Neutrino democracy, fermion mass hierarchies, and proton decay from 5D SU(5)

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The explanation of various observed phenomena such as large angle neutrino oscillations, hierarchies of charged fermion masses and CKM mixings, and apparent baryon number conservation may have a common origin. We show how this could occur in 5D SUSY SU(5) supplemented by a $\mathcal{U}(1)$ flavor symmetry and additional matter supermultiplets called "copies." In addition, the proton decays into $p\to K\nu$, with an estimated lifetime of the order of $10^{33}-10^{36}$ yr. Other decay channels include Ke and $K\mu$ with comparable rates. We also expect that $\mathrm{BR}(\mu\to e\gamma)\sim\mathrm{BR}(\tau\to\mu\gamma)$.

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The neutrino sector of electroweak interactions is one of the windows which sheds light on physics beyond the standard model. The SuperKamiokande (SK) experiments provide credible evidence for atmospheric [1] and solar [2] neutrino oscillations. The atmospheric neutrino anomaly suggests $\nu_\mu\to\nu_\tau$ oscillations with $\Delta m^2_{\mathrm{atm}}\simeq 2\times10^{-3}$ eV^2 and a nearly maximal mixing angle $\sin^2 2\theta_{\mu\tau}\simeq 1$. For solar neutrinos the preferred solution seems to be the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) one with the parameters $\Delta m^2_{\mathrm{sol}}\simeq 6\times10^{-5}$ eV^2, $\sin^2 2\theta_{e\mu,\tau}\approx 0.8$. In contrast with this, the quark Cabibbo-Kobayashi-Maskawa (CKM) mixing angles are small and there are noticeable intergeneration hierarchies between the charged fermion masses. It is tempting to think that there is some underlying framework responsible for the generation of fermion masses and mixings. A particularly attractive possibility is an Abelian $\mathcal{U}(1)$ flavor symmetry [3] which can be quite effective in the charged fermion [4] and neutrino [5] sectors.

With a $\mathcal{U}(1)$ (or for that matter any flavor) symmetry perhaps the most intriguing and challenging task is to understand the origin of large (for $\nu_\mu-\nu_\tau$ even maximal) neutrino mixing versus the small CKM mixing angles. One promising scheme is the *democratic approach* to neutrinos [6,7] in which the left handed lepton doublets l are not distinguished by $\mathcal{U}(1)$ and consequently can mix strongly. In contrast with l, the quarks and right handed leptons have distinct $\mathcal{U}(1)$ charges so that the mass hierarchies between them can be realized.

In a recent work [7] we examined the democratic approach within the minimal supersymmetric standard model (MSSM) and the grand unified theory (GUT) framework. It was shown that it is difficult to realize neutrino "democracy" in SU(5) in a straightforward way, whereas an extended version of flipped SU(5) allows such an implementation. The difficulties faced in SU(5) mainly arise because the $(10+\overline{5})_\alpha$ ($\alpha=1,2,3$ is a generation index) multiplets unify the quark and lepton fields, leaving us with less freedom in their $\mathcal{U}(1)$ charge assignments. This leads to an unacceptably small value for V_{us} ($\sim 1/125$), and a 4D scenario for resolving this is still missing. To overcome this, one can think of some reasonable extension in such a way as to "split" the fermion fragments of GUT multiplets and thereby relax the unwanted constraints on $\mathcal{U}(1)$ charges. This may not be a trivial task in SU(5) from one's earlier experience with the doublet-triplet (DT) splitting problem in the scalar sector which ends up requiring a rather complicated extension. However, these arguments and also the discussions of Ref. [7] are valid for four-dimensional constructions. Recent developments in higher-dimensional orbifold constructions [8–14] have shown that many outstanding problems of GUTs can be resolved in an extra-dimensional setting. Namely, the orbifold setting can be exploited to yield natural DT splitting, GUT symmetry breaking, and baryon number conservation (to the desired level). Note that the concept of "split" multiplets has previously appeared in the framework of superstring theories [15].

