Supersymmetric one-family model without Higgsinos

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The Higgs potential and the mass spectrum of the $N=1$ supersymmetric extension of a recently proposed one family model based on the local gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, which is a subgroup of the electroweak-strong unification group E_6, is analyzed. In this model the slepton multiplets play the role of the Higgs scalars and no Higgsinos are needed, with the consequence that the sneutrino, the selectron, and six other sleptons play the role of the Goldstone bosons. We show how the μ problem is successfully addressed in the context of this model which also predicts the existence of a light CP-odd scalar.

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I. INTRODUCTION

In spite of the remarkable experimental success of our leading theory of fundamental interactions, the so-called standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ [1], it fails to explain several issues such as hierarchical fermion masses and mixing angles, charge quantization, CP violation, and replication of families, among others. These well known theoretical puzzles of the SM have led to the strong belief that the model is still incomplete and that it must be regarded as a low-energy effective field theory originating from a more fundamental one. Among the unsolved questions of the SM, the elucidation of the nature of the electroweak symmetry breaking remains one of the most challenging issues. If the electroweak symmetry is spontaneously broken by Higgs scalars, the determination of the value of the Higgs boson mass M_H becomes a key ingredient of the model. By direct search, CERN e^+e^- collider LEP-II has set an experimental lower bound of $M_H \geqslant 114.4$ GeV [2].

After the proposal of the SM many scenarios for a more fundamental theory have been advocated in several attempts for answering the various open questions of the model. All those scenarios introduce theoretically well motivated ideas associated with physics beyond the SM [3]. Supersymmetry (SUSY) is considered as a leading candidate for new physics. Even though SUSY does not solve many of the open questions, it has several attractive features, the most important one being that it protects the electroweak scale from destabilizing divergences, that is, SUSY provides an answer to why the scalars remain massless down to the electroweak scale when there is no symmetry protecting them (the "hierarchy problem"). This has motivated the construction of the minimal supersymmetric standard model (MSSM) [4], the supersymmetric extension of the SM, that is defined by the minimal field content and minimal superpotential necessary to account for the known Yukawa mass terms of the SM. At present, however, there is no experimental evidence for Nature to be supersymmetric.

In the MSSM it is not enough to add the Higgsino to construct the left chiral Higgs supermultiplet. Because of the holomorphicity of the superpotential and the requirement of anomaly cancellation, a second Higgs doublet together with its superpartner must be introduced. The two Higgs doublets mix via a mass parameter (the so-called μ-parameter) whose magnitude remains to be explained. In addition, since the quartic Higgs self-couplings are determined by the gauge couplings, the mass of the lightest Higgs boson h is constrained very stringently; in fact, the upper limit $m_h \leqslant 128$ GeV has been established [5] (the tree level limit is $m_h \leqslant m_Z$, the mass of the SM neutral gauge boson [6]).

Since at present there are not many experimental facts pointing toward what lies beyond the SM, the best approach may be to depart from it as little as possible. In this regard, $SU(3)_L \otimes U(1)_X$ as a flavor group has been considered several times in the literature; first as a family independent theory [7], and then with a family structure [8,9]. Some versions of the family structure provide a solution to the problem of the number N of families, in the sense that anomaly cancellation is achieved when N is a multiple of three; further, from the condition of $SU(3)_c$ asymptotic freedom which is valid only if the number of families is less than five, it follows that in those models N is equal to 3 [8].

Over the past decade two three family models based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge group (hereafter the 3-3-1 structure) have received special attention. In one of them the three known left-handed lepton components for each family are associated to three $SU(3)_L$ triplets [8] as $(\nu_l, l^-, l^+)_L$, where l_L^+ is related to the right-handed isospin singlet of the charged lepton l_L^- in the SM. In the other model the three $SU(3)_L$ lepton triplets are of the form $(\nu_l, l^-, \nu_l^c)_L$ where ν_{lL}^c is related to the right-handed component of the neutrino field ν_{lL} [9]. In the first model anomaly cancellation implies quarks with exotic electric charges $-4/3$ and 5/3, while in the second one the exotic particles have only ordinary electric charges.

All possible 3-3-1 models without exotic electric charges are presented in Ref. [10], where it is shown that there are just a few anomaly free models for one or three families, all of which have in common the same gauge-boson content.

In this paper we are going to present the supersymmetric version of the one-family 3-3-1 model introduced in Ref. [11]. The non-SUSY version has the feature that the fermion states in the model are just the 27 states in the fundamental representation of the electroweak-strong unification group E_6

[12]. Besides, the scale of new physics for the non-SUSY version of this model is in the range of 1–5 TeV [11,13], so it is just natural to link this new scale with the SUSY scale.

Our main motivation lies in the fact that in the non-SUSY model, the three left-handed lepton triplets and the three Higgs scalars [needed to break the symmetry down to $SU(3)_c \otimes U(1)_Q$ in two steps] transform as the $\bar{3}$ representation of $SU(3)_L$ and have the same quantum numbers under the 3-3-1 structure. This becomes interesting when the supersymmetric $N=1$ version of the model is constructed, because the existing scalars and leptons in the model can play the role of superpartners of each other. As a result four main consequences follow: first, the reduction of the number of free parameters in the model as compared to supersymmetric versions of other 3-3-1 models in the literature [14]; second, the result that the sneutrino, selectron and six other sleptons do not acquire masses in the context of the model constructed playing the role of the Goldstone bosons; third, the absence of the μ problem, in the sense that the μ term is absent at the tree level, arising only as a result of the symmetry breaking, and fourth, the existence of a light CP-odd scalar which may have escaped experimental detection [15].

This paper is organized as follows. In Sec. II we briefly review the non-supersymmetric version of the model; in Sec. III we comment on its supersymmetric extension and calculate the superpotential; in Sec. IV we calculate the mass spectrum (excluding the squark sector) and in Sec. V we present our conclusions.

II. THE NONSUPERSYMMETRIC MODEL

Let us start by describing the fermion content, the scalar sector and the gauge boson sector of the nonsupersymmetric one-family 3-3-1 model in Ref. [11].

We assume that the electroweak gauge group is $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$, that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform as the 3 and $\bar{3}$ representations of $SU(3)_L$ respectively, that $SU(3)_c$ is vectorlike, and that anomaly cancellation takes place family by family as in the SM. If we begin with $Q_L^T=(u,d,D)_L$, where $(u,d)_L$ is the usual isospin doublet of quarks in the SM and D_L is an isospin singlet exotic down quark of electric charge $-1/3$, then the restriction of having particles without exotic electric charges and the condition of anomaly cancellation produce the following multiplet structure for one family [11]:

$$Q_L=\begin{pmatrix} u \\ d \\ D \end{pmatrix}_L \sim (3,3,0), \quad u_L^c \sim \left(\bar{3},1,-\frac{2}{3}\right), \tag{1}$$

$$d_L^c \sim \left(\bar{3},1,\frac{1}{3}\right), \quad D_L^c \sim \left(\bar{3},1,\frac{1}{3}\right),$$

$$L_{1L}=\begin{pmatrix} e^- \\ \nu_e \\ N_1^0 \end{pmatrix}_L \sim \left(1,\bar{3},-\frac{1}{3}\right),$$

$$L_{2L}=\begin{pmatrix} E^- \\ N_2^0 \\ N_3^0 \end{pmatrix}_L \sim \left(1,\bar{3},-\frac{1}{3}\right),$$

$$L_{3L}=\begin{pmatrix} N_4^0 \\ E^+ \\ e^+ \end{pmatrix}_L \sim \left(1,\bar{3},\frac{2}{3}\right),$$

where N_1^0 and N_3^0 are $SU(2)_L$ singlet exotic leptons of electric charge zero, and $(E^-,N_2^0)_L \cup (N_4^0,E^+)_L$ is an $SU(2)_L$ doublet of exotic leptons, vectorlike with respect to the SM as far as we identify $N_{4L}^0=N_{2L}^{0c}$. The numbers inside the parenthesis refer to the $[SU(3)_c, SU(3)_L, U(1)_X]$ quantum numbers respectively.

