

Choice of heavy baryon currents in QCD sum rules

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In this paper we investigate the effects due to the mixing of two interpolating currents for ground-state baryons within the framework of heavy quark effective theory using the QCD sum rule approach. Both two-point and three-point sum rules, and thus the mass, coupling constant, and Isgur-Wise function sum rules are considered. It is interesting to contrast those results with each other. Based on the Isgur-Wise functions obtained in this paper, we also analyze the effects of current mixing to Λ type and Σ type semileptonic decays $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$, $\Sigma_b \rightarrow \Sigma_c \ell \bar{\nu}$, and $\Sigma_b \rightarrow \Sigma_c^* \ell \bar{\nu}$. Decay widths corresponding to various mixing parameters are obtained and can be compared to the experimental data.

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I. INTRODUCTION

Strong interactions between quarks can be described well by QCD in the standard model. Recently important progress in the theoretical description of hadrons containing a heavy quark has been achieved with the development of the heavy quark effective theory (HQET) [1–3]. Based on the spin-flavor symmetry of QCD, exactly valid in the infinite heavy quark mass limit, $m_Q \rightarrow \infty$, this framework provides a systematic expansion of heavy hadron spectra and both the strong and weak transition amplitudes in terms of the leading contribution, plus corrections decreasing as powers of $1/m_Q$. HQET has been applied successfully to learn about the properties of mesons and baryons made of both heavy and light quarks.

Because of the asymptotic freedom of QCD, the nonperturbative effect plays an important role in hadronic physics. Thus it is inevitable to employ some nonperturbative technique in strong interaction related problems. The QCD sum rule [4] formulated in the framework of HQET [5] is a desirable approach and proves to be predictive [6]. This method allows one to relate hadronic observable to the QCD parameter *via* the operator product expansion (OPE) of the correlator. The choice of the interpolating current for a state with given spin and parity is the first step in the application of the QCD sum rule method. Principally, if a current is chosen within the framework of HQET, the QCD sum rule can be applied to many fields without ambiguity and successfully. But the real situation is not so simple. The main problem lies just on the choice of the interpolating currents. On the heavy meson side, the current interpolating a given spin and parity ground state is unique for it is constituted of one heavy and one light quark. However, on the heavy baryon side, the interpolating current is not unique [7–9]. For a given state there exist two commonly adopted interpolating currents. Both bear the general form as [8,10]

$$j^v = \epsilon_{abc} (q_1^{Ta} C \Gamma \tau q_2^b) \Gamma' h_v^c, \quad (1)$$

in which C is the charge conjugation matrix, τ is the flavor matrix which is antisymmetric for Λ_Q baryon and symmetric for $\Sigma_Q^{(*)}$ baryon, Γ and Γ' are some gamma matrices, and a, b, c denote the color indices. One kind of currents Γ and Γ' can be chosen covariantly as

$$\Gamma = \gamma_5, \quad \Gamma' = 1, \quad (2)$$

for Λ_Q baryon,

$$\Gamma = \gamma_\mu, \quad \Gamma' = (\gamma_\mu + v_\mu) \gamma_5, \quad (3)$$

for Σ_Q baryon, and

$$\Gamma = \gamma_\nu,$$

$$\Gamma' = -g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3} (\gamma_\mu v_\nu - \gamma_\nu v_\mu) + \frac{2}{3} v_\nu v_\mu, \quad (4)$$

for Σ_Q^* baryon. Another kind of current can be obtained by inserting a factor \not{v} behind the Γ matrices defined by Eqs. (2)–(4). We denote them as j_1^v and j_2^v , respectively. In QCD sum rule applications those two currents are usually used separately [7,8,11–15]. The constituent quark-type current, which is the linear combination of the two previously defined currents with the same coefficient, can also be found in application [16]. But generally speaking, the interpolating current should be the linear combination of the two currents with arbitrary coefficients. And also there have been many papers treating just the question of the choice of baryon currents both in full QCD and in HQET sum rules [17,18]. In this paper we adopt the general form $j^v = a j_1^v + b j_2^v$, in

which the coefficients a and b can be arbitrary real numbers, to investigate the effects of a different choice of baryon currents on physical observable.

The baryon coupling constants in HQET are defined through the vacuum-to-baryon matrix element of the interpolating current as follows:

$$\begin{aligned}\langle 0|j^v|\Lambda(v)\rangle &= F_\Lambda u, \\ \langle 0|j^v|\Sigma(v)\rangle &= F_\Sigma u, \\ \langle 0|j^v|\Sigma^*(v)\rangle &= \frac{1}{\sqrt{3}} F_{\Sigma^*} u^\alpha,\end{aligned}\quad (5)$$

where u is the spinor and u_α is the Rarita-Schwinger spinor in the HQET, respectively. The coupling constants F_Σ and F_{Σ^*} are equivalent since Σ_Q and Σ_Q^* belong to the doublet with the same spin parity of the light degrees of freedom.

The remainder of this paper is organized as follows. In Sec. II we focus our emphasis on two-point and three-point correlators and thus obtained sum rules for ground-state baryons. In Sec. II A mass sum rules and in Sec. II B sum rules for Isgur-Wise functions are presented. Section III is devoted to numerical results and our conclusions.

