Primordial nucleosynthesis constraints on Z' properties

Vernon Barger
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

Paul Langacker
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104
and Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

Hye-Sung Lee
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706
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In models involving new TeV-scale Z' gauge bosons, the new $U(1)'$ symmetry often prevents the generation of Majorana masses needed for a conventional neutrino seesaw mechanism, leading to three superweakly interacting "right-handed" neutrinos ν_R, the Dirac partners of the ordinary neutrinos. These can be produced prior to big bang nucleosynthesis by the Z' interactions, leading to a faster expansion rate and too much ^{4}He. We quantify the constraints on the Z' properties from nucleosynthesis for Z' couplings motivated by a class of E_6 models parametrized by an angle θ_{E6}. The rate for the annihilation of three approximately massless right-handed neutrinos into other particle pairs through the Z' channel is calculated. The decoupling temperature, which is higher than that of ordinary left-handed neutrinos due to the large Z' mass, is evaluated, and the equivalent number of new doublet neutrinos ΔN_ν is obtained numerically as a function of the Z' mass and couplings for a variety of assumptions concerning the Z-Z' mixing angle and the quark-hadron transition temperature T_c. Except near the values of θ_{E6} for which the Z' decouples from the right-handed neutrinos, the Z' mass and mixing constraints from nucleosynthesis are much more stringent than the existing laboratory limits from searches for direct production or from precision electroweak data, and are comparable to the ranges that may ultimately be probed at proposed colliders. For the case $T_c = 150$ MeV with the theoretically favored range of Z-Z' mixings, $\Delta N_\nu \lesssim 0.3$ for $M_{Z'} \gtrsim 4.3$ TeV for any value of θ_{E6}. Larger mixing or larger T_c often lead to unacceptably large ΔN_ν except near the ν_R decoupling limit.

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I. INTRODUCTION

Additional heavy Z' gauge bosons [1] are predicted in many superstring [2] and grand unified [3] theories, and also in models of dynamical symmetry breaking [4]. If present at a scale of a TeV or so they could provide a solution to the μ problem [5] and other problems of the minimal supersymmetric standard model (MSSM) [6]. Current limits from collider [7,8] and precision [9] experiments are model dependent, but generally imply that $M_{Z'} > (500–800)$ GeV and that the Z-Z' mixing angle is smaller than a few $\times 10^{-3}$. There are even hints of deviations in atomic parity violation [10,11] and the NuTeV experiment [13], which could be an early indication of a Z' [14]. A Z' lighter than a TeV or so should be observable at run II at the Fermilab Tevatron. Future colliders should be able to observe a Z' with mass up to around 5 TeV and perform diagnostics on the couplings up to a few TeV [15].

An electroweak or TeV-scale Z' would have important implications for theories of neutrino mass. If the right-handed neutrinos carry a non-zero $U(1)'$ charge, then the $U(1)'$ symmetry forbids them from obtaining a Majorana mass much larger than the $U(1)'$-breaking scale, and, in particular, would forbid a conventional neutrino seesaw model [16]. In this case, it might still be possible to generate small Majorana masses for the ordinary (active) neutrinos by some sort of TeV-scale seesaw mechanism in which there are additional mass suppressions [17]. However, another possibility is that there are no Majorana mass terms, and that the neutrinos have Dirac masses which are small for some reason, such as higher dimensional operators [18] or volume suppressions in theories with large extra dimensions [19]. In this case, the model would contain three additional right-handed partners of the ordinary neutrinos, which would be almost massless. Such light Dirac neutrinos (i.e., with mass less than an eV or so) in the standard model or MSSM are essentially sterile, except for the tiny effects associated with their masses and Higgs couplings, which are too much small to produce them in significant numbers prior to nucleosynthesis or in a supernova. However, the superweak interactions of these states due to their coupling to a heavy Z' (or a heavy W' in the $SU(2)_L \times SU(2)_R \times U(1)$ extension of the standard model [20]) might be sufficient to create them in large numbers in the early universe [21–23] or in a supernova [24]. In this paper, we consider the constraints following from big bang nucleosynthesis on Z' properties in a class of E_6-motivated models.

It is well known that any new relativistic particle species that were present when the temperature T was a few MeV would increase the expansion rate, leading to an earlier freeze-out of the neutron to proton ratio and therefore to a higher ^{4}He abundance [25,26]. Their contribution is usually parametrized by the number ΔN_ν of additional neutrinos with full-strength weak interactions that would yield the same contribution to the energy density. The primordial ^{4}He abundance is still rather uncertain, but typical estimates of the upper limit on ΔN_ν are in the range [27] $\Delta N_\nu < (0.3-1)$ [26,28]. Of course, the Z width does not allow more than 3 light active neutrinos [29], so ΔN_ν should be interpreted as an effective parameter describing degrees of freedom that do not couple with full strength to the Z.

In 1979, Steigman, Olive, and Schramm [21,22] described the implications of a superweakly interacting light particle, such as a right-handed neutrino coupling to a heavy Z'. Because of their superweak interactions, such particles decoupled earlier than ordinary neutrinos. As the temperature dropped further, massive particles such as quarks, pions, and muons subsequently annihilated, reheating the ordinary neutrinos and other particles in equilibrium, but not the superweak particles. One must also take into account the transition between the quark-gluon phase and the hadron phase.

A simple estimate of the decoupling temperature is obtained as follows [21,22]. Ordinary neutrinos have cross-sections $\sigma_W \propto G_W^2 T^2$, where G_W is the Fermi constant, and interaction rates

$$\Gamma_W(T) = n\langle \sigma_W v\rangle \propto G_W^2 T^5, \tag{1.1}$$

where n is the density of target particles. The Hubble expansion parameter varies as $H \propto T^2/M_P$, where M_P is the Planck scale, so the decoupling temperature T_d at which Γ is equal to H becomes

$$T_d \propto (G_W^2 M_P)^{-1/3}. \tag{1.2}$$

Putting in the coefficients, $T_d(\nu_L) \approx 1$ MeV for the ordinary neutrinos [30]. Similarly, a superweakly interacting particle such as a right-handed neutrino with a cross-section $\sigma_{SW} \propto G_{SW}^2 T^2$, would decouple at

$$T_d(\nu_R) \sim \left(\frac{G_W}{G_{SW}}\right)^{2/3} T_d(\nu_L). \tag{1.3}$$

If in the specific model, the effective superweak coupling constant G_{SW} is proportional to M_{SW}^{-2}, where M_{SW} is the mass of a superweak gauge boson, the decoupling temperature can be written as

$$T_d(\nu_R) \sim \left(\frac{M_{SW}}{M_W}\right)^{4/3} T_d(\nu_L), \tag{1.4}$$

where M_W is the W mass. It is then straightforward to calculate the dilution by the subsequent quark-hadron transition and the annihilations of heavy particles, and the corresponding ΔN_ν from the superweak particles.

