

Finite theories and the supersymmetry flavor problem

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(Received 26 December 2002; published 24 April 2003)

We study a finite $SU(5)$ grand unified model based on the non-Abelian discrete symmetry A_4 . This model leads to the democratic structure of the mass matrices for the quarks and leptons. In the soft supersymmetry breaking sector, the scalar trilinear couplings are aligned and the soft scalar masses are degenerate, thus solving the SUSY flavor problem.

DOI: 10.1103/PhysRevD.67.075018

PACS number(s): 12.60.Jv, 11.10.Hi, 11.30.Hv, 12.10.Kt

I. INTRODUCTION

Supersymmetry (SUSY) is broken in nature. It is widely accepted that the effects of supersymmetry breaking appear as soft supersymmetry breaking (SSB) terms [1]. However, if only renormalizability is used to guide the SSB parameters, it is possible to introduce more than 100 new parameters into the minimal supersymmetric standard model (MSSM) [2]. The problem is not only this large number of the independent parameters, but also the fact that one has to highly fine-tune these parameters so that they do not cause problems with experimental observations on the flavor changing neutral current (FCNC) processes and CP -violation phenomena [3–7]. This problem, called the SUSY flavor problem, is not new, but has existed ever since supersymmetry found phenomenological applications [8].

There are several approaches to overcome this problem. The most well-known one [1] is to simply assume that the SSB parameters have a universal form, independent of the flavor structure of the standard model (SM) at, say, grand unified theory (GUT) scale M_{GUT} . This is the so-called minimal supergravity model. In this model, supersymmetry breaking occurs in a sector that is hidden to the MSSM sector, and supersymmetry breaking is mediated to the MSSM sector by gravity. There exist other ideas of mediation: gauge mediation [9], anomaly mediation [10] and gaugino mediation [11]. Their common feature is the assumption that there exists a hidden sector that is separated from the MSSM by cleverly chosen interactions or it is separated in space-time (for which one needs extra dimensions¹), or both. Another type of idea to overcome the SUSY flavor problem is to use the infrared attractive force of the gauge interactions [13], in four dimensions [14–18] as well as in extra dimensions [19].

In these scenarios, it is not necessary to assume that the supersymmetry is broken in a sector that is separated from the MSSM.

Although it is attractive to find dynamical mechanisms that suppress the dangerous FCNC processes and CP -violating phases, it is also worthwhile to look for other attractive possibilities. In fact, it has been argued [20–23] that finiteness² of softly broken supersymmetric Yang-Mills theories [28–32] may play an important role to understand the universality of the SSB parameters. However, it has turned out [32,33] that the universality is not a necessary condition for finiteness: It has been found [32,33] that more relaxed conditions, sum rules among the soft scalar masses, are sufficient. Clearly, the sum rules do not automatically ensure the degeneracy of the soft scalar masses. In fact, in the unified models of [29–32] in which the hierarchical structure of the Yukawa couplings emerges, the sum rules cannot sufficiently constrain the individual soft scalar masses in the first two generations of the squarks and sleptons.

Recently, a class of finite models based on $SU(5)$ with certain discrete symmetries has been considered in [34], and it has been found that some of these models yield a democratic structure of the Yukawa couplings. As we will see, the democratic structure is essential to obtain sum rules of the soft scalar masses from which their degeneracy in generation follows. Therefore, the exact finiteness in the models of [34] ensures the absence of the SUSY flavor problem.

In Sec. II we will recapitulate the finiteness conditions in softly broken supersymmetric Yang-Mills theories [28–32]. The unified model of [34] based on $SU(5) \times A_4$, including the SSB sector, is investigated in Sec. III, where A_4 is the group of even permutations of four objects. As we will see, the model has a strong predictive power, and we will calculate various low-energy parameters such as the top quark mass and the spectrum of the superpartners that are predicted from the model. In Sec. IV we conclude.

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¹There is a problem associated with this approach, the problem of sequestering branes between the visible sector and the hidden SSB sector [12].

²See [24–27] for earlier references on finite theories.