II. TWO-POINT AND THREE-POINT CORRELATORS

A. Two-point correlator and mass sum rule

Two-point correlators are the T -product of interpolating currents saturated between vacuum

$$i \int dx e^{ik \cdot x} \langle 0|T\{j^v(x)\bar{j}^v(0)\}|0\rangle = \Gamma' \frac{1+\not{x}}{2} \bar{\Gamma}' \text{Tr}[\tau\tau^+] \Pi(\omega), \quad (6)$$

where k is the residual momentum and $\omega = 2v \cdot k$. The QCD sum-rule determination of baryon coupling constants can be achieved by analyzing the two-point correlator. These diagonal correlators of the single interpolating currents have been obtained long ago by many authors and are of the same form for both the interpolating currents j_1^v and j_2^v . Nondiagonal correlators have been analyzed in Ref. [8] in the leading order in α_s and in next to leading order in α_s in Ref. [14]. In our previous works [11–13] we adopted the diagonal correlator as the starting point since for the nondiagonal correlator there is no perturbative contribution under the usual assumption of quark-hadron duality, let alone to be the dominant part to the sum rules derived. Here we only have one unique interpolating current and there is only one diagonal correlator and no nondiagonal case to be analyzed. Our theoretical result thus is the combination of the previously called diagonal and nondiagonal results. It should be noted that the nondiagonal one is merely treated as power corrections of operator product expansion (OPE) in our choice of current.

In our calculations condensates with a dimension higher than 6 are not included for lack of information and radiative corrections are out of consideration contemporarily. In order to obtain an estimate of the dimension 3 nonlocal quark con-

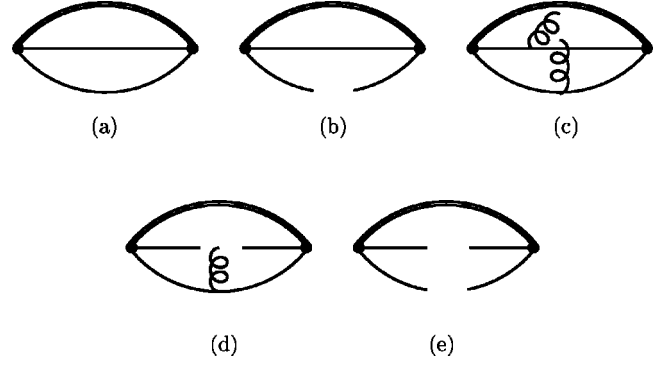


FIG. 1. Nonvanishing diagrams for the two-point correlator: (a) perturbative contribution, (b) quark condensate, (c) gluon condensate, (d) mixed condensate, (e) four-quark condensate contributions. The interpolating baryonic currents are denoted by black circles. Heavy-quark propagators are drawn as double curves.

densate we adopt the Gaussian ansatz $\langle \bar{q}(0)q(x) \rangle = \langle \bar{q}q \rangle \exp(m_0^2 x^2/16)$. Relevant Feynman diagrams are presented in Fig. 1. Then it is straightforward to obtain the two-point sum rules:

$$\begin{aligned}2 F_\Lambda^2 e^{-2\bar{\Lambda}_\Lambda/T} &= (a^2 + b^2) \left[\frac{3T^6}{2^4 \pi^4} \delta_5(\omega_c/T) \right. \\ &\quad \left. + \frac{T^2}{2^6 \pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{1}{3} \langle \bar{q}q \rangle^2 e^{-m_0^2/2T^2} \right] \\ &\quad - \frac{ab}{\pi^2} \langle \bar{q}q \rangle \left[T^3 e^{-m_0^2/4T^2} + T \frac{m_0^2}{8} \right],\end{aligned}\quad (7)$$

$$\begin{aligned}\frac{2}{3} F_\Sigma^2 e^{-2\bar{\Lambda}_\Sigma/T} &= (a^2 + b^2) \left[\frac{3T^6}{2^4 \pi^4} \delta_5(\omega_c/T) \right. \\ &\quad \left. - \frac{T^2}{32^6 \pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{1}{3} \langle \bar{q}q \rangle^2 e^{-m_0^2/2T^2} \right] \\ &\quad - \frac{ab}{\pi^2} \langle \bar{q}q \rangle \left[T^3 e^{-m_0^2/4T^2} - T \frac{m_0^2}{24} \right].\end{aligned}$$

Our two-point sum rules do agree with results previously obtained in Refs. [8,11]. The functions $\delta_n(\omega_c/T)$ arise from the continuum subtraction and are given by

$$\delta_n(x) = \frac{1}{n!} \int_0^x dt t^n e^{-t} = 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}. \quad (8)$$

The second term of the last equation is assigned to the continuum mode, which can be much larger than the ground-state contribution for the typical value of parameter T if the dimension of the spectral densities are very high.