In this article we will apply the orbifold approach for realizing the observed fermion mass pattern consistent with neutrino democracy within 5D SUSY SU(5), supplemented by a $\mathcal{U}(1)$ flavor symmetry. We show how the problems discussed above can be nicely avoided by invoking a fifth dimension, with additional supermultiplets (so-called "copies") playing an essential role. The $\mathcal{U}(1)$ symmetry also helps ensure sufficient proton stability. Due to democracy in the left handed lepton sector, the decays $p\to Ke$, $p\to K\mu$ are expected to proceed at comparable rates. Similarly, the branching rates in the lepton sector satisfy $\mathrm{BR}(\mu\to e\gamma)\sim\mathrm{BR}(\tau\to\mu\gamma)$, which may be testable in the future.

Consider 5D $N=1$ SUSY SU(5) compactified on an $S^{(1)}/Z_2\times Z_2'$ orbifold, such that the "low" energy theory has the MSSM field content [8]. In 4D $N=1$ superfield notation, the 5D gauge supermultiplet is $V_{N=2}=(V,\Sigma)$, where V and Σ respectively are vector and chiral superfields, both in the adjoint **24** representation of SU(5). In terms of $SU(3)_c\times SU(2)_L\times U(1)_Y\equiv G_{321}$, $V(24)=V_c(8,1)_0+V_w(1,3)_0+V_s(1,1)_0+V_X(3,\overline{2})_5+V_Y(\overline{3},2)_{-5}$, where the subscripts denote the hypercharge $Y=(1/\sqrt{60})\mathrm{diag}(2,2,2,-3,-3)$ in the units of $1/\sqrt{60}$. Similar decomposition holds for

$\Sigma(24)$. It is assumed that the fifth space like dimension y describes a compact $S^{(1)}$ circle with radius R. Under the $Z_2\times Z_2'$ symmetry Z_2: $y\to -y$, Z_2': $y'\to -y'$ ($y'=y+\pi R/2$), all states should have definite $Z_2\times Z_2'$ parities (P,P'). With the following parity assignments for the fragments from $V_{N=2}(24)$:

$$(V_c,V_w,V_s)\sim(+,+),\quad (V_X,V_Y)\sim(-,+),$$
$$(\Sigma_c,\Sigma_w,\Sigma_s)\sim(-,-),\quad (\Sigma_X,\Sigma_Y)\sim(+,-), \tag{1}$$

on the fixed point $y=0$ (identified with our 4D world) we will have $N=1$ SUSY with massless G_{321} gauge bosons. The remaining states in $SU(5)/G_{321}$ acquire large (GUT scale) masses. Thus, states with parities $(\pm,\pm)$, $(\pm,\mp)$ respectively, have masses $(2n+2)/R$, $(2n+1)/R$, where n denotes the quantum number in the Kaluza-Klein (KK) mode expansion.

The SU(5) "Higgs" superfields, which contain the pair of MSSM Higgs doublets, are also introduced in the bulk. Namely, there are two $N=2$ supermultiplets $\mathcal{H}_{N=2}(5)=(H,\overline{H}')$, $\overline{\mathcal{H}}_{N=2}(\overline{5})=(\overline{H},H')$, where H, $\overline{H}$ are 5, $\overline{5}$ plets of SU(5) and $\overline{H}'$, H', respectively, are their mirrors. In terms of G_{321}, $H(5)=h_u(1,2)_{-3}+T(3,1)_2$, $\overline{H}(\overline{5})=h_d(1,\overline{2})_3+\overline{T}(\overline{3},1)_{-2}$, and similarly for H', $\overline{H}'$. With $Z_2\times Z_2'$ parity assignments

$$(h_u,h_d)\sim(+,+),\quad (h_d{}',h_u{}')\sim(-,-),$$
$$(T,\overline{T})\sim(-,+),\quad (\overline{T}',T')\sim(+,-), \tag{2}$$

only h_u, h_d have zero modes and can be identified with the MSSM doublets. All colored triplet partners become superheavy and in this way the DT splitting occurs naturally. Note that since h_u, h_d arise from different $N=2$ supermultiplets, the mass term $M_h h_u h_d$ is not allowed in 5D. This can be considered a good starting point for obtaining an adequately suppressed μ term (however, at 4D level additional care must be exercised [12,14] for avoiding a large μ term).

