Instability of the hedgehog shape for the octet baryon in the chiral quark soliton model

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In this paper the stability of the hedgehog shape of the chiral soliton is studied for the octet baryon with the SU(3) chiral quark soliton model. The strangeness degrees of freedom are treated by a simplified bound-state approach, which omits the locality of the kaon wave function. The mean field approximation for the flavor rotation is applied to the model. The classical soliton changes shape according to the strangeness. The baryon appears as a rotational band of the combined system of the deformed soliton and the kaon.

I. INTRODUCTION

In the limit of a large number of colors N_c of QCD [1] the baryon appears as a soliton [2,3]. By now, various effective models of QCD in the low energy region have employed the soliton picture of the baryon: the Skyrme model [4–9], the Nambu–Jona-Lasinio (NJL) model [10–12], and the chiral quark soliton model (CQSM) [13–16]. The static properties (mass, magnetic moment) [17–19] and the quark distribution functions [20–22] of the baryon have been studied by means of these models. Here, the baryon appears as a rotational band of the soliton [5]. The soliton takes the hedgehog shape and adiabatically rotates as a rigid body in flavor and real space. Although there is also an approach using the harmonic approximation of the meson fluctuations off the shape [23], in any case the stability of the shape is assumed.

From the point of view of the constituent quark, the assumption is justified in the case of the nucleon (N), since the hedgehog shape of the chiral meson fields is caused by the S state of the u,d quarks, which have equal mass [24]. However, it may be not so for the strange baryons (Λ, Σ, Ξ, etc.) consisting of quarks with different masses. The inertial force in the body fixed frame of the soliton depends on the mass. Therefore, the hedgehog shape would be unstable.

The baryon as a nonrigid rotator has been studied with the Skyrmion [25]. There, the effects of the Coriolis force were neglected in the shape of the static soliton and included perturbatively in the state vector for the collective rotation. For the excited states of the baryon, there are several studies of this problem in the nonrelativistic quark model [26].

In this article we shall argue this problem for the octet baryon by means of the CQSM. This model provides the simplest foothold to study the above problem from the point of view of the relativistic quark model. The shape of the soliton can be determined self-consistently as the mean field potential for the quark. Thus the model can eliminate the influence of the artificial assumption that the mean field potential takes a spherical shape.

In Sec. II, after a brief review of the SU(3) chiral quark soliton model, we show the cranking method for the deformed soliton and the mean field approximation for the rotated system. Section III deals with the perturbative expan-

sion of the effective action. At the the lowest order expansion, we show the profile functions and the energy of the deformed classical soliton. For the higher order expansion, we construct the Hamiltonian due to the collective motion of the soliton and the canonical quntization of the collective variables. In Sec. IV, first we deal with the kaon in the background soliton and the stability of the combined system of the soliton and the kaon. Next we give an outline of the diagonalization of the collective Hamiltonian and show the results for the octet baryon. Finally, in Sec. V we summarize the results and discuss the relation to other papers.

II. THE SU(3) CHIRAL QUARK SOLITON MODEL AND ITS MEAN FIELD APPROXIMATION

The chiral quark model in the case of flavor SU(3) is given by the following path integral [13,14,27]:

$$Z = \int D\psi D\bar{\psi} DU \exp\left[i \int d^4x \, \bar{\psi}(i\not{\partial} - MU^{\gamma_5} - \hat{m})\psi\right] \tag{1}$$

with

$$U^{\gamma_5}(x) = \frac{1+\gamma_5}{2}U(x) + \frac{1-\gamma_5}{2}U^\dagger(x), \tag{2}$$

where ψ is the quark field and U is the chiral meson field $\in$ SU(3). Furthermore, M is the dynamical quark mass and $\hat{m}$ is the current quark mass matrix given by

$$\hat{m} = m_0\lambda_0 + m_3\lambda_3 + m_8\lambda_8$$

$$= \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \tag{3}$$

where λ_μ ($\mu = 1,2,\ldots,8$) are the Gell-Mann matrices, $\lambda_0 = (\sqrt{2/3})\mathbf{1}$, and $m_3 \approx 0$ because $m_u \approx m_d \ll m_s$. In this article, we set $M = 400$ MeV, $m_u = m_d = 6$ MeV, and $m_s = 200$ MeV.

Using the path integral formula for the quark field, we find

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$$Z = \int DU e^{iS_F[U]}, \tag{4}$$

$$iS_F[U] = N_c \log \det(i\partial_t - H), \tag{5}$$

where N_c is the number of colors and

$$H = \frac{1}{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta(MU^{\gamma_5} + \hat{m}) \tag{6}$$

is the quark Hamiltonian.

For the chiral field $U(x)$, we postulate the so called cranking form [5,28]:

$$U(x) = \mathcal{A}(t)B^{\dagger}(t)U_0(\mathbf{r})B(t)\mathcal{A}^{\dagger}(t), \tag{7}$$

where $U_0(\mathbf{r})$ is the static chiral meson field, $\mathcal{A}(t)$ describes the adiabatic rotation of the system in SU(3) flavor space, and $B(t)$ describes the spatial rotation. We write $U_0(\mathbf{r})$ as

$$U_0(\mathbf{r}) = \begin{pmatrix} e^{iF(\mathbf{r})\hat{\Lambda}(\mathbf{r})\cdot\boldsymbol{\tau}} & 0 \\ 0^{\dagger} & 1 \end{pmatrix}, \tag{8}$$

where F is the radial component of the profile function, $\hat{\Lambda}$ is a unit vector in the isospin space, and $\boldsymbol{\tau}$ represents Pauli matrices. For the flavor rotation we write [29]

$$\mathcal{A}(t) = \begin{pmatrix} A(t) & 0 \\ 0^{\dagger} & 1 \end{pmatrix} A_s(t), \tag{9}$$

where A is the flavor SU(2) rotation operator and A_s rotates U_0 into the strange directions. In particular, we parametrize $A_s(t)$ as

$$A_s(t) = \exp i\mathcal{D}(t), \tag{10}$$

where

$$\mathcal{D}(t) = \begin{pmatrix} 0 & \sqrt{2}D(t) \\ \sqrt{2}D^{\dagger}(t) & 0 \end{pmatrix} \tag{11}$$

and $D = (D^1, D^2)^T$ is the isodoublet spinor. Although Eq. (9) was motivated by the bound-state approach [30], we will not treat the locality of the kaon wave function in this article.