Of course, the estimate in Eq. (1.4) is very rough. In particular, the detailed couplings of the Z' to the ν_R and to all of the other relevant particles must be considered for a precise estimate [31]. In this paper, we do this for a class of Z' models with couplings motivated by E_6 grand unification [36]. (The full structure of E_6 is not required.) We define the $U(1)'$ model in Sec. II. The implications of superweakly coupled particles for nucleosynthesis and the uncertainties from the quark-hadron transition temperature T_c are summarized in Sec. III. Section IV deals with the calculation of the decoupling temperature. We present our results and numerical analysis for T_d and ΔN_ν for three right-handed neutrinos as a function of the Z' mass and couplings for various assumptions concerning the Z-Z' mixing and T_c in Sec. V. The discussion and conclusion follows in Sec. VI.

TABLE I. The (family-universal) charges of the $U(1)_\chi$ and the $U(1)_\psi$.

Fields	Q_χ	Q_ψ
u_L	$-1/2\sqrt{10}$	$1/2\sqrt{6}$
u_R	$1/2\sqrt{10}$	$-1/2\sqrt{6}$
d_L	$-1/2\sqrt{10}$	$1/2\sqrt{6}$
d_R	$-3/2\sqrt{10}$	$-1/2\sqrt{6}$
e_L	$3/2\sqrt{10}$	$1/2\sqrt{6}$
e_R	$1/2\sqrt{10}$	$-1/2\sqrt{6}$
ν_L	$3/2\sqrt{10}$	$1/2\sqrt{6}$
ν_R	$5/2\sqrt{10}$	$-1/2\sqrt{6}$

II. Z' IN E_6-MOTIVATED MODELS

A general model with an extra Z' is characterized by the Z' mass; the Z-Z' mixing angle; the $U(1)'$ gauge coupling; the $U(1)'$ chiral charges for all of the fermions and scalars, which in general may be family nonuniversal, leading to flavor changing neutral currents [37]; and an additional parameter associated with mixing between the Z and Z' kinetic terms [38]. Furthermore, most concrete Z' models involve additional particles with exotic standard model quantum numbers, which are required to prevent anomalies. It is difficult to work with the most general case, so many studies make use of the $U(1)'$ charges and exotic particle content associated with the E_6 model, as an example of a consistent anomaly-free construction [39]. Explicit string constructions [40] often lead to other patterns of couplings and exotics, but these are very model dependent.

E_6 actually yields two additional $U(1)'$ factors when broken to the standard model [or to $SU(5)$], i.e.,

$$E_6 \to SO(10)\times U(1)_\psi \to SU(5)\times U(1)_\chi \times U(1)_\psi. \tag{2.1}$$

It is usually assumed that only one linear combination survives at low energies, parametrized by a mixing angle θ_{E6}. The resultant $U(1)'$ charge is then [41]

$$Q = Q_\chi \cos\theta_{E6} + Q_\psi \sin\theta_{E6}. \tag{2.2}$$

A special case that is often considered is $U(1)_\eta$, which corresponds to $\theta_{E6} = 2\pi - \tan^{-1}\sqrt{(5/3)} = 1.71\pi$. We list the charges of $U(1)_\chi$ and $U(1)_\psi$ that we need in Table I. The quantum numbers of the associated exotic particles are given in [36]. It is conventional to choose θ_{E6} to be in the range $(0,\pi)$, since the charges merely change sign for $\theta_{E6} \to \theta_{E6} + \pi$. With this convention one must allow both positive and negative values for the Z-Z' mixing angle δ. In this paper, we find it convenient to choose a different convention in which θ_{E6} varies from 0 to 2π, but for which $\delta \leqslant 0$. That is, the range $0-\pi$ corresponds to the E_6 models with negative mixing, while π–2π corresponds to positive mixing. The ν_R charge is nonzero, precluding an ordinary seesaw mechanism, except for $\theta_{E6} \sim 0.42\pi$ and 1.42π. We will always assume that the neutrinos are Dirac and that the three right-handed neutrinos are therefore very light. (In fact, the nonzero Dirac masses play no role in the analysis.) There could be additional sterile states, such as the $SO(10)$-singlet states occurring in the 27-plet of E_6. If these involve nearly massless fermions they could also contribute to the expansion rate prior to nucleosynthesis. We assume that these additional neutralinos acquire electroweak scale masses from the gauge symmetry breaking [1].

Let Z and Z' represent the standard model and $U(1)'$ gauge bosons, respectively, and $Z_{1,2}$ the mass eigenstate bosons, related by

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}, \tag{2.3}$$

where δ is the Z-Z' mixing angle. As stated in the Introduction, the limits on $M_{Z_2} \sim M_{Z'}$ depend on θ_{E6} and also on the masses of any exotics and superpartners to which the Z' couples, but are typically in the range $M_{Z'} > (500 - 800)$ GeV. The limits on δ are correlated with those for $M_{Z'}$ and are asymmetric under $\delta \to -\delta$. However, for $M_{Z'} \sim 1$ TeV the constraints are less sensitive to θ_{E6} and are approximately symmetric, with $|\delta| < 0.002$ giving a reasonable approximation for all θ_{E6}. For larger $M_{Z'}$ there are two theoretical constraints on the mixing, corresponding to Eqs. (6) and (5) of [42]. The first is a theoretical relation between the mass and mixing,

$$\delta = C \frac{g'_Z}{g_Z} \frac{M^2_{Z_1}}{M^2_{Z_2}}, \tag{2.4}$$

where $g_Z \equiv \sqrt{g_1^2 + g_2^2}$ and g'_Z is the $U(1)'$ gauge coupling constant. The value of g'_Z depends on the embedding and breaking of the underlying theory. We will choose $g'_Z = \sqrt{(5/3)} g_Z \sin\theta_W$, which corresponds to a unification of g'_Z with the other gauge couplings for the exotic particle quantum numbers of supersymmetric E_6. In Eq. (2.4) C depends on the charges of the scalar fields which lead to the mixing (see Table III of [42]). However, for the typical cases in which the mixing is induced by scalars in an E_6 27 or $\overline{27}$-plet, it is a reasonable approximation to take $-1 < C < 1$ for all θ_{E6}. (One can have a slightly more restrictive range for some θ_{E6}.) The assumption $|C| < 1$ corresponds to $|\delta| < 0.0051/M^2_{Z_2}$, where M_{Z_2} is in TeV. The second theoretical constraint is the requirement that the mixing should not change the mass of the lighter Z more than is allowed by the data. It is equivalent to

$$|\delta| \sim \sqrt{\rho_0 - 1}\, \frac{M_{Z_1}}{M_{Z_2}}, \tag{2.5}$$

where $M_{Z_1} = M_Z$, and the ρ_0 parameter, defined precisely in [43], should be exactly 1 in the standard model. The precision data imply $\rho_0 < 1.001$. Hence, $|\delta| < 0.0029/M_{Z_2}$, where M_{Z_2} is again in TeV. We will consider the following cases:

$$\begin{aligned} &(\mathrm{A0}) \quad \delta = 0 \quad (\text{no mixing}) \\ &(\mathrm{A1}) \quad |\delta| < 0.0051/M^2_{Z_2} \\ &\qquad\qquad (\text{theoretical mass-mixing relation}) \\ &(\mathrm{A2}) \quad |\delta| < 0.0029/M_{Z_2} \quad (\rho_0 \ \text{constraint}) \\ &(\mathrm{A3}) \quad |\delta| = 0.002 \\ &\qquad\qquad (\text{maximal mixing allowed for} M_{Z_2} \sim 1 \ \text{TeV}). \end{aligned} \tag{2.6}$$

A1 is more stringent than A2 and A3 in the large mass range, so we will mainly focus on A0 and A1.

