-\alpha}-4\lambda_U c^3_{\beta-\alpha}s_{\beta-\alpha}+\lambda_V c^4_{\beta-\alpha}] \\
& +4h^3H[\lambda s_{\beta-\alpha}c_{\beta-\alpha}(3c^2_{\beta-\alpha}-1)-\hat{\lambda}s^2_{\beta-\alpha}(4c^2_{\beta-\alpha}-1)-(2\lambda_A-\lambda_T)s_{\beta-\alpha}c_{\beta-\alpha}(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})+\lambda_U c^2_{\beta-\alpha}(4s^2_{\beta-\alpha}-1) \\
& -\lambda_V s_{\beta-\alpha}c^3_{\beta-\alpha}]+2h^2H^2[\lambda(2-9s^2_{\beta-\alpha}c^2_{\beta-\alpha})-6(\hat{\lambda}-\lambda_U)s_{\beta-\alpha}c_{\beta-\alpha}(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})-(2\lambda_A-\lambda_T)(1-6s^2_{\beta-\alpha}c^2_{\beta-\alpha}) \\
& +3\lambda_V s^2_{\beta-\alpha}c^2_{\beta-\alpha}]+4hH^3[\lambda s_{\beta-\alpha}c_{\beta-\alpha}(3s^2_{\beta-\alpha}-1)+\hat{\lambda}c^2_{\beta-\alpha}(4s^2_{\beta-\alpha}-1)+(2\lambda_A-\lambda_T)s_{\beta-\alpha}c_{\beta-\alpha}(c^2_{\beta-\alpha}-s^2_{\beta-\alpha}) \\
& -\lambda_U s^2_{\beta-\alpha}(4c^2_{\beta-\alpha}-1)-\lambda_V c_{\beta-\alpha}s^3_{\beta-\alpha}]+H^4[\lambda c^2_{\beta-\alpha}(3s^2_{\beta-\alpha}+1)+4\hat{\lambda}c^3_{\beta-\alpha}s_{\beta-\alpha}-2(2\lambda_A-\lambda_T)c^2_{\beta-\alpha}s^2_{\beta-\alpha} \\
& +4\lambda_U c_{\beta-\alpha}s^3_{\beta-\alpha}+\lambda_V s^4_{\beta-\alpha}]+2h^2A^2[\lambda_T s^2_{\beta-\alpha}-2\lambda_U s_{\beta-\alpha}c_{\beta-\alpha}+\lambda_V c^2_{\beta-\alpha}]+4h^2AG[2(\lambda-\lambda_A)s_{\beta-\alpha}c_{\beta-\alpha}-\hat{\lambda}s^2_{\beta-\alpha} \\
& -\lambda_U c^2_{\beta-\alpha}]+2h^2G^2[\lambda s^2_{\beta-\alpha}-2\hat{\lambda}s_{\beta-\alpha}c_{\beta-\alpha}+\lambda_T c^2_{\beta-\alpha}]+4h^2H^+H^-[(\lambda_T+\lambda_F)s^2_{\beta-\alpha}-2\lambda_U s_{\beta-\alpha}c_{\beta-\alpha}+\lambda_V c^2_{\beta-\alpha}] \\
& +4(h^2H^+G^-+h^2H^-G^+)[(2\lambda-2\lambda_A-\lambda_F)s_{\beta-\alpha}c_{\beta-\alpha}-\hat{\lambda}s^2_{\beta-\alpha}-\lambda_U c^2_{\beta-\alpha}]+4h^2G^+G^-[\lambda s^2_{\beta-\alpha}-2\hat{\lambda}s_{\beta-\alpha}c_{\beta-\alpha} \\
& +(\lambda_T+\lambda_F)c^2_{\beta-\alpha}]+4hHA^2[(\lambda_T-\lambda_V)s_{\beta-\alpha}c_{\beta-\alpha}-\lambda_U(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})]+8hHAG[(\lambda-\lambda_A)(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})-(\hat{\lambda} \\
& -\lambda_U)s_{\beta-\alpha}c_{\beta-\alpha}]+4hHG^2[(\lambda-\lambda_T)s_{\beta-\alpha}c_{\beta-\alpha}-\hat{\lambda}(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})]+8hHH^+H^-[(\lambda_T-\lambda_V+\lambda_F)s_{\beta-\alpha}c_{\beta-\alpha} \\
& -\lambda_U(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})]+4(hHH^+G^-+hHH^-G^+)[(2\lambda-2\lambda_A-\lambda_F)(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})-2(\hat{\lambda}-\lambda_U)s_{\beta-\alpha}c_{\beta-\alpha}] \\
& +8hHG^+G^-[(\lambda-\lambda_T-\lambda_F)s_{\beta-\alpha}c_{\beta-\alpha}-\hat{\lambda}(c^2_{\beta-\alpha}-s^2_{\beta-\alpha})]+2H^2A^2[\lambda_T c^2_{\beta-\alpha}+2\lambda_U s_{\beta-\alpha}c_{\beta-\alpha}+\lambda_V s^2_{\beta-\alpha}] \\
& +4H^2AG[2(\lambda_A-\lambda)s_{\beta-\alpha}c_{\beta-\alpha}-\hat{\lambda}c^2_{\beta-\alpha}-\lambda_U s^2_{\beta-\alpha}]+2H^2G^2[\lambda c^2_{\beta-\alpha}+2\hat{\lambda}s_{\beta-\alpha}c_{\beta-\alpha}+\lambda_T s^2_{\beta-\alpha}]+4H^2H^+H^-[(\lambda_T \\
& +\lambda_F)c^2_{\beta-\alpha}+2\lambda_U s_{\beta-\alpha}c_{\beta-\alpha}+\lambda_V s^2_{\beta-\alpha}]+4(H^2H^+G^-+H^2H^-G^+)[(2\lambda_A-2\lambda+\lambda_F)s_{\beta-\alpha}c_{\beta-\alpha}-\hat{\lambda}c^2_{\beta-\alpha}-\lambda_U s^2_{\beta-\alpha}] \\
& +4H^2G^+G^-[\lambda c^2_{\beta-\alpha}+2\hat{\lambda}s_{\beta-\alpha}c_{\beta-\alpha}+(\lambda_T+\lambda_F)s^2_{\beta-\alpha}]+4i[hAH^+G^--hAH^-G^++HGH^+G^- \\
& -HGH^-G^+]\lambda_F s_{\beta-\alpha}-4i[hGH^+G^--hGH^-G^+-HAH^+G^-+HAH^-G^+]\lambda_F c_{\beta-\alpha},
\end{aligned} \tag{G1}
$$