From Eqs. (5) and (7), we obtain

$$iS_F = N_c \log \det(i\partial_t + i\mathcal{A}^{\dagger}\dot{\mathcal{A}} + iB\dot{B}^{\dagger} - H'), \tag{12}$$

where

$$H' = \mathcal{A}^{\dagger}BHB^{\dagger}\mathcal{A} = \frac{1}{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta(MU_0^{\gamma_5} + \hat{m}') \tag{13}$$

is the rotated Hamiltonian and $\hat{m}'$ is given by

$$\hat{m}' = \mathcal{A}^{\dagger}\hat{m}\mathcal{A} = m_0\lambda_0 + m_8 D^{(8)}_{8\mu}(A_s)\lambda_{\mu}. \tag{14}$$

Here, the term proportional to m_8 breaks the SU(3) symmetry and $D^{(8)}_{8\mu}(A_s)$ is the Wigner D function of A_s in the adjoint representation. We show some important components:

$$D^{(8)}_{83} = -\frac{\sqrt{3}}{2}\frac{\sin^2\sqrt{\kappa_0}}{\kappa_0}\kappa_3, \tag{15}$$

$$D^{(8)}_{88} = 1 - \frac{3}{2}\sin^2\sqrt{\kappa_0}, \tag{16}$$

where

$$\kappa_0 = 2D^{\dagger}D, \tag{17}$$

$$\kappa_3 = 2D^{\dagger}\tau_3 D. \tag{18}$$

We now wish to discuss this model in the mean field approximation. Suppose that the collective motion $\mathcal{A}$ and B is quantized and $|B\rangle$ as an eigenstate of the collective Hamiltonian. The operators D and $D^{\dagger}$ convert a non-strange quark into a strange one. Therefore, if $|B\rangle$ points to a specific direction in the isospin space,

$$\kappa_{B0} = \langle B|\kappa_0|B\rangle, \tag{19}$$

$$\kappa_{B3} = \langle B|\kappa_3|B\rangle \tag{20}$$

have nonzero values. For a fixed κ_{B0}, we have the following constraint for κ_{B3}:

$$-\kappa_{B0} \leqslant \kappa_{B3} \leqslant +\kappa_{B0}. \tag{21}$$

Intuitively, κ_{B0} and κ_{B3} represent the quantity of the strangeness and the asymmetry of the strangeness, respectively, coupled to the isodoublet in $|B\rangle$. The expectation value $\langle B|\hat{m}'|B\rangle$ may be approximated by

$$\hat{m}_B = m_0\lambda_0 + m_{B3}\lambda_3 + m_{B8}\lambda_8$$

$$= \begin{pmatrix} m_{Bu} & 0 & 0 \\ 0 & m_{Bd} & 0 \\ 0 & 0 & m_{Bs} \end{pmatrix} \tag{22}$$

with

$$m_{B\mu} = m_8 \lim_{\kappa_{0,3}\to\kappa_{B0,3}} D^{(8)}_{8\mu}(A_s) \quad (\mu = 3, 8). \tag{23}$$

Equation (22) indicates that the u, d, s quarks mix with each other by rotation in the strange direction, and their masses are renormalized in the body fixed frame. Figure 1 shows the κ_{B0} and κ_{B3} dependencies of the effective quark masses $m_{Bu,d,s}$.

III. PERTURBATION AND CANONICAL QUANTIZATION

We separate H' into an unperturbed part H'_0 and a perturbing part $\Delta H'$ as follows:

$$H' = H'_0 + \Delta H', \tag{24}$$
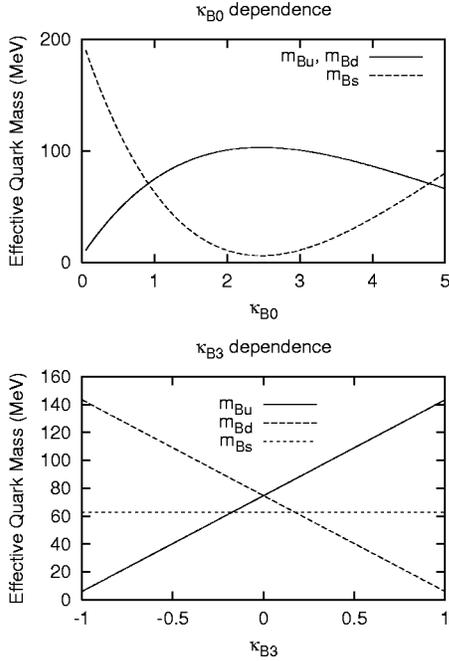
FIG. 1. The κ_{B0} and κ_{B3} dependencies of the effective quark mass. For κ_{B0} dependence, we set $\kappa_{B3}=0$, at which m_{Bu} and m_{Bd} take the same value. For κ_{B3}, we set $\kappa_{B0}=1$.

$$H_0' = \frac{1}{i}\boldsymbol{\alpha}\cdot\nabla + \beta(MU^{\gamma_5}+\hat{m}_B), \tag{25}$$

$$\Delta H' = \beta(\hat{m}'-\hat{m}_B). \tag{26}$$

We expand iS_F in powers of $i\mathcal{A}^\dagger\mathcal{A}$, $iB\dot{B}^\dagger$, and $\Delta H'$ around the eigenstate of $(i\partial_t - H_0')$ [15].

Since H_0' contains the $\hat{m}_B$, H_0' breaks not only the SU(3) symmetry but also SU(2). Thus it has only rotation symmetry around the third axis in isospin space. When the isospin space is mapped into the real space, the configuration of the system has an axial symmetry with respect to the specific axis in real space.

The grand spin operator in the quark space is given by

$$\mathbf{K}^{(q)} = \mathbf{J}^{(q)} + \mathbf{I}^{(q)} = \mathbf{L}^{(q)} + \mathbf{S}^{(q)} + \mathbf{I}^{(q)}, \tag{27}$$

where $\mathbf{J}^{(q)}$, $\mathbf{L}^{(q)}$, and $\mathbf{S}^{(q)}$ are the total, orbital, and spin angular momenta, respectively, and $\mathbf{I}^{(q)}$ denotes the isospin. The above argument is expressed as

$$[K_3^{(q)}, U_0(\mathbf{r})]=0, \tag{28}$$

where $K_3^{(q)}$ is the third component of $\mathbf{K}^{(q)}$ and we choose the z axis as the specific one. We call this equation "K_3 symmetry."

The isosinglet part of Eq. (28) means that the radial component of the profile function F depends only on r and θ of the polar coordinates. On the other hand, the isotriplet part means that the unit vector $\hat{\boldsymbol{\Lambda}}$ has axial symmetry about the z axis. Thus, the deviation from the hedgehog shape takes place only in the r and θ directions.

In actual calculation, we introduce the tensor operators $Z_{KK_3}^{JLSI}(\hat{\mathbf{r}})$ defined by the following equations, where $r=|\mathbf{r}|$ and $\hat{\mathbf{r}}=\mathbf{r}/r$ [31]:

$$[K_3^{(q)}, Z_{KK_3}^{JLSI}]=K_3 Z_{KK_3}^{JLSI}, \tag{29}$$

$$[\mathbf{K}^{(q)2}, Z_{KK_3}^{JLSI}]=K(K+1)Z_{KK_3}^{JLSI}, \tag{30}$$

$$[\mathbf{J}^{(q)2}, Z_{KK_3}^{JLSI}]=J(J+1)Z_{KK_3}^{JLSI}, \tag{31}$$

$$[\mathbf{L}^{(q)2}, Z_{KK_3}^{JLSI}]=L(L+1)Z_{KK_3}^{JLSI}, \tag{32}$$

$$[\mathbf{S}^{(q)2}, Z_{KK_3}^{JLSI}]=S(S+1)Z_{KK_3}^{JLSI}, \tag{33}$$

$$[\mathbf{I}^{(q)2}, Z_{KK_3}^{JLSI}]=I(I+1)Z_{KK_3}^{JLSI}. \tag{34}$$

Then the chiral fields can be expanded in series of $Z_{KK_3}^{JLSI}$:

$$U_0(\mathbf{r})=\sum_L \sum_I \sum_K \sum_{K_3} U_{KK_3}^{LL0I}(r) Z_{KK_3}^{LL0I}(\hat{\mathbf{r}}). \tag{35}$$

Equation (28) means that Eq. (35) contain only $K_3=0$ components.

Similarly, the quark fields are expanded in the Kana-Ripka bases $\phi_{KK_3\alpha}$ [32]:

$$\psi(\mathbf{r})=\sum_{KK_3\alpha} C_{KK_3\alpha}\phi_{KK_3\alpha}(\mathbf{r}), \tag{36}$$

where the subscript α indicates quantum numbers other than K and K_3. The matrix elements of H_0' with $\phi_{KK_3\alpha}$ have a contribution only from the bases with the same K_3.

A. The self-consistent classical soliton

The lowest order perturbative expansion of the action iS_F is given by

$$iS_{F0}=N_c\log\det(i\partial_t-H_0')=-iE_{cl}T, \tag{37}$$

where T is a sufficiently large time interval and E_{cl} is the classical soliton energy. With the eigenvalues ϵ_n of H_0' and ϵ_n^0 of

$$K'=\lim_{U_0\to 1}H_0'=\frac{1}{i}\boldsymbol{\alpha}\cdot\nabla+\beta(M+\hat{m}_B), \tag{38}$$

we obtain

$$E_{cl}=\sum_n [N_c\eta_n^{val}+\rho_\Lambda^{vac}(\epsilon_n)]\epsilon_n - \sum_n \rho_\Lambda^{vac}(\epsilon_n^0)\epsilon_n^0, \tag{39}$$

where η_n^{val} is the occupation number of the valence quark level ϵ_n and $\rho_\Lambda^{vac}(\epsilon_n)$ is the cutoff function of the vacuum energy for level ϵ_n with cutoff parameter Λ (Appendix). In this article, we use Schwinger's proper time regularization
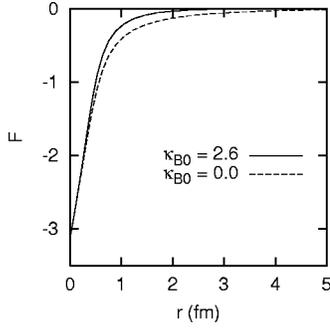
FIG. 2. The radial component of the profile function F with $|\kappa_{B3}|=0$. The solid line represents the function at $\kappa_{B0}=2.6$ and the dashed line at $\kappa_{B0}=0$.

[11,13,33] and set $\Lambda=700$ MeV, and we assume that the valence quark is in the lowest positive energy state.

To subtract E_{cl}, we use K' but

$$K=\lim_{U\to 1}H=\frac{1}{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta(M+\hat{m}). \tag{40}$$

The reason is the following. Physically, E_{cl} is the static soliton energy in the body fixed frame and its subtraction point is the Dirac sea in the absence of a static soliton. We employ the body fixed frame in which κ_{B0} and κ_{B3} are nonzero. In this frame the Hamiltonian without the static soliton is K'.

The equations of motion for the profile functions [14] are obtained by the extremum conditions for the action S_{F0} with respect to the radial component F and the direction of the unit vector $\hat{\boldsymbol{\Lambda}}$:

$$S(\mathbf{r})\sin F(\mathbf{r})=\mathbf{P}(\mathbf{r})\cdot\hat{\boldsymbol{\Lambda}}(\mathbf{r})\cos F(\mathbf{r}), \tag{41}$$

$$\hat{\boldsymbol{\Lambda}}(\mathbf{r})=\mathrm{sgn}[S(\mathbf{r})\sin 2F(\mathbf{r})]\frac{\mathbf{P}(\mathbf{r})}{|\mathbf{P}(\mathbf{r})|}. \tag{42}$$

Here, $S(\mathbf{r})$ and $\mathbf{P}(\mathbf{r})$ are the scalar-isoscalar and pseudoscalar-isovector densities, respectively, and are defined by
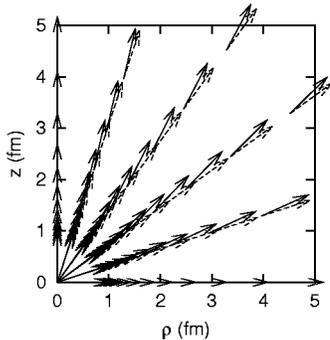
FIG. 3. The unit vector $\hat{\boldsymbol{\Lambda}}$ in isospin space with $|\kappa_{B3}|=\kappa_{B0}$ and $\rho=\sqrt{x^2+y^2}$. The solid arrow represents the vector at $\kappa_{B0}=2.6$ and the dashed arrow at $\kappa_{B0}=0$.
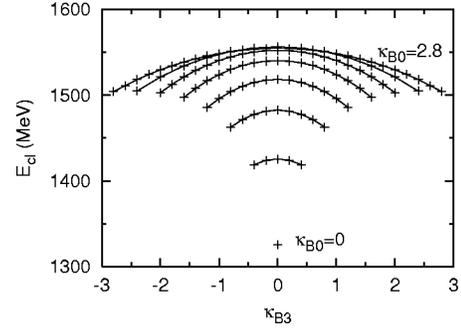
FIG. 4. Classical soliton energy. Each curve corresponds to a value of $\kappa_{B0}=0$–2.8. The range of κ_{B3} is restricted to $-\kappa_{B0}\leq\kappa_{B3}\leq+\kappa_{B0}$.

$$S(\mathbf{r})=\sum_n \rho_\Lambda^R(\epsilon_n)\bar{\psi}_n(\mathbf{r})\psi_n(\mathbf{r}), \tag{43}$$

$$\mathbf{P}(\mathbf{r})=\sum_n \rho_\Lambda^R(\epsilon_n)\bar{\psi}_n(\mathbf{r})i\gamma_5\boldsymbol{\tau}\psi_n(\mathbf{r}), \tag{44}$$

where $\psi_n(\mathbf{r})=\langle\mathbf{r}|n\rangle$, $|n\rangle$ is the eigenvector belonging to the eigenvalue ϵ_n of H_0', and $\rho_\Lambda^R(\epsilon_n)$ is the cutoff function shown in the Appendix. Using the boundary conditions

$$\lim_{r\to 0}F(\mathbf{r})=-\pi, \tag{45}$$

$$\lim_{r\to\infty}F(\mathbf{r})=0, \tag{46}$$

Eqs. (41),(42) and the Dirac equation with the Hamiltonian Eq. (25) are self-consistently solved [14].