The lagrangian for the massive neutral current coupling to fermion f is [42]

$$\begin{aligned} -\mathcal{L}_{int} = {} & g_Z Q_Z(f_L) \overline{f_L} \gamma^\mu f_L Z_\mu + g_Z Q_Z(f_R) \overline{f_R} \gamma^\mu f_R Z_\mu \\ & + g'_Z Q(f_L) \overline{f_L} \gamma^\mu f_L Z'_\mu + g'_Z Q(f_R) \overline{f_R} \gamma^\mu f_R Z'_\mu \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} Q_Z(f_L) &\equiv T^3_f - q_f \sin^2\theta_W, \\ Q_Z(f_R) &\equiv -q_f \sin^2\theta_W, \end{aligned} \tag{2.8}$$

and $Q(f_{L,R})$ is given by Eq. (2.2). The annihilation cross-section through Z' has both (light) Z_1 and (heavy) Z_2 contributions unless $\delta = 0$ and is calculated in Sec. IV.

III. NUCLEOSYNTHESIS

As described in the Introduction, the observed ^{4}He abundance constrains the energy density at the time of big bang nucleosynthesis (BBN) [25], with most recent estimates [26,28] of the number of equivalent new active neutrino types in the range $\Delta N_\nu < (0.3 - 1)$.

The contribution of new relativistic species can be written as

$$\Delta N_\nu = \frac{8}{7} \sum_B \frac{g_B}{2} \left(\frac{T_B}{T_{BBN}} \right)^4 + \sum_F \frac{g_F}{2} \left(\frac{T_F}{T_{BBN}} \right)^4, \tag{3.1}$$

where g_B and g_F are degrees of freedom of new bosons B and new fermions F, respectively, $T_{B,F}$ are their effective temperatures, and $T_{BBN} \sim 1$ MeV is the temperature at the

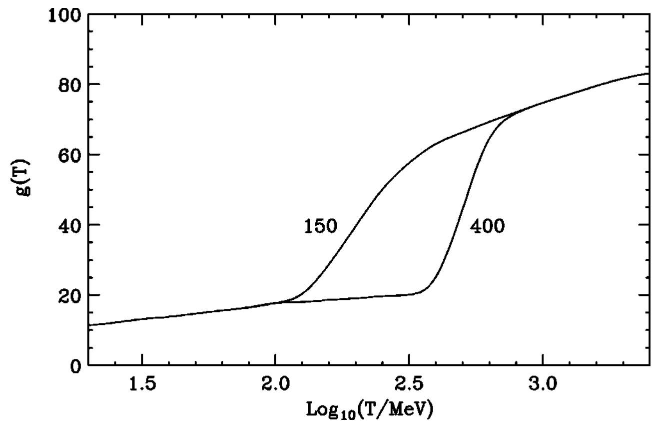

FIG. 1. The effective number of degrees of freedom as a function of temperature for the quark-hadron transition temperature $T_c = 150$ MeV and 400 MeV, from [46]. $g(T)$ does not include contributions from the three right-handed neutrinos, which are added separately in the expansion rate formula.

time of the freeze-out of the neutron to proton ratio. In particular, the contribution of three types of right-handed neutrinos is

$$\Delta N_\nu = 3\cdot 1\cdot\left(\frac{T_{\nu_R}}{T_{BBN}}\right)^4 = 3\left(\frac{g(T_{BBN})}{g(T_d(\nu_R))}\right)^{4/3} \tag{3.2}$$

where $T_d(\nu_R)$ is the decoupling temperature of the right-handed neutrinos. $g(T)$ is the effective number of degrees of freedom at temperature T. Neglecting finite mass corrections, it is given by $g_B(T)+\frac{7}{8}g_F(T)$, where $g_{B,F}(T)$ are the number of bosonic and fermionic relativistic degrees of freedom in equilibrium at temperature T [21,22]. In particular, $g(T_{BBN})=43/4$ from the three active neutrinos, $e^\pm$, and γ, and $g(T)$ increases (in this approximation) as a series of step functions at higher temperature as more particles are in equilibrium. The second equality in Eq. (3.2) comes from entropy conservation [21] in the heavy particle decouplings and quark-hadron transition subsequent to the ν_R decoupling. Therefore, the ν_R are not included in our definition of $g(T)$. (They will be included in the expansion rate formula prior to decoupling.)

In calculating $g(T)$ one must also take into account the QCD phase transition at temperature T_c. Above T_c, the u and d (and possibly s) quarks and the gluons were the relevant hadronic degrees of freedom, while below T_c they are replaced by pions [21,22]. The value of T_c is poorly known, but is usually estimated to be in the range $(150-400)$ MeV [44]. This range is estimated in quark and hadron potential models as the temperature above which hadrons start to overlap (lower end) or as the temperature below which the quark gas in no longer ideal (upper end). A related uncertainty is whether to use current or constituent quark masses. At very high temperatures the quarks can be considered as asymptotically free and current masses are appropriate, while around T_c constituent effects become important [45]. The range of estimates for T_c is essentially unchanged if one simply fixes the quark masses at either value [44].

Figure 1 shows the explicit values of $g(T)$ from the more detailed analysis of Ref. [46], which includes finite mass and other corrections, and uses the two values $T_c=150$ MeV and 400 MeV. We will also use these values for our numerical analysis. The sharp increase in $g(T)$ above T_c (because of the large number of quark and gluon degrees of freedom) is extremely important for relaxing the constraints on the Z' mass.

The QCD phase transition does not occur instantaneously or at one temperature but rather smoothly (meaning both quarks and hadrons exist at the same temperature) for a period of time around T_c, as illustrated by the smooth curves in Fig. 1. Risking a small inconsistency, we approximate our calculation of the interaction rate by simply switching from quarks to hadrons for temperatures below T_c. We will take the values $T_c=150$ and 400 MeV to illustrate the range of hadronic uncertainties. Above T_c, the interaction rate depends in principle on the quark masses, especially for low T_c. However, we have found in practice that the results are almost identical for constituent and current masses, so we will mainly display them for the constituent case (both will be shown for the η model).

The calculation of the right-handed neutrino decoupling temperature $T_d(\nu_R)$ in terms of the Z' parameters is discussed in the next section.

IV. THE EXPANSION AND INTERACTION RATES

A particle is decoupled from the background when its interaction rate drops below the expansion rate of the universe. In this section, we present the the cosmological expansion rate $H(T)$ along with the explicit form of the interaction rate $\Gamma(T)$ for $\overline{\nu_R}\nu_R$ annihilating into all open channels [47], and estimate the decoupling temperature T_d of a right-handed neutrino by $\Gamma(T_d)\sim H(T_d)$.