B. Three-point sum rules

For a heavy-heavy velocity changing current $\mathcal{J}=\bar{h}\Gamma\bar{h}'$ we can define a three-point correlator in which J is inserted between two interpolating currents as below:

$$i^2 \int dx_1 dx_2 e^{i(k \cdot x_1 - k' \cdot x_2)} \langle 0 | T j^v(x_1) \mathcal{J}(0) \bar{j}^{v'}(x_2) | 0 \rangle$$

$$= \Gamma' \frac{1+\not{v}}{2} \Gamma \frac{1+\not{v}'}{2} \bar{\Gamma}' \text{Tr}[\tau\tau^+] T(\omega, \omega', y), \quad (9)$$

where $T(\omega, \omega', y)$ is an analytic function in the ‘‘off-shell energies’’ $\omega=2v \cdot k$ and $\omega'=2v' \cdot k'$ with discontinuities for positive values of these variables. It furthermore depends on the velocity transfer $y=v \cdot v'$, which is fixed at its physical region for the process under consideration. In the heavy quark limit, the matrix element of current \mathcal{J} can be parametrized by one or two scalar functions of y . Those scalar functions are called Isgur-Wise functions [19] and can be defined as

$$\langle \Lambda_Q | \bar{h}\Gamma h' | \Lambda_{Q'} \rangle = \xi(y) \bar{u} \Gamma u', \quad (10)$$

for the Λ -type baryon, and

$$\langle \Sigma_Q | \bar{h}\Gamma h' | \Sigma_{Q'} \rangle = [-\xi_1(y) g_{\mu\nu} + \xi_2(y) v'_\mu v_\nu] \bar{\Psi}_\mu \Gamma \Psi'_\nu, \quad (11)$$

for the Σ -type baryon, in which u is the Dirac spinor as defined in Eq. (5) and Ψ_μ is the covariant representation of the spin-1/2 doublets $\Psi_\mu = u_\mu + (1/\sqrt{3})(v_\mu + \gamma_\mu)u$. Both $\xi(y)$ and $\xi_1(y)$ are normalized to unity at the zero recoil $y=1$ due to the heavy quark symmetry. However, one cannot invoke symmetry arguments to predict the normalization at $y=1$ of $\xi_2(y)$.

Saturating the three-point correlator with complete set of baryon states, one can divide it into two parts. One is the part of interest, the contribution of the lowest-lying baryon states associated with the heavy-heavy currents, as one having poles in both the variables ω and ω' at the value $\omega=\omega'=2\bar{\Lambda}$. The other contribution to the correlator comes from higher resonant states. For the little knowledge of this part of contribution it is common to resort to the quark-hadron duality, which insures that continuum contribution can be approximated by the integral of the perturbative spectral density over a continuum threshold, to get a predictive result.

On theoretical side the scalar function $T(\omega, \omega', y)$ can be calculated using quark and gluon language with vacuum condensates. Dispersion relation enables one to express the correlator in the form of integrals of the double spectral density as

$$T(\omega, \omega', y) = \int_0^\infty \int_0^\infty \frac{ds}{s-\omega} \frac{ds'}{s'-\omega'} \rho(s, s', y). \quad (12)$$

With the redefinition of the integral variables [7,13,20,21]

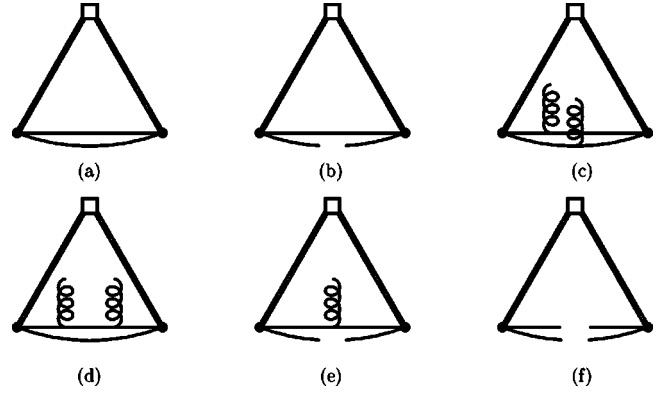


FIG. 2. Nonvanishing diagrams for the three-point correlator: (a) perturbative contribution, (b) quark condensate, (c) and (d) gluon condensate, (e) mixed condensate, and (f) four-quark condensate. The velocity-changing current operator is denoted by a white square, and the interpolating baryonic currents by black circles.

$$s_+ = \frac{s+s'}{2},$$

$$s_- = \left(\frac{y-1}{y+1} \right)^{1/2} \frac{s-s'}{2}, \quad (13)$$

the integration domain becomes

$$\int_0^\infty ds \int_0^\infty ds' = 2 \left(\frac{y-1}{y+1} \right)^{1/2} \int_0^\infty ds_+ \int_{-s_+}^{s_+} ds_-. \quad (14)$$

It is in variable s_+ that the commonly adopted quark-hadron duality is assumed [21,22],

$$\text{res.} = 2 \left(\frac{y-1}{y+1} \right)^{1/2} \int_{\omega'_c}^\infty ds_+ \int_{-s_+}^{s_+} ds_- \frac{\rho(s, s', y)}{(s-\omega)(s'-\omega')}, \quad (15)$$

and for simplicity we take ω'_c to be equal to the two-point continuum threshold ω_c : $\omega'_c = \omega_c$.

In our theoretical calculations of the three-point correlator only condensates with dimension of no more than 6 are included. Order $1/m_Q$ power corrections and radiative corrections are not included in present calculations, either, for their contributions to the correlator only amount to several percents and do not change the numerical result dramatically. Also the Gaussian ansatz for the nonlocal quark condensate is adopted. Feynman diagrams related to the calculations of a three-point correlator are shown in Fig. 2.