In order to break the symmetry following the pattern

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_Q, \tag{2}$$

and give, at the same time, masses to the fermion fields in the nonsupersymmetric model, the following set of Higgs scalars is introduced [11]:

$$\phi_1=\begin{pmatrix} \phi_1^- \\ \phi_1^0 \\ \phi_1'^0 \end{pmatrix} \sim \left(1,\bar{3},-\frac{1}{3}\right),$$

$$\phi_2=\begin{pmatrix} \phi_2^- \\ \phi_2^0 \\ \phi_2'^0 \end{pmatrix} \sim \left(1,\bar{3},-\frac{1}{3}\right),$$

$$\phi_3=\begin{pmatrix} \phi_3^0 \\ \phi_3^+ \\ \phi_3'^+ \end{pmatrix} \sim \left(1,\bar{3},\frac{2}{3}\right); \tag{3}$$

with vacuum expectation values (VEVs) given by

$$\langle \phi_1 \rangle^T=(0,0,W), \quad \langle \phi_2 \rangle^T=(0,v,0),$$
$$\langle \phi_3 \rangle^T=(v',0,0), \tag{4}$$

with the hierarchy $W>v\sim v' \sim 174$ GeV, the electroweak breaking scale. From Eqs. (1) and (3) we can see that the three left-handed lepton triplets and the three Higgs scalars have the same quantum numbers under the 3-3-1 gauge group, so they can play the role of superpartners. Also, the isospin doublet in ϕ_2 plays the role of ϕ_d and the isospin doublet in ϕ_3 plays the role of ϕ_u in extensions of the SM with two Higgs doublets (2HDM), in which ϕ_d couples only to down type quarks and ϕ_u couples only to up type quarks (2HDM type II).

There are a total of 17 gauge bosons in this 3-3-1 model. One gauge field B^μ associated with $U(1)_X$, the 8 gluon fields G^μ associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 8 gauge fields A^μ associated with $SU(3)_L$ and that we write for convenience in the following way:

$$\frac{1}{2}\lambda_\alpha A_\alpha^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} D_1^\mu & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D_2^\mu & K^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D_3^\mu \end{pmatrix},$$

where $D_1^\mu=A_3^\mu/\sqrt{2}+A_8^\mu/\sqrt{6}$, $D_2^\mu=-A_3^\mu/\sqrt{2}+A_8^\mu/\sqrt{6}$, and $D_3^\mu=-2A_8^\mu/\sqrt{6}$. λ_i, $i=1,2,\ldots,8$ are the eight Gell-Mann matrices normalized as $\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij}$.

The covariant derivative for this model is given by the expression $D^\mu=\partial^\mu-i(g_3/2)\lambda^\alpha G_\mu^\alpha-i(g_2/2)\lambda^\alpha A_\mu^\alpha-ig_1XB^\mu$, where g_i, $i=1,2,3$ are the gauge coupling constants for $U(1)_X$, $SU(3)_L$ and $SU(3)_c$ respectively.

The sine of the electroweak mixing angle is given by $S_W^2=3g_1^2/(3g_2^2+4g_1^2)$. The photon field is thus

$$A_0^\mu=S_WA_3^\mu+C_W\left[\frac{T_W}{\sqrt{3}}A_8^\mu+\sqrt{(1-T_W/3)}B^\mu\right], \tag{5}$$

where C_W and T_W are the cosine and tangent of the electroweak mixing angle.

Finally, the two neutral currents in the model are defined as

$$Z_0^\mu=C_WA_3^\mu-S_W\left[\frac{T_W}{\sqrt{3}}A_8^\mu+\sqrt{(1-T_W/3)}B^\mu\right],$$

$$Z_0'^\mu=-\sqrt{(1-T_W/3)}A_8^\mu+\frac{T_W}{\sqrt{3}}B^\mu, \tag{6}$$

where Z^μ coincides with the weak neutral current of the SM, with the gauge boson associated with the Y hypercharge given by

$$Y^\mu=\left[\frac{T_W}{\sqrt{3}}A_8^\mu+\sqrt{(1-T_W/3)}B^\mu\right].$$

The consistency of the model requires the existence of eight Goldstone bosons in the scalar spectrum, out of which four are charged and four are neutral (one CP-even state and three CP-odd) [13] in order to provide with masses for $W^\pm$, $K^\pm$, K^0, $\bar{K}^0$, Z^0, and Z'^0.

III. THE SUPERSYMMETRIC EXTENSION

When we introduce supersymmetry in the SM, the entire spectrum of particles is doubled as we must introduce the superpartners of the known fields, besides two scalar doublets ϕ_u and ϕ_d must be used in order to cancel the triangle anomalies; then the superfields $\hat{\phi}_u$, and $\hat{\phi}_d$, related to the two scalars, may couple via a term of the form $\mu\hat{\phi}_u\hat{\phi}_d$ which is gauge and supersymmetric invariant, and thus the natural value for μ is expected to be much larger than the electroweak and supersymmetry breaking scales. This is the so-called μ problem.

However, in a nonsupersymmetric model as the one presented in the former section, in which the Higgs fields and the lepton fields transform identically under the symmetry group, we can have (as far as we take proper care of the mass generation and the symmetry breaking pattern) the three lepton triplets and the three Higgs triplets as the superpartners of each other. Consequently, we can construct the supersymmetric version of our model without the introduction of Higgsinos, with the supersymmetric extension automatically free of chiral anomalies.

For one family we thus end up with the following seven chiral superfields: $\hat{Q}$, $\hat{u}$, $\hat{d}$, $\hat{D}$, $\hat{L}_1$, $\hat{L}_2$, and $\hat{L}_3$, plus gauge bosons and gauginos. The identification of the gauge bosons eigenstates in the SUSY version follows the non-SUSY analysis as we will show below.

A. The superpotential

Let us now write the most general $SU(3)_c\otimes SU(3)_L\otimes U(1)_X$ invariant superpotential

$$\begin{aligned}U=\sum_a\;&(h^u\hat{Q}_a\hat{u}\hat{L}_3^a+\lambda^{(1)}\hat{Q}_a\hat{d}\hat{L}_1^a+h^d\hat{Q}_a\hat{d}\hat{L}_2^a\\&+\lambda^{(2)}\hat{Q}_a\hat{D}\hat{L}_1^a+h^D\hat{Q}_a\hat{D}\hat{L}_2^a)+\lambda^{(3)}\hat{u}\hat{d}\hat{D}\\&+\sum_{abc}\epsilon_{abc}(h^e\hat{L}_1^a\hat{L}_2^b\hat{L}_3^c+\lambda^{(4)}\hat{Q}^a\hat{Q}^b\hat{Q}^c),\end{aligned} \tag{7}$$

where $a,b,c=1,2,3$ are $SU(3)_L$ tensor indices and the chirality and color indices have been omitted. Notice the absence of terms bilinear in the superfields, so a bare μ term is absent in the superpotential U, but it can be generated, after symmetry breaking, by one of the terms in Eq. (7); as a matter of fact it is proportional to $h^e(\langle\tilde{N}_1^0\rangle\tilde{N}_2^0+\langle\tilde{N}_3^0\rangle\tilde{\nu})\tilde{N}_4^0$, where $\langle\cdots\rangle$ stands for the VEV of the neutral scalar field inside the brackets and the tilde denotes the superpartner of the respective field. This effective μ term is at most of the order of the supersymmetry breaking scale, but as we will show in the next section $h^e\approx 0$ in order to have a consistent supersymmetric model. This is how the supersymmetric μ problem is avoided in the context of the model in this paper.

The $\hat{u}\hat{d}\hat{D}$ and $\hat{Q}\hat{Q}\hat{Q}$ terms violate baryon number and can possibly lead to rapid proton decay. We may forbid these interactions by introducing the following baryon-parity:

$$\begin{aligned}(\hat{Q},\hat{u},\hat{d},\hat{D})&\to-(\hat{Q},\hat{u},\hat{d},\hat{D}),\\(\hat{L}_1,\hat{L}_2,\hat{L}_3)&\to+(\hat{L}_1,\hat{L}_2,\hat{L}_3).\end{aligned} \tag{8}$$

This protects the model from too fast proton decay, but the superpotential still contains operators inducing lepton number violation. This is desirable if we want to describe Majorana masses for the neutrinos in our model.

Another discrete symmetry worth considering is $L_{1L}\leftrightarrow L_{2L}$, which implies $h^e=0$, $\lambda^{(1)}=h^d$, and $\lambda^{(2)}=h^D$.

As we will see in the next section, a very small value of h^e is mandatory for having a neutrino with a very small tree-level mass.

B. The scalar potential

The scalar potential is written as

$$V_{SP}=V_F+V_D+V_{\rm soft}, \tag{9}$$

where the first two terms come from the exact SUSY sector, while the last one is the sector of the theory that breaks SUSY explicitly.

We now display the different terms in Eq. (9):

$$\begin{aligned}V_F&=\sum_i \left|\frac{\partial U}{\partial \phi_i}\right|^2\\ &=|h^e|^2(|\tilde L_1|^2|\tilde L_2|^2+|\tilde L_1|^2|\tilde L_3|^2+|\tilde L_2|^2|\tilde L_3|^2-|\tilde L_1^\dagger \tilde L_2|^2\\ &\quad -|\tilde L_1^\dagger \tilde L_3|^2-|\tilde L_2^\dagger \tilde L_3|^2),\end{aligned} \tag{10}$$

$$V_D=\frac{1}{2}D^\alpha D^\alpha+\frac{1}{2}D^2,$$

where

$$D^\alpha=g_2\sum_{i=1}^{3}\sum_{a,b=1}^{8}L^*_{i,a}\left(\frac{-\lambda^{\alpha *}}{2}\right)_{ab}L_{i,b}\quad(\alpha=1,\dots,8),$$

and

$$D=g_1\sum_{i=1}^{3}\sum_{a=1}^{8}L^*_{i,a}X(L)L_{i,a}$$

[$a,b=1,2,\dots,8$ are $SU(3)_L$ tensor indices]. Then we have

$$\begin{aligned}V_D&=\frac{1}{2}g_2^2\Bigg[\frac{1}{3}\{(\tilde L_1^\dagger \tilde L_1)^2+(\tilde L_2^\dagger \tilde L_2)^2+(\tilde L_3^\dagger \tilde L_3)^2-(\tilde L_1^\dagger \tilde L_1)(\tilde L_2^\dagger \tilde L_2)\\ &\quad -(\tilde L_1^\dagger \tilde L_1)(\tilde L_3^\dagger \tilde L_3)-(\tilde L_2^\dagger \tilde L_2)(\tilde L_3^\dagger \tilde L_3)\}\\ &\quad +|\tilde L_1^\dagger \tilde L_2|^2+|\tilde L_1^\dagger \tilde L_3|^2+|\tilde L_2^\dagger \tilde L_3|^2\Bigg]\\ &\quad +\frac{1}{18}g_1^2[(\tilde L_1^\dagger \tilde L_1)^2+(\tilde L_2^\dagger \tilde L_2)^2\\ &\quad +4(\tilde L_3^\dagger \tilde L_3)^2+2(\tilde L_1^\dagger \tilde L_1)(\tilde L_2^\dagger \tilde L_2)-4(\tilde L_1^\dagger \tilde L_1)(\tilde L_3^\dagger \tilde L_3)\\ &\quad -4(\tilde L_2^\dagger \tilde L_2)(\tilde L_3^\dagger \tilde L_3)].\end{aligned} \tag{11}$$

(On deriving V_F and V_D we have used the identities $\epsilon_{ijk}\epsilon^{ilm}=\delta^l_j\delta^m_k-\delta^m_j\delta^l_k$, and $\lambda^\alpha_{ij}\lambda^\alpha_{kl}=2\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}$.)

Finally, the soft SUSY-breaking potential is given by

$$\begin{aligned}V_{\rm soft}&=m^2_{L_1}\tilde L_1^\dagger \tilde L_1+m^2_{L_2}\tilde L_2^\dagger \tilde L_2+m^2_{L_3}\tilde L_3^\dagger \tilde L_3\\ &\quad +m^2_{L_1L_2}\,\mathrm{Re}\,(\tilde L_1^\dagger \tilde L_2)+h'\,\mathrm{Re}(\epsilon^{abc}\tilde L_a\tilde L_b\tilde L_c)\\ &\quad +\frac{M_1}{2}\tilde B^0\tilde B^0+\frac{M_2}{2}\sum_{a=1}^{8}\tilde A_a\tilde A^a+\cdots,\end{aligned} \tag{12}$$

where M_1 is the soft mass parameter of the $U(1)_X$ gaugino and M_2 refers to the soft mass parameter of the $SU(3)_L$ gauginos.

