Forward-backward asymmetry in $K^+ \to \pi^+ l^+ l^-$

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We study the forward-backward asymmetries in the decays of $K^+ \to \pi^+ l^+ l^-$ ($l = e$ and μ) in the presence of scalar or tensor terms. We find that with the scalar [tensor] type interaction the asymmetry can be up to $O(10^{-3})$ [$O(10^{-1})$] and arbitrarily large for the electron and muon modes, respectively, without conflict with the experimental data. We also discuss the cases in the minimal supersymmetric standard model where the scalar terms can be induced. In particular, we show that the asymmetry in $K^+ \to \pi^+ \mu^+ \mu^-$ can be as large as $O(10^{-3})$ in the large $\tan\beta$ limit, which can be tested in future experiments such as the CKM experiment at Fermilab.

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I. INTRODUCTION

The flavor-changing neutral current processes of $K^\pm \to \pi^\pm l^+ l^-$ ($l = e, \mu$) are suppressed and dominated by the long distance contributions involving one photon exchange [1–4] in the standard model (SM). The decays have been successfully described within the framework of chiral perturbation theory (ChPT) [5] including electroweak interactions at $O(p^6)$ [6] in terms of a vector interaction form factor fixed by experiments. However, it is important to compare the measurements in the two decays to see if there are differences in the form factors, since they would indicate new physics. Recently, the vector form factor has been determined by a high precision measurement on the electron mode by the BNL E865 Collaboration [7] at the Brookhaven Alternating Gradient Synchrotron (AGS) with a sample of 10300 events and branching ratio (BR) of $[2.94 \pm 0.05(\mathrm{stat}) \pm 0.13(\mathrm{syst}) \pm 0.05(\mathrm{theor})] \times 10^{-7}$. For the muon channel, it was first observed by BNL E787 [8] at the AGS with the measured branching ratio being $(5.0 \pm 0.4 \pm 0.9) \times 10^{-8}$, which is too small to accommodate within the SM. However, two subsequent experiments of BNL E865 [9] and HyperCP (E871) [10] measured $\mathrm{BR}(K^+ \to \pi^+ \mu^+ \mu^-) = (9.22 \pm 0.60 \pm 0.49) \times 10^{-8}$ and $(9.8 \pm 1.0 \pm 0.5) \times 10^{-8}$, respectively, which are all consistent with a model-independent analysis based on the data for $\mathrm{BR}(K^+ \to \pi^+ e^+ e^-)$. However, there is still room for new physics, particularly in the muon mode.

In Refs. [11], [12], P and T violating muon polarization effects in $K^+ \to \pi^+ \mu^+ \mu^-$ were discussed in various theoretical models. In this paper, we study the forward-backward asymmetry (FBA) in the decay of $K^+ \to \pi^+ l^+ l^-$ with $l = e$ or μ. It is known that the FBA in $K^+ \to \pi^+ l^+ l^-$ violates P like the longitudinal lepton polarization but it vanishes in the SM and can exist only if there is a scalar-type interaction. In the multi-Higgs-doublet models such as the most popular two-Higgs-doublet model (2HDM) of type II [13], where two Higgs scalar doublets (H_u and H_d) are coupled to up- and down-type quarks, respectively, the scalar type of four fermion operators $\overline{s_R} d_L \overline{l} l$ can be generated at the loop level [14,15]. This type of operator is particularly interesting in the minimal supersymmetric standard model (MSSM) [16] since it receives an enormous enhancement for a large ratio of $v_u / v_d = \tan\beta$ where $v_{u(d)}$ is the vacuum expectation value of the Higgs doublet $H_{u(d)}$. Recently, there has been considerable interest in the large $\tan\beta$ effects in B decays such as $B \to \mu^+ \mu^-$ and $B \to K \mu^+ \mu^-$ [14,15,17]. In the report, we will discuss the large $\tan\beta$ scenario in the MSSM for $K^+ \to \pi^+ l^+ l^-$.

The paper is organized as follows. In Sec. II, we present the general analysis for the forward-backward asymmetries in $K^+ \to \pi^+ l^+ l^-$ ($l = e$ and μ). In Sec. III, we discuss the experimental constraints on the asymmetries. We estimate the asymmetries in the MSSM in Sec. IV. We present our conclusions in Sec. V.