Then following the standard procedure we resort to the Borel transformation $B_\tau^\omega, B_{\tau'}^{\omega'}$ to suppress the contributions of the excited states. Considered the symmetry of the correlation function it is natural to set the parameters τ, τ' to be the same and equal to $2T$, where T is the Borel parameter of the two-point sum rules. Finally, we obtain the sum rules for the Isgur-Wise functions as

$$\begin{aligned}
4F_{\Lambda}^2 \xi(y) e^{-2\bar{\Lambda}_{\Lambda}/T} &= \frac{3(a^2+b^2)}{\pi^4(1+y)^3} T^6 \delta_5(\omega_c/T) + \frac{[a^2(2y+1)+b^2y(y+2)]T^2}{24(1+y)^2\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{2(a^2+b^2y)}{3} \langle \bar{q}q \rangle^2 e^{-(1+y)m_0^2/4T^2} \\
&\quad - \frac{4ab\langle \bar{q}q \rangle}{(1+y)\pi^2} \left(T^3 e^{-m_0^2(1+y)/8T^2} + T \frac{m_0^2}{24}(y+2) \right), \\
\frac{4}{3}F_{\Sigma}^2 \xi_1(y) e^{-2\bar{\Lambda}_{\Sigma}/T} &= \frac{3(a^2+b^2)}{\pi^4(1+y)^3} T^6 \delta_5(\omega_c/T) - \frac{(a^2+b^2)T^2}{24(1+y)^2\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{2(a^2+b^2y)}{3} \langle \bar{q}q \rangle^2 e^{-(1+y)m_0^2/4T^2} \\
&\quad - \frac{4ab\langle \bar{q}q \rangle}{(1+y)\pi^2} \left(T^3 e^{-m_0^2(1+y)/8T^2} - T \frac{m_0^2}{24}y \right), \\
\frac{4}{3}F_{\Sigma}^2 \xi_2(y) e^{-2\bar{\Lambda}_{\Sigma}/T} &= \frac{3(a^2+b^2y)}{\pi^4(1+y)^4} T^6 \delta_5(\omega_c/T) + \frac{(a^2y-b^2)T^2}{24(1+y)^3\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \frac{2b^2}{3} \langle \bar{q}q \rangle^2 e^{-(1+y)m_0^2/4T^2} \\
&\quad - \frac{4ab\langle \bar{q}q \rangle}{(1+y)^2\pi^2} \left(T^3 e^{-m_0^2(1+y)/8T^2} + T \frac{m_0^2}{24}(y-2) \right). \tag{16}
\end{aligned}$$

The unitary normalization of flavor matrix $\text{Tr}[\tau\tau^+]=1$ has been applied to get those sum rules. It is obvious that the normalization of the Isgur-Wise functions $\xi(y)$ and $\xi_1(y)$ at zero recoil is satisfied automatically.

III. NUMERICAL RESULTS AND CONCLUSIONS

It is obvious that in the expressions of two-point and three-point sum rules the relative value of the two parameters but not the absolute value plays an important role. So in this section we change the current mixing parameters a, b to one angular variable θ through relations $b/a = \tan \theta$ where θ can be restrained to the range $\theta \in [-\pi/2, \pi/2]$, in which $\theta=0, \pm\pi/2$ correspond to the diagonal cases. In the numerical analysis we will investigate the current mixing effects in those sum rules. The standard values of the vacuum condensates are

$$\begin{aligned}
\langle \bar{q}q \rangle &= -(0.23\text{GeV})^3, \\
\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle &= 0.012\text{GeV}^4, \tag{17} \\
\langle \bar{q}g\sigma_{\mu\nu}G^{\mu\nu}q \rangle &= m_0^2\langle \bar{q}q \rangle, \quad m_0^2 = 0.8\text{GeV}^2.
\end{aligned}$$

They will be used in the following numerical analysis of the sum rules.

A. Mass sum rules

In the analysis of the coupling constant sum rules we need the effective mass of the baryons in consideration as the input parameter. One way to obtain this parameter is to extract it from the experimental data assuming the heavy quark mass to be the commonly recognized value from the outset

of the analysis. For the aim of consistency we adopt another way of obtaining the effective-mass parameter from Eq. (7) which is based on the QCD sum-rule method entirely. The effective mass can be expressed through the derivative of Borel variable T in the coupling constant sum rules as

$$\bar{\Lambda} = -\frac{1}{2} \frac{\partial}{\partial T^{-1}} \ln K(T), \tag{18}$$

in which $K(T)$ denotes the right-hand side of Eq. (7). So the first step of our numerical analysis of those two-point sum rules is to find the value of the effective mass. But the second step, which is the analysis of coupling constant sum rules, will be omitted here as the focus of our interest is on the mass sum rules entirely. Our main idea in the consideration of the effect of the current mixing parameter to the sum rules is to see if there exists a reasonable stability window of the Borel parameter T under the variation of θ in the range from $-\pi/2$ to $\pi/2$. For the analysis of the coupling constant and mass sum rules it is enough to take θ from 0 to $\pi/2$ for on the range of θ from $-\pi/2$ to 0 the mass sum rules oscillate for the Borel parameter $T \sim 1$ GeV so sharply that it is impossible to find a desirable stability window. Thus we do not take into account that half part of θ .