Both the profile functions and the classical soliton energy are some even functions of κ_{B3} because of the isospin symmetry of the model. The value of κ_{B0} mainly affects the r dependence of the profile functions and the value of κ_{B3} affects the θ dependence of the profile functions. It was found from the calculation that F hardly depends on $|\kappa_{B3}|$. Thus, we can treat F as a function of r only. The profile $F(\mathbf{r})$ is shown for the cases of $\kappa_{B0}=0,2.6$ with $|\kappa_{B3}|=0$ in Fig. 2. It is shown below that the range $\kappa_{B0}=0$–2.6 corresponds to the octet baryon. On the other hand, the unit vector $\hat{\boldsymbol{\Lambda}}$ has axial symmetry about the z axis and plane symmetry with respect to the xy plane. The deviation from the hedgehog shape is an increasing function of $|\kappa_{B3}|$ and takes a maximum at $\theta=\pi/4,3\pi/4$ and a minimum at $\theta=0,\pi/2,\pi$ in real space. We show $\hat{\boldsymbol{\Lambda}}$ at $\kappa_{B0}=\kappa_{B3}=0,2.6$ in Fig. 3.

Figure 4 displays the κ_{B3} dependence of E_{cl}. Each curve corresponds to a value of κ_{B0}, and the range of κ_{B3} is restricted by Eq. (21). The classical soliton energy E_{cl} is an increasing function of κ_{B0} around $\kappa_{B0}=0$ and a decreasing function of $|\kappa_{B3}|$ at a fixed κ_{B0}. Because the deviation from the hedgehog shape increases with $|\kappa_{B3}|$, the soliton takes a stable deformed shape at $\kappa_{B0}\neq 0$.

B. Effective Lagrangian and Hamiltonian

There are two expansion parameters for the effective action S_F: the number of colors N_c and the SU(3) symmetry breaking m_8. We expand S_F up to second order in powers of $1/N_c$ and first order in powers of m_8.

First, we define the local variables α, a, and b of the rotation $\mathcal{A}$, A, and B, respectively, and the fluctuation σ by

$$\dot{\alpha}^\mu T_\mu = -i\mathcal{A}^\dagger \dot{\mathcal{A}}$$

$$= A_s^\dagger \begin{pmatrix} -iA^\dagger \dot{A} & 0 \\ 0^\dagger & 0 \end{pmatrix} A_s - iA_s^\dagger \dot{A}_s, \tag{47}$$

$$\dot{a}^j \frac{\tau_j}{2} = -iA^\dagger \dot{A}, \tag{48}$$

$$\dot{b}^j J_j^{(q)} = -iB\dot{B}^\dagger, \tag{49}$$

$$\sigma^\mu T_\mu = \hat{m}' - \hat{m}_B, \tag{50}$$

where $T_\mu = \lambda_\mu/2$, $\mu = 1,2,\ldots,8$, and $j = 1,2,3$. Since in the large N_c limit [1,2]

$$E_{cl} \sim N_c, \tag{51}$$

$$iA^\dagger \dot{A} \sim 1/N_c, \tag{52}$$

$$iB\dot{B}^\dagger \sim 1/N_c, \tag{53}$$

$$D, D^\dagger \sim 1/\sqrt{N_c}, \tag{54}$$

we expand S_F up to second order in $\dot{\alpha}$ and $\dot{b}$, and first order in σ. Thus, we get the Lagrangian [12,15]

$$\mathcal{L} = \frac{S_F}{T} = -E_{cl} + \frac{1}{2}\dot{\alpha}^\mu \dot{\alpha}^\nu U_{\mu\nu} + \frac{1}{2}\dot{b}^i \dot{b}^j V_{ij} - \dot{\alpha}^\mu \dot{b}^j W_{\mu j}$$

$$+ \dot{\alpha}^\mu \sigma^\nu \Delta_{\mu\nu} - \dot{b}^i \sigma^\nu \tilde{\Delta}_{i\mu} - \dot{\alpha}^\mu B_\mu + \dot{b}^i \tilde{B}_i - \sigma^\mu \Gamma_\mu, \tag{55}$$

Since $\dot{\alpha}$ and σ in Eq. (55) are functions of D, $D^\dagger$ and $iA^\dagger \dot{A}$, we should further expand these quantities in the actual calculation.

Next, we would like to express the Hamiltonian in terms of a, b, D and their canonically conjugate momenta. The ca-

where the coefficients are defined as

$$U_{\mu\nu} = \sum_{m \neq n} \rho_\Lambda^R(\epsilon_m, \epsilon_n) \langle m|T_\mu|n\rangle \langle n|T_\nu|m\rangle,$$

$$V_{ij} = \sum_{m \neq n} \rho_\Lambda^R(\epsilon_m, \epsilon_n) \langle m|J_i^{(q)}|n\rangle \langle n|J_j^{(q)}|m\rangle,$$

$$W_{\mu j} = -\sum_{m \neq n} \rho_\Lambda^R(\epsilon_m, \epsilon_n) \langle m|T_\mu|n\rangle \langle n|J_j^{(q)}|m\rangle,$$

$$B_\mu = \sum_m \rho_\Lambda^I(\epsilon_m) \langle m|T_\mu|m\rangle,$$

$$\tilde{B}_i = -\sum_m \rho_\Lambda^I(\epsilon_m) \langle m|J_i^{(q)}|m\rangle,$$

$$\Delta_{\mu\nu} = \sum_{m \neq n} \rho_\Lambda^I(\epsilon_m, \epsilon_n) \langle m|T_\mu|n\rangle \langle n|\beta T_\nu|m\rangle,$$

$$\tilde{\Delta}_{i\nu} = -\sum_{m \neq n} \rho_\Lambda^I(\epsilon_m, \epsilon_n) \langle m|J_i^{(q)}|n\rangle \langle n|\beta T_\nu|m\rangle,$$

$$\Gamma_\mu = \sum_m \rho_\Lambda^R(\epsilon_m) \langle m|\beta T_\mu|m\rangle. \tag{56}$$

Here, $\rho_\Lambda^{R,I}(\epsilon_m)$ and $\rho_\Lambda^{R,I}(\epsilon_m, \epsilon_n)$ are the cutoff functions shown in the Appendix. The indices R and I denote the origin of the vacuum part of the coefficients from the real and imaginary parts of the action S_F in the imaginary time prescription. We subtract the vacuum contribution of the eigenstates of K' [Eq. (38)] from these quantities in a similar way to E_{cl}.

By K_3 symmetry, Eq. (55) becomes

$$\mathcal{L} = -E_{cl} + \frac{1}{2}(\dot{\alpha}^1 \dot{\alpha}^1 + \dot{\alpha}^2 \dot{\alpha}^2)U_{11} + \frac{1}{2}(\dot{b}^1 \dot{b}^1 + \dot{b}^2 \dot{b}^2)V_{11} - (\dot{\alpha}^1 \dot{b}^1 + \dot{\alpha}^2 \dot{b}^2)W_{11} + (\dot{\alpha}^1 \sigma^1 + \dot{\alpha}^2 \sigma^2)\Delta_{11} - (\dot{b}^1 \sigma^1 + \dot{b}^2 \sigma^2)\tilde{\Delta}_{11}$$

$$+ \frac{1}{2}(\dot{\alpha}^3 - \dot{b}^3)^2 U_{33} - (\dot{\alpha}^3 - \dot{b}^3)(B_3 - \sigma^3 \Delta_{33} - \sigma^8 \Delta_{38}) - \dot{\alpha}^8 B_8 - \sigma^3 \Gamma_3 - \sigma^8 \Gamma_8 + \frac{1}{2}(\dot{\alpha}^4 \dot{\alpha}^4 + \dot{\alpha}^5 \dot{\alpha}^5)U_{44}$$

$$+ \frac{1}{2}(\dot{\alpha}^6 \dot{\alpha}^6 + \dot{\alpha}^7 \dot{\alpha}^7)U_{66} + (\dot{\alpha}^4 \sigma^4 + \dot{\alpha}^5 \sigma^5)\Delta_{44} + (\dot{\alpha}^6 \sigma^6 + \dot{\alpha}^7 \sigma^7)\Delta_{66}. \tag{57}$$

nonical momenta conjugate to these coordinates are defined by

$$I_i = \frac{\partial \mathcal{L}}{\partial \dot{a}^i}, \tag{58}$$

$$J_i = \frac{\partial \mathcal{L}}{\partial \dot{b}^i}, \tag{59}$$

$$P^\gamma = \frac{\partial \mathcal{L}}{\partial \dot{D}^\dagger_\gamma} \quad (\gamma = 1, 2), \tag{60}$$

and satisfy the following commutation relations:

$$[I_i, a^j] = [J_i, b^j] = \frac{1}{i} \delta^j_i, \tag{61}$$

$$[P^\gamma, D^\dagger_\beta] = [P^\dagger_\beta, D^\gamma] = \frac{1}{i} \delta^\gamma_\beta. \tag{62}$$

The collective Hamiltonian derived from Eq. (57),

$$\mathcal{H} = \dot{a} \cdot \mathbf{I} + \dot{b} \cdot \mathbf{J} + P^\dagger \dot{D} + \dot{D}^\dagger P - \mathcal{L}, \tag{63}$$

is calculated up to order $1/N_c$. Because of the K_3 symmetry of the chiral fields, there is the following constraint on the canonical momenta:

$$I_3 + J_3 + I_{K3} = 0. \tag{64}$$

Here I_{K3} is the third component of the isospin carried by the kaon,

$$\mathbf{I}_K = i \left(D^\dagger \frac{\boldsymbol{\tau}}{2} P - P^\dagger \frac{\boldsymbol{\tau}}{2} D \right). \tag{65}$$

If we assume the hedgehog shape for the chiral fields, the constraint becomes

$$\mathbf{I} + \mathbf{J} + \mathbf{I}_K = 0. \tag{66}$$

IV. DIAGONALIZATION OF THE COLLECTIVE HAMILTONIAN

A. The mean field approximation

Before diagonalizing $\mathcal{H}$ [Eq. (63)], we will describe $\mathcal{H}_0$, which is the $O(1)$ Hamiltonian of $\mathcal{H}$ in the large N_c limit in the mean field approximation:

$$\mathcal{H}_0 = E_{cl} + E_{ind} + \frac{1}{4\Phi_{(+)}} P^\dagger P + \frac{1}{4\Phi_{(-)}} P^\dagger \tau_3 P + 3\left(\frac{B_8^2}{4\Phi_{(+)}} + 2m_8(\Delta_{(+)}B_8 - \Gamma_8) \right) D^\dagger D + \sqrt{3}\left(\frac{B_8}{4\Phi_{(+)}} + m_8 \Delta_{(+)} \right)$$

$$\times i(D^\dagger P - P^\dagger D) + 3\left[\frac{B_8^2}{4\Phi_{(-)}} + 2m_8\left(\Delta_{(-)}B_8 - \frac{\Gamma_3}{\sqrt{3}} \right) \right] D^\dagger \tau_3 D + \sqrt{3}\left(\frac{B_8}{4\Phi_{(-)}} + m_8 \Delta_{(-)} \right) i(D^\dagger \tau_3 P - P^\dagger \tau_3 D), \tag{67}$$

where

$$E_{ind} = 3m_8\left(\kappa_{B0}\Gamma_8 + \kappa_{B3}\frac{\Gamma_3}{\sqrt{3}} \right) \frac{\sin^2\sqrt{\kappa_{B0}}}{\kappa_{B0}}, \tag{68}$$

$$\frac{1}{\Phi_{(\pm)}} = \frac{1}{2}\left(\frac{1}{U_{44}} \pm \frac{1}{U_{66}} \right), \tag{69}$$

$$\Delta_{(\pm)} = \frac{1}{2}\left(\frac{\Delta_{44}}{U_{44}} \pm \frac{\Delta_{66}}{U_{66}} \right). \tag{70}$$

The eigenstate of $\mathcal{H}_0$ describes the kaon in the background soliton. The energy E_{ind} is induced by the deviation to the strange and isospin direction from the SU(2) hedgehog soliton. Here the Γ_8 term in E_{ind} gives a negative contribution to $\mathcal{H}_0$ and Γ_3 a positive one.

The Hamiltonian $\mathcal{H}_0$ is in a bilinear form of the isodoublets D and P, and the type of separation of variables with respect to the individual component. For the individual components, the Hamiltonian except for the constant terms is given by

$$\mathcal{H}_\gamma = h_{1\gamma}P^\dagger_\gamma P^\gamma + h_{2\gamma}D^\dagger_\gamma D^\gamma + h_{3\gamma}(P^\dagger_\gamma D^\gamma + D^\dagger_\gamma P^\gamma)$$

$$+ h_{4\gamma}i(P^\dagger_\gamma D^\gamma - D^\dagger_\gamma P^\gamma) \quad (\gamma = 1, 2), \tag{71}$$

where $h_{i\gamma}$ are constants for each component. If $h_{i\gamma}$ satisfy

$$h_{1\gamma}h_{2\gamma} - (h_{3\gamma})^2 \geqslant 0, \tag{72}$$

$\mathcal{H}_0$ can be diagonalized exactly using the following transformations [34]:

$$D^\gamma = c_\gamma(\xi_\gamma + \bar{\xi}^\dagger_\gamma), \tag{73}$$

$$P^\gamma = \frac{1}{2ic_\gamma \cos\delta_\gamma}(e^{-i\delta_\gamma}\xi_\gamma - e^{+i\delta_\gamma}\bar{\xi}^\dagger_\gamma), \tag{74}$$

where c_γ and δ_γ are constants depending on $h_{i\gamma}$. The quantities $\xi^\dagger_\gamma$ (ξ_γ) and $\bar{\xi}^\dagger_\gamma$ ($\bar{\xi}_\gamma$) are the creation (annihilation) operators for states with the same quantum numbers as the kaon and antikaon, respectively. These satisfy the commutation relations

$$[\xi_\gamma, \xi^\dagger_\beta] = [\bar{\xi}_\gamma, \bar{\xi}^\dagger_\beta] = \delta_{\gamma\beta}, \tag{75}$$

$$[\xi_\gamma, \bar{\xi}_\beta] = [\xi_\gamma, \bar{\xi}^\dagger_\beta] = 0. \tag{76}$$