II. GENERAL ANALYSIS

We write the decay as

$$K^+(p_K) \to \pi^+(p_\pi) l^+(p) l^-(\overline{p}), \tag{1}$$

where p_K, p_π, p, and $\overline{p}$ are the four-momenta of K^+, π^+, l^+, and l^-, respectively. The most general invariant amplitude for the decay can be written as [11,12,18]

$$\mathcal{M} = F_S \overline{l} l + i F_P \overline{l} \gamma_5 l + F_V p_K^\mu \overline{l} \gamma_\mu l + F_A p_K^\mu \overline{l} \gamma_\mu \gamma_5 l, \tag{2}$$

where F_S, F_P, F_V, and F_A are scalar, pseudoscalar, vector, and axial-vector form factors, respectively. The differential decay rate in the K^+ rest frame is given by [11]

$$\frac{d^2\Gamma}{dE\,d\overline{E}}=\frac{1}{2^4\pi^3 m_K}\Bigg[|F_S|^2\frac{1}{2}(s-4m_l^2)+|F_P|^2\frac{1}{2}s+|F_V|^2 m_K^2\left(2E\overline{E}-\frac{1}{2}s\right)+|F_A|^2 m_K^2\left(2E\overline{E}-\frac{1}{2}s+2m_l^2\right)+2\,\mathrm{Re}(F_S F_V^*)m_l m_K(E-\overline{E})+\mathrm{Im}(F_P F_A^*)m_l(m_\pi^2-m_K^2-s)\Bigg], \tag{3}$$

where m_l is the lepton mass, E ($\overline{E}$) is the energy of μ^+ (μ^-), and $s=(p+\overline{p})^2=2(m_l^2+E\overline{E}-\mathbf{p}\cdot\overline{\mathbf{p}})$ is the invariant mass of the dilepton system. In terms of the invariant mass and the angle θ between the three-momentum of the kaon and the three-momentum of the l^- in the dilepton rest frame, we can rewrite Eq. (3) as

$$\frac{d^2\Gamma}{ds\,d\cos\theta}=\frac{1}{2^8\pi^3 m_K^3}\beta_l\lambda^{1/2}(s)\Bigg\{|F_S|^2 s\beta_l^2+|F_P|^2 s+|F_V|^2\frac{1}{4}\lambda(s)(1-\beta_l^2\cos^2\theta)+|F_A|^2\left[\frac{1}{4}\lambda(s)(1-\beta_l^2\cos^2\theta)+4m_K^2 m_l^2\right]+\mathrm{Re}(F_S F_V^*)2m_l\beta_l\lambda^{1/2}(s)\cos\theta+\mathrm{Im}(F_P F_A^*)2m_l(m_\pi^2-m_K^2-s)\Bigg\}, \tag{4}$$

where $\lambda(s)=m_K^4+m_\pi^4+s^2-2m_\pi^2 s-2m_K^2 s-2m_\pi^2 m_K^2$ and $\beta_l=(1-4m_l^2/s)^{1/2}$ with s and $\cos\theta$ bounded by

$$4m_l^2\leqslant s\leqslant(m_K-m_\pi)^2,\quad -1\leqslant\cos\theta\leqslant 1. \tag{5}$$

Here, we have used that

$$E=\frac{s+m_K^2-m_\pi^2+\beta_l\lambda^{1/2}(s)\cos\theta}{4m_K},$$

$$\overline{E}=\frac{s+m_K^2-m_\pi^2-\beta_l\lambda^{1/2}(s)\cos\theta}{4m_K}. \tag{6}$$

By integrating the angle θ in Eq. (4), we obtain

$$\frac{d\Gamma}{ds}=\frac{1}{2^8\pi^3 m_K^3}\beta_l\lambda^{1/2}(s)\Bigg\{|F_S|^2 2s\beta_l^2+|F_p|^2 2s+|F_V|^2\frac{1}{3}\lambda(s)\left(1+\frac{2m_l^2}{s}\right)+|F_A|^2\left[\frac{1}{3}\lambda(s)\left(1+\frac{2m_l^2}{s}\right)+8m_K^2 m_l^2\right]+\mathrm{Im}(F_P F_A^*)4m_l(m_\pi^2-m_K^2-s)\Bigg\}. \tag{7}$$

