Infrared renormalons and analyticity structure in perturbative QCD

Gorazd Cvetič*
Department of Physics, Universidad Técnica Federico Santa María, Valparaíso, Chile
(Received 30 December 2002; published 23 April 2003)

The relation between the infrared renormalons, the Borel resummation prescriptions, and the analyticity structure of Green's functions in perturbative QCD is investigated. A specific recently suggested Borel resummation prescription resulted in a principal value and an additional power-suppressed correction that is consistent with operator product expansion. Arguments requiring the finiteness of the result for any power coefficient of the leading infrared renormalon, and consistency in the case of the absence of that renormalon, require that this prescription be modified. The apparently most natural modification leads to the result represented by the principal value. The analytic structure of the amplitude in the complex coupling plane, obtained in this way, is consistent with that obtained in the literature by other methods.

DOI: 10.1103/PhysRevD.67.074022 PACS number(s): 12.38.Bx, 12.38.Aw, 12.38.Cy

Green's functions in QCD are quantities which, in general, are known to possess renormalons, i.e., singularities of the Borel transform on the real axis [1]. They appear as a consequence of the specific asymptotic behavior of the high-order perturbation coefficients. The singularities on the positive axis, called infrared (IR) renormalons, represent an obstacle to the Borel integration and lead to the well-known renormalon-induced ambiguity for the observable. The most widely used prescription for fixing this ambiguity has been to perform the Borel integration parallel to the positive real axis and take the real part of the resulting integral (principal value; see, e.g., Refs. [2,3]). This prescription in perturbative QCD (PQCD) was shown to be favored by or to be consistent with analyticity requirements in the momentum plane [4] and in the coupling parameter plane [5]. Further, this prescription is attractive due to its mathematical simplicity. If the QCD quantity under consideration can be presented via operator product expansion (OPE), then any additional, nonperturbative, terms are expected to have the power-suppressed form of the higher-twist terms. Perturbative QCD involving the quark and gluon degrees of freedom is usually expected to be unable to predict the strength of such terms.[1]

Recently, a specific prescription, based on IR renormalon considerations, has been proposed [6] to fix the strength of such higher-twist terms, and this method gave encouraging numerical results in the case of the resummation of the Gross–Lewellyn-Smith sum rule and of the heavy-quark potential [7]. The main observation was that the IR renormalon induces in the Borel-integrated quantity a nonphysical cut along the positive axis in the complex plane of the coupling parameter z, and that this cut structure can be naturally eliminated by subtracting a cut function proportional to $(-z)^{\nu}$ where ν is related to the power coefficient of the renormalon singularity. Further, the energy dependence of the subtraction term, when the coupling parameter z is positive, is consistent with the predictions of OPE for the corresponding higher-twist term. For positive z, the result is the principal value of the Borel integration minus the aforementioned term. A somewhat speculative interpretation of this result suggests that in this way the dominant part of the genuine nonperturbative higher-twist effect is obtained (the correction term to the principal value), although the method is based on perturbative (PQCD+renormalons) knowledge only. A more conservative rephrasing of this would be that this result represents "the most that we can get" out of PQCD, i.e., the natural basis to which one should eventually add other contributions to the aforementioned higher-twist term; such genuine nonperturbative contributions would involve the vacuum expectation values of the higher-twist operators appearing in OPE.

In the present work, the aforementioned method is scrutinized. As a result, more support is given to the second of the two mentioned interpretations of the PQCD renormalon resummation results. Even more, the results suggest that the natural resummed perturbative contribution is just the principal value of the Borel integral, without the aforementioned higher-twist term. As a by-product, some insights into the role of the IR renormalons in the analyticity structure of the PQCD amplitudes in the coupling plane are obtained.

Let us consider a Euclidean QCD amplitude $\Delta[a(Q)]$, where the quark mass effects are neglected. Therefore, it can be regarded as depending on the energy $Q \equiv \sqrt{-q^2}$ of the corresponding process only via the QCD coupling parameter $a(Q) \equiv \alpha_s(Q;\overline{\mathrm{MS}})/\pi$ whose running is determined by the [modified minimal subtraction scheme ($\overline{\mathrm{MS}}$)] renormalization group equation

$$\frac{\partial a(\mu)}{\partial \ln \mu^2} = -\beta_0 a^2 (1 + c_1 a + c_2 a^2 + c_3 a^3 + \cdots). \qquad (1)$$

For simplicity of argument, we will consider only the effects of the leading IR renormalon and will neglect the subleading IR renormalons. We will also assume that such an amplitude can be described by OPE.

First, let us review the method of Ref. [6] in detail. The presentation of the method is here somewhat different. The Borel transform $B(b)$ of $\Delta[a(Q)]$ has a singularity at $b \geqslant n$ ($n=1$, or 2, or 3, . . .) of the form

*Electronic address: gorazd.cvetic@fis.utfsm.cl

[1]The IR renormalons predict the energy dependence of such terms.

$$B(b)=\frac{C}{(1-b/n)^{1+\nu}}[1+\kappa_1(1-b/n)+\kappa_2(1-b/n)^2\cdots]$$
$$+\text{analytic part.} \tag{2}$$

Here $\nu=(nc_1-\gamma_{2n})/\beta_0$, where γ_{2n} is the one-loop coefficient of the anomalous dimension of the corresponding higher-twist operator (with dimension $d=2n$) in OPE for $\Delta[a(Q)]$. In the following, we will ignore the contributions of the terms with coefficients κ_j ($j\geqslant 1$), since their inclusion is straightforward ($\nu\mapsto\nu-j$) and does not affect the conclusions. For definiteness, the renormalization scale μ is taken to be $\mu=Q$, and the renormalization scheme is also regarded as fixed, e.g., $\overline{\text{MS}}$. We will regