Extra dimensions and invisible decay of orthopositronium

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(Received 18 October 2002; published 21 April 2003)

We point out that some models with infinite additional dimension(s) of Randall-Sundrum type predict the disappearance of orthopositronium (o-Ps) into additional dimension(s). The experimental signature of this effect is the o-Ps$\rightarrow$invisible decay of orthopositronium which may occur at a rate within three orders of magnitude of the present experimental upper limit. This result enhances existing motivations for a more sensitive search for this decay mode and suggests additional directions for testing extra dimensions in nonaccelerator experiments.

DOI: 10.1103/PhysRevD.67.075012 PACS number(s): 14.80.$-$j, 12.60.$-$i, 13.20.Cz, 13.35.Hb

Positronium (Ps), the positron-electron bound state, is the lightest known atom, which is bounded and self-annihilates through the same electromagnetic interaction. At the current levels of experimental and theoretical precision this is the only interaction present in this system; see, e.g., [1]. This feature has made positronium an ideal system for testing the accuracy of QED calculations for bound states, in particular for the triplet ($1\,^3S_1$) state of Ps, orthopositronium (o-Ps). Because of the odd parity under C transformation, o-Ps decays predominantly into three photons. As compared with singlet ($1\,^1S_0$) state (parapositronium), the "slowness" of o-Ps decay, due to the phase-space and additional α suppression factors, gives an enhancement factor $\simeq 10^3$, making it more sensitive to an admixture of new interactions which are not accommodated in the standard model (SM); see, e.g., [2].

Within the SM orthopositronium can decay invisibly into a neutrino-antineutrino pair. The o-Ps$\rightarrow \nu_e \overline{\nu_e}$ decay occurs through W exchange in the t channel and e^+e^- annihilation via Z. The decay width is [3]

$$\Gamma(o\text{-Ps}\rightarrow \nu_e \overline{\nu_e}) \approx 6.2\times 10^{-18}\Gamma(o\text{-Ps}\rightarrow 3\gamma). \tag{1}$$

For other neutrino flavors only the Z diagram contributes. For $l\neq e$ the decay width is [3]

$$\Gamma(o\text{-Ps}\rightarrow \nu_l \overline{\nu_l}) \approx 9.5\times 10^{-21}\Gamma(o\text{-Ps}\rightarrow 3\gamma). \tag{2}$$

Thus, in the SM the o-Ps$\rightarrow \nu\overline{\nu}$ decay width is very small and its contribution to the total decay width can be neglected.

Presently there is a big interest in models with additional dimensions which might provide solution to the gauge hierarchy problem [4–9], for a recent review see, e.g., [10]. For instance, as has been shown in the five-dimensional model, so-called RS two-model [4], there exists a thin-brane solution to the five-dimensional Einstein equations which has flat four-dimensional hypersurfaces,

$$ds^2 = a^2(z)\,\eta_{\mu\nu}dx^\mu dx^\nu - dz^2. \tag{3}$$

Here

$$a(z) = \exp(-k|z|) \tag{4}$$

and the parameter $k>0$ is determined by the five-dimensional Planck mass and bulk cosmological constant.

Recently, a peculiar feature of massive matter in the brane world has been reported [11]. It has been shown that tunneling of massive matter into extra dimensions is generic to fields that can have bulk modes. The massive matter becomes unstable; namely, the discrete zero modes turn into quasilocalized states with finite four-dimensional mass and finite width [11]. For massive scalar particle Φ with the mass m the transition rate into additional dimension is given by [11]

$$\Gamma(\Phi\rightarrow \text{add dim}) = \frac{\pi m}{16}\left(\frac{m}{k}\right)^2. \tag{5}$$

It should be noted that even for a massless scalar particle the nonzero transition rate into additional dimension(s) results in a nonzero imaginary part of the corresponding scalar propagator [11]

$$D(p^2) = \frac{1}{p^2 - i\,\theta(p^2)\sqrt{p^2}\,\Gamma(p^2)}, \tag{6}$$

where $\Gamma(p^2)\approx(\pi/4)\sqrt{p^2}(p^2/k^2)$. It means that even for the massless case, when the transition rate on mass shell is zero, the virtual scalar particle has a nonzero transition rate into additional dimension. The explicit expression for the transition rate into additional dimension depends on the concrete model.