In orbifold constructions with a minimal setting, the introduction of fermions in the bulk is not straightforward. For example, the d^c and l states, which come from the same $\overline{5}$ plet, are involved in a 5D kinetic coupling $l^+V_Xd^c$ and, since the V_X boson has parity $(-,+)$ [see Eq. (1)], either d^c or l should have parity $(-,+)$. This would mean loss of some (zero mode) MSSM chiral states. For overcoming this difficulty, one could attempt to introduce chiral states not in the bulk but directly on a brane with no KK excitations. Although the theory would be fully self-consistent, we follow a different procedure here and introduce in the bulk additional supermultiplets [10] called "copies" (see first and third citations in Ref. [11]), denoted by $10'+\overline{5}'$ (per generation). By suitable prescription of $Z_2\times Z_2'$ parities, these states allow one to realize at low energies complete three generations of MSSM massless chiral states. It will turn out that the introduction of these copies enables us to realize the democratic approach for neutrinos and obtain a nice picture for the charged fermion sector. From this point of view, the motivation for introducing copies therefore becomes twofold.

In the bulk we introduce three generations of $N=2$ supermultiplets $\mathcal{X}_{N=2}=(10,\overline{10})$, $\mathcal{V}_{N=2}=(\overline{5},5)$ together with their copies $\mathcal{X}'_{N=2}$, $\mathcal{V}'_{N=2}$. Recall that in terms of G_{321} $10=e^c(1,1)_{-6}+q(3,2)_{-1}+u^c(\overline{3},1)_4$, $\overline{5}=l(1,\overline{2})_3+d^c(1,\overline{3})_{-2}$ and likewise for $10'$, $\overline{5}'$. The $\overline{10}$, 5, $\overline{10}'$, $5'$ are mirrors and their fragments have conjugate transformation properties under G_{321}. The following orbifold parity assignments:

$$(q',l,u^c,d^{c\prime},e^c)\sim(+,+),$$
$$(q,l',u^{c\prime},d^c,e^{c\prime})\sim(-,+), \tag{3}$$

with opposite parities for the corresponding mirrors (we assume generation independent parities), are consistent with the prescriptions in Eq. (1), and it is easy to verify that all $N=2$ SUSY invariant terms also possess $Z_2\times Z_2'$ invariance. From Eq. (3) we see that the states $e^c,q',u^c,l,d^{c\prime}$ contain zero modes which we identify with the three chiral quark-lepton families of MSSM.

In addition, we introduce a $\mathcal{U}(1)$ flavor symmetry (on whose origin we will comment later) and a singlet superfield X carrying $\mathcal{U}(1)$ charge $Q_X=-1$. We assume $\langle X\rangle/M\equiv\epsilon\simeq 0.2$ (M is some cut off close to the fundamental scale). Because of the fact that the "matter" states come from different SU(5) multiplets, the constraints on $\mathcal{U}(1)$ charge assignments are more relaxed (this turns out to be sufficient to obtain a nice and consistent picture). We have only one constraint

$$Q[u^c{}_\alpha]=Q[e^c{}_\alpha]. \tag{4}$$

The $\mathcal{U}(1)$ charges for matter states are chosen as follows:

$$Q[d_1^{c}{}']=b+c-a+k+2,$$
$$Q[d_2^{c}{}']=Q[d_3^{c}{}']=b+c-a+k,$$
$$Q[u_1^c]=Q[e_1^c]=b+5,\quad Q[u_2^c]=Q[e_2^c]=b+2,$$
$$Q[u_3^c]=Q[e_3^c]=b,\quad Q[l_1]=Q[l_2]=Q[l_3]=c+k,$$
$$Q[q_1']=a+3,\quad Q[q_2']=a+2,\quad Q[q_3']=a \tag{5}$$