II. SOFTLY BROKEN $N=1$ SUPERSYMMETRIC FINITE UNIFIED THEORIES

We start by considering a generic form of the superpotential³

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j, \quad (1)$$

along with the Lagrangian for the SSB terms

$$-\mathcal{L}_{\text{SB}} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_i^j \phi^*{}^i \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.}, \quad (2)$$

where ϕ_i is the scalar component of Φ_i , and λ stands for gaugino. Since we consider only finite theories, we assume that the one-loop β function of the gauge coupling g vanishes, i.e.,

$$\sum_i T(R_i) - 3C(G) = 0, \quad (3)$$

where $T(R_i)$ is the Dynkin index of the representation R_i and $C(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G . We also assume that the gauge group is a simple group, and that the theory is free from the gauge anomaly, of course. According to the finiteness theorem of [28], the theory is finite to all orders in perturbation theory,⁴ if (i) the reduction equation [36–38]

$$\beta_Y^{ijk} = \beta_g dY^{ijk}/dg \quad (4)$$

admits a unique power series solution

$$Y^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}, \quad (5)$$

where β_g and β_Y^{ijk} are the β functions of g and Y^{ijk} , respectively,⁵ and (ii) the one-loop anomalous dimensions vanish, that is,

$$\frac{1}{2} \sum_{p,q} \rho_{ipq(0)} \rho_{(0)}^{jpq} - 2\delta_i^j C(R_i) = 0, \quad (6)$$

where $\rho_{ipq(0)} = \rho_{(0)}^{*jpk}$. We would like to recall that if the condition (ii) is satisfied, the two-loop expansion coefficients in Eq. (5), $\rho_{(1)}^{ijk}$, vanish [22], and that if (i) and (ii) are satisfied, the anomalous dimensions γ_i^j vanish to all orders [28]. Field theories that satisfy (i) and (ii) possess the exact scale invariance.

In the presence of the SSB terms, the exact scale invariance is broken by them in a strict sense. However, it is expected that the couplings, masses, etc., in a unified field theory without gravity are vacuum expectation values (VEV's) of certain fields in a more fundamental theory. Therefore, it would be natural to transform them under the scale transformation, too. Then the scale invariance of a one particle irreducible (1PI) function Γ means

$$\begin{aligned} \Gamma[e^t p, e^t h, e^t \mu, e^{2t} B, e^{2t} m^2, Y, g] \\ = e^{d_\Gamma t} \Gamma[p, h, \mu, B, m^2, Y, g], \end{aligned} \quad (7)$$

where p stand for momenta, and d_Γ is the canonical dimension of Γ . Clearly, Eq. (7) is correct, only if the theory is finite. Finiteness in the SSB sector can be achieved by using the relations among the renormalization of the SSB parameters and those of an unbroken supersymmetric gauge theory [23,33,35,40–47].⁶ Accordingly, the β functions of the M, h and m^2 parameters can be written as [42,43]⁷

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right), \quad (8)$$

$$\begin{aligned} \beta_h^{ijk} = \gamma^j_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl} - 2\gamma^j_l Y^{ljk} - 2\gamma^j_l Y^{ilk} \\ - 2\gamma^k_l Y^{ijl}, \end{aligned} \quad (9)$$

$$(\beta_{m^2})^i_j = \left[\Delta + Xg \frac{\partial}{\partial g} \right] \gamma^i_j, \quad (10)$$

$$\mathcal{O} = \left(Mg^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial Y^{lmn}} \right), \quad (11)$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{Y}_{lmn} \frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn} \frac{\partial}{\partial Y^{lmn}}, \quad (12)$$

where $(\gamma_1)^i_j = \mathcal{O}\gamma^i_j$, $Y_{lmn} = (Y^{lmn})^*$, and

$$\tilde{Y}^{ijk} = (m^2)_i^l Y^{ljk} + (m^2)_j^l Y^{ilk} + (m^2)_k^l Y^{ijl}. \quad (13)$$

X in Eq. (10) has been first explicitly calculated in the lowest order [22,49], and later its all order form [33,45–48]

$$X = \frac{-|M|^2 C(G) + \sum_l m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \quad (14)$$

has been found in the renormalization scheme of Novikov *et al.*, in which the β function of the gauge coupling g is given by [50]

³We follow the notation of [35].

⁴Finiteness here means only for dimensionless couplings g and Y^{ijk} .

⁵See [39] for further references on reduction of couplings.

⁶In [48] this matter is reviewed in a transparent way. See also [16].

⁷We do not consider B^{ij} in the following discussions, because they do not enter into the β functions of the other quantities [42,43]. Moreover, they are automatically finite if the other quantities are finite [23].

$$\beta_g^{\text{NSVZ}} = \frac{g^3}{16\pi^2} \left[\frac{\sum_l T(R_l)(1-2\gamma_l) - 3C(G)}{1-g^2C(G)/8\pi^2} \right]. \quad (15)$$

The key point in [23,33,35] is the assumption that the differential operators \mathcal{O} and Δ given in Eqs. (11) and (12) become total derivative operators on the renormalization group (RG) invariant surface which is defined by the solution of the reduction equations for the SBB parameters. It has been shown in [23,33,35] that if the trilinear couplings are expressed in terms of M and g as [23,35]⁸

$$h^{ijk} = -M \frac{dY^{ijk}(g)}{d \ln g}, \quad (16)$$

and the soft scalar masses satisfy the sum rules [33]