To be specific, let us consider a model for the localization of gauge fields suggested in Refs. [12,13]. One begins with the solution to $(4+n+1)$-dimensional Einstein equations

$$ds^2 = \frac{1}{(1+k|\xi|)^2}\left(dt^2 - dx^2 - \sum_{i=1}^{n} R_i^2 d\theta_i^2 - d\xi^2\right), \tag{7}$$

where $\theta_i\in[0,2\pi]$ are compact coordinates, R_i are radii of compact dimensions, and k is the inverse anti-de Sitter (adS) radius determined by the bulk cosmological constant. There is a single brane located at $\xi=0$. The only difference between this metric and the Randall-Sundrum metric is the presence of extra compact dimensions θ_i. These dimensions are added for obtaining a localized zero mode of the gauge field. In what follows we assume that their radii R_i are the smallest length scales involved, so all fields are taken independent of θ_i. The inverse adS radii k is assumed to be the largest energy scale involved. For the model of Refs. [12,13] with additional $(n+1)$ dimensions and metric of Randall-Sundrum type the imaginary part of the propagator of massless scalar particle is $\Gamma \sim \sqrt{p^2}(p^2/k^2)^{1+n/2}$.

The case of the electromagnetic field propagating in the Randall-Sundram type of metric of Eq. (7) has been considered in Ref. [13]. It was shown that the transition rate of a virtual photon with the virtual mass $m_{\gamma^*}=\sqrt{p^2}$ into additional dimensions is different from zero and it is equal to

$$\Gamma = k(n) m_{\gamma^*}\left(\frac{m_{\gamma^*}}{k}\right)^n, \tag{8}$$

where $k(n)$ is a numerical coefficient.

Consider now o-Ps→invisible decay, which is a good candidate for the searching for effect of disappearance into additional dimension(s) since o-Ps has specific quantum numbers similar to those of vacuum and is a system which allows its constituents a rather long interaction time. To make a quantitative estimate we take $n=2$ [$(4+2+1)$-dimensional space-time]. In this case the disappearance rate of a virtual photon into additional dimensions is given by

$$\Gamma(\gamma^* \to \text{add dim}) = \frac{\pi m_{\gamma^*}}{4}\left(\frac{m_{\gamma^*}}{k}\right)^2. \tag{9}$$

Using Eq. (9), for the branching ratio of orthopositronium invisible decay into additional dimension(s) through single-photon annihilation o-Ps→γ^*→ add dim one gets as an estimate

$$\frac{\Gamma(o\text{-Ps}\to\gamma^*\to \text{add dim})}{\Gamma(o\text{-Ps}\to 3\gamma)} = \frac{9\pi}{4(\pi^2-9)}\frac{1}{\alpha^2}\frac{\pi}{4}\left(\frac{m_{o\text{-Ps}}}{k}\right)^2 \approx 1.2\times 10^5\left(\frac{m_{o\text{-Ps}}}{k}\right)^2. \tag{10}$$

To solve the gauge hierarchy problem models with additional dimension(s) one may expect $k \lesssim O(10)$ TeV. It means that

$$\text{Br}(o\text{-Ps}\to \text{add dim}) \gtrsim O(10^{-9}). \tag{11}$$

Important bounds on the parameter k and Br(o-Ps → add dim) arise from the combined LEP result on the precise measurements of the total and partial Z widths [14]. We can write the Z invisible width in the following form:

$$\Gamma_{inv} = \Gamma^{SM}(Z\to\nu\bar{\nu}) + \Delta\Gamma_{inv}, \tag{12}$$

where $\Gamma^{SM}(Z\to\nu\bar{\nu})$ is the SM contribution and $\Delta\Gamma_{inv}$ contains the effects beyond the SM. Assuming that each neutrino type contributes the same amount to the invisible Z width, one has, numerically [15],

$$\Gamma^{SM}(Z\to\nu\bar{\nu}) = 3\Gamma(Z\to\nu_i\bar{\nu_i}) = 3\times(167.06\pm 0.22)\ \text{MeV}. \tag{13}$$