After redefining $(\tilde E^-,\tilde N_2)$ as ϕ_d and $(\tilde N_4,\tilde E^+)$ as ϕ_u, the parts of $V=V_F+V_D$ containing the sleptons are given by

$$\begin{aligned}V&=\delta[(\phi_d^\dagger\phi_d+\tilde N_3^\dagger\tilde N_3)^2+(\tilde e^+\tilde e^-+\tilde\nu^\dagger\tilde\nu+\tilde N_1^\dagger\tilde N_1)^2]\\ &\quad +\eta(\phi_u^\dagger\phi_u+\tilde e^+\tilde e^-)^2+\gamma(\phi_d^\dagger\phi_u+\tilde N_3^\dagger\tilde e^+)(\phi_u^\dagger\phi_d+\tilde e^-\tilde N_3)\\ &\quad +\beta(\phi_d^\dagger\phi_d+\tilde N_3^\dagger\tilde N_3)(\phi_u^\dagger\phi_u+\tilde e^+\tilde e^-)\\ &\quad +\alpha(\phi_d^\dagger\phi_d+\tilde N_3^\dagger\tilde N_3)(|\tilde e|^2+|\tilde\nu|^2+|\tilde N_1|^2)\\ &\quad +\beta(\phi_u^\dagger\phi_u+\tilde e^+\tilde e^-)(|\tilde e|^2+|\tilde\nu|^2+|\tilde N_1|^2)\\ &\quad +\gamma(\tilde e^+\tilde E^-+\tilde\nu^\dagger\tilde N_2+\tilde N_1^\dagger\tilde N_3)(\tilde E^+\tilde e^-+\tilde N_2^\dagger\tilde\nu_2+\tilde N_3^\dagger\tilde N_1)\\ &\quad +\gamma(\tilde e^+\tilde N_4^-+\tilde\nu^\dagger\tilde E^++\tilde N_1^\dagger\tilde e^+)(\tilde N_4^\dagger\tilde e^-+\tilde E^-\tilde\nu+\tilde e^-\tilde N_1),\end{aligned} \tag{13}$$

where $\delta=(g_2^2/6+g_1^2/18)$, $\eta=(g_2^2/6+2g_1^2/9)$, $\gamma=(g_2^2/2-|h^e|^2)$, $\beta=(|h^e|^2-g_2^2/6-2g_1^2/9)$, and $\alpha=(|h^e|^2-g_2^2/6+g_1^2/9)$.

IV. MASS SPECTRUM

Masses for the particles are generated in this model from the VEV of the scalar fields and from the soft terms in the superpotential.

For simplicity we assume that the VEVs are real, which means that spontaneous CP violation through the scalar exchange is not considered in this work. Now, for convenience in reading we rewrite the expansion of the scalar fields acquiring VEVs as

$$\tilde N_1^0=\langle\tilde N_1^0\rangle+\frac{\tilde N_{1R}^0+i\tilde N_{1I}^0}{\sqrt2}, \tag{14}$$

$$\tilde N_2^0=\langle\tilde N_2^0\rangle+\frac{\tilde N_{2R}^0+i\tilde N_{2I}^0}{\sqrt2},$$

$$\tilde N_3^0=\langle\tilde N_3^0\rangle+\frac{\tilde N_{3R}^0+i\tilde N_{3I}^0}{\sqrt2},$$

$$\tilde N_4^0=\langle\tilde N_4^0\rangle+\frac{\tilde N_{4R}^0+i\tilde N_{4I}^0}{\sqrt2},$$

$$\tilde{\nu}_e = \langle\tilde{\nu}\rangle + \frac{\tilde{\nu}_R + i\tilde{\nu}_I}{\sqrt{2}},$$

in an obvious notation taken from Eq. (1), where $\tilde{N}^0_{iR}$ ($\tilde{\nu}_R$) and $\tilde{N}^0_{iI}$ ($\tilde{\nu}_I$) $i=1,2,3,4$ refer, respectively, to the real sector and to the imaginary sector of the sleptons. In general, $\langle\tilde{\nu}_e\rangle$ and $\langle\tilde{N}^0_i\rangle$, $i=1,2,3,4$ can all be different from zero, but as we will see in the following analysis there are some constraints relating them. Also $\langle\tilde{\nu}\rangle \leqslant 0.2$ TeV, $\langle\tilde{N}_i\rangle \leqslant 0.2$ TeV, for $i=2,4$ in order to respect the SM phenomenology, and $\langle\tilde{N}_j\rangle \geqslant 1$ TeV, for $j=1,3$ in order to respect the phenomenology of the 3-3-1 model in Refs. [11,13].

Our approach will be to look for consistency in the sense that the mass spectrum must include a light spin-1/2 neutral particle (the neutrino) with the other spin-1/2 neutral particles having masses larger than or equal to half of the Z^0 mass, to be in agreement with experimental bounds. Also we need eight spin zero Goldstone bosons, four charged and four neutral ones, out of which one neutral must be related to the real sector of the sleptons and three neutrals to the imaginary sector, in order to produce masses for the gauge bosons after the breaking of the symmetry.

As we will show in this section, a consistent set of VEVs is provided by $\langle\tilde{\nu}_e\rangle = v$, $\langle\tilde{N}^0_3\rangle = V$, $\langle\tilde{N}^0_2\rangle = v_d$, and $\langle\tilde{N}^0_4\rangle = v_u$, with the hierarchy $V > v_u \sim v_d \sim v$, and the constraint $\langle\tilde{N}_1\rangle = -v v_d/V$. This situation implies a symmetry breaking pattern of the form $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \rightarrow SU(3)_c \otimes U(1)_Q$, instead of the chain in Eq. (2). So, we cannot claim that the MSSM is an effective theory of the model presented here; rather the model here is an alternative to the MSSM so well analyzed in the literature [4–6].

Playing with the VEV and the other parameters in the superpotential, special attention must be paid to the several constraints coming from the minimization of the scalar potential, which at the tree level are

$$\langle\tilde{N}^0_2\rangle\langle\tilde{\nu}\rangle = -\langle\tilde{N}^0_3\rangle\langle\tilde{N}_1\rangle, \tag{15}$$

$$m^2_{L_1L_2} = h'\langle\tilde{N}^0_4\rangle \frac{\langle\tilde{N}^0_3\rangle\langle\tilde{N}^0_1\rangle + \langle\tilde{\nu}\rangle\langle\tilde{N}^0_2\rangle}{\langle\tilde{N}^0_3\rangle\langle\tilde{\nu}\rangle - \langle\tilde{N}^0_2\rangle\langle\tilde{N}^0_1\rangle} = 0,$$

$$m^2_{L_1} = -\alpha(\langle\tilde{N}^0_2\rangle^2 + \langle\tilde{N}^0_3\rangle^2) - \beta\langle\tilde{N}^0_4\rangle^2 - 2\delta(\langle\tilde{\nu}\rangle^2 + \langle\tilde{N}^0_1\rangle^2) - h'\frac{\langle\tilde{N}^0_4\rangle\langle\tilde{N}^0_3\rangle}{2\langle\tilde{\nu}\rangle},$$

$$m^2_{L_2} = -\beta\langle\tilde{N}^0_4\rangle^2 - \alpha(\langle\tilde{\nu}\rangle^2 + \langle\tilde{N}^0_1\rangle^2) - 2\delta(\langle\tilde{N}^0_2\rangle^2 + \langle\tilde{N}^0_3\rangle^2) - h'\frac{\langle\tilde{N}^0_4\rangle\langle\tilde{N}^0_1\rangle}{2\langle\tilde{N}^0_2\rangle},$$

$$m^2_{L_3} = -\beta(\langle\tilde{\nu}\rangle^2 + \langle\tilde{N}^0_1\rangle^2 + \langle\tilde{N}^0_2\rangle^2 + \langle\tilde{N}^0_3\rangle^2) - 2\eta\langle\tilde{N}^0_4\rangle^2 - h'\frac{\langle\tilde{N}^0_3\rangle\langle\tilde{\nu}\rangle - \langle\tilde{N}^0_1\rangle\langle\tilde{N}^0_2\rangle}{2\langle\tilde{N}^0_4\rangle},$$

where α, β, δ and η were defined above. The result $m_{L_1L_2} = 0$ comes from the first constraint and has important consequences as we will see in what follows.

A. Spectrum in the gauge boson sector

With the most general VEV structure presented in Eq. (14), the charged gauge bosons $W^\pm_\mu$ and $K^\pm_\mu$ mix up and the diagonalization of the corresponding squared-mass matrix yields the masses [13]

$$M^2_{W'} = \frac{g_2^2}{2}(\langle\tilde{N}^0_4\rangle^2 + \langle\tilde{\nu}\rangle^2 + \langle\tilde{N}^0_1\rangle^2),$$
$$M^2_{K'} = \frac{g_2^2}{2}(\langle\tilde{N}^0_3\rangle^2 + \langle\tilde{N}^0_4\rangle^2 + \langle\tilde{N}^0_2\rangle^2), \tag{16}$$

related to the physical fields $W'_\mu = \eta(\langle\tilde{N}^0_2\rangle K_\mu - \langle\tilde{N}^0_3\rangle W_\mu)$ and $K'_\mu = \eta(\langle\tilde{N}^0_3\rangle K_\mu + \langle\tilde{N}^0_2\rangle W_\mu)$ associated with the known charged current $W'^\pm_\mu$, and the new one $K'^\pm_\mu$ predicted in the context of this model ($\eta^{-2} = \langle\tilde{N}^0_2\rangle^2 + \langle\tilde{N}^0_3\rangle^2$ is a normalization factor). Notice that with the hierarchy $\langle\tilde{N}^0_3\rangle \gg \langle\tilde{N}^0_2\rangle \sim \langle\tilde{N}^0_4\rangle \sim \langle\tilde{\nu}\rangle$, the mixing between $W^\pm_\mu$ and $K^\pm_\mu$ is well under control due to fact that the physical $W'^\pm$ is mainly the $W^\pm$ of the weak basis, with a small component along $K^\pm$ of the order of $\langle\tilde{N}^0_2\rangle/\langle\tilde{N}^0_3\rangle$.

The expression for the $W'^\pm$ mass combined with the minimization conditions in Eq. (15) implies $(\langle\tilde{N}^0_4\rangle^2 + \langle\tilde{\nu}\rangle^2 + \langle\tilde{\nu}\rangle^2\langle\tilde{N}^0_2\rangle^2/\langle\tilde{N}^0_3\rangle^2)^{1/2} \approx 174$ GeV.

For the five electrically neutral gauge bosons we get first that the imaginary part of K^0_μ decouples from the other four electrically neutral gauge bosons, acquiring a mass $M^2_{K^0_I} = (g_2^2/4)(\langle\tilde{\nu}\rangle^2 + \langle\tilde{N}^0_2\rangle^2 + \langle\tilde{N}^0_3\rangle^2 + \langle\tilde{N}^0_4\rangle^2)$ [13]. Now, in the basis $(B^\mu, A^\mu_3, A^\mu_8, K^{0\mu}_R)$, the obtained squared-mass matrix has determinant equal to zero which implies that there is a zero eigenvalue associated to the photon field with eigenvector A^μ_0 as given in Eq. (5).

The mass matrix for the neutral gauge boson sector can now be written in the basis $(Z'^\mu_0, Z^\mu_0, K^{0\mu}_R)$, where the fields Z'^μ_0 and Z^μ_0 have been defined in Eqs. (6). We can diagonalize this mass matrix in order to obtain the physical fields, but the mathematical results are not very illuminating. Since $\langle\tilde{N}^0_3\rangle \gg \langle\tilde{N}^0_2\rangle \sim \langle\tilde{N}^0_4\rangle \sim \langle\tilde{\nu}\rangle \sim 174$ GeV, we perform a perturbation analysis for the particular case $\langle\tilde{N}^0_2\rangle = \langle\tilde{N}^0_4\rangle = \langle\tilde{\nu}\rangle \equiv v$ using $q = v/\langle\tilde{N}^0_3\rangle$ as the expansion parameter. In this way we obtain one eigenvalue of the form

$$M^2_{Z_1} \approx g_2^2 C_W^{-2} v^2 \left(1 + \frac{1}{8}q^2(7 + 6T_W^2 - 9T_W^4)\right), \tag{17}$$

and another two of the order $\langle\tilde{N}_3^0\rangle^2$ [13]. So, we have a neutral current associated to a mass scale $v\simeq 174$ GeV which may be identified with the known SM neutral current, and two new electrically neutral currents associated to a mass scale $\langle\tilde{N}_3^0\rangle \gg v$.

Now, using the expressions for $M_{W'}$ and M_{Z_1} we obtain for the ρ parameter at the tree-level [16]

$$\rho = M_{W'}^2/(M_{Z_1}^2 C_W^2) \simeq 1 - \frac{3}{8}q^2(1+2T_W^2-3T_W^4), \quad (18)$$

so that the global fit $\rho = 1.0012^{+0.0023}_{-0.0014}$ [17] provides us with the lower limit $\langle\tilde{N}_3^0\rangle \geq 8.7$ TeV [where we are using $S_W^2 = 0.23113$ [18] and neglecting loop corrections which depend on the splitting of the $SU(2)_L$ doublets].

This result justifies both the imposition of the hierarchy $\langle\tilde{N}_3^0\rangle \gg \langle\tilde{N}_2^0\rangle \sim \langle\tilde{N}_4^0\rangle \sim \langle\tilde{\nu}\rangle$ and the existence of the expansion parameter $q \leq 0.02$. This in turn shows first that the small component $(\langle\tilde{N}_2^0\rangle/\sqrt{\langle\tilde{N}_2^0\rangle^2+\langle\tilde{N}_3^0\rangle^2})K_\mu$ of the eigenstate W'_μ will contaminate tree-level physical processes at most at the level of 2% (by the way, such a mixing can contribute to the $\Delta I = 1/2$ enhancement in nonleptonic weak processes), and second that the estimated order of the masses of the new charged and neutral gauge bosons in the model are not in conflict neither with constraints on their mass scale calculated from a global fit of data relevant to electron-quark contact interactions [19], nor with the bounds obtained in $p\bar{p}$ collisions at the Tevatron [20].

B. Masses for the quark sector

Let us assume in the following analysis that we are working with the third family. The first term in the superpotential produces for the up type quark a mass $m_t = h^u\langle\tilde{N}_4^0\rangle = 174$ GeV, which implies $\langle\tilde{N}_4^0\rangle \approx 10^2$ GeV and $h^u \sim 1$, while for the down type quarks the second to fifth terms generate, in the basis (d,D) [(d_R,D_R) column and (d_L,D_L) row], the mass matrix

$$M_{dD} = \begin{pmatrix} \lambda^{(1)}\langle\tilde{\nu}\rangle + h^d\langle\tilde{N}_2^0\rangle & \lambda^{(1)}\langle\tilde{N}_1^0\rangle + h^d\langle\tilde{N}_3^0\rangle \\ \lambda^{(2)}\langle\tilde{\nu}\rangle + h^D\langle\tilde{N}_2^0\rangle & \lambda^{(2)}\langle\tilde{N}_1^0\rangle + h^D\langle\tilde{N}_3^0\rangle \end{pmatrix}, \quad (19)$$

which produces a mass of the order of $\langle\tilde{N}_3^0\rangle$ for the exotic quark D, and for the ordinary quark d a mass of the order of $(\langle\tilde{\nu}\rangle + \langle\tilde{N}_2^0\rangle)$, suppressed by differences of Yukawa couplings (it is zero for $\lambda^{(1)} = h^d$ and $\lambda^{(2)} = h^D$).

Using the former results and the expression for the $W^\pm$ mass it follows that $\langle\tilde{N}_4^0\rangle \approx \langle\tilde{\nu}\rangle \approx \langle\tilde{N}_2^0\rangle \approx 10^2$ GeV.

It is worth noticing that the isospin doublet in $\tilde{L}_{3L}$ couples only to up type quarks, while the isospin doublets in $\tilde{L}_{1L}$ and $\tilde{L}_{2L}$ couple only to down type quarks.

C. Masses for neutralinos

The neutralinos are linear combinations of neutral gauginos and neutral leptons (there are no Higgsinos). For this model and in the basis $(\nu_e, N_1, N_2, N_3, N_4, \tilde{B}^0, \tilde{A}_3, \tilde{A}_8, \tilde{K}^0, \tilde{\bar{K}}^0)$, their mass matrix is given by

$$M_{ntns} = \begin{pmatrix} M_N & M_{gN}^T \\ M_{gN} & M_g \end{pmatrix}, \quad (20)$$

where M_N is the matrix

$$M_N = \frac{h^e}{2}\begin{pmatrix} 0 & 0 & 0 & \langle\tilde{N}_4^0\rangle & \langle\tilde{N}_3^0\rangle \\ 0 & 0 & -\langle\tilde{N}_4^0\rangle & 0 & -\langle\tilde{N}_2^0\rangle \\ 0 & -\langle\tilde{N}_4^0\rangle & 0 & 0 & -\langle\tilde{N}_1^0\rangle \\ \langle\tilde{N}_4^0\rangle & 0 & 0 & 0 & \langle\tilde{\nu}\rangle \\ \langle\tilde{N}_3^0\rangle & -\langle\tilde{N}_2^0\rangle & -\langle\tilde{N}_1^0\rangle & \langle\tilde{\nu}\rangle & 0 \end{pmatrix}, \quad (21)$$

M_{gN} is given by

$$M_{gN} = \begin{pmatrix} -g_1\frac{\sqrt{2}}{3}\langle\tilde{\nu}\rangle & -g_1\frac{\sqrt{2}}{3}\langle\tilde{N}_1^0\rangle & -g_1\frac{\sqrt{2}}{3}\langle\tilde{N}_2^0\rangle & -g_1\frac{\sqrt{2}}{3}\langle\tilde{N}_3^0\rangle & g_1 2\frac{\sqrt{2}}{3}\langle\tilde{N}_4^0\rangle \\ g_2\frac{1}{\sqrt{2}}\langle\tilde{\nu}\rangle & 0 & g_2\frac{1}{\sqrt{2}}\langle\tilde{N}_2^0\rangle & 0 & -g_2\frac{1}{\sqrt{2}}\langle\tilde{N}_4^0\rangle \\ -g_2\frac{1}{\sqrt{6}}\langle\tilde{\nu}\rangle & g_2\frac{2}{\sqrt{6}}\langle\tilde{N}_1^0\rangle & -g_2\frac{1}{\sqrt{6}}\langle\tilde{N}_2^0\rangle & g_2\frac{2}{\sqrt{6}}\langle\tilde{N}_3^0\rangle & -g_2\frac{1}{\sqrt{6}}\langle\tilde{N}_4^0\rangle \\ -g_2\langle\tilde{N}_1^0\rangle & 0 & -g_2\langle\tilde{N}_3^0\rangle & 0 & 0 \\ 0 & -g_2\langle\tilde{\nu}\rangle & 0 & -g_2\langle\tilde{N}_2^0\rangle & 0 \end{pmatrix}, \quad (22)$$

and from the soft terms in the superpotential we read $M_g = \mathrm{Diag}(M_1, M_2, M_2, A_{2\times 2})$, where $A_{2\times 2}$ is a 2×2 matrix with entries zero in the main diagonal and M_2 in the secondary diagonal.

Now, in order to have a consistent model, one of the eigenvalues of this mass matrix must be very small (corresponding to the neutrino field), with the other eigenvalues larger than half of the Z^0 mass. It is clear that for h^e very small and simultaneously M_i, $i=1,2$ very large, we have a seesaw type mass matrix; but M_i, $i=1,2$ very large is inconvenient because it restores the hierarchy problem.

A detailed analysis shows that M_{ntns} contains two Dirac neutrinos and six Majorana neutral fields, and that for $M_i \leqslant 10$ TeV, $i=1,2$ we have a mass spectrum consistent with the low energy phenomenology only if $h^e \approx 0$. By imposing $h^e = 0$, a zero tree-level Majorana mass for the neutrino is obtained, with the hope that the radiative corrections should produce a small mass. (The symmetry $L_{1L} \leftrightarrow L_{2L}$ implies $h^e = 0$.)

To diagonalize M_{ntns} analytically is a hopeless task, so we propose a controlled numerical analysis using fixed values for some parameters as suggested by the low energy phenomenology [for example g_1 (TeV)≈ 0.38 and g_2 (TeV) ≈ 0.65] and leaving free other parameters, but in a range of values bounded by theoretical and experimental restrictions. With this in mind we use 0.1 TeV$\leqslant M_i \leqslant 10$ TeV, $i=1,2$ (in order to avoid the hierarchy problem) and $h^e \approx 0$ (in order to have a consistent mass spectrum).

The random numerical analysis with the constraints stated above shows that for $M_1 \approx 0.35$ TeV, $M_2 \approx 3.1$ TeV, $\langle \tilde{N}_3^0 \rangle \approx 10$ TeV, $\langle \tilde{N}_4^0 \rangle \approx 150$ GeV, $\langle \tilde{N}_2^0 \rangle = \langle \tilde{\nu} \rangle \approx 80$ GeV, $\langle \tilde{N}_1^0 \rangle$ calculated from the constraints coming from the minimum of the scalar potential [see Eq. (15)], and $h^e \approx 0$, we get a neutrino mass of a few electron volts, while all the other neutral fields acquire masses above 45 GeV as desired. Also, the analysis is quite insensitive to the variation of the parameters, with the peculiarity that an increase in M_1 and M_2 implies an increase in $\langle \tilde{N}_3^0 \rangle$.