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ \frac{1}{1-g^2C(G)/(8\pi^2)} \times \frac{d \ln Y^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln Y^{ijk}}{d(\ln g)^2} \right\} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d \ln Y^{ijk}}{d \ln g}, \quad (17)$$

the differential operators \mathcal{O} and Δ become total derivatives with respect to g :

$$\mathcal{O} = \frac{M}{2} \frac{d}{d \ln g}, \quad (18)$$

$$\Delta + Xg \frac{\partial}{\partial g} = |M|^2 \left\{ \frac{1}{1-g^2C(G)/(8\pi^2)} \frac{d}{d \ln g} + \frac{1}{2} \frac{d^2}{d(\ln g)^2} \right\} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d}{d \ln g}. \quad (19)$$

Note that in the derivations from Eqs. (16) to (19), it has been assumed that

$$\gamma_i^j = \gamma_i \delta_i^j, \quad (20)$$

$$(m^2)^j_i = m_i^2 \delta_i^j, \quad (21)$$

$$Y^{ijk} \frac{\partial}{\partial Y^{ijk}} = Y^{*ijk} \frac{\partial}{\partial Y^{*ijk}} \quad \text{on the space of the RG functions.} \quad (22)$$

Therefore, if the anomalous dimensions γ_i vanish to all orders [which is ensured if (i) and (ii) given in Eqs. (5) and (6) are satisfied], we have $\beta_M = \beta_h^{ijk} = (\beta_{m^2})_j^i = 0$.

We see from Eq. (17) that the universal choice

$$m_i^2 = \frac{|M|^2}{3} \quad (23)$$

also ensures the finiteness to two-loop order in accord with [21–23]. Note that $C(G) = \sum_l T(R_l)/3$ and $d \ln Y^{ijk}/d \ln g = 1 + O(g^4)$. Similarly, the $N=4$ supersymmetric case [$T(R_l) = C(G)$] with the SSB parameters [20] can be simply derived from Eqs. (16) and (17).

To summarize, finiteness in the SSB sector is guaranteed if h^{ijk} are expressed according to Eq. (16), and the sum rules (17) are satisfied. The trilinear couplings h^{ijk} , unless they are aligned, contribute to δ_{RL} [7] which are strongly constrained from FCNC processes and dangerous CP -violating phenomena. The explicit form of h^{ijk} in finite theories is known to two-loop order [21,22]:

$$h^{ijk} = -MY^{ijk}(g) + O(g^5). \quad (24)$$

The higher order terms depend on the renormalization scheme. In fact, it is possible [38] to make vanish all the expansion coefficients $\rho_{(n)}^{ijk}$ of Eq. (5) except the lowest order one $\rho_{(0)}^{ijk}$ by a suitable redefinition of the Yukawa couplings Y^{ijk} . The redefinition does not modify the form of β function β_g^{NSVZ} (15), because only the anomalous dimensions change in β_g^{NSVZ} . Therefore, in finite theories, h^{ijk} are aligned to all orders, and therefore h^{ijk} introduce no extra CP -violating phases:

$$h^{ijk} = -MY^{ijk}(g). \quad (25)$$

In contrast to this case, the sum rules (17) of the soft scalar masses do not automatically ensure their degeneracy in the space of generation. However, as we will see in a concrete model, the sum rules (17) can yield the degeneracy of the soft scalar masses. The exact finiteness does not automatically yield a solution to the SUSY flavor problem. But a solution to the SUSY flavor problem can result from the quantum scale invariance.

Since superstring theories are scale invariant theories, a solution to the SUSY flavor problem based on the exact scale invariance may be realized. In fact, in a certain class of orbifold models of superstrings, in which the massive string states are organized into $N=4$ supermultiplets [52] (see also [53]), so that they do not contribute to the quantum modification of the kinetic function, the sum rules

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{1}{1-g^2C(G)/(8\pi^2)} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \quad (26)$$

along with $h^{ijk} = -MY^{ijk}$ are satisfied [32]. (See also [54–60].) Therefore, the finiteness conditions (25) and (17) coincide with those of the above superstrings models to all orders.

⁸Reduction of massive parameters has been first proposed in [51].

III. MODEL BASED ON $SU(5) \times A_4$

There exist various unified models that are all-order finite at least in the dimensionless sector [28–32]. In all the models, only such solutions of the reduction equations (4) have been considered that admit the hierarchal structure of the Yukawa couplings. As a result, the sum rules (17) are indeed satisfied, but it cannot strongly constrain the individual soft scalar masses in the first two generations (see [32]). In contrast to the previous models, the $SU(5)$ models (two of three models) proposed in [34] yield a democratic structure of the Yukawa couplings. As we will see, the democratic structure (which follows as a consequence of certain discrete symmetries) is essential to obtain sum rules of the soft scalar masses from which their degeneracy follows.

Three generations of quarks and leptons are accommodated in $\mathbf{10}_i$ and $\bar{\mathbf{5}}_i$, where i runs over the three generations. A Σ in $\mathbf{24}$ is used to break $SU(5)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, and there are four pairs of Higgs supermultiplets H_a and \bar{H}_a ($a=1-4$). The starting superpotential is

$$W = \sum_{i,j=1}^3 \sum_{a=1}^4 \left(\frac{1}{2} u_{ij}^a \mathbf{10}_i \mathbf{10}_j H_a + d_{ij}^a \mathbf{10}_i \bar{\mathbf{5}}_j \bar{H}_a \right) + \sum_{a,b=1}^4 \kappa^{ab} \bar{H}_a \Sigma H_b + \frac{\lambda}{3} \Sigma^3 + \frac{\mu_\Sigma}{2} \Sigma^2 + \mu_H^{ab} \bar{H}_a H_b, \quad (27)$$

and the SSB Lagrangian is

$$-\mathcal{L}_{\text{SSB}} = \sum_{a=1}^4 [m_{H_a}^2 \hat{H}_a^* \hat{H}_a + m_{\bar{H}_a}^2 \hat{\bar{H}}_a^* \hat{\bar{H}}_a] + m_\Sigma^2 \hat{\Sigma}^\dagger \hat{\Sigma} + \sum_{i=1}^3 [m_{\mathbf{5}_i}^2 \hat{\mathbf{5}}_i^* \hat{\mathbf{5}}_i + m_{\mathbf{10}_i}^2 \hat{\mathbf{10}}_i^* \hat{\mathbf{10}}_i] + \left\{ \frac{1}{2} M \lambda \lambda + B_\Sigma \hat{\Sigma}^2 + \sum_{a,b=1}^4 [B_H^{ab} \hat{H}_a \hat{H}_b + h_f^{ab} \hat{H}_a \hat{\Sigma} \hat{H}_b] + \frac{h_\lambda}{3} \hat{\Sigma}^3 + \sum_{i,j=1}^3 \sum_{a=1}^4 \left(\frac{h_{uij}^a}{2} \hat{\mathbf{10}}_i \hat{\mathbf{10}}_j \hat{H}_a + h_{dij}^a \hat{\mathbf{10}}_i \hat{\bar{\mathbf{5}}}_j \hat{H}_a \right) + \text{H.c.} \right\}, \quad (28)$$

where a hat is used to denote the scalar component of each chiral supermultiplet. The resulting theory has an unbroken R parity along with the conservation of $B-L$. Note that we assumed the diagonal soft scalar masses, because non-diagonal soft masses would not satisfy the assumption (6) as well as (20), and hence violates finiteness.

A. The degeneracy of the soft scalar masses from their sum rules

To be specific, we consider the model based on $SU(5) \times A_4$ symmetry [34], where A_4 is the group of even

permutations.⁹ A_4 has three irreducible representations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ and $\mathbf{3}$ [61], and the matter supermultiplets belong to its representation according to [34]

$$\mathbf{10}_i : \mathbf{3}, \quad \bar{\mathbf{5}}_i : \mathbf{3}$$

$$(H_i, H_4) : \mathbf{3} + \mathbf{1}', \quad (\bar{H}_i, \bar{H}_4) : \mathbf{3} + \mathbf{1}'' \quad \Sigma : \mathbf{1},$$

where $i=1 \sim 3$. Then the cubic part of the superpotential (27) invariant under A_4 becomes

$$W_3 = \frac{a}{2} (\mathbf{10}_1 \mathbf{10}_1 + \omega \mathbf{10}_2 \mathbf{10}_2 + \omega^2 \mathbf{10}_3 \mathbf{10}_3) H_4 + c (\mathbf{10}_1 \bar{\mathbf{5}}_1 + \omega^2 \mathbf{10}_2 \bar{\mathbf{5}}_2 + \omega \mathbf{10}_3 \bar{\mathbf{5}}_3) \bar{H}_4 + b \mathbf{10}_1 \mathbf{10}_2 H_3 + d (\mathbf{10}_1 \bar{\mathbf{5}}_2 + \mathbf{10}_2 \bar{\mathbf{5}}_1) \bar{H}_3 + b \mathbf{10}_3 \mathbf{10}_1 H_2 + d (\mathbf{10}_3 \bar{\mathbf{5}}_1 + \mathbf{10}_1 \bar{\mathbf{5}}_3) \bar{H}_2 + b \mathbf{10}_2 \mathbf{10}_3 H_1 + d (\mathbf{10}_2 \bar{\mathbf{5}}_3 + \mathbf{10}_3 \bar{\mathbf{5}}_2) \bar{H}_1 + k (\bar{H}_1 H_1 + \bar{H}_2 H_2 + \bar{H}_3 H_3) \Sigma + \frac{\lambda}{3} \Sigma^3, \quad (29)$$