From Eq. (4) and the definition of the forward-backward asymmetry

$$A_{\mathrm{FB}}(s)\equiv\frac{\int_0^1 d\cos\theta\, d^2\Gamma/ds\,d\cos\theta-\int_{-1}^0 d\cos\theta\, d^2\Gamma/ds\,d\cos\theta}{\int_0^1 d\cos\theta\, d^2\Gamma/ds\,d\cos\theta+\int_{-1}^0 d\cos\theta\, d^2\Gamma/ds\,d\cos\theta}, \tag{8}$$

we find that

$$A_{\mathrm{FB}}(s)=\frac{1}{2^8\pi^3 m_K^3}2m_l\beta_l^2\lambda(s)\mathrm{Re}(F_S F_V^*)\left(\frac{d\Gamma}{ds}\right)^{-1}. \tag{9}$$

As seen from Eq. (9), to get a nonzero value of A_{FB}, it is necessary to have a scalar interaction. However, in the SM the contributions from F_S to the decay widths of $K^+\to\pi^+e^+e^-$ and $K^+\to\pi^+\mu^+\mu^-$ are about seven and four orders of magnitude smaller than those from F_V [19,20], respectively, and therefore the forward-backward asymmetries are expected to be vanishingly small.

III. EXPERIMENTAL CONSTRAINTS

To study the experimental constraints on A_{FB} in $K^+\to\pi^+l^+l^-$, we consider the amplitude adopted in Ref. [7]:

$$\mathcal{M}=\frac{\alpha G_F}{4\pi}f_V P^\mu\overline{l}\gamma_\mu l+G_F m_K f_S\overline{l}l+G_F f_T\frac{P^\mu q^\nu}{m_K}\overline{l}\sigma_{\mu\nu}l, \tag{10}$$

where $f_{V,S,T}$ are dimensionless form factors of vector, scalar, and tensor interactions, respectively, $P=p_K+p_\pi$, and $q=p_K-p_\pi$. It is clear that, in Eq. (10), the vector term arises from the one-photon exchange in the SM, which gives the dominant contribution to the decay rate, whereas the scalar and tensor ones come from some new physics beyond the SM [21].

For the form factor f_V, we take the form derived in the ChPT [6], given by

$$f_V(s)=a_++b_+\frac{s}{m_K^2}+\omega^{\pi\pi}(s), \tag{11}$$

where a_+ and b_+ are free parameters and $\omega^{\pi\pi}$ is the contribution from the pion loop diagram given in Ref. [6]. The experimental measurement of $K^+\to\pi^+e^+e^-$ at BNL E865 [7] has determined the parameters of a_+ and b_+ to be -0.587 ± 0.010 and -0.655 ± 0.044, respectively. The scalar and tensor form factors in Eq. (10) for $K^+\to\pi^+e^+e^-$ are also constrained by the experiment [7] and the results are that

$$|f_S|<6.6\times10^{-5}\quad\text{or}\quad|f_T|<3.7\times10^{-4} \tag{12}$$

for the existence of either a scalar or tensor interaction. We note that so far there are no similar constraints on $f_{S,T}$ for $K^+\to\pi^+\mu^+\mu^-$ and they can be quite different for the two channels in theoretical models.

It is easy to see that the amplitude in Eq. (10) can be simplified to

$$\mathcal{M}=\frac{\alpha G_F}{4\pi}f_V' P^\mu \bar{l}\gamma_\mu l + G_F m_K f_s' \bar{l} l \tag{13}$$

with

$$f_V'=f_V-\frac{8\pi i m_l}{\alpha m_K}f_T, \quad f_S'=f_S-\frac{i\beta_l\lambda^{1/2}(s)\cos\theta}{m_K^2}f_T. \tag{14}$$

By comparing the amplitude in Eq. (13) with the general one in Eq. (2), we get

$$F_V=\frac{\alpha G_F}{2\pi}f_V', \quad F_S=G_F m_K f_S', \quad F_{P,A}=0. \tag{15}$$