The Hubble expansion parameter is given by

$$H(T)=\sqrt{\frac{8\pi G_N\rho(T)}{3}}=\sqrt{\frac{4\pi^3 G_N g'(T)}{45}}T^2 \tag{4.1}$$

where $G_N=M_P^{-2}$ is the Newton constant and $\rho(T)$ is the energy density. We define $g'(T)=g(T)+\frac{21}{4}$, where the 21/4 reflects the 3 massless right-handed neutrinos.

The cross-section $\sigma_i(s)\equiv\sigma(\overline{\nu_R}\nu_R\to\overline{f_i}f_i)$ for a massless right-handed neutrino pair to annihilate into a fermion pair through the Z' channel is

$$\sigma_i(s)=N_C^i\frac{s\beta_i}{16\pi}\left\{\left(1+\frac{\beta_i^2}{3}\right)[(G_{RL}^i)^2+(G_{RR}^i)^2] + 2(1-\beta_i^2)G_{RL}^iG_{RR}^i\right\} \tag{4.2}$$

where (for $s\ll M_{Z_1}^2, M_{Z_2}^2$)

$$G_{RX}^{i}=g_{Z}^{\prime 2}Q(\nu_R)Q(f_{iX})\left(\frac{\sin^2\delta}{M_{Z_1}^2}+\frac{\cos^2\delta}{M_{Z_2}^2}\right)-g_Z^{\prime}g_ZQ(\nu_R)Q_Z(f_{iX})\left(\frac{\sin\delta\cos\delta}{M_{Z_1}^2}-\frac{\sin\delta\cos\delta}{M_{Z_2}^2}\right), \tag{4.3}$$

where $X=L$ or R, $\beta_i\equiv\sqrt{1-4m_{f_i}^2/s}$ is the relativistic velocity for the final particles, and N_C^i is the color factor of particle f_i.

In the limit of no-mixing ($\delta=0$) and massless final particles ($\beta_i=1$), the cross-section simplifies to

$$\sigma_i(s)\to N_C^i\frac{s}{12\pi}\left(\frac{g_Z^{\prime 2}}{M_{Z'}^2}\right)^2Q(\nu_R)^2[Q(f_{iL})^2+Q(f_{iR})^2], \tag{4.4}$$

consistent with the earlier estimate $\sigma_{SW}\propto G_{SW}^2T^2$ with $G_{SW}\propto g_Z^{\prime 2}/M_{Z'}^2$, and $T\propto\sqrt{s}$.

For temperatures less than the quark-hadron transition temperature $T_c=150-400$ MeV, we replace the quark degrees of freedom with hadrons. The only relevant annihilation channels are into charged pions. We approximate the cross-section of $\overline{\nu_R}\nu_R$ annihilating into $\pi^+\pi^-$ by using the ρ dominance model [48],

$$\sigma_\pi(s)\equiv\sigma(\overline{\nu_R}\nu_R\to\pi^+\pi^-)=\frac{s\beta_\pi^3}{96\pi}|F_\pi(s)|^2(G_{RL}^u+G_{RL}^{\bar{d}}+G_{RR}^u+G_{RR}^{\bar{d}})^2 \tag{4.5}$$

which is basically obtained by using $Q(f_{iL})=Q(u_L)+Q(\overline{d}_L)$ and $Q_Z(f_{iL})=Q_Z(u_L)+Q_Z(\overline{d}_L)$ for G_{RL}^i and likewise for G_{RR}^i. The pion form factor [49] is

$$F_\pi(s)=\frac{m_\rho^2}{s-m_\rho^2+im_\rho\Gamma_\rho}, \tag{4.6}$$

with $m_\rho=771$ MeV and $\Gamma_\rho=149$ MeV.

The interaction rate per ν_R is

$$\Gamma(T)=\sum_i\Gamma_i(T)=\sum_i\frac{n_{\nu_R}}{g_{\nu_R}}\langle\sigma v(\overline{\nu_R}\nu_R\to\overline{f_i}f_i,\pi^+\pi^-)\rangle, \tag{4.7}$$

where n_{ν_R} is the number density of a single flavor of massless right-handed neutrinos plus antineutrinos, $g_{\nu_R}=2$ is the number of degrees of freedom, and $\langle\sigma v\rangle$ is the thermal average of the cross-section times velocity.

We use the same masses (Table II) used in the calculation [22,46] of $g(T)$ in Fig. 1, except for the value $m_b=4200$ MeV of the b quark current mass [29]. We include the contributions of all particles up to the b quarks. The contributions from the top quark and heavy particles from new physics, such as squarks, sleptons, and exotics would only be relevant when the decoupling temperature is close to the electroweak scale or higher. This only occurs when θ_{E6} is extremely close to the values for which the ν_R decouples from the Z'.

TABLE II. The masses (in MeV) used for the numerical analysis.

Quarks	Current (constituent) masses	Others	Masses
u	4.2(340)	ν	0
d	7.5(340)	e	0.511
s	150(540)	μ	105
c	1150(1500)	τ	1800
b	4200(4500)	π	137

For a massless right-handed neutrino pair colliding with 4-momenta $p^\mu\equiv(p,\mathbf{p})$ and $k^\mu\equiv(k,\mathbf{k})$ with relative angle θ, the interaction rate per neutrino is [51]

$$\Gamma_i(T)=\frac{g_{\nu_R}}{n_{\nu_R}(T)}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d^3\mathbf{k}}{(2\pi)^3}f_\nu(p)f_\nu(k)\sigma_i(s)v_M=\frac{g_{\nu_R}}{8\pi^4n_{\nu_R}}\int_0^\infty p^2dp\int_0^\infty k^2dk\times\int_{-1}^{1}d\cos\theta\,\frac{(1-\cos\theta)}{(e^{k/T}+1)(e^{p/T}+1)}\sigma_i(s), \tag{4.8}$$

where $f_\nu(k)=(e^{k/T}+1)^{-1}$ is the Fermi-Dirac distribution with

$$n_{\nu_R}(T)=g_{\nu_R}\int\frac{d^3\mathbf{k}}{(2\pi)^3}f_\nu(k)=2\cdot\frac{3}{4\pi^2}\zeta(3)T^3, \tag{4.9}$$

$v_M=p\cdot k/pk=1-\cos\theta$ is the Mϕller velocity, and $s=2pk(1-\cos\theta)$ is the square of the center-of-mass energy.

A root-finding method was used to calculate the decoupling temperature, for which $H=\Gamma$. A several percent error was allowed in the numerical result to calculate the roots efficiently. Finite temperature effects, such as changes in the phase space due to interactions with the thermal bath, can increase the ordinary neutrino decoupling temperature by several percent [52]. Analogous effects for the ν_R are too small to significantly affect our results.

V. NUMERICAL RESULTS

In this section, we present the numerical results from the calculation. The marked points in Figs. 2–5 are the results of the actual calculation, while the curves interpolate.