where $w = \exp(i2\pi/3)$ can be removed by field redefinition. The lowest order solution to the reduction equation (4) is [34]:

$$a^2 = b^2 = \frac{8}{15} g^2, \quad c^2 = d^2 = e^2 = \frac{2}{5} g^2,$$

$$k^2 = \frac{1}{3} g^2, \quad \lambda^2 = \frac{15}{7} g^2. \quad (30)$$

It can be shown that the power series solution (5) exists uniquely, so that the dimensionless sector can be made finite to any finite order in perturbation theory. At this point we assume that a suitable redefinition of the Yukawa couplings [38] has been performed so that the above solution (30) is exact. Note the mass term for the H and \bar{H} is not invariant under A_4 for an arbitrary mass matrix μ_H . The choice of μ_H is, however, very important to make the model phenomenologically viable, because the Cabibbo-Kobayashi-Maskawa (CKM) mixing of the quarks in this model basically originates from μ_H . So, the A_4 invariance has to be broken by the mass term, already at the GUT scale M_{GUT} . Therefore, it is natural to assume that the operators with dimension less than four do not have to respect the A_4 invariance. Since the SSB terms consist of such operators, we should not impose the A_4 invariance on the SSB Lagrangian (28). We proceed with this remark in mind.

Equation (25) means

$$h_{uij}^a = -M u_{ij}^a, \quad h_{dij}^a = -M d_{ij}^a. \quad (31)$$

⁹The S_4 model [34] can be treated similarly. We have found that as far as the SSB sector is concerned, it is exactly the same as the A_4 model.

Further, the right-hand side of Eq. (17) (which we denote by \tilde{M}^2) can be written as

$$\tilde{M}^2 = |M|^2 \frac{1}{1 - g^2 C(G)/(8\pi^2)} + \sum_I \frac{m_I^2 T(R_I)}{C(G) - 8\pi^2/g^2}. \quad (32)$$

Using \tilde{M}^2 above, we write down all the sum rules (17):

$$\tilde{M}^2 = 2m_{10_1}^2 + m_{H_4}^2 = 2m_{10_2}^2 + m_{H_4}^2 = 2m_{10_3}^2 + m_{H_4}^2, \quad (33)$$

$$\begin{aligned} \tilde{M}^2 &= m_{10_1}^2 + m_{10_2}^2 + m_{H_1}^2 = m_{10_1}^2 + m_{10_3}^2 + m_{H_2}^2 \\ &= m_{10_2}^2 + m_{10_3}^2 + m_{H_3}^2, \end{aligned} \quad (34)$$

$$\begin{aligned} \tilde{M}^2 &= m_{10_1}^2 + m_{5_1}^2 + m_{\bar{H}_4}^2 = m_{10_2}^2 + m_{5_2}^2 + m_{\bar{H}_4}^2 \\ &= m_{10_3}^2 + m_{5_3}^2 + m_{\bar{H}_4}^2, \end{aligned} \quad (35)$$

$$\tilde{M}^2 = m_{10_1}^2 + m_{5_2}^2 + m_{\bar{H}_1}^2 = m_{10_2}^2 + m_{5_1}^2 + m_{\bar{H}_1}^2, \quad (36)$$

$$\tilde{M}^2 = m_{10_1}^2 + m_{5_3}^2 + m_{\bar{H}_2}^2 = m_{10_3}^2 + m_{5_1}^2 + m_{\bar{H}_2}^2, \quad (37)$$

$$\tilde{M}^2 = m_{10_2}^2 + m_{5_3}^2 + m_{\bar{H}_3}^2 = m_{10_3}^2 + m_{5_2}^2 + m_{\bar{H}_3}^2, \quad (38)$$

$$\begin{aligned} \tilde{M}^2 &= m_{H_1}^2 + m_{\bar{H}_1}^2 + m_{\Sigma}^2 = m_{H_2}^2 + m_{\bar{H}_2}^2 + m_{\Sigma}^2 \\ &= m_{H_3}^2 + m_{\bar{H}_3}^2 + m_{\Sigma}^2, \end{aligned} \quad (39)$$

$$\tilde{M}^2 = 3m_{\Sigma}^2. \quad (40)$$

The sum rules (33) require the degeneracy of $m_{10_i}^2$, and the degeneracy of $m_{5_i}^2$ follows from Eq. (35). Similarly, one can easily derive the degeneracy of $m_{\bar{H}_a}^2$ as well as that of $m_{H_a}^2$:

$$\begin{aligned} m_{10_i}^2 &= m_{10}^2, \quad m_{5_i}^2 = \frac{4}{3}\tilde{M}^2 - 3m_{10}^2, \\ m_{H_a}^2 &= \tilde{M}^2 - 2m_{10}^2, \quad m_{\bar{H}_a}^2 = -\frac{1}{3}\tilde{M}^2 + 2m_{10}^2, \end{aligned} \quad (41)$$

where $i = 1-3$ and $a = 1-4$. As we can see from (41), There are only two independent parameters in the SSB sector, m_{10}^2 and the gaugino mass M , for instance, as we have indicated in Eq. (41). To express \tilde{M}^2 in terms of M , we have to compute the trace in Eq. (32). We find

$$\begin{aligned} m_I^2 T(R_I) &= \frac{1}{2} \left[\sum_{i=1}^3 m_{5_i}^2 + \sum_{a=1}^4 (m_{H_a}^2 + m_{\bar{H}_a}^2) \right] \\ &\quad + \frac{3}{2} \sum_{i=1}^3 m_{10_i}^2 + 5m_{\Sigma}^2 \\ &= C(SU(5))\tilde{M}^2, \end{aligned} \quad (42)$$

where we have used Eqs. (33)–(40). Using this, we then obtain

$$\tilde{M}^2 = |M|^2. \quad (43)$$

Note that the democratic structure for the quark mass matrices is essential to obtain the set of the sum rules (35) and (36)–(38) that yields the universal soft masses (41). To summarize, finiteness requires that the trilinear couplings have to be aligned (31) and the soft scalar masses have to have the universal form (41). Before the diagonalization of μ_H^{ab} , the Yukawa couplings u_{ij} and d_{ij} are real numbers. Note that there are no restrictions on μ_H^{ab} and B_H^{ab} from finiteness. The diagonalization of μ_H^{ab} and an appropriate phase rotation of H_a and \bar{H}_a will introduce phases into the Yukawa couplings, which yields the ordinary CKM phase. The redefinition of the superfields above does not destroy the alignment of the trilinear couplings (31) and the universality of the soft masses (41). Then only the gaugino mass M and B_H^{ab} are complex numbers and contain CP -violating phases in this model. They may contribute to the electric dipole moment (EDM) of the neutron, for instance [5]. Nevertheless, the SUSY flavor problem is drastically reduced in this finite unified model.

B. Predictions at low energy

Since there are four pairs of the Higgs supermultiplets, it is not all automatic that there is only one pair of light Higgs doublets at low energies after a fine tuning at M_{GUT} . Furthermore, the Yukawa couplings are of order $O(g)$, and so we have to worry about the problem of fast proton decay [63] via dimension five operators [64]. These problems are related to the choice of the supersymmetric Higgs mass matrix μ_H^{ab} in the superpotential (29), which we would like to leave for future problems. Note that there are no constraints on μ_H^{ab} from finiteness. In what follows, we simply assume that there is one pair of light Higgs doublets and the proton decay can be sufficiently suppressed.

The finiteness conditions (30), (31) and (41) do not restrict the renormalization property at low energies, because the gauge symmetry is spontaneously broken below M_{GUT} . This should be contrasted to the case of the anomaly mediated supersymmetry breaking [10]. Therefore, the conditions (30), (31) and (41) are just boundary conditions at M_{GUT} in our case. By assumption, the evolution of the parameters below M_{GUT} is governed by the MSSM [65]. We further assume a unique supersymmetry breaking scale M_{SUSY} , which is identified with $\sqrt{(m_{t_1}^2 + m_{t_2}^2)/2}$, where $m_{t_i}^2$ are two stop masses, so that below M_{SUSY} the SM is the correct

effective theory. We recall that $\tan\beta$ is usually determined in the Higgs sector. However, in the case at hand, it is convenient to define $\tan\beta$ by using the matching condition at M_{SUSY} ,

$$\alpha_t^{\text{SM}} = \alpha_t \sin^2 \beta, \quad \alpha_b^{\text{SM}} = \alpha_b \cos^2 \beta, \quad \alpha_\tau^{\text{SM}} = \alpha_\tau \cos^2 \beta,$$

$$\alpha_\lambda = \frac{1}{4} \left(\frac{3}{5} \alpha_1 + \alpha_2 \right) \cos^2 2\beta, \quad (44)$$

where α_i^{SM} ($i = t, b, \tau$) are the SM Yukawa couplings and α_λ is the Higgs coupling ($\alpha_I = g_I^2/4\pi^2$). The matching conditions (44) and the boundary conditions at M_{GUT} can be satisfied only for a specific value of $\tan\beta$. This is the reason of why it is possible without knowing the details of the scalar sector of the MSSM to predict various parameters such as the top and quark masses [29,31,62].