From Eqs. (4), (7), and (15), we obtain

$$\begin{aligned}\frac{d^2\Gamma}{ds\, d\cos\theta}=&\frac{G_F^2}{2^8\pi^3 m_K^3}\beta_l\lambda^{1/2}(s)\Bigg\{|f_V|^2\frac{\alpha^2}{16\pi^2}\lambda(s)\\ &\times(1-\beta_l^2\cos^2\theta)+|f_S|^2 s\beta_l^2 m_K^2\\ &+|f_T|^2\frac{s\lambda(s)}{m_K^2}\left(\cos^2\theta+\frac{4m_l^2}{s}\sin^2\theta\right)\\ &+\mathrm{Re}(f_V^* f_S)\frac{\alpha m_l m_K}{\pi}\beta_l\lambda^{1/2}(s)\cos\theta\\ &-\mathrm{Im}(f_V f_T^*)\frac{\alpha\lambda(s)}{\pi}\frac{m_l}{m_K}\\ &-\mathrm{Im}(f_S f_T^*)2s\beta\lambda^{1/2}(s)\cos\theta\Bigg\}\end{aligned} \tag{16}$$

and

$$\begin{aligned}\frac{d\Gamma}{ds}=&\frac{G_F^2}{2^8\pi^3 m_K^3}\beta_l\lambda^{1/2}(s)\Bigg\{|f_V|^2\frac{\alpha^2\lambda(s)}{4\pi^2}\frac{1}{3}\left(1+\frac{2m_l^2}{s}\right)\\ &+2|f_S|^2 s\beta_l^2 m_K^2+|f_T|^2\frac{2s\lambda(s)}{3m_K^2}\left(1+\frac{8m_l^2}{s}\right)\\ &-\mathrm{Im}(f_V f_T^*)\frac{2\alpha\lambda(s)}{\pi}\frac{m_l}{m_K}\Bigg\}.\end{aligned} \tag{17}$$

Similarly, from Eq. (9) we find

$$\begin{aligned}A_{\rm FB}(s)=&\frac{G_F^2}{2^8\pi^3 m_K^3}\beta_l^2\lambda(s)\Bigg[\mathrm{Re}(f_V^* f_S)\frac{\alpha m_l m_K}{\pi}\\ &-\mathrm{Im}(f_S f_T^*)2s\Bigg]\left(\frac{d\Gamma}{ds}\right)^{-1}.\end{aligned} \tag{18}$$

From Eq. (17), one can check that the bound for f_S or f_T in Eq. (12) yields at most a few percent of the decay rate in $K^+\to\pi^+e^+e^-$. Moreover, the last term in Eq. (17) is negligible for the electron channel no matter whether f_T is real or imaginary, due to the electron mass suppression. However, for the muon case this term could be large and spoil the vector dominant mechanism if the imaginary part of f_T is not small. In Fig. 1, we show the differential decay rate and forward-backward asymmetry as functions of $\hat{s}=s/m_K^2$ for the decay of $K^+\to\pi^+e^+e^-$ by using the upper value of f_S in Eq. (12) and $f_T=0$. In Fig. 2, we display them by assuming that $f_S\sim-4\times10^{-5}i$ and $f_T\sim2\times10^{-4}$. As illustrations, in Figs. 3 and 4 we also give $d\Gamma/d\hat{s}$ and $A_{\rm FB}$ in $K^+\to\pi^+\mu^+\mu^-$ with the same sets of parameters as those in Figs. 1 and 2, respectively. It is clear that, as mentioned earlier, since there is no direct strict experimental constraint on f_S or f_T in the muon mode, $A_{\rm FB}(K^+\to\pi^+\mu^+\mu^-)$ can be arbitrarily large.