This then finally leads to the diagonalized form

$$\mathcal{H}_0 = E_{cl} + E_{ind} + \sum_{\gamma=1}^{2} (\omega_\gamma \xi_\gamma^\dagger \xi_\gamma + \bar{\omega}_\gamma \bar{\xi}_\gamma^\dagger \bar{\xi}_\gamma), \qquad (77)$$

where

$$\omega_\gamma = \mathrm{sgn}(h_{1\gamma}) \sqrt{h_{1\gamma} h_{2\gamma} - (h_{3\gamma})^2} - h_{4\gamma}, \qquad (78)$$

$$\bar{\omega}_\gamma = \mathrm{sgn}(h_{1\gamma}) \sqrt{h_{1\gamma} h_{2\gamma} - (h_{3\gamma})^2} + h_{4\gamma}. \qquad (79)$$

There are also two physical quantities diagonalized by the creation and annihilation operators: the third component of the isospin carried by the kaon [Eq. (65)],

$$I_{K3} = \frac{1}{2}(\xi_1^\dagger \xi_1 - \bar{\xi}_1^\dagger \bar{\xi}_1 - \xi_2^\dagger \xi_2 + \bar{\xi}_2^\dagger \bar{\xi}_2), \qquad (80)$$

and the strangeness carried by the kaon,

$$S = i(D^\dagger P - P^\dagger D) = \sum_{\gamma=1}^{2} (\xi_\gamma^\dagger \xi_\gamma - \bar{\xi}_\gamma^\dagger \bar{\xi}_\gamma). \qquad (81)$$

The Fock space is generated by successive operation of the creation operators on the vacuum state $|0\rangle$:

$$|n_1, \bar{n}_1, n_2, \bar{n}_2\rangle = \prod_{\gamma=1}^{2} \frac{1}{\sqrt{n_\gamma! \bar{n}_\gamma!}} (\xi_\gamma^\dagger)^{n_\gamma} (\bar{\xi}_\gamma^\dagger)^{\bar{n}_\gamma} |0\rangle, \quad (82)$$

where n_γ and $\bar{n}_\gamma$ are some positive integers.

In the mean field approximation, κ_{B0} and κ_{B3} should be self-consistently determined by Eqs. (19) and (20). Then the stability of the approximation should be checked. Because the coefficients of the individual terms of $\mathcal{H}_0$ are evaluated using a soliton depending on κ_{B0} and κ_{B3}, the potential term is physically meaningful only in the vicinity of the expectation values. It is difficult to draw a potential diagram over a wide range of D and $D^\dagger$. The reason is that $\mathcal{H}_0$ describes the combined system of the classical soliton and the kaon.

In order to investigate the stability of the mean field approximation we will treat κ_{B0} and κ_{B3} as parameters at first. When κ_{B0} and κ_{B3} are different from the expectation values of Eqs. (19) and (20), the differences

$$\kappa_{B0} - \langle B|\kappa_0|B\rangle, \qquad (83)$$

$$\kappa_{B3} - \langle B|\kappa_3|B\rangle \qquad (84)$$

act on the system as a kind of external field. Therefore, by investigating the κ_{B0} and κ_{B3} dependencies of the lowest eigenvalue E_0 of $\mathcal{H}_0$, we can study the stability of the system against external perturbation. Figure 5 shows the behavior of E_0 as a function of κ_{B0} and κ_{B3} in the cases of $S = 0, -1, -2$. E_0 is an even function of κ_{B3} like E_{cl}.

We further investigate the characteristic behavior of E_0 in the three cases. Figure 6 shows the κ_{B0} dependence of E_0. For $S = 0$, the κ_{B0} dependence of E_0 is similar to that of the SU(2) sector quark mass $3(m_{Bu} + m_{Bd})/2$. It is due to the Γ_8 term in E_{ind} of Eq. (67) that E_0 is flat compared with E_{cl} or $3(m_{Bu} + m_{Bd})/2$. In the case of $S = -1$, E_0 shows a similar
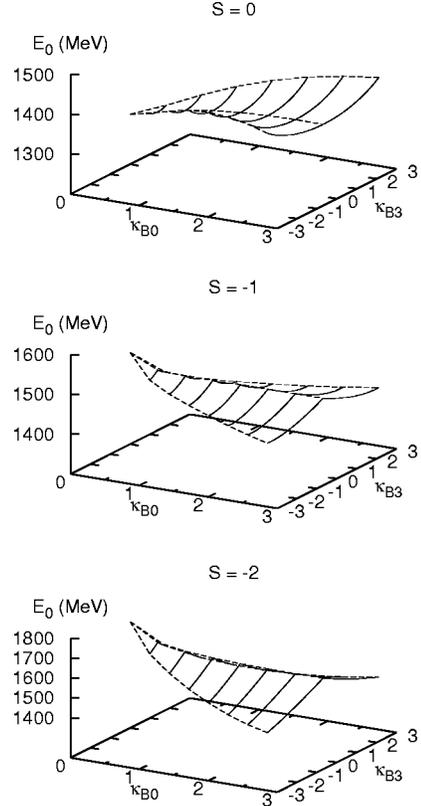
FIG. 5. The lowest eigenvalues of $\mathcal{H}_0$, where κ_{B0} and κ_{B3} are treated as parameters.

tendency to $m_{Bu} + m_{Bd} + m_{Bs}$. In the case of $S = -2$, E_0 is similar to $(m_{Bu} + m_{Bd})/2 + 2m_{Bs}$.

Figure 7 shows the κ_{B3} dependence of E_0. It is due to the Γ_3 term in E_{ind} that E_0 is convex downward as a function of κ_{B3} for $S = 0$. For $S = -1, -2$, since E_{ind} is canceled by the potential of D and $D^\dagger$, E_0 is convex upward due to the κ_{B3} dependence of $m_{Bu,d}$ (Fig. 1).

Therefore, in the case of $S = 0$ it is energetically forbidden that the soliton deviate from the hedgehog shape, and the harmonic analysis around $\kappa_{B0} = \kappa_{B3} = 0$ is justified. On the other hand, in cases of $S = -1, -2$ E_0 is energetically unstable at $\kappa_{B3} = 0$. Thus the soliton deviates from the hedgehog shape.
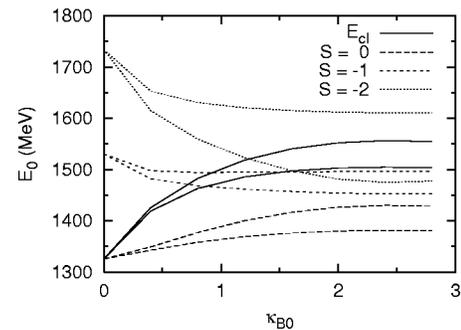
FIG. 6. The lowest eigenvalues of $\mathcal{H}_0$ and the classical soliton energy in the cases of $|\kappa_{B3}| = 0$ and $|\kappa_{B3}| = \kappa_{B0}$. Between the two curves with $S = 0$, the case of $|\kappa_{B3}| = 0$ corresponds to lower energy. For $S = -1, -2$ it corresponds to higher energy.
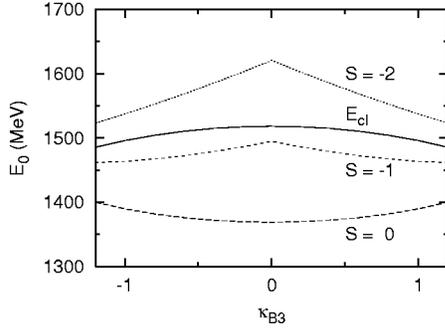
FIG. 7. The lowest eigenvalues of $\mathcal{H}_0$ and the classical soliton energy at $\kappa_{B0}=1.0$.