For the Λ -type baryon mass sum rule we find that there is no agreeable stability window except around the vicinity of $\theta=0$ or $\theta=\pi/2$, i.e., $a=1, b=0$ or $a=0, b=1$. So there is no or at most little space left for the mixing of currents and what we obtained is the diagonal sum rule. It is reasonable to assume that this result does indicate that there exists some mechanism which forbids the mixing of the two sector interpolating currents in the mass sum rule in the leading order. The diagonal sum rule result can be checked with previous work: When ω_c lies between 1.9 and 2.5 GeV there exists the stability window $0.35 < T < 0.65$ GeV. The effective mass

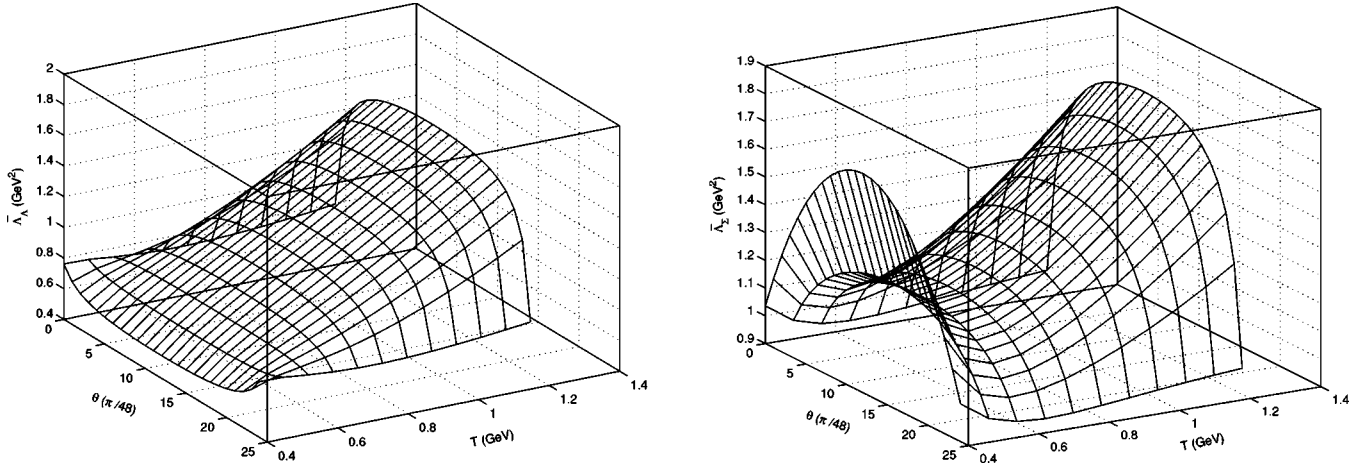


FIG. 3. Sum rules for effective mass parameter $\bar{\Lambda}$: The left one is for the Λ -type baryon, in which $\omega_c = 2.2$ GeV, and the right one is for the Σ -type baryon, in which $\omega_c = 2.8$ GeV.

thus obtained is $\bar{\Lambda}_\Lambda = 0.73 \pm 0.07$ GeV, in which the error only reflects the variation of Borel parameter T and continuum threshold ω_c .

For the Σ -type baryon all sum-rule windows are narrower than that of Λ -type baryon and the stability is not as good as that of Λ -type baryon, either [8]. With the increasing of θ the stability falls drastically so that the optional space left for the variation of θ is smaller than that of Λ -type baryon. When ω_c lies between 2.8 and 3.3 GeV there exists the stability window $0.4 < T < 0.7$ GeV for the diagonal sum rules, which appear to be the only surviving result with respect to the mixing of currents. The effective mass thus obtained is $\bar{\Lambda}_\Sigma = 0.90 \pm 0.14$ GeV, in which the error only reflects the variation of Borel parameter T and continuum threshold ω_c , too. Those results can be checked with Refs. [8,11]. It is also worth noting that the constituent quark-type interpolating current cannot be distinguished from the currents with arbitrary mixing parameters from the stability point of view. Both Λ -type and Σ -type mass sum rules are presented in Fig. 3.

B. Sum rules for the three-point correlators

1. Isgur-Wise functions

In order to get the numerical results for the Isgur-Wise functions, we divide our three-point sum rules by two-point functions to obtain ξ , ξ_1 , and ξ_2 as functions of the continuum threshold ω_c and the Borel parameter T . This procedure can eliminate the systematic uncertainties and cancel the dependence on mass parameter $\bar{\Lambda}$. As for the mixing parameter θ in this part, we take it varying from $-\pi/2$ to $\pi/2$ to determine the stability of the sum rules.

For the Isgur-Wise function of the Λ -type baryon $\xi(y)$, we find that it is not sensitive to the mixing parameter θ . Almost in the gamut of θ there exists a stability window, and the stability does not change rapidly when θ goes far beyond the vicinity of the diagonal θ 's. The continuum threshold is the same as that for the two-point sum rule for the Λ -type baryon. When ω_c lies between 1.9 and 2.5 GeV there exists

the stability window. The stability window for $\xi(y=1.2)$ is a much narrower one, $0.4 < T < 0.8$ GeV. The numerical results are shown in Fig. 4. In the numerical analysis it is interesting that there seems to exist a more stable window for the constituent quark-type current with $\theta = \pi/4$, though the tendency is not so predominant.