($k\geqslant 0$ is an integer and a, b, c are some phases undetermined for the time being). Note that Eq. (4) is satisfied for each generation and l_α all have the same $\mathcal{U}(1)$ charge. Assuming $Q(h_u)=-a-b$, $Q(h_d)=-b-c$, the relevant couplings generating the up and down quark and charged lepton masses, respectively, are

$$\begin{array}{c} \\ q_1' \\ q_2' \\ q_3' \end{array}\begin{array}{c} \begin{matrix} u_1^c & u_2^c & u_3^c \end{matrix} \\ \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix} \end{array} h_u, \quad \begin{array}{c} \\ q_1' \\ q_2' \\ q_3' \end{array}\begin{array}{c} \begin{matrix} d_1^{c\prime} & d_2^{c\prime} & d_3^c \end{matrix} \\ \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \end{pmatrix} \end{array} \epsilon^k h_d, \tag{6}$$

$$\begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array}\begin{array}{c} \begin{matrix} e_1^c & e_2^c & e_3^c \end{matrix} \\ \begin{pmatrix} \epsilon^5 & \epsilon^2 & 1 \\ \epsilon^5 & \epsilon^2 & 1 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix} \end{array} \epsilon^k h_d. \tag{7}$$

The expressions (6), (7) and all other relevant couplings presented below are assumed to be written on $y=0$ fixed point in terms of 4D superfields (after rescaling from 5D is performed). The entries in textures (6) and (7) are taken for simplicity to be real and are accompanied by factors of order unity (here we will not concern ourselves with CP violating phases). Diagonalization of Eqs. (6), (7) yields for the Yukawa couplings

$$\lambda_t \sim 1, \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^8 : \epsilon^4 : 1, \tag{8}$$

$$\lambda_b \sim \lambda_\tau \sim \epsilon^k, \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^5 : \epsilon^2 : 1, \tag{9}$$

$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1, \tag{10}$$

which have the desired hierarchical pattern. More precisely, $\lambda_t \sim (\mu_0/M)^{3/2}$, where μ_0 and M are the compactification and fundamental scales, respectively. If $\mu_0 \sim M$, we obtain $\lambda_t \sim 1$. However, $\mu_0 < M$ is also possible if there is an infrared fixed point solution for the top Yukawa coupling. From Eq. (6), we find

$$V_{us} \sim \epsilon, \quad V_{cb} \sim \epsilon^2, \quad V_{ub} \sim \epsilon^3, \tag{11}$$

values that are consistent with the observations.

From Eq. (7) the expected values for the lepton mixing angles are

$$\sin^2 2\theta_{\mu\tau} \sim 1, \quad \sin^2 2\theta_{e\mu,\tau} \sim 1, \tag{12}$$

which nicely fit with the SK data. To generate neutrino masses we introduce two right handed neutrinos N, N' [in 5D they are accompanied by appropriate mirrors $\overline{N}$, $\overline{N}'$ with parities $(-,-)$] with $\mathcal{U}(1)$ charges $Q(N)=p+1/2$, $Q(N')=q+1/2$ (p,q are positive integers). With

$$a+b=\frac{1}{2}, \quad c=0, \tag{13}$$

and taking into account Eq. (5) the relevant couplings are

$$\epsilon^{k+p}(\lambda_1 l_1+\lambda_2 l_2+\lambda_3 l_3)Nh_u+\epsilon^{2p+1}M_N N^2+\epsilon^{k+q}(\lambda_1' l_1 + \lambda_2' l_2+\lambda_3' l_3)N' h_u+\epsilon^{2q+1}M_N' N'^2+\epsilon^{p+q+1}M_{NN'}NN', \tag{14}$$