However, it is necessary to keep in mind that under the condition of Eq. (72) the potential term of $\mathcal{H}_0$ is convex downward and instability for $D, D^\dagger$ is not caused. Thus, the instability is due to the classical soliton. If the degrees of freedom for the κ_{B3} direction are not included in the soliton, the instability does not occur.

In Table I we show κ_{B0}, $|\kappa_{B3}|$, and E_0 ($\equiv E_{mf}$), which are self-consistently determined in the mean field approximation.

B. Baryon as the rotational band

The Hamiltonian $\mathcal{H}$ [Eq. (63)] is a highly complicated function of U, V, etc. Thus we focus on the algebraic point of view and do not show its exact form at this point. The actual calculation will be performed numerically.

For the diagonalization of $\mathcal{H}$, we introduce the eigenstates of the body fixed operators $\hat{\mathbf{J}}^2, \hat{J}_3, \hat{\mathbf{I}}^2, \hat{I}_3$ defined by Eqs. (58) and (59) and the space fixed spin $\hat{j}_3$ and isospin $\hat{i}_3$ operators. Hereafter we denote the operators by a character with a caret, and a character without a caret denotes its eigenvalue:

$$\hat{\mathbf{J}}^2|J,j_3,J_3;I,i_3,I_3\rangle = J(J+1)|J,j_3,J_3;I,i_3,I_3\rangle,$$

$$\hat{j}_3|J,j_3,J_3;I,i_3,I_3\rangle = j_3|J,j_3,J_3;I,i_3,I_3\rangle,$$

$$\hat{J}_3|J,j_3,J_3;I,i_3,I_3\rangle = J_3|J,j_3,J_3;I,i_3,I_3\rangle,$$

$$\hat{\mathbf{I}}^2|J,j_3,J_3;I,i_3,I_3\rangle = I(I+1)|J,j_3,J_3;I,i_3,I_3\rangle,$$

$$\hat{i}_3|J,j_3,J_3;I,i_3,I_3\rangle = i_3|J,j_3,J_3;I,i_3,I_3\rangle,$$

$$\hat{I}_3|J,j_3,J_3;I,i_3,I_3\rangle = I_3|J,j_3,J_3;I,i_3,I_3\rangle \tag{85}$$

with

$$-J \leq j_3, J_3 \leq J, \tag{86}$$

$$-I \leq i_3, I_3 \leq I. \tag{87}$$

By using the constraint of Eq. (64) between J_3, I_3, and I_{K3}, the basis for the whole space is given by

$$|J,j_3,J_3;I,i_3,-(J_3+I_{K3})\rangle|n_1,\bar{n}_1,n_2,\bar{n}_2\rangle, \tag{88}$$

TABLE I. κ_{B0}, $|\kappa_{B3}|$, and E_{mf} for $S=0,-1,-2$ in the mean field approximation.

S	κ_{B0}	$|\kappa_{B3}|$	E_{cl} (MeV)	E_{mf} (MeV)
0	0	0	1326	1326
-1	0.8	0.8	1463	1468
-2	2.6	2.6	1505	1475

where $I_{K3}=(n_1-\bar{n}_1-n_2+\bar{n}_2)/2$ from Eqs. (80) and (82). Because $\mathcal{H}$ commutes with $\hat{S}$ [Eq. (81)], the diagonalization is well performed in the subspace with fixed eigenvalues (J,j_3), (I,i_3), $S=n_1-\bar{n}_1+n_2-\bar{n}_2$. For the octet baryon, $J=|j_3|=1/2$.

Corresponding to the expansion of $\mathcal{H}$ up to order $1/N_c$, we truncate the Fock space of Eq. (82) on the condition that

$$n_1+\bar{n}_1+n_2+\bar{n}_2 \leq |S|+2. \tag{89}$$

Here we introduce the symbols $|K\rangle$ for the states created by $\xi_\gamma^\dagger$ and $|\bar{K}\rangle$ for $\bar{\xi}_\gamma^\dagger$. The particular states with the same S consist of $\|S|\bar{K}\rangle$ (valence kaon), $\|S|\bar{K}+K\bar{K}\rangle$ (valence kaon +kaon-antikaon pair), etc. Since we determined κ_{B0} and κ_{B3} for the lowest eigenstate $\|S|\bar{K}\rangle$ of $\mathcal{H}_0$ in the mean field approximation, the state $\|S|\bar{K}+K\bar{K}\rangle$ may be far from the mean field. Thus, we should not treat equally $\|S|\bar{K}\rangle$ and $\|S|\bar{K}+K\bar{K}\rangle$. First, we diagonalize $\mathcal{H}$ in the subspace $\|S|\bar{K}\rangle$. Next $\|S|\bar{K}+K\bar{K}\rangle$ is treated as a virtual state in the perturbation.

For example, the bases for the Σ particle are given by

$$\begin{pmatrix} |J,j_3,+1/2;I,i_3,-1\rangle|0,0,0,1\rangle \\ |J,j_3,-1/2;I,i_3,0\rangle|0,0,0,1\rangle \\ |J,j_3,+1/2;I,i_3,0\rangle|0,1,0,0\rangle \\ |J,j_3,-1/2;I,i_3,+1\rangle|0,1,0,0\rangle \\ |J,j_3,+1/2;I,i_3,-1\rangle|0,0,1,2\rangle \\ |J,j_3,-1/2;I,i_3,0\rangle|0,0,1,2\rangle \\ |J,j_3,+1/2;I,i_3,0\rangle|0,1,1,1\rangle \\ |J,j_3,-1/2;I,i_3,+1\rangle|0,1,1,1\rangle \\ |J,j_3,+1/2;I,i_3,+1\rangle|0,2,1,0\rangle \\ |J,j_3,-1/2;I,i_3,-1\rangle|1,0,0,2\rangle \\ |J,j_3,+1/2;I,i_3,-1\rangle|1,1,0,1\rangle \\ |J,j_3,-1/2;I,i_3,0\rangle|1,1,0,1\rangle \\ |J,j_3,+1/2;I,i_3,0\rangle|1,2,0,0\rangle \\ |J,j_3,-1/2;I,i_3,+1\rangle|1,2,0,0\rangle \end{pmatrix}. \tag{90}$$

Here the top four bases span the valence kaon states $\|S|\bar{K}\rangle$, and the others are bases of $\|S|\bar{K}+K\bar{K}\rangle$.

Diagonalization of $\mathcal{H}$ in the basis of Eq. (88) gives an estimation of the fluctuation for D, J, I around the mean field approximation. The results of the calculation are displayed in

TABLE II. Baryon masses E_{B1} (without $|K\bar{K}\rangle$) and E_{B2} (with $|K\bar{K}\rangle$). Expt. denotes experimental value.