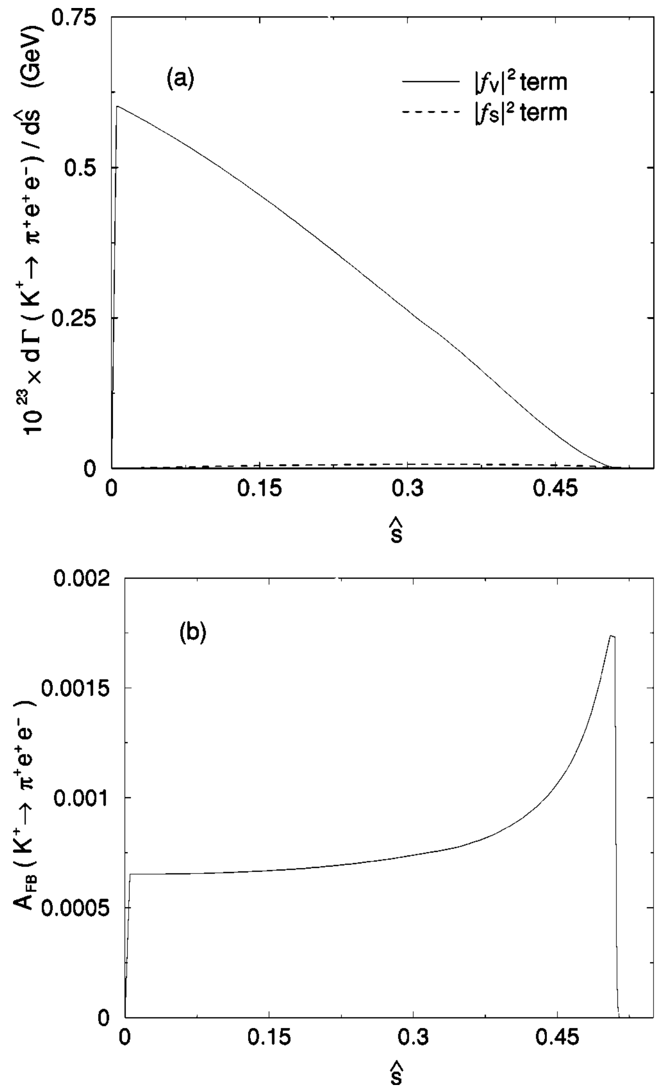

FIG. 1. (a) Differential decay rate and (b) forward-backward asymmetry for $K^+\to\pi^+e^+e^-$ as functions of $\hat{s}=s/m_K^2$ with $f_S=6.6\times10^{-5}$ and $f_T=0$.

IV. SUPERSYMMETRY

In the MSSM, the one-loop effective down-type Yukawa interaction is given by

$$\mathcal{L}^{\rm eff}=\bar{d}_R Y_d[H_d+(\epsilon_0+\epsilon_Y Y_u^\dagger Y_u)H_u^*]Q_L+\text{H.c.}, \tag{19}$$

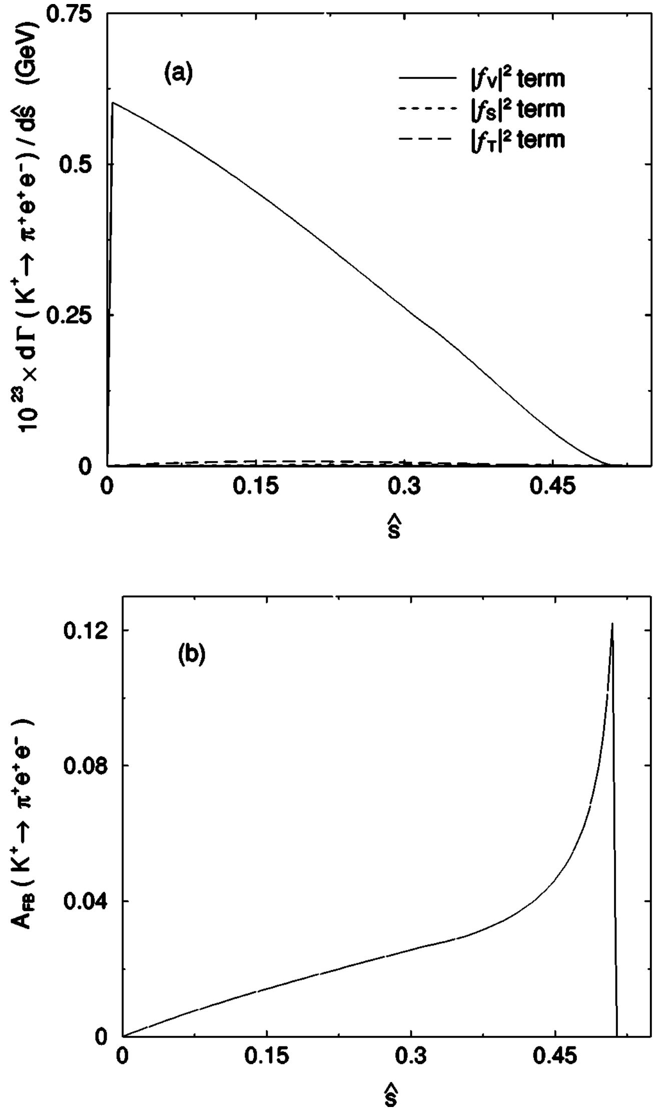

FIG. 2. Same as Fig. 1 but $f_S \sim -4\times 10^{-5} i$ and $f_T \sim 2\times 10^{-4}$.