The numerical analysis of the two Isgur-Wise functions of the Σ -type baryon can be compared with each other. For the function ξ_1 , the existence of the stability window can only allow for the appearance of two diagonal sum rules and one constituent quark-type sum rule with mixing parameter $\theta = -\pi/4$. As for the function ξ_2 , the existence of a stability window can allow for the appearance of two diagonal sum rules besides one constituent quark-type sum rule with mixing parameter $\theta = \pi/4$. When the continuum threshold ω_c lies between 2.5 and 3.3 GeV there exist the stability windows for both functions with the allowed mixing angles. The numerical results of two Isgur-Wise functions are shown in Fig. 5. Due to the lack of a stability window of those two constituent quark-type sum rules with the mixing parameters

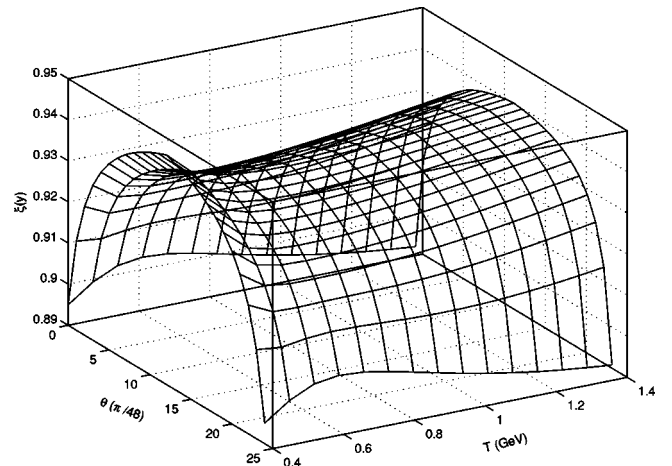


FIG. 4. Sum rules of the Isgur-Wise function $\xi(y)$ with various mixing parameters. The threshold in this figure is $\omega_c = 2.2$ GeV and the momentum transfer is $y = 1.1$.

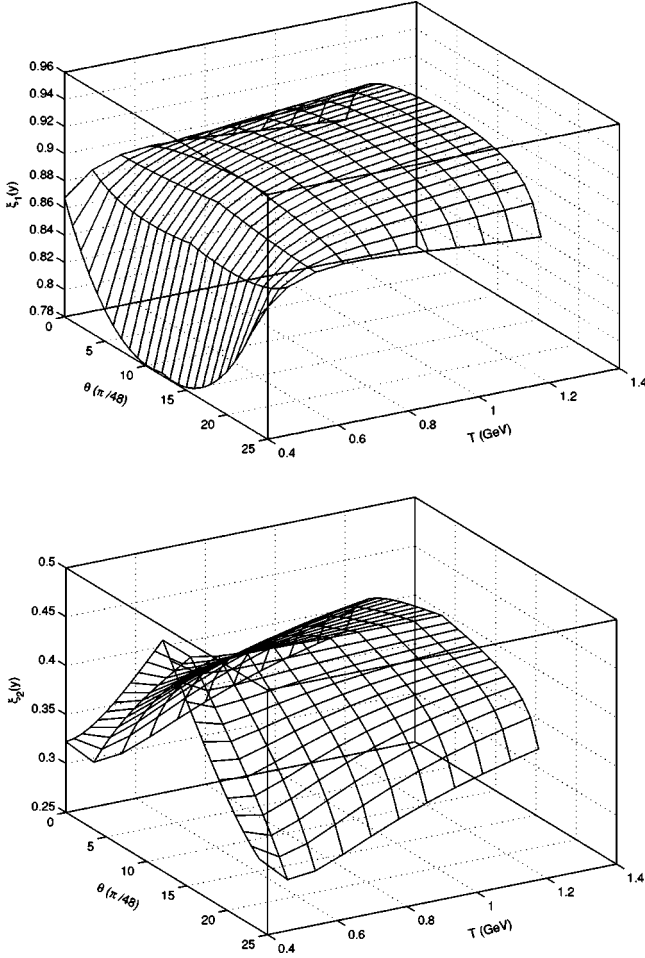


FIG. 5. Sum rules of Isgur-Wise function $\xi_1(y)$ and $\xi_2(y)$ with various mixing parameters. The threshold in this figure is $\omega_c=2.8$ GeV and the momentum transfer is $y=1.1$.

$\theta = \pm \pi/4$ for ξ_1 and ξ_2 , both numerical results related to those two sum rules are taken from the range $0.8 < T < 1.2$, the continuum threshold is the same as that of the diagonal sum rules. We also present our results for the function $\xi_2(1)$ in Table I. Our results of the two constituent quark-type currents are approximately equal to 0.5 at the zero recoil, which is consistent with the value obtained from constituent quark model and large N_c limit in Refs. [23,24].

If we put the two Isgur-Wise functions which are normalized to unity at the zero recoil into the linear form $\xi_{(1)}(y) = 1 - \rho_{(1)}^2(y-1)$, in which the parameters ρ^2 and ρ_1^2 are the slopes (or charge radii) of the Isgur-Wise functions, we can obtain the slopes ρ^2 and ρ_1^2 via a linear fit for $\xi(y)$ and $\xi_1(y)$ near the zero recoil. The final results of the slopes are presented in Table II. Many predictions have been made on the value of the charge radii, and the results vary greatly from each other [9,12,25–28].

TABLE I. The value of Isgur-Wise function ξ_2 at the zero recoil.