where λ_α, λ_α' are dimensionless coefficients of order unity. With $p>q$, $\epsilon^{2q}M_N' \gg \epsilon^{2p}M_{NN'}$, $M_{NN'}^2 \ll M_N M_N'$ (these assumptions are needed for the correct scales of lepton number violations whose origin is still unexplained in this setting), integration of N, N' states leads to the neutrino mass matrix

$$m_{\alpha\beta}^\nu = \lambda_\alpha \lambda_\beta m + \lambda_\alpha' \lambda_\beta' m', \tag{15}$$

where $m=\epsilon^{2k-1}h_u^2/M_N$, $m'=\epsilon^{2k-1}h_u^2/M_N'$. For $M_N/\epsilon^{2k-1} \simeq 2\times 10^{14}$ GeV and $M_N'/\epsilon^{2k-1} \simeq 1.2\times 10^{15}$ GeV we have $m \simeq 5\times 10^{-2}$ eV, $m' \simeq 8\times 10^{-3}$ eV. Ignoring the subleading term in Eq. (15), one finds that only m_{ν_3} acquires a mass $m_{\nu_3}=(\lambda_1^2+\lambda_2^2+\lambda_3^2)m$, while in this limit $m_{\nu_1}=m_{\nu_2}=0$. Therefore, $\Delta m_{\rm atm}^2 \simeq m_{\nu_3}^3 \sim 10^{-3}$ eV^2. The second term in Eq. (15) gives rise to a mass $m_{\nu_2} \sim m'$, so that $\Delta m_{\rm sol}^2 \sim m'^2 \simeq 6\times 10^{-5}$ eV^2, the scale relevant for LAMSW solution. We therefore conclude that the desirable $\nu_e \to \nu_{\mu,\tau}$, $\nu_\mu \to \nu_\tau$ oscillation scenarios are realized within our 5D framework.

Let us now turn to the issue of baryon number and matter parity violation. With the selections (5), (13), it is easy to verify that matter parity violating operators lh_u, $e^c ll$, $q' l d^{c\prime}$, $u^c d^{c\prime} d^{c\prime}$ are forbidden to all orders if a is either an integer, or $a>2k+1/2$. The $\mathcal{U}(1)$ symmetry can also forbid dimension three and four baryon number violating operators. As far as the $d=5$ operators are concerned, because of the absence of zero modes in colored triplet "scalar" states, potential nucleon decay through their exchange does not arise in orbifold SUSY SU(5). The nonrenormalizable $d=5$ operators $q'q'q'l$ and $u^c u^c d^{c\prime} e^c$ are eliminated if both $3a$ and $5a$ are nonintegers (this choice is also compatible with matter parity conservation).

Dimension five nucleon decay at measurable rates could occur through the couplings $(\overline{\lambda}/M)q_1' q_1' q_2' l_\alpha$ if $a=(3m+1)/3$, such that $\overline{\lambda} \sim \epsilon^{9+3m+k}$. For $M \sim 10M_G$ and $(k,m)=(0,1),(1,1),(2,1),(3,0)$, we find $\tau(p\to K\nu_\alpha) \sim 10^{33}-10^{36}$ yr [note that $\tan\beta \sim (m_t/m_b)\epsilon^k$]. The above couplings also lead to proton decay with emission of charged leptons. However, their rates are smaller by a factor of ~ 10. The democratic scenario predicts that $p\to Ke$ and $p\to K\mu$ proceed with nearly equal rates, such that $\tau(p\to Ke) \sim \tau(p\to K\mu) \sim 10^{34}-10^{37}$ yr.