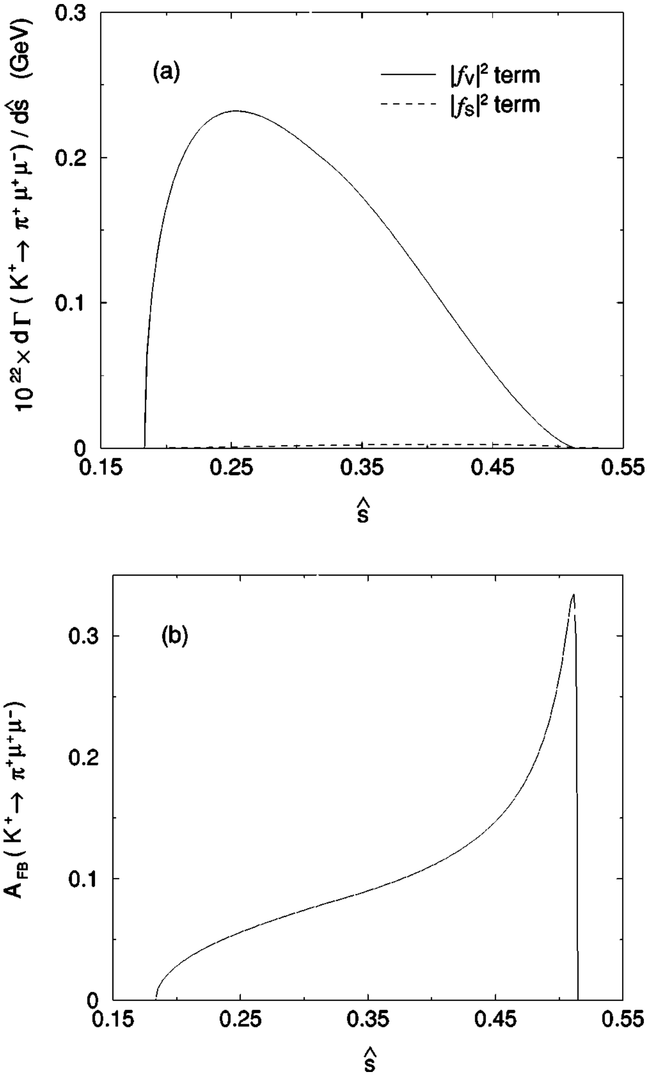

FIG. 3. Same as Fig. 1 but for $K^+ \to \pi^+ \mu^+ \mu^-$.

where $Y_{u,d}$ are 3×3 Yukawa coupling matrices and $\epsilon_{0,Y}$, defined in Ref. [15], are typically $O(10^{-2})$. In the diagonal Y_d basis of $(Y_d)_{ij} = y_i^d \delta_{ij}$, the interaction in Eq. (19) becomes

$$\mathcal{L}_{\rm mass}^{\rm eff} = v_d \overline{d_R^i} y_i^d [(1+\epsilon_0 \tan\beta)\delta_{ij} + \epsilon_Y V_{ik}^\dagger (y_k^u)^2 V_{kj}] d_L^j + \text{H.c.}, \tag{20}$$

where V is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. By writing the effective Hamiltonian in the transition $s \to d l^+ l^-$ induced by the scalar type of interaction as

$$\mathcal{H}_S^{\rm eff} = (C_S \overline{s_R} d_L + C_S' \overline{d_R} s_L)\overline{l} l, \tag{21}$$

from Eq. (20) one has that [15]

$$C_S = -\frac{G_F^2}{4\pi^2}\,\frac{m_s m_l m_t^2 \overline{\lambda_{21}^t} \tan^3\beta}{(1+\epsilon_0 \tan\beta)}\,\frac{1}{M_A^2}\,\frac{\mu A f(x_{\mu L}, x_{\mu R})}{M_{\tilde{t}_L}^2},$$