θ ($\tan \theta = b/a$)	$\theta=0$	$\theta=\pi/4$	$\theta=\pi/2$	$\theta=-\pi/4$	Refs. [23,24]
$\xi_2(1)$	0.40 ± 0.07	0.51 ± 0.01	0.62 ± 0.07	0.48 ± 0.01	0.5

2. Semileptonic decay rates

With the appropriate forms for the Isgur-Wise functions ξ , ξ_1 , and ξ_2 as we have derived in Eq. (16), we can discuss various semileptonic decays involving the heavy-to-heavy transition $b \rightarrow c$. As some illustrative examples here we shall only consider three types of semileptonic decays: $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$, $\Sigma_b \rightarrow \Sigma_c \ell \bar{\nu}$, and $\Sigma_b \rightarrow \Sigma_c^* \ell \bar{\nu}$.

The semileptonic decay $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$ can be analyzed directly after obtaining the Isgur-Wise function from the QCD sum rules. By neglecting the lepton mass, the differential decay rate is [12]

$$\frac{1}{\sqrt{y^2-1}} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu})}{dy} = \frac{G_F^2 |V_{cb}|^2 m_{\Lambda_b}^2 m_{\Lambda_c}^3}{(2\pi)^3} \left((1-2ry+r^2)[(y-1)F_1^2 + (y+1)G_1^2] + \frac{y^2-1}{3}(Ar^2+2Br+C) \right), \quad (19)$$

where $r = m_{\Lambda_c}/m_{\Lambda_b}$. In the above equation,

$$A = 2F_1F_2 + (y+1)F_2^2 + 2G_1G_2 + (y-1)G_2^2,$$

$$B = F_1^2 + F_1F_2 + F_2F_3 + F_3F_1 + yF_2F_3 + G_1^2 - G_1G_2 - G_2G_3 + G_3G_1 + yG_2G_3,$$

$$C = (y+1)F_3^2 + 2F_1F_3 + (y-1)G_3^2 - 2G_1G_3. \quad (20)$$

To the next to leading order of $1/m_Q$, the form factors F_i and G_i bear the simple form

$$F_1 = C(\mu)\xi(y) + \left(\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b} \right) [2\chi(y) + \xi(y)],$$

$$G_1 = C(\mu)\xi(y) + \left(\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b} \right) \left(2\chi(y) + \frac{y-1}{y+1}\xi(y) \right),$$

$$F_2 = G_2 = -\frac{\bar{\Lambda}}{m_c(y+1)}\xi(y),$$

$$F_3 = -G_3 = -\frac{\bar{\Lambda}}{m_b(y+1)}\xi(y), \quad (21)$$

where $C(\mu)$ is the perturbative QCD coefficient, $\chi(y)$ is the subleading order Isgur-Wise function, which only amounts to the order of a few percents to the leading function and can be safely neglected [12,29]. With the form of the leading order

TABLE II. Charge radii ρ^2 and ρ_1^2 for the Isgur-Wise functions $\xi(y)$ and $\xi_1(y)$.

Radii	$\theta=0$	$\theta=\pi/4$	$\theta=\pi/2$	$\theta=-\pi/4$
ρ^2	0.66 ± 0.08	0.46 ± 0.03	0.35 ± 0.13	0.41 ± 0.10
ρ_1^2	0.80 ± 0.07	0.58 ± 0.08	0.57 ± 0.18	0.30 ± 0.11

Isgur-Wise function, the differential decay rate of $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ is shown in Fig. 6. In this analysis, we choose those related heavy quark masses to be $m_b=4.77$ GeV, $m_c=1.41$ GeV [11], and parameters $m_{\Lambda_b}=5.641$ GeV, $m_{\Lambda_c}=2.285$ GeV can be found in Ref. [30]; the renormalization point is $\mu=470$ MeV. It seems to be inconsistent to use the quark masses obtained in Ref. [11] using factorization instead of Gaussian ansatz to parametrize the nonlocal quark condensate as done in Ref. [7]. But the fact that the decay width is not sensitive to the heavy quark masses allows us to use either pair of parameters without varying the width significantly. The decay widths corresponding to four typical mixing variables are listed in Table III. Also listed in Table III are some predictions made by the QCD sum rule and other phenomenological approaches. The averages of the decay widths are taken between $0.8 < T < 1.1$ GeV and $1.9 \leq \omega_c \leq 2.5$ GeV. Our results are in good consistence with the experimental value, $\Gamma=(4.0\pm 1.0)\times 10^{-14}$ GeV [30].

As to the two decays between Σ -type baryons, the decay widths have simple and easy-to-interpret forms when expressed with helicity amplitudes. Related formulas can be found in many references [3,26,31] and the decay widths corresponding to various mixing parameters are listed in Table III. In this part of the numerical analysis we only take

into account the contributions of the leading-order Isgur-Wise functions and omit the higher-order effects. The masses of the heavy baryons are taken to be $m_{\Sigma_b}=5.80$ GeV [11], $m_{\Sigma_c^*}=2.52$ GeV, and $m_{\Sigma_c}=2.455$ GeV [30]. For comparison we list the results for those three types of decays predicted by the QCD sum rule and other phenomenological approaches in Table III, too. It should be noted here that the function ξ_1 is the predominant part in the decay rates, so even though for the mixing parameter $\theta=-\pi/4$ there exists no stability window for function ξ_2 , the total decay rates still have a mild stability window for $\theta=-\pi/4$.