Figures 2 and 3 show how the right-handed neutrino decoupling temperature T_d and the equivalent number of extra neutrino species ΔN_ν change with M_{Z_2} for $\theta_{E6}=2\pi-\tan^{-1}\sqrt{(5/3)}\sim1.71\pi$ (the η model) for constituent and current masses, respectively, for $T_c=$ 150 and 400 MeV and the various assumptions concerning the Z-Z' mixing listed in

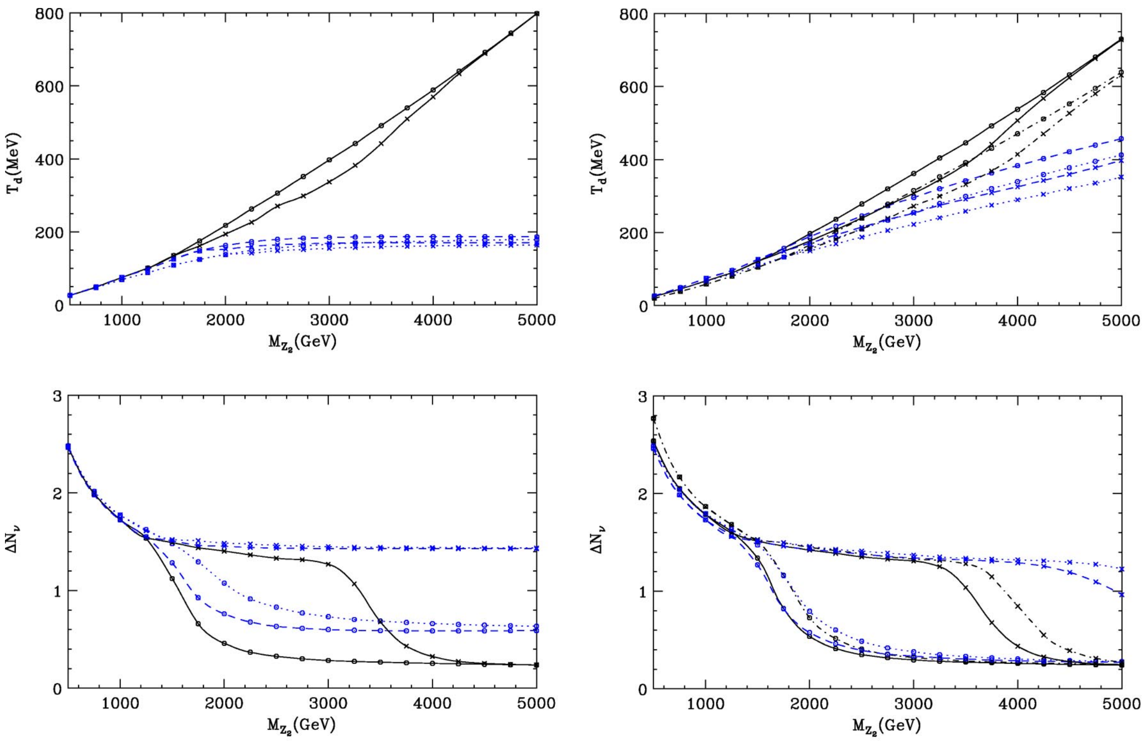

FIG. 2. The decoupling temperature T_d (top) and the equivalent number of extra neutrinos ΔN_ν (bottom) for the η model as a function of the Z_2 mass M_{Z_2} for constituent quark masses, for a quark-hadron transition temperature $T_c=150$ MeV (circles) and 400 MeV (crosses). The left two figures are for the cases A0 and A3 defined in Eq. (2.6), i.e., the solid, dashed and dotted lines represent zero mixing ($\delta=0$), and positive and negative maximal mixing ($\delta=\pm 0.002$), respectively. The $T_c=150$ MeV case has higher T_d and lower ΔN_ν for the same M_{Z_2} than $T_c=400$ MeV. The right figures are for the intermediate mixing assumptions A1 and A2. The solid and dash-dot curves are for the mass-mixing relations $\delta=\pm 0.0051/M_{Z_2}^2$, while the dashed and dotted curves are for the ρ_0 constraints $\delta=\pm 0.0029/M_{Z_2}$.

Eq. (2.6). The no-mixing curves A0 exhibit an approximate $T_d\sim(M_{Z_2}/M_Z)^{4/3}$ dependence, in agreement with the simple estimate in the Introduction [21,22]. This is to be roughly expected because of the $M_{Z_2}^{-4}$ dependence of the cross section for no mixing, but is not exact because additional channels which affect both the expansion and interaction rates open up at higher temperatures. The no-mixing curves in Figures 2 and 3 are reasonably described by Eq. (1.4) for $T_d(\nu_L)\sim 3$ MeV for the η model, but the coefficients in front of $(M_{Z_2}/M_Z)^{4/3}$ are strongly model dependent, as is apparent in Figs. 4 and 5. T_d is usually lower in the cases involving Z-Z' mixing, because the Z annihilation channel yields a contribution proportional to δ^2 even for infinite M_{Z_2}. That is why the (theoretically unrealistic) curves A3 for fixed $|\delta|=0.002$ are asymptotically flat for large M_{Z_2}. Case A1, in which $|\delta|\sim 0.0051/M_{Z_2}^2$, also has $T_d\sim(M_{Z_2}/M_Z)^{4/3}$, though with a smaller coefficient than for no mixing [53], while A2, with $|\delta|=0.0029/M_{Z_2}$, has $T_d\sim(M_{Z_2}/M_Z)^{2/3}$. For case A1, T_d is asymmetric under $\delta\rightarrow-\delta$ for all M_{Z_2}, as is apparent from Eqs. (2.6) and (4.3). The difference vanishes asymptotically for A2 and A3, but even for $M_{Z_2}=5$ TeV there is still a difference, especially for A2.

The decoupling temperature is slightly lower for $T_c=400$ MeV than for 150 MeV, provided it is in the range for which the two curves in Fig. 1 differ. Both the expansion and annihilation rates are smaller for $T_c=400$ MeV, but the effect on the expansion rate is more important because of the gluonic degrees of freedom. Similarly, T_d is smaller for current quark masses than for constituent masses, provided $T_d>T_c$, because of the larger annihilation rate [54].

The ΔN_ν curves change rapidly when T_d reaches the quark-hadron phase transition temperature T_c, where $g(T)$ changes significantly. That is why ΔN_ν is so much larger for $T_c=400$ MeV than for 150 MeV. For the no-mixing case, the difference is significant for $M_{Z_2}\lesssim 4$ TeV, and it persists to even higher masses for the mixing cases (and to infinite mass for maximal mixing). The only significant difference between the constituent and current quark masses is in the maximal mixing case with $T_c=150$ MeV. That is because T_d is very close to T_c, and even a small change in T_d leads to a significant change in $g(T)$, as can be seen in Fig. 1.