We are going to use from now on the notation $\langle \tilde{\nu} \rangle = v$, $\langle \tilde{N}_4 \rangle = v_u$, $\langle \tilde{N}_2^0 \rangle = v_d$, $\langle \tilde{N}_3^0 \rangle = V$, with $\langle \tilde{N}_1^0 \rangle = -v_d v/V$ as constrained by the minimization conditions in Eq. (15).

Another possibility with $h^e \neq 0$ but very small demands for $\langle \tilde{\nu} \rangle = \langle \tilde{N}_1^0 \rangle = 0$, and produces a lightest neutralino only in the KeV scale, which may be adequate for the second and third family, but not for the first one. The advantage of this particular case is that it reduces to the study of the scalar potential presented in Ref. [13] for the nonsupersymmetric case, with an analysis of the mass spectrum similar to the one in that paper.

D. Masses for the scalar sector

For the scalars we have three sectors, one charged and two neutrals (one real and the other one imaginary) which do not mix, so we can consider them separately.

1. The charged scalars sector

For the charged scalars in the basis $(\tilde{e}_1^-, \tilde{e}_2^-, \tilde{E}_1^-, \tilde{E}_2^-)$, we get the squared-mass matrix:

$$\begin{pmatrix} 2\gamma v_u^2 - h' v_u V/v & 2\gamma\langle \tilde{N}_1^0\rangle v_u + h' v_d & \gamma(v v_d + \langle \tilde{N}_1^0 \rangle V) & 2\gamma v v_u - h' V \\ 2\gamma\langle \tilde{N}_1^0\rangle v_u + h' v_d & 2\gamma(\langle \tilde{N}_1^0\rangle + V^2) + h' \dfrac{\langle \tilde{N}_1^0\rangle v_d - vV}{v_u} & 2\gamma v_u V - h' v & 2\gamma(v\langle \tilde{N}_1^0\rangle + v_d V) \\ \gamma(v v_d + \langle \tilde{N}_1^0\rangle V) & 2\gamma v_u V - h' v & 2\gamma v_u^2 & 2\gamma v_u v_d + h'\langle \tilde{N}_1^0\rangle \\ 2\gamma v v_u - h' V & 2\gamma(v\langle \tilde{N}_1^0\rangle + v_d V) & 2\gamma v_d v_u + h'\langle \tilde{N}_1^0\rangle & 2\gamma(v^2 + v_d^2) + h' \dfrac{v_d\langle \tilde{N}_1^0\rangle - vV}{v_u} \end{pmatrix}.$$

The analysis shows that only for $h'=0$ this matrix has two eigenvalues equal to zero which correspond to the four Goldstone bosons needed to produce masses for $W^\pm$ and $K^\pm$. So, $h'=0$ is mandatory ($h'=0$ is a consequence of the symmetry $L_{1L} \leftrightarrow L_{2L}$). For the other two eigenvalues one is in the TeV scale and the other one at the electroweak mass scale.

2. The neutral real sector

For the neutral real sector and in the basis $(\tilde{\nu}_R, \tilde{N}_{1R}, \tilde{N}_{2R}, \tilde{N}_{3R}, \tilde{N}_{4R})$ we get the following mass matrix:

$$M_{real}^2 = \begin{pmatrix} M_{2\times 2} & M_{2\times 3} \\ M_{2\times 3}^T & M_{3\times 3} \end{pmatrix}, \tag{23}$$

where the submatrices are

$$M_{2\times 2} = \begin{pmatrix} \gamma v_d^2 + 4\delta v^2 - \dfrac{h' v_u V}{2v} & \gamma v_d V + 4\delta v\langle \tilde{N}_1^0\rangle \\ \gamma v_d V + 4\delta v \langle \tilde{N}_1^0\rangle & \gamma V^2 + 4\delta\langle \tilde{N}_1^0\rangle^2 - \dfrac{h' v_u V}{2v} \end{pmatrix}, \tag{24}$$

$$M_{2\times 3}=\begin{pmatrix} v_d v(4\delta-\gamma) & 2\alpha v V+\gamma v_d\langle\tilde{N}_1^0\rangle+h' v_u/2 & 2\beta v v_u+h' V/2 \\ 2\alpha\langle\tilde{N}_1^0\rangle v_d+\gamma v V-h' v_u/2 & (\gamma-4\delta)v v_d & 2\beta\langle\tilde{N}_1^0\rangle v_u-h' v_d/2 \end{pmatrix}, \tag{25}$$

$$M_{3\times 3}=\begin{pmatrix} \gamma v^2+4\delta v_d^2-\frac{h' v v_u}{2V} & \gamma v\langle\tilde{N}_1^0\rangle+4\delta v_d V & 2\beta v_u v_d-h'\langle\tilde{N}_1^0\rangle/2 \\ \gamma v\langle\tilde{N}_1^0\rangle+4\delta v_d V & \gamma\langle\tilde{N}_1^0\rangle^2+4\delta V^2-\frac{h' v v_u}{2V} & 2\beta v_u V+h' v/2 \\ 2\beta v_u v_d-h'\langle\tilde{N}_1^0\rangle/2 & 2\beta v_u V+h' v/2 & 4\eta v_u^2+h'\frac{\langle\tilde{N}_1^0\rangle v_d-v V}{2v_u} \end{pmatrix}. \tag{26}$$

Using the constraints in Eqs. (15), this mass matrix has one eigenvalue equal to zero which identifies one real Goldstone boson needed to produce a mass for $K_I^{0\mu}$. Now, using $h^e\approx 0$, $h'=0$ and with the other values as given before, we get for the remaining four eigenvalues that two of them are in the TeV scale, another one is at the electroweak mass scale, while for the lightest CP-even scalar h we get a tree-level mass smaller than the one obtained in the MSSM. This result, which is strongly dependent on the value of h^e, is not realistic due to the fact that the radiative corrections have not been taken into account, but such analysis is not in the scope of the present work.

3. The neutral imaginary sector

For the neutral imaginary sector and in the basis $(\tilde{\nu}_I,\tilde{N}_{1I},\tilde{N}_{2I},\tilde{N}_{3I},\tilde{N}_{4I})$ we get the following mass matrix:

$$M_{imag}^2=\begin{pmatrix} M'_{2\times 2} & M'_{2\times 3} \\ M'^{T}_{2\times 3} & M'_{3\times 3} \end{pmatrix}, \tag{27}$$

where the submatrices are

$$M'_{2\times 2}=\begin{pmatrix} \gamma v_d^2-\frac{h' v_u V}{2v} & \gamma v_d V \\ \gamma v_d V & \gamma V^2-\frac{h' v_u V}{2v} \end{pmatrix}, \tag{28}$$

$$M'_{2\times 3}=\begin{pmatrix} \gamma\langle\tilde{N}_1^0\rangle V & -\gamma\langle\tilde{N}_1^0\rangle v_d-h'\frac{v_u}{2} & -h'\frac{V}{2} \\ -\gamma v V+h'\frac{v_u}{2} & -\gamma\langle\tilde{N}_1^0\rangle V & h'\frac{v_d}{2} \end{pmatrix}, \tag{29}$$

$$M'_{3\times 3}=\begin{pmatrix} \gamma v^2-\frac{h' v v_u}{2V} & \gamma v\langle\tilde{N}_1^0\rangle & h'\langle\tilde{N}_1^0\rangle/2 \\ \gamma v\langle\tilde{N}_1^0\rangle+ & \gamma\langle\tilde{N}_1^0\rangle^2-\frac{h' v v_u}{2V} & h' v/2 \\ h'\langle\tilde{N}_1^0\rangle/2 & -h' v/2 & h'\frac{\langle\tilde{N}_1^0\rangle v_d-v V}{2v_u} \end{pmatrix}. \tag{30}$$

Using the constraints in Eqs. (15), this mass matrix has three eigenvalues equal to zero which identify three real Goldstone bosons (two of them CP-odd), needed to produce masses for Z_0^μ, Z'^μ_0 and $K_R^{0\mu}$.