and

$$
\begin{aligned}
8\mathcal{V}_B = {} & \lambda_V(A^4+4A^2H^+H^-+4H^+H^-H^+H^-)-4\lambda_U(A^3G+A^2H^+G^-+A^2H^-G^++2AGH^+H^-+2H^+H^-H^+G^- \\
& +2H^+H^-H^-G^+)+2[2(\lambda-\lambda_A)+\lambda_T]A^2G^2+4(\lambda_T+\lambda_F)(A^2G^+G^-+G^2H^+H^-)-4\hat{\lambda}(AG^3+2AGG^+G^- \\
& +G^2H^+G^-+G^2H^-G^++2H^+G^+G^-G^-+2H^-G^-G^+G^+)+4[2(\lambda-\lambda_A)-\lambda_F](AGH^+G^-+AGH^-G^+)+\lambda(G^4 \\
& +4G^2G^+G^-+4G^+G^-G^+G^-)+4(\lambda-\lambda_A)(H^+H^+G^-G^-+H^-H^-G^+G^+)+8(\lambda-\lambda_A+\lambda_T)H^+H^-G^+G^-.
\end{aligned} \tag{G2}
$$

The quartic Higgs boson couplings are now easily obtained by including the appropriate symmetry factors. For example, the h^4 and H^4 couplings are given by

$$
\begin{aligned}
g_{hhhh} = {} & -3[\lambda s^2_{\beta-\alpha}(1+3c^2_{\beta-\alpha})-4\hat{\lambda}c_{\beta-\alpha}s^3_{\beta-\alpha}-2(2\lambda_A \\
& -\lambda_T)c^2_{\beta-\alpha}s^2_{\beta-\alpha}-4\lambda_U c^3_{\beta-\alpha}s_{\beta-\alpha}+\lambda_V c^4_{\beta-\alpha}],
\end{aligned} \tag{G3}
$$

$$
\begin{aligned}
g_{HHHH} = {} & -3[\lambda c^2_{\beta-\alpha}(1+3s^2_{\beta-\alpha})+4\hat{\lambda}c^3_{\beta-\alpha}s_{\beta-\alpha}-2(2\lambda_A \\
& -\lambda_T)c^2_{\beta-\alpha}s^2_{\beta-\alpha}+4\lambda_U c_{\beta-\alpha}s^3_{\beta-\alpha}+\lambda_V s^4_{\beta-\alpha}].
\end{aligned} \tag{G4}
$$

Note the first appearance of physical observables that depend on λ_V.