It is apparent from Figs. 2 and 3 that the η model leads to a significant ΔN_ν for all of the cases and parameter ranges considered. Even the very conservative constraint $\Delta N_\nu<1$ implies $M_{Z_2}>1.5-2.2$ TeV for $T_c=150$ MeV, or, limiting ourselves to the most realistic cases A0 and A1, $M_{Z_2}>1.5-1.9$ TeV. For $T_c=400$ MeV one finds $M_{Z_2}>3.3-4$ TeV for A0 and A1, $M_{Z_2}>5$ TeV for A2 and no allowed values for A3. All of these are much more stringent than the direct laboratory limit of 620 GeV [7] or the indirect limits from

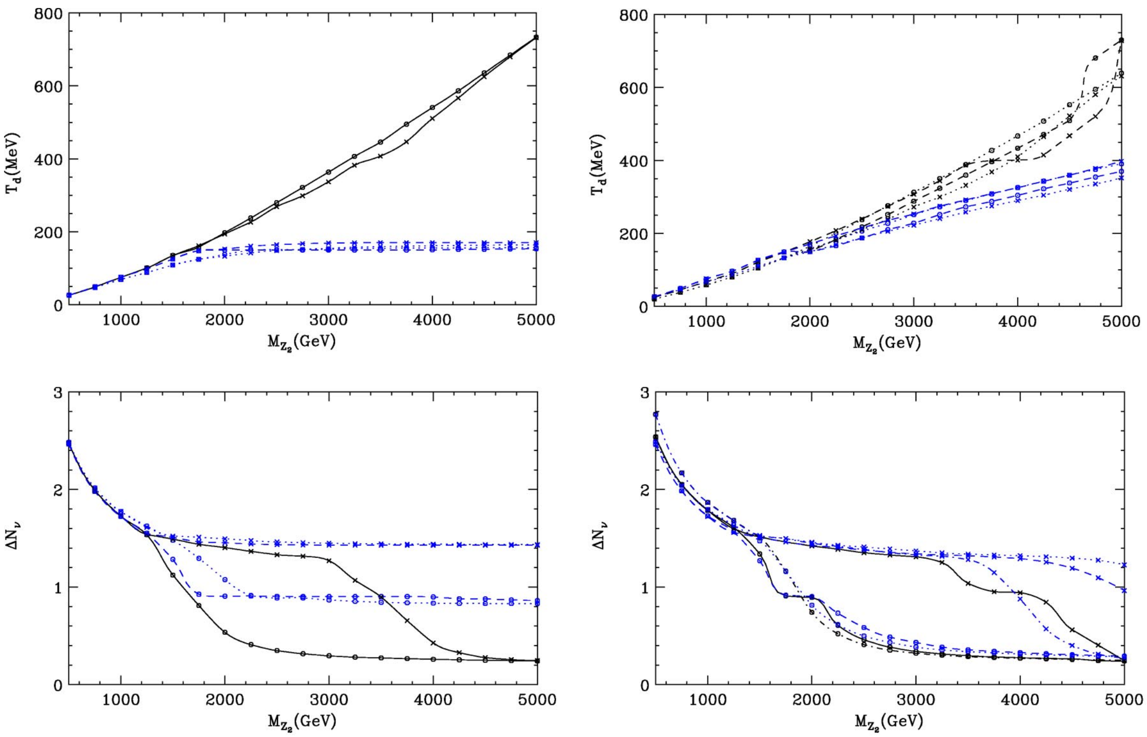

FIG. 3. Same as Fig. 2 except that current quark masses are used. The upper graphs share most features with the constituent mass case except that T_d can be slightly lower when $T_d > T_c$. The only significant change in ΔN_ν is for the $T_c = 150$ MeV maximal mixing case (see text).

precision electroweak data [9]. The more stringent limit $\Delta N_\nu < 0.3$ is satisfied for cases A0 and A1 for $M_{Z_2} > 2.5-3.2$ TeV for $T_c = 150$ MeV, and $M_{Z_2} > 4.0-4.9$ TeV for $T_c = 400$ MeV. It is not satisfied for case A2 with $T_c = 400$ MeV until extremely high masses, and never for (fixed) maximal mixing unless one takes a mixing much smaller than the present accelerator limit ($|\delta| < 0.0024$) [8].

Figures 4 and 5 display the results for the class of E_6 models parametrized by the angle θ_{E6} defined in Eq. (2.2), for constituent masses and $T_c = 150$ MeV and 400 MeV, respectively. Each figure includes the no-mixing case and the mixing assumption A1 defined in Eq. (2.6), which is the most stringent and realistic. The limits in the presence of Z-Z' mixing are asymmetric under $\delta \to -\delta$. This is represented in the right-handed graphs by taking $\delta < 0$ but allowing θ_{E6} to run from 0 to 2π, so that the $(\pi - 2\pi)$ range for $\delta < 0$ is equivalent to $(0-\pi)$ with $\delta > 0$. The top graphs display T_d as a function of θ_{E6} for fixed values $M_{Z_2} = 500$, 1000, 1500, 2000, 2500, 3500, 4000, and 5000 GeV, with larger M_{Z_2} corresponding to higher T_d. The middle graphs show ΔN_ν as a function of θ_{E6} for the same values of M_{Z_2}, with larger M_{Z_2} corresponding to smaller ΔN_ν. The bottom figures show the lower bounds on M_{Z_2} for $\Delta N_\nu < 0.3$, 0.5, 1.0 and 1.2, with larger ΔN_ν corresponding to smaller M_{Z_2}.

It is seen that T_d becomes very large and the M_{Z_2} limits essentially disappear as θ_{E6} approaches $\theta_{E6} \sim 0.42\pi$ or 1.42π, for which ν_R decouples completely $[Q(\nu_R) = 0]$, but the details depend on the new physics at the electroweak and higher scales (we only explicitly included particles up to the b quark). $\theta_{E6} = 1.71\pi$ corresponds to the η model with $\delta < 0$, while $\theta_{E6} = 0.71\pi$ corresonds to $\delta > 0$. It is seen from the figures that ΔN_ν is larger for values of θ_{E6} closer to 0 (the χ model), but are weaker near $\theta_{E6} = \pi/2$ (the ψ model).

From the figures it is apparent that requiring $\Delta N_\nu \leqslant 1$ excludes much of the interesting parameter space for $T_c = 150$ MeV, except for large Z_2 masses or regions very close to the ν_R decoupling angles $\sim 0.42\pi$ and 1.42π. In particular, the $\Delta N_\nu \leqslant 1$ constraint is satisfied for all values of θ_{E6} for $M_{Z_2} \gtrsim 2.2$ TeV if there is no mixing, with a slightly more stringent constraint $M_{Z_2} \gtrsim 2.4$ TeV for mixing assumption A1. The corresponding M_{Z_2} limits for $\Delta N_\nu \leqslant 0.3$ are 3.8 and 4.3 TeV. The constraints for $T_c = 400$ MeV are even more stringent, essentially requiring ν_R decoupling or very large Z_2 masses. One has $\Delta N_\nu \leqslant 1(0.3)$ for all θ_{E6} for cases A0 and A1 for $M_{Z_2} \gtrsim 5.1(6.1)$ TeV.