$$C_S' \simeq \frac{m_d}{m_s} C_S, \tag{22}$$

where A is the coupling of the soft-breaking trilinear term and

$$\overline{\lambda_{21}^t} = \lambda_{21}^t \left[\frac{1+\tan\beta(\epsilon_0 + \epsilon_Y y_t^2)}{1+\epsilon_0 \tan\beta}\right]^2,$$

$$f(x,y) = \frac{1}{x-y}\left[\frac{x\ln x}{1-x} - \frac{y \ln y}{1-y}\right], \quad f(1,1) = \frac{1}{2}, \tag{23}$$

with

$$x_{\mu L} = \frac{\mu^2}{M_{\tilde{t}_L}^2}, \quad x_{\mu R} = \frac{M_{\tilde{t}_R}^2}{M_{\tilde{t}_L}^2}, \quad \lambda_{21}^t = V_{ts}^* V_{td}, \tag{24}$$

and y_t being the top quark Yukawa coupling. By comparing Eq. (21) with Eq. (10) and using

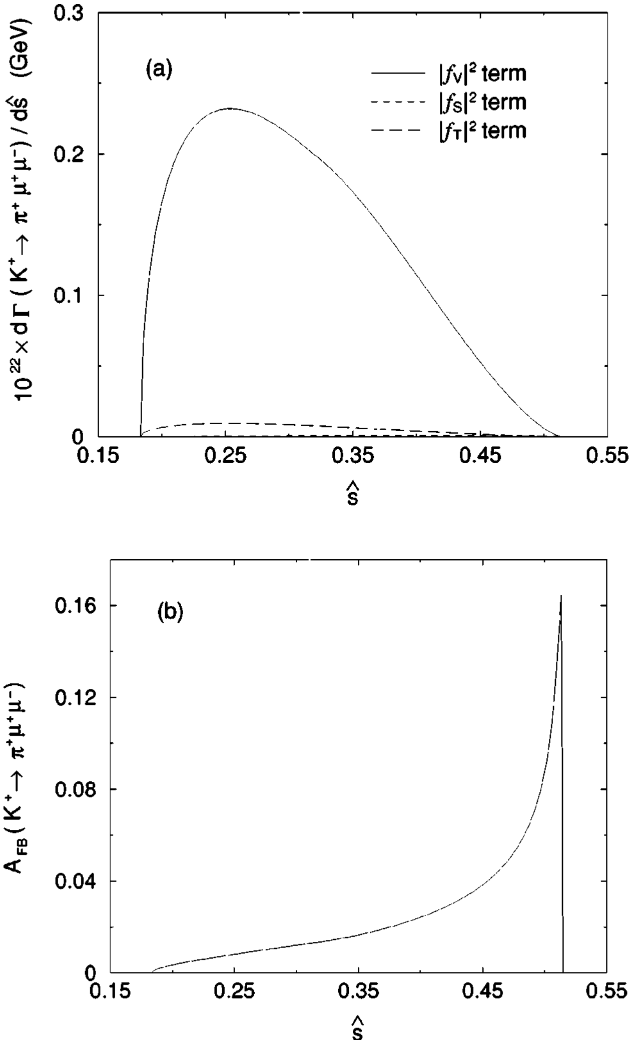

FIG. 4. Same as Fig. 2 but for $K^+\to\pi^+\mu^+\mu^-$.

$$\langle\pi|\overline{d}(1+\gamma_5)s|K\rangle\simeq\frac{m_K^2}{m_s}f_+, \tag{25}$$

we find

$$f_S^{\mathrm{MSSM}}\simeq-\frac{G_F^2}{8\pi^2}\frac{m_K m_l m_t^2\overline{\lambda}_{21}^t\tan^3\beta}{(1+\epsilon_0\tan\beta)^2}\frac{1}{M_A^2}\frac{\mu A f(x_{\mu L},x_{\mu R})}{M_{\tilde{t}_L}^2}, \tag{26}$$

where we have neglected the small terms related to $y_{1,2}^u$ and used $f_+\simeq 1$.