Let us denote the Feynman rule for the $ABCD$ vertex by ig_{ABCD}. In the approach to the decoupling limit, the four-Higgs-boson vertices simplify considerably as exhibited in Table II. Here, we have listed all couplings in the form

$$
g_{ABCD} = (X_{ABCD}+Y_{ABCD}c_{\beta-\alpha}), \tag{G5}
$$

where the coefficients X and Y are given in terms of various linear combinations of the invariant coupling parameters. Note that the terms contained in $\mathcal{V}_B$ are not affected by the decoupling limit since these terms are independent of $\beta-\alpha$.

The four-Higgs-boson couplings can be rewritten in terms of $\lambda_1,\ldots,\lambda_7$, α, and β. The resulting expressions are generally more complex, with a few notable exceptions. For example, the quartic couplings in $\mathcal{V}_A$ that depend only on h and H are independent of β:

$$\mathcal{V}_A \ni \tfrac{1}{8} h^4[\lambda_1 s_\alpha^4+\lambda_2 c_\alpha^4+\tfrac{1}{2}\lambda_{345}s_{2\alpha}^2-2s_{2\alpha}(\lambda_6 s_\alpha^2+\lambda_7 c_\alpha^2)]+\tfrac{1}{2}h^3H[\tfrac{1}{2}s_{2\alpha}(-\lambda_1 s_\alpha^2+\lambda_2 c_\alpha^2-\lambda_{345}c_{2\alpha})+\lambda_6 s_\alpha s_{3\alpha}+\lambda_7 c_\alpha c_{3\alpha}]$$
$$+\tfrac{1}{4}h^2H^2[\tfrac{3}{4}s_{2\alpha}^2(\lambda_1+\lambda_2-2\lambda_{345})+\lambda_{345}-3s_{2\alpha}c_{2\alpha}(\lambda_6-\lambda_7)]+\tfrac{1}{2}hH^3[\tfrac{1}{2}s_{2\alpha}(-\lambda_1 c_\alpha^2+\lambda_2 s_\alpha^2+\lambda_{345}c_{2\alpha})+\lambda_6 c_\alpha c_{3\alpha}$$
$$+\lambda_7 s_\alpha s_{3\alpha}]+\tfrac{1}{8}H^4[\lambda_1 c_\alpha^4+\lambda_2 s_\alpha^4+\tfrac{1}{2}\lambda_{345}s_{2\alpha}^2+2s_{2\alpha}(\lambda_6 c_\alpha^2+\lambda_7 s_\alpha^2)], \tag{G6}$$

and in this form these results are somewhat simpler than the corresponding expressions in terms of the invariant coupling parameters given in Eq. (G1). One can check that the latter can be obtained from Eq. (G6) by rotating to the Higgs basis (see the discussion in Appendix E). That is, in Eq. (G6), let $\alpha\to\alpha-\beta$, $\lambda_1\to\lambda$, $\lambda_2\to\lambda_V$, $\lambda_{345}\to 2(\lambda-\lambda_A)+\lambda_T$, $\lambda_6\to-\hat{\lambda}$, and $\lambda_7\to-\lambda_U$ [cf. Eq. (E6)].

One can also express the four-Higgs-boson vertices in terms of the Higgs boson masses by using Eqs. (D20)–(D23). For example [12],

$$g_{hhhh}=-3\left[\frac{m_h^2}{v^2}\left(s_{\beta-\alpha}-\frac{c_{\beta+\alpha}c_{\beta-\alpha}^2}{s_\beta c_\beta}\right)^2+\frac{m_H^2}{v^2}\left(\frac{s_\alpha c_\alpha c_{\beta-\alpha}}{s_\beta c_\beta}\right)^2-\frac{m_A^2+\lambda_5 v^2}{v^2}\left(\frac{c_{\beta+\alpha}c_{\beta-\alpha}}{s_\beta c_\beta}\right)^2-\frac{2(\lambda_6 s_\alpha^2+\lambda_7 c_\alpha^2)c_{\beta-\alpha}^2}{s_\beta c_\beta}\right]. \tag{G7}$$

Note that the decoupling limit result [Eq. (58)] follows trivially, after using Eqs. (29) and (31) to obtain the $\mathcal{O}(c_{\beta-\alpha})$ correction. Expressions for other four-Higgs-boson couplings in terms of the Higgs boson masses can be found in Ref. [12] (see also Ref. [48] for the case of $\lambda_6=\lambda_7=0$ and Ref. [51] for other special cases). However, in the most general case, such expressions are less useful.

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