VI. DISCUSSION AND CONCLUSION

Many theories beyond the standard model predict the existence of additional Z' gauge bosons at the TeV scale. The associated $U(1)'$ gauge symmetry often prevents the large Majorana masses needed for an ordinary neutrino seesaw model. One possibility is that the neutrino masses are Dirac and small. In that case, there is a possibility of producing the sterile right-handed neutrino partners ν_R via Z' interactions prior to nucleosynthesis [21,22], leading to a faster expansion and additional ^{4}He.

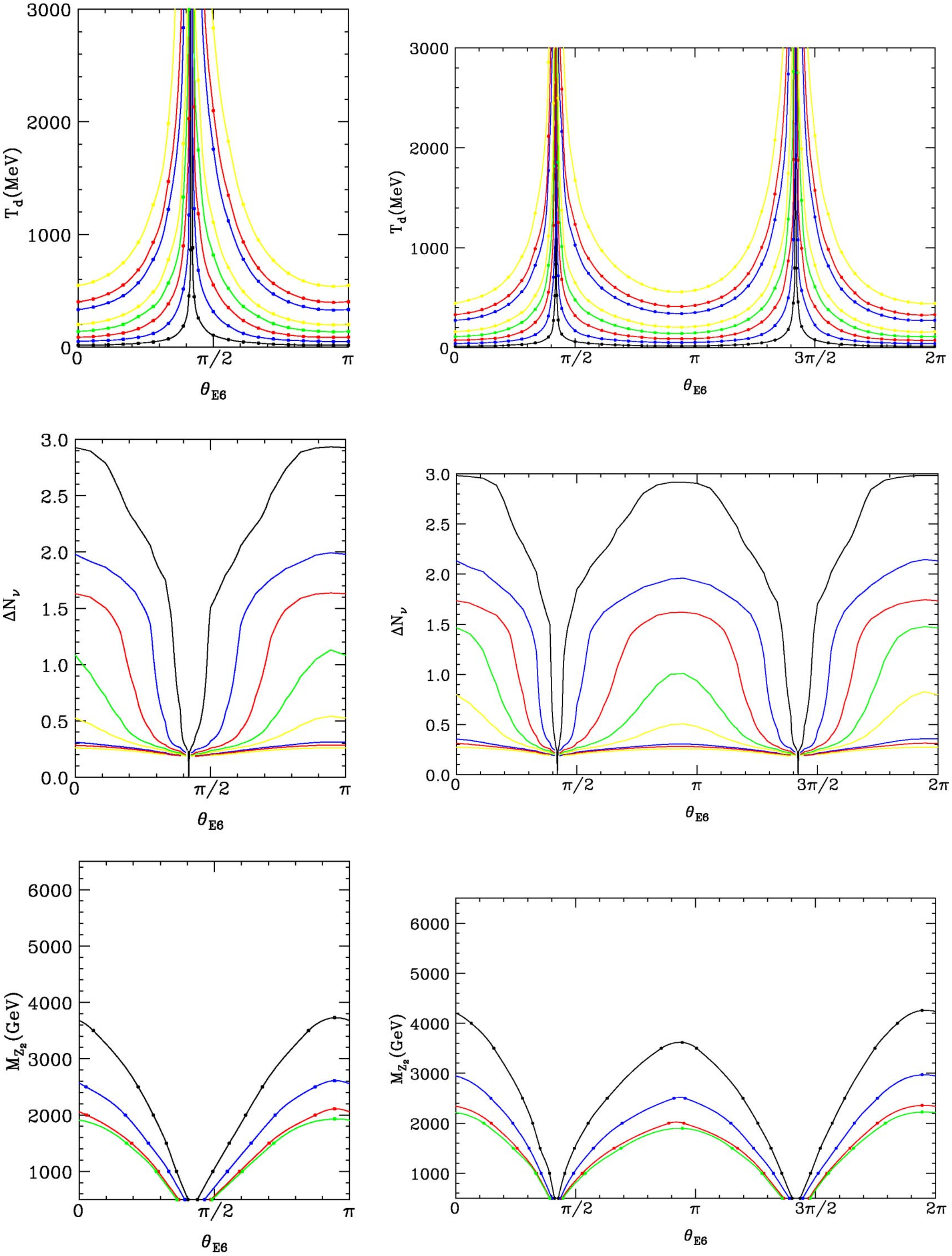

FIG. 4. T_d (top) and ΔN_ν (middle) for $M_{Z_2}=500$, 1000, 1500, 2000, 2500, 3500, 4000, and 5000 GeV, for $T_c=150$ MeV and constituent masses. Larger M_{Z_2} corresponds to higher T_d and smaller ΔN_ν. The graphs on the left are for no mixing [case A0 in Eq. (2.6)], while the right-hand graphs are for the mass-mixing relation $|\delta|<0.0051/M_{Z_2}^2$ (case A1). The bottom graphs are M_{Z_2} corresponding to $\Delta N_\nu=0.3$, 0.5, 1.0, and 1.2, with larger ΔN_ν corresponding to smaller M_{Z_2}.

We have studied the right-handed neutrino decoupling temperature T_d in a class of E_6-motivated $U(1)'$ models as a function of the Z' mass and couplings (determined by an angle θ_{E6}) for a variety of assumptions concerning the Z-Z' mixing angle δ, the quark-hadron transition temperature T_c, and the nature (constituent or current) of the quark masses. We have taken all relevant channels (quark, gluon, lepton, and hadron) into account, not only in the expansion rate $H(T)$ and entropy, but also in the rate $\Gamma(T)$ for a massless right-handed neutrino pair to annihilate into a fermion or pion pair via the ordinary or heavy Z bosons. We therefore obtain a larger annihilation rate, and thus a lower decoupling