Particle	E_{B1} (MeV)	E_{B2} (MeV)	Expt. (MeV)
N	1382	1369	939
Λ	1195	1187	1116
Σ	1218	1200	1193
Ξ	1437	1330	1318

Table II and compared with the hedgehog results at $\kappa_{B0} = \kappa_{B3} = 0$ in Fig. 8. Here E_{B1} is the eigenvalue of the collective Hamiltonian $\mathcal{H}$ in only $\|S|\bar{K}\rangle$. The mass E_{B2} is the eigenvalue in the state which incorporate $|K\bar{K}\rangle$ states as a perturbation. Except for N, the energy eigenvalues E_{B2} are smaller than the hedgehog results and close to the experimental values.

In the eigenstates of every baryon, there are large transitions among the bases with different values of $J_3 + I_3$, that is, remains of the Clebsch-Gordan series for the constraint Eq. (66) which is maintained only in the hedgehog shape. In the case of N, the energy eigenvalues are increased by the transition. The situation does not change on incorporating the $|K\bar{K}\rangle$ states. For Λ and Σ, an interaction induced by the deformation of the soliton picks up the transition. Then the energy shows a decrease. That is caused only in the space with $\|S|\bar{K}\rangle$. For Ξ, there is a similar decrease in the space that takes account of $|K\bar{K}\rangle$ perturbatively. The contributions of $|K\bar{K}\rangle$ to the energy take negative values for every baryon due to the second order perturbation formula.

V. DISCUSSION

The stability of the hedgehog shape has been investigated for the octet baryon by means of the chiral quark soliton model.

In the mean field approximation, it was shown that the stable form of the soliton changes according to the strangeness of the baryon. In the case of the soliton without strange-
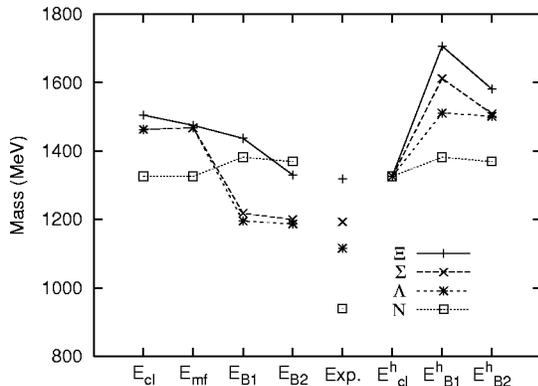
FIG. 8. Comparison of the energy between the deformed soliton and the hedgehog one ($\kappa_{B0} = \kappa_{B3} = 0$). The superscript h of E^h represent the results of the hedgehog soliton. Exp. denotes the experimental values.

ness, the hedgehog shape is stable. On the other hand, for the soliton with strangeness, an instability occurs not only in the strange direction but also in the isotopic one. The instability has a global nature due to the inertial force, and one cannot find it from a study of the curvature of the local potential. It is necessary to incorporate the degrees of freedom for the deviation in the ansatz of the soliton.

After the collective quantization has been performed, the states identified as the strange baryons (Λ, Σ, Ξ) also energetically favor the deformed soliton. The strange baryon masses are in good agreement with both the absolute value and the difference among the experimental values. These become small compared with the results of the mean field approximation due to the interaction caused by the deformation of the soliton. On the other hand, the approach with the hedgehog soliton reproduces the mass difference among the baryons well but the absolute value. Also, the collective states have higher energy than the classical soliton.

Weigel *et al.* investigated the quantum correction due to the zero modes off the hedgehog soliton in the NJL model [35,36]. The correction gives a large negative contribution to the N and Δ masses, and the results are in good agreement with the experimental values. In our approach, the mass of N is somewhat too large due to the hedgehog shape of the soliton. Therefore, both approaches cooperatively would be in agreement with the experimental value for N. On the other hand, the deformed soliton corresponding to the strange baryon has less symmetry than the hedgehog soliton. Because of the estimation [35,36] relied on the hedgehog shape [35], the relation between the two approaches is obscure at present.

The eigenvalue of the Hamiltonian up to order 1 in the large N_c limit is an even function of κ_{B3}. Therefore the mean field energy at κ_{B3} is degenerate with that at $-\kappa_{B3}$. A more accurate calculation of the fluctuation off the mean field should treat the tunnel effect.

A description of the decuplet baryon incorporating the local variation of the kaon wave function are in progress.

ACKNOWLEDGMENTS

APPENDIX: CUTOFF FUNCTION

The cutoff function of the vacuum energy for the energy level ϵ_n is given by

$$\rho_\Lambda^{vac}(\epsilon_n) = N_c \text{sgn}(\epsilon_n) \bar{\mathcal{N}}_\Lambda(\epsilon_n) \qquad (A1)$$

with the cutoff parameter Λ, where, using the Schwinger proper time regularization, $\bar{\mathcal{N}}_\Lambda$ may be written as

$$\bar{\mathcal{N}}_\Lambda(\epsilon_m) = \frac{1}{4\sqrt{\pi}} \int_{(\epsilon_m/\Lambda)^2}^{\infty} \frac{e^{-\rho}}{\rho^{3/2}} d\rho. \qquad (A2)$$

The cutoff function for the first order matrix element is given by

$$\rho_\Lambda^{R,I}(\epsilon_m) = N_c \, \eta_m^{val} + N_c \mathrm{sgn}(\epsilon_m) \mathcal{N}_\Lambda^{R,I}(\epsilon_m) \qquad \text{(A3)}$$

with

$$\mathcal{N}_\Lambda^R(\epsilon_m) = -\frac{1}{2} \, \mathrm{erfc}(|\epsilon_m/\Lambda|), \qquad \text{(A4)}$$

$$\mathcal{N}_\Lambda^I(\epsilon_m) = -\frac{1}{2}. \qquad \text{(A5)}$$

The indices R and I denote the origin of the function from the real and imaginary parts of the action S_F in the imaginary time prescription. Similarly, for the second order matrix elements one obtain

$$\rho_\Lambda^{R,I}(\epsilon_m, \epsilon_n) = N_c \, \frac{\eta_m^{val} - \eta_n^{val}}{\epsilon_n - \epsilon_m} + \frac{N_c}{2} f_\Lambda^{R,I}(\epsilon_m, \epsilon_n) \quad \text{(A6)}$$

where $f_\Lambda^{R,I}$ are given by

$$f_\Lambda^R(\epsilon_m, \epsilon_n) = \frac{\mathrm{sgn}(\epsilon_m)\mathrm{erfc}(|\epsilon_m/\Lambda|) - \mathrm{sgn}(\epsilon_n)\mathrm{erfc}(|\epsilon_n/\Lambda|)}{\epsilon_m - \epsilon_n}$$
$$- \frac{2\Lambda}{\sqrt{\pi}} \frac{e^{(\epsilon_m/\Lambda)^2} - e^{(\epsilon_n/\Lambda)^2}}{\epsilon_m^2 - \epsilon_n^2}, \qquad \text{(A7)}$$

$$f_\Lambda^I(\epsilon_m, \epsilon_n) = \frac{\mathrm{sgn}(\epsilon_m) - \mathrm{sgn}(\epsilon_n)}{\epsilon_m - \epsilon_n}. \qquad \text{(A8)}$$

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