To estimate the scalar form factor in Eq. (26) in the MSSM with large $\tan\beta$, we take $\epsilon_0\sim 1/100\gg\epsilon_Y y_t^2$ and $\tan\beta=50r$, and we get

$$f_S^{\mathrm{MSSM}}|_{l=e}\sim 1.1\times10^{-9}(1-\overline{\rho}-i\overline{\eta})\frac{r^3}{(1+r/2)^2}\times\left(\frac{200\ \mathrm{GeV}}{M_A}\right)^2\left(\frac{\mu A f(x_{\mu L},x_{\mu R})}{M_{\tilde{t}_L}^2}\right),$$

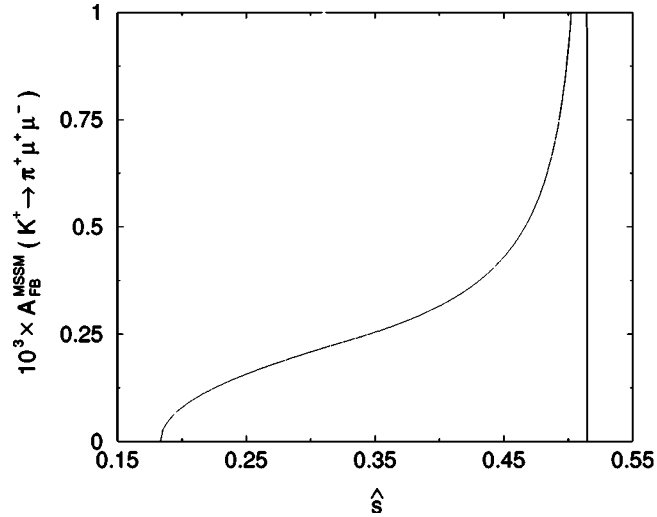

FIG. 5. Forward-backward asymmetry in $K^+\to\pi^+\mu^+\mu^-$ as a function of $\hat{s}$ in the MSSM with large $\tan\beta$.

$$f_S^{\mathrm{MSSM}}|_{l=\mu}\sim 2.3\times10^{-7}(1-\overline{\rho}-i\overline{\eta})\frac{r^3}{(1+r/2)^2}\times\left(\frac{200\ \mathrm{GeV}}{M_A}\right)^2\left(\frac{\mu A f(x_{\mu L},x_{\mu R})}{M_{\tilde{t}_L}^2}\right), \tag{27}$$

where $\overline{\rho}=\rho(1-\lambda^2/2)$ and $\overline{\eta}=\eta(1-\lambda^2/2)$ with λ, ρ, and η being the Wolfenstein parameters of the CKM matrix V. Since the values of $f_S^{\mathrm{MSSM}}|_{l=e,\mu}$ in Eq. (27) are about three and one orders of magnitude smaller than the experimental bound in Eq. (12), the scalar contributions to the decay rates in the MSSM are negligible. Moreover, the scalar contribution to the FBA in $K^+\to\pi^+e^+e^-$ is also suppressed. However, in $K^+\to\pi^+\mu^+\mu^-$ the FBA can be as large as 10^{-3} as shown in Fig. 5 using $\overline{\rho}\sim0.2$ [22] and assuming $r\sim1$, $M_A\sim200$ GeV, and $\mu A f(x_{\mu L},x_{\mu R})/M_{\tilde{t}_L}^2\sim2$. We note that $A_{\mathrm{FB}}(K^+\to\pi^+\mu^+\mu^-)=O(10^{-3})$ is accessible to future experiments such as the CKM experiment at Fermilab, where on the order of 10^5 events can be produced [23].

V. CONCLUSIONS

We have studied the forward-backward asymmetries in the decays of $K^+\to\pi^+l^+l^-$ ($l=e$ and μ) in the most general amplitudes. In particular, we have explored the experimental constraints on the asymmetries by including the scalar and tensor interactions. We have found that with the scalar [tensor] term the asymmetry can be up to $O(10^{-3})$ [$O(10^{-1})$] and arbitrarily large for the electron and muon channels, respectively, without conflict with the experimental data. We have also discussed the asymmetries in the minimal supersymmetric standard model where the scalar terms can be explicitly induced. We have shown that the FBA in $K^+\to\pi^+e^+e^-$ is negligibly small due to the electron mass suppression, but in $K^+\to\pi^+\mu^+\mu^-$ it can be as large as $O(10^{-3})$ with large $\tan\beta$, which can be tested in future experiments such as the CKM experiment at Fermilab.

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