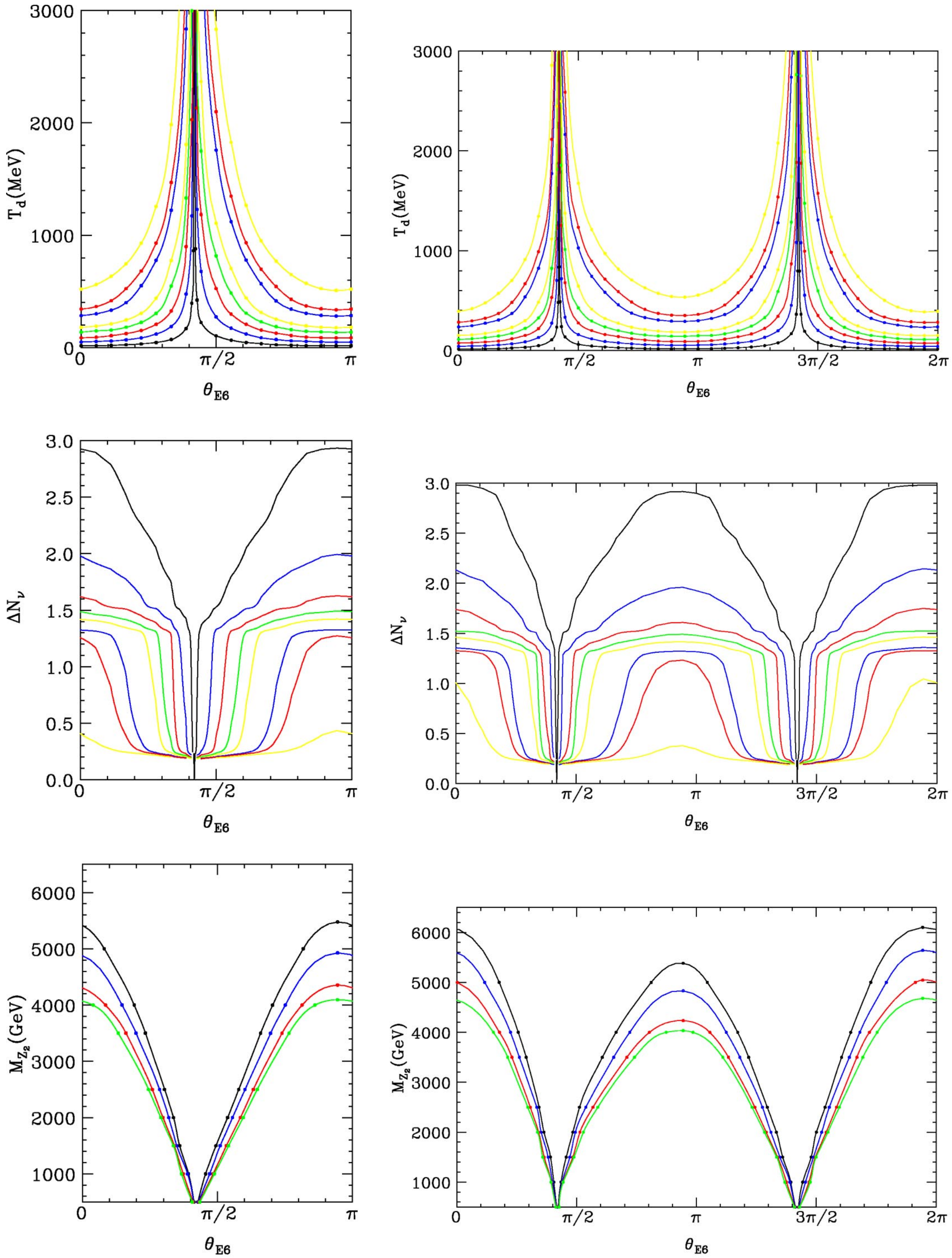

FIG. 5. Same as Fig. 4, except $T_c = 400$ MeV. T_d is slightly smaller (for $T_d > 150$ MeV) for fixed M_{Z_2} and θ_{E6}, while ΔN_ν and the bound on M_{Z_2} for fixed ΔN_ν are increased.

temperature and more stringent constraints, than earlier calculations, which only included annihilation into e^+e^- and $\nu_L \overline{\nu_L}$.

From the decoupling temperature and entropy conservation as quarks and gluons are confined or as various heavy particle types decouple and annihilate, one can obtain the number of right-handed neutrinos at nucleosynthesis, expressed in terms of the equivalent number ΔN_ν of new ordinary neutrino species, for various sets of model parameters M_{Z_2}, δ, θ_{E6}, and T_c. Most recent studies of the primordial abundances obtain upper limits on ΔN_ν in the range (0.3–1) [26,28]. As can be seen in Figs. 4 and 5, this implies rather stringent constraints on the Z' parameters for most values of θ_{E6}. For $T_c = 150$ MeV, the constraint $\Delta N_\nu < 0.3(1)$ is satisfied for all θ_{E6} for $M_{Z_2}\gtrsim 3.8(2.2)$ TeV for no Z-Z' mixing, and for $M_{Z_2}\gtrsim 4.3(2.4)$ TeV allowing the range of mixing angles δ obtained approximately when one assumes that the scalar fields responsible for the mixing are contained in the 27 or $\overline{27}$-plet of E_6 [case A1 in Eq. (2.6)]. For $T_c=400$ MeV the constraints are much stronger, $M_{Z_2}\gtrsim 6.1(5.1)$ TeV for $\Delta N_\nu<0.3(1)$. The strong dependence on T_c is due to the large increase in the number of degrees of freedom for temperatures $\gtrsim T_c$ [Fig. 1], so that the number density of ν_R is strongly diluted for $T_d\gtrsim T_c$. The constraints are strongest for θ_{E6} close to 0 or π, i.e., near the χ model, which corresponds to $SO(10)\rightarrow SU(5)\times U(1)_\chi$, and are very weak near the ψ model corresponding to $E_6\rightarrow SO(10)\times U(1)_\psi$, $\theta_{E6}=\pi/2$. They disappear entirely at the values $\theta_{E6}=0.42\pi$ and 1.42π, for which the ν_R decouple from the Z'. The often considered η model, $\theta_{E6}=2\pi-\tan^{-1}\sqrt{(5/3)}=1.71\pi$ (or 0.71π for $-Z_\eta$) is somewhere in between, with the constraints shown in more detail in Figs. 2 and 3.

Except near the ν_R decoupling angles, the Z' mass and mixing constraints from nucleosynthesis are much more stringent than the existing laboratory limits from searches for direct production or from precision electroweak data, and are comparable to the ranges that may ultimately be probed at proposed colliders. They are qualitatively similar to the limits from energy emission from supernova 1987A [24], but somewhat more stringent for $\Delta N_\nu<0.3$, and have entirely different theoretical and systematic uncertainties.

There are several ways to evade the nucleosynthesis constraints on an extra Z'. One possibility is to generate small Majorana neutrino masses for the ordinary neutrinos by invoking an extended seesaw model [17], in which the extra sterile neutrinos are typically at the TeV scale. Another possibility is that the ν_R decouple from the Z', in which case the constraints disappear. This can in fact occur naturally in classes of models in which one combination of the χ and ψ charges is broken at a large scale associated with an F and D-flat direction [55], leaving a light Z' which decouples from the ν_R [56]. Yet another possibility is to weaken the observational constraint on ΔN_ν by allowing a large excess [57] of ν_e with respect to $\bar{\nu}_e$. This would, however, require a somewhat fine-tuned cancellation between the effects of the ν_R and the ν_e-$\bar{\nu}_e$ asymmetry.

Similar constraints on the W' and Z' properties in $SU(2)_L\times SU(2)_R\times U(1)$ models [20] are under investigation [59].

The precision Wilinson Microwave Anisotropy Probe (WMAP) data [60] on cosmic microwave background (CMB) anisotropies were announced shortly after the submission of this paper. Several authors have shown that these give improved bounds on N_ν [61–64]. For example, for a flat universe, [61] finds $\Delta N_\nu<6$ at 95% C.L. from the WMAP data alone and $-2.6<\Delta N_\nu<4$ with a prior on the Hubble constant of $H_0=72\pm 8$ km s^{-1} Mpc^{-1}. A more significant improvement on N_ν is obtained by combining the CMB constraint on $\Omega_b h^2$ with BBN constraints. For example, [62] finds $N_\nu=2.5\pm 0.5$ at 95% C.L. from a combined CMB plus BBN analysis. The condition that $N_\nu\leqslant 3$ would very severely constrains the Z' mass in our model with three right-handed neutrinos.

ACKNOWLEDGMENTS

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