In conclusion, we have investigated the mixing of currents interpolating ground heavy baryon state within the framework of HQET using the QCD sum-rule approach. For the two-point sum rules there can only survive the diagonal ones and the constituent quark-type current is not preferable from the stability point of view for both Λ -type and Σ -type baryons. As for the three-point sum rules, Isgur-Wise function $\xi(y)$ for Λ -type baryon is not sensitive to the mixing parameter and the stability window exists almost for all the range of mixing parameters; on the other hand, the Isgur-Wise function $\xi_1(y)$ and $\xi_2(y)$ allows for two diagonal, one constituent ($\theta=\pi/4$), and one anticonstituent ($\theta=-\pi/4$)

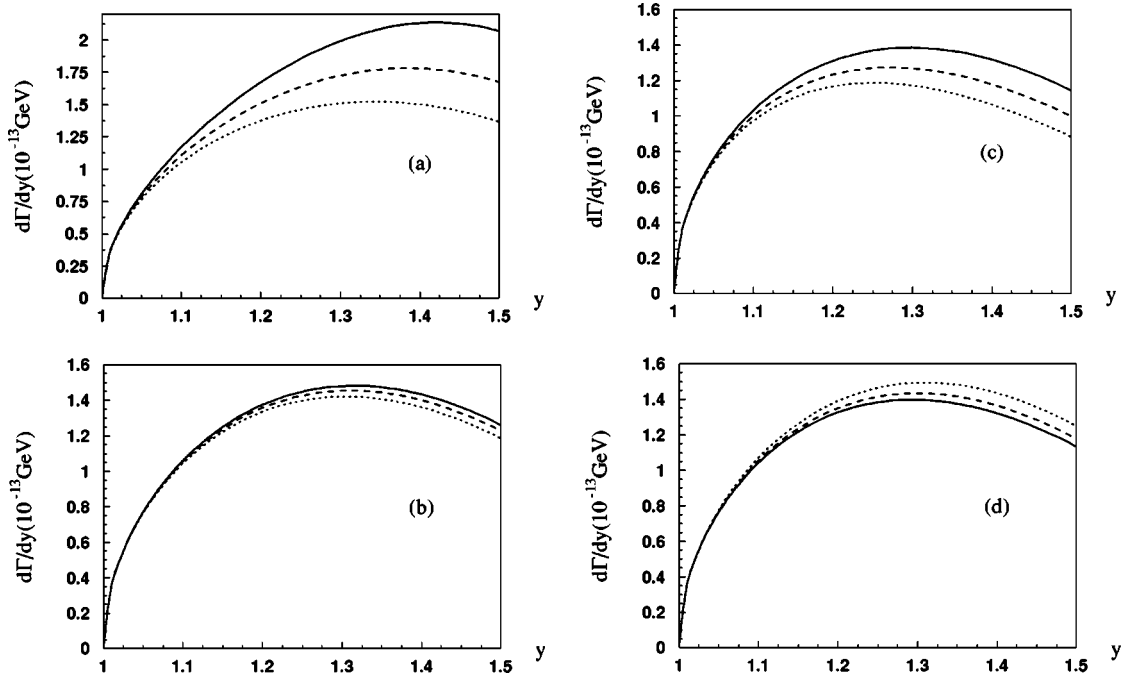


FIG. 6. Differential decay ratio of semileptonic decay $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$ with various mixing parameters as below: (a) $\theta=0$, (b) $\theta=\pi/4$, (c) $\theta=\pi/2$, and (d) $\theta=-\pi/4$. The solid, dashed, and dotted curves correspond to the threshold $\omega_c=1.9, 2.2, 2.5$ GeV, respectively. And the Borel parameter is $T=0.85$ GeV in this figure.

TABLE III. Decay widths Γ (in 10^{-14} GeV) for the semi-leptonic decays $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$, $\Sigma_b \rightarrow \Sigma_c l \bar{\nu}$, and $\Sigma_b \rightarrow \Sigma_c^* l \bar{\nu}$. Also presented in this table are some phenomenological predictions using the relativistic three-quark model (RTQM), the quark confinement model (QCM), and the spectator quark model (SQM).

Refs.	$\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$	$\Sigma_b \rightarrow \Sigma_c l \bar{\nu}$	$\Sigma_b \rightarrow \Sigma_c^* l \bar{\nu}$	
This paper	$\theta=0$	4.57 ± 0.62	1.38 ± 0.15	2.89 ± 0.16
	$\theta=\pi/4$	3.98 ± 0.07	1.66 ± 0.11	2.78 ± 0.06
	$\theta=\pi/2$	3.60 ± 0.29	1.50 ± 0.25	2.94 ± 0.22
	$\theta=-\pi/4$	3.97 ± 0.13	2.09 ± 0.24	3.27 ± 0.38
RTQM [9]	4.08	1.51	2.48	
RTQM [26]	3.521	1.468	3.001	
RTQM [28]	3.769	0.946	2.171	
QCM [31]	6.582	3.226		
SQM [32]	3.883	2.830		

quark-type sum rules. The effect of different currents to semileptonic decays $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}$, $\Sigma_b \rightarrow \Sigma_c l \bar{\nu}$, and $\Sigma_b \rightarrow \Sigma_c^* l \bar{\nu}$ has also been analyzed in this paper. We find that the current mixing effects in those processes are not significant.

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