Since $\tan\beta$ is fixed in the dimension-zero sector and the soft scalar masses have to satisfy the boundary conditions (41) at M_{GUT} , it is by no means trivial that the electroweak symmetry is correctly broken at low energies [66]. Fortunately, the supersymmetric mass parameter μ_H for the pair of the light Higgs doublets and the corresponding B term are not constrained by the finiteness conditions. Therefore, we use this freedom to fix μ_H and B to trigger the electroweak symmetry breaking. To proceed we write down the up-quark mass matrix at M_{GUT} which can be read off from Eqs. (29) and (30):

$$M^u = \sqrt{\frac{8}{15}} g \langle H_4 \rangle \begin{pmatrix} 1 & 1 + \epsilon_1 & 1 + \epsilon_2 \\ 1 + \epsilon_1 & 1 & 1 + \epsilon_3 \\ 1 + \epsilon_2 & 1 + \epsilon_3 & 1 \end{pmatrix}, \quad (45)$$

where

$$\epsilon_i = \frac{\langle H_i \rangle}{\langle H_4 \rangle} - 1 \quad \text{with} \quad i = 1, 2, 3. \quad (46)$$

As we can see also from Eqs. (29) and (30), the down-type quark mass matrix has the same structure. It has been found [67] that the above democratic mass matrix with $\epsilon_i \ll 1$ agree with experimental data. Therefore, H_a have to have almost equal VEV's, although there is only one pair of light Higgs doublets.

After so many remarks, we are now in position to compute low energy quantities. We use the renormalization group equations of two-loop order for dimensionless parameters and those of one-loop order for dimensional ones [65]. To see the gross nature of the low energy predictions of the present model, we, however, neglect ϵ_i in the mass matrix (45) and the threshold corrections at M_{GUT} as well as M_{SUSY} , while we take into account the SM correction to the physical mass of the top quark m_t and to m_b , which is the running bottom mass at m_b . Under this simplification, the top and bottom Yukawa couplings at M_{GUT} are given by

$$y_t = \sqrt{6/5} g, \quad y_b = \sqrt{9/10} g. \quad (47)$$

In Table I we present the predictions for $\alpha_3(M_Z), \tan\beta, m_t$

TABLE I. The predictions for different M .

M (TeV)	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	m_b (GeV)	m_t (GeV)
1	0.118	52	4.6	179
1.5	0.117	52	4.6	179

and m_b for different choices of the unified gaugino mass M . Comparing, for instance, the m_t prediction above with the most recent experimental value [68],

$$m_t = (174.3 \pm 5.1) \text{ GeV}, \quad (48)$$

and recalling that we have neglected ϵ_i in Eq. (45) and the threshold corrections, we see that the prediction can be consistent with the experimental data.

Next we turn to the SSB sector with the finiteness conditions (31) and (41). As we can see from Eqs. (41), we may treat M and m_{10} as independent parameters. The nice feature of Eqs. (41) is that the soft scalar masses of H_a and \bar{H}_a are degenerate. Therefore, one pair of the light Higgs doublets, H_u and H_d , which can be obtained after the diagonalization by an appropriate unitary matrix, has exactly the same soft scalar mass. Then we look for the parameter space in the $m_{10} - M$ plane, in which a successful radiative electroweak symmetry breaking occurs and the lightest neutralino is the LSP. In Fig. 1 we show the result where the region with dots and open squares leads to a successful radiative electroweak symmetry breaking. In the region with dots, the lightest neutralino is the LSP, while in the region with open squares the LSP is the stau. So the phenomenologically viable parameter space in the $M - m_{10}$ plane is very restricted; m_{10} has to lie approximately on the straight line given by $m_{10} = 5/8M$. That is, for a given unified gaugino mass M , the spectrum of the superpartners is basically fixed. In Table II we present the results for $M = 1$ TeV and 1.5 TeV in an obvious notation.

The dotted region in Fig. 1 is interesting also from the cosmological viewpoint. In the dotted region, the lightest

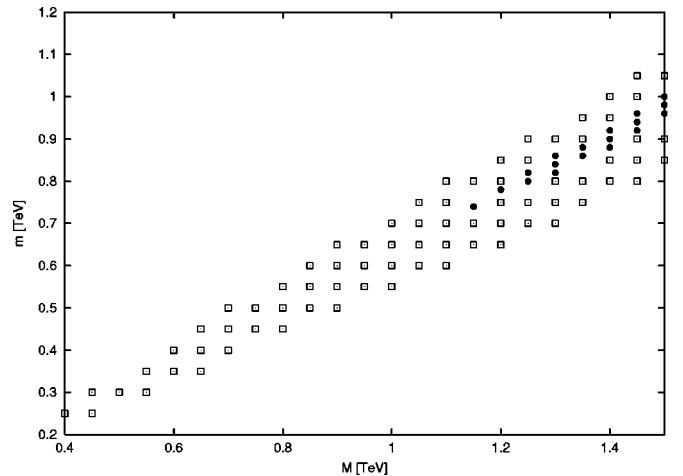


FIG. 1. The parameter space with a successful radiative electroweak symmetry breaking. The LSP is the lightest neutralino (the stau) in the region with dots (open squares).

TABLE II. The predictions of the spectrum of the superpartners for $M = 1$ TeV with $m_{10} = 0.63$ TeV and $M = 1.5$ TeV with $m_{10} = 0.94$ TeV.

m_{χ_1} (TeV)	0.45	0.69	$m_{\tilde{s}_1} = m_{\tilde{d}_1}$ (TeV)	1.9	2.8
m_{χ_2} (TeV)	0.84	1.3	$m_{\tilde{s}_2} = m_{\tilde{d}_2}$ (TeV)	2.2	3.2
m_{χ_3} (TeV)	1.3	1.9	$m_{\tilde{\tau}_1}$ (TeV)	0.43	0.69
m_{χ_4} (TeV)	1.3	1.9	$m_{\tilde{\tau}_2}$ (TeV)	0.72	1.0
$m_{\chi_1^\pm}$ (TeV)	0.84	1.3	$m_{\tilde{\mu}_1} = m_{\tilde{e}_1}$ (TeV)	0.78	1.2
$m_{\chi_2^\pm}$ (TeV)	1.3	1.9	$m_{\tilde{\mu}_2} = m_{\tilde{e}_2}$ (TeV)	1.1	1.6
$m_{\tilde{t}_1}$ (TeV)	1.5	2.2	$m_{\tilde{\nu}_\tau}$ (TeV)	0.68	1.0
$m_{\tilde{t}_2}$ (TeV)	1.7	2.5	$m_{\tilde{\nu}_\mu} = m_{\tilde{\nu}_e}$ (TeV)	0.78	1.2
$m_{\tilde{b}_1}$ (TeV)	1.5	2.2	m_A (TeV)	0.62	0.93
$m_{\tilde{b}_2}$ (TeV)	1.7	2.5	m_{H^\pm} (TeV)	0.63	0.94
$m_{\tilde{c}_1} = m_{\tilde{u}_1}$ (TeV)	2.1	3.1	m_H (TeV)	0.63	0.93
$m_{\tilde{c}_2} = m_{\tilde{u}_2}$ (TeV)	2.2	3.2	m_h (TeV)	0.13	0.13

neutralino χ_1 and the light stau $\tilde{\tau}_1$ are nearly degenerate in mass, that is, $m_{\tilde{\tau}_1} - m_{\chi_1} < 25$ GeV, where the light stau $\tilde{\tau}_1$ is the next-to-LSP. With this type of spectrum, neutralino-stau co-annihilation can occur and that reduces the relic LSP density [69]. Thus, this parameter region is quite interesting for the LSP dark matter scenario.

IV. CONCLUSION

Although it was suggested in the past [20–23] that the finiteness of softly broken supersymmetric Yang-Mills theories may play an important role in understanding the universality of the SSB parameters, there has been so far no finite model based on softly broken $N = 1$ supersymmetry in which the universality of the SSB parameters follows solely from finiteness. The simple reason is the relaxed finiteness condition, the sum rule (17).

In this paper we considered the finite model of [34], which is based on the discrete symmetry A_4 and has the democratic structure of the mass matrices for the quarks and leptons. We included the SSB sector to this model and required that this sector does not destroy finiteness. A_4 symmetry in the SSB sector was not assumed, because A_4 sym-

metry has to be broken at M_{GUT} by operators with dimension less than four. We found that finiteness in this model requires that the trilinear couplings have to be aligned (31) and the soft scalar masses have to have the universal form (41). The democratic structure of the mass matrices played the essential role to obtain the universality of the soft scalar masses. Therefore, this model shows that finiteness can offer a solution to the SUSY flavor problem, and indicates that the SUSY flavor problem is closely related to the exact scale invariance.

ACKNOWLEDGMENTS

We would like to thank M. Mondragon for useful discussions. This work is supported by the Grants-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (JSPS) (No. 14540252, 13135210, 14540256). The work of K.B. is supported in part by U.S. DOE Grant No. DE-FG03-98ER-41076, a grant from the Research Corporation and by U.S. DOE Grant No. DE-FG02-01ER-45684. This work was partially conducted by way of a grant awarded by the Government of Mexico in the Secretariat of Foreign Affairs.

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