The invisible width Γ_{inv} can be obtained from the Z total width and its partial width into hadrons and leptons using the equation

$$\Gamma_{tot} = \Gamma_{had} + \Gamma_{lept} + \Gamma_{inv}. \tag{14}$$

The value of Γ_{inv} derived from the LEP measurements of Γ_{tot}, Γ_{had}, and Γ_{lept} [14,16] is

$$\Gamma_{inv} = 499.0 \pm 1.5\ \text{MeV}. \tag{15}$$

Using Eqs. (13) and (15) we obtain

$$\Delta\Gamma_{inv} = -2.7 \pm 1.6\ \text{MeV}. \tag{16}$$

If a conservative approach is taken to constrain the result to only positive values $\Delta\Gamma_{inv}$ and renormalizing the probability for $\Delta\Gamma_{inv} \geqslant 0$ to be unity, then the resulting 95% C.L. upper limit on additional invisible decay of Z is [16]

$$\Delta\Gamma_{inv} < 2.0\ \text{MeV}. \tag{17}$$

Assuming $\Delta\Gamma_{inv} = \Gamma(Z\to \text{add dim})$ and using Eqs. (9),(10) for the estimate leads to $k \geqslant 17$ TeV and to the corresponding bound

$$\text{Br}(o\text{-Ps}\to \text{add dim}) \leqslant 0.4\times 10^{-9}. \tag{18}$$

Note that combined result on direct CERN e^+e^- collider LEP measurements of the invisible width, $\Gamma_{inv}=503 \pm 16$ MeV [14], gives a less stringent limit

$$\text{Br}(o\text{-Ps}\to \text{add dim}) \lesssim 10^{-8}. \tag{19}$$

These estimates, giving only an order of magnitude for the corresponding branching ratio, show that this decay may occur at a rate within roughly three orders of magnitude of the best present experimental limit [17]:

$$\text{Br}(o\text{-Ps}\to \text{invisible}) < 2.8\times 10^{-6}. \tag{20}$$

Thus, the region Br(o-Ps→ invisible)$\simeq 10^{-9}$ is of great interest for possible observation of effect of extra dimensions. Interestingly, that for $n=1$ the bound Br(o-Ps→ add dim) $\lesssim 10^{-4}$ obtained from Eq. (17) is weaker than that of Eq. (20) obtained from the direct measurement. We believe these results strengthen current motivations related to the orthopositronium decay rate puzzle and mirror world [18], millicharged particle [19], and light gauge boson [2] searches and justify efforts for a more sensitive search for the o-Ps → invisible decay in a near future experiment [20].

The experimental signature of the o-Ps→ invisible decay is the absence of an energy deposition of $\simeq 1$ MeV, which is expected from the ordinary o-Ps annihilation in a 4π Hermetic calorimeter surrounding the o-Ps formation region [21]. Our first Monte Carlo simulations, based on the results of the recent search for $o\text{-Ps}\rightarrow\gamma+\text{invisible}$ decay [22], show that for the branching ratio one may expect a limit $\text{Br}(o\text{-Ps}\rightarrow\text{add dim})\lesssim 10^{-8}$ if the calorimeter has a mass of $\simeq 0.5$ ton. Larger simulation statistics and better background evaluation are required in order to see if the sensitivity to the branching ratio as low as 10^{-9} is experimentally reachable.

N.V.K. is indebted to V.A. Rubakov and P.G. Tinyakov for many useful discussions on models with additional infinite dimensions. The work of N.V.K. has been supported by RFFI Grant No. N02-01-00601.

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