In the limit $h'=0$, this mass matrix has one eigenvalue in the TeV scale and four eigenvalues equal to zero that correspond to the three Goldstone bosons identified for the case $h'\neq 0$, plus an extra CP-odd scalar of zero mass at the tree level.

E. Masses for charginos

The charginos in the model are linear combinations of the charged leptons and charged gauginos. In the gauge eigenstate basis $\psi^{\pm}=(e_1^+,E_1^+,\widetilde{W}^+,\widetilde{K}^+,e_1^-,E_1^-,\widetilde{W}^-,\widetilde{K}^-)$ the chargino mass terms in the Lagrangian are of the form $(\psi^{\pm})^T M \psi^{\pm}$, where

$$M=\begin{pmatrix} 0 & M_C^T \\ M_C & 0 \end{pmatrix},$$

and

$$M_C=\begin{pmatrix} h^e v_d & -h^e v & 0 & -g_2 v_u \\ -h^e V & h^e\langle\widetilde{N}_1^0\rangle & -g_2 v_u & 0 \\ -g_2 v & -g_2 v_d & M_2 & 0 \\ -g_2\langle\widetilde{N}_1^0\rangle & -g_2 V & 0 & M_2 \end{pmatrix}. \quad (31)$$

In the limit $h^e=0$ and M_2 very large, this mass matrix is a seesaw type matrix. The numerical evaluation using the parameters as stated before produces a tree-level mass for the τ lepton of the order of 1 GeV, with all the other masses above 90 GeV.

V. GENERAL REMARKS AND CONCLUSIONS

We have built the complete supersymmetric version of the 3-3-1 model in Ref. [11] which, like the MSSM, has two Higgs doublets at the electroweak energy scale (the isospin doublets in $\widetilde{L}_{1L}$ and $\widetilde{L}_{3L}$). Since the MSSM is not an effective theory of the model constructed, exploring the Higgs sector at the electroweak energy scale it is important to realize that the MSSM is not the only possibility for two low energy Higgs doublets.

For the model presented here the slepton multiplets play the role of the Higgs scalars and no Higgsinos are required, which implies a reduction of the number of free parameters compared to other models in the literature [14].

The absence of bilinear terms in the bare superpotential avoids the presence of possible unwanted μ terms; in this way the so-called μ problem is absent in the construction developed in this paper.

The sneutrino, selectron and other six sleptons do not acquire masses in the context of the model, and they play the role of the Goldstone bosons needed to produce masses for the gauge fields. The right number of Goldstone bosons is obtained by demanding $h'=m_{L_1L_2}=0$ in V_{soft}.

$h'=0$ in V_{soft} has as a consequence the existence of a zero mass CP-odd Higgs scalar at tree level. Once radiative corrections are taken into account we expect it acquires a mass of a few (several?) GeV, which in any case is not troublesome because, as discussed in Ref. [15], a light CP-odd Higgs scalar not only is very difficult to detect experimentally, but also it has been found that in the two Higgs doublet model type II and, when a two-loop calculation is used, a very light ($\sim$10 GeV) CP-odd scalar A_0 can still be compatible with precision data such as the ρ parameter, $BR(b\rightarrow s\gamma)$, R_b, A_b, and $BR(\Upsilon\rightarrow A_0\gamma)$ [21].

$h^e=0$ or very small is a necessary condition in order to have a consistent model, in the sense that it must include a very light neutrino, with masses for the other spin-1/2 neutral particles larger than half the Z^0 mass. There is no problem with this constraint, because due to the existence of heavy leptons in the model, h^e is not the only parameter controlling the charged lepton masses.

We have also analyzed the mass value at the tree level for h, the lightest CP-even Higgs scalar in this model, which is smaller than the lower bound of the lightest CP-even Higgs scalar in the MSSM, although strongly dependent on the radiative corrections. This fact is not in conflict with experimental results due to the point that the coupling hZZ and hA_0Z are suppressed because of the mixing of the $SU(2)_L$ doublet sleptons with the singlets $\widetilde{N}_1^0$ and $\widetilde{N}_3^0$.

The recent experimental results announced by the Muon $(g-2)$ Collaboration [22] show a small discrepancy between the SM prediction and the measured value of the muon anomalous spin precession frequency, which only under special circumstances may be identified with the muon's anomalous magnetic moment a_μ [23], a quantity related to loop corrections.

Immediately following the experimental results a number of papers appeared analyzing the reported value, in terms of various forms of new physics, starting with the simplest extension of the SM to two Higgs doublets [21], or by using supersymmetric extensions, technicolor models, leptoquarks, exotic fermions, extra gauge bosons, extra dimensions, etc., in some cases extending the analysis even at two loops (for a complete bibliography see the various references in [24]). More challenging, although not in complete agreement between the different authors, are the analyses presented in Refs. [25] and [26] where it is shown how the MSSM parameter space gets constrained by the experimental results.

Our model, even though different from the MSSM, shares with it the property that very heavy superpartners decouple from the a_μ value yielding a negligible contribution. Nevertheless, the model in this paper includes many interesting new features that may be used for explaining the measured value of the muon's anomalous precession frequency, as for example a light CP-odd and a light CP-even scalars which get very small masses at the tree level, but that the loop radiative corrections may raise these masses up to values ranging from a few GeV to the electroweak mass scale. But an analysis similar to the one presented in Refs. [25] and [26] is outside the scope of the present study, because in our case it depends crucially on the predicted values of the Higgs scalar masses, an obscure matter in supersymmetry. (For example, a_μ^{exp} can be understood in the context of our model if the CP-odd scalar has a mass of the order of a few GeV [21], with all the other scalars and supersymmetric particles acquiring masses larger than the electroweak mass scale. Similarly, the light CP-even Higgs boson h with enough suppressed hZZ and hA_0Z couplings can contribute significantly to a_μ [24].)

The idea of using sleptons as Goldstone bosons is not new in the literature [27], but as far as we know there are just a few papers where this idea is developed in the context of specific models, all of them related to one family structure [28].

The model can be extended to three families, but the price is high since nine $SU(3)_L$ triplets of leptons with their corresponding sleptons are needed, which implies the presence of nine $SU(2)_L$ doublets of Higgs scalars. An alternative is to work with the three family structures presented in Refs. [8,9].

In conclusion, the present model has a rich phenomenology and it deserves to be studied in more detail.

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