As far as dimension 6 nucleon decay is concerned, with all matter introduced in the bulk and due to the copies, the 5D bulk kinetic terms are irrelevant for nucleon decay [10]. This is because through the exchange of V_X, V_Y bosons, the light quark-lepton states are converted into heavy states with masses of order $1/R$. The only source for nucleon decay could be some brane localized nondiagonal kinetic operators allowed by G_{321} and orbifold symmetries. Such operators have the form [13] $\delta(y)\psi_1^+(\partial_5 e^{2\hat{V}} - \hat{\Sigma} e^{2\hat{V}} - e^{2\hat{V}}\hat{\Sigma})\psi_2$, where

ψ_1 and ψ_2 denote quark and lepton superfields, respectively, and $\hat{V}$, $\hat{\Sigma}$ are fragments from the coset $SU(5)/G_{321}$. In order for these operators be invariant under $\mathcal{U}(1)$, the multiplier $(X^+)^{Q_1}X^{Q_2}$ will be present, where Q_1, Q_2 are the $\mathcal{U}(1)$ charges of ψ_1, ψ_2. If either Q_1 or Q_2 is not an integer, the corresponding operator is not allowed. From Eqs. (5),(13) we verify that either $Q[q_\alpha{}']$ or $Q[u^c_\alpha]=Q[e^c_\alpha]$ is not an integer, and consequently the couplings $(q'^+e^c+u^{c+}q')\partial_5 V_X$, $(e^{c+}q'+q'^+u^c)\partial_5 V_Y$ are absent. For $a=(2m+1)/4$ (m is an integer), the couplings $l^+d^{c\prime}\partial_5 V_X(X^+)^k X^{m-k}$ will appear, but these terms alone do not induce nucleon decay. Thus, thanks to the $\mathcal{U}(1)$ symmetry, $d=6$ nucleon decay is absent.

We conclude with some observations.

(a) The leptonic mixing angles θ_{12}, θ_{23} receive contributions both from the charged lepton and neutrino sectors. In the absence of cancellations between these contributions we expect Eq. (12) to hold naturally. However, the CHOOZ data [16] requires $\theta_{13}\lesssim 0.2(\simeq\epsilon)$, so that some cancellation between contributions from the two sectors is needed. If future measurements turn out to favor a much smaller ($\ll\epsilon$) θ_{13}, then some new explanation would be required.

(b) The democratic approach also has important implications for lepton flavor violating rare proccesses. Since the neutrino Dirac Yukawa couplings in Eq. (14) for different families are all of the same order, one can expect that $\mathrm{BR}(\mu\to e\gamma)\sim\mathrm{BR}(\tau\to\mu\gamma)$. For universal (at high scale) sparticle masses $\sim$300–500 GeV and $\tan\beta=25-50$, the constraint $\mathrm{BR}\lesssim 10^{-14}$ (the most stringent expected bound for $\mu\to e\gamma$ [17]) requires $p>q=2,3$. For $\tan\beta\sim 1-5$ we can have $p>q=0$.

(c) The origin of lepton number violation scale (masses of right handed neutrinos) is unexplained in this setting. This is not surprising in SU(5), but in GUTs such as SO(10) or even $SU(4)_c\times SU(2)_L\times SU(2)_R$, the violation of lepton number is directly related to the $B-L$ breaking scale which, in a minimal setting, is close to 10^{16} GeV. Thus, it would be interesting to extend the present discussion to such models.

(d) While the $\mathcal{U}(1)$ flavor symmetry can provide an understanding of why proton decay has so far not been seen, it remains to be seen if dimension five operators should be expunged or not. Hopefully, future measurements will shed more light on this fundamental question, help determine some of the $\mathcal{U}(1)$ charges and test the democratic approach by comparing decays with emission of charged leptons.

(e) The $\mathcal{U}(1)$ flavor symmetry can be global or even can be substituted by some discrete $\mathcal{Z}_N$ symmetry which arises in the fermion sector from some more fundamental theory. If $\mathcal{U}(1)$ is introduced in 5D as a vector-like gauge symmetry [18], after compactification it can cause localized anomalies on the orbifold fixed points [19]. Their cancellation could occur through bulk Chern-Simons term, with possibly some additional states playing an essential role [20,12,18].

(f) One could imagine extending the $\mathcal{U}(1)$ flavor symmetry to non-Abelian flavor groups such as $SU(2)_H$ or $SU(3)_H$ within the orbifold constructions. Such flavor symmetries, apart from providing an expanation of hierarchical structures, may yield additional predictions and relations between fermion masses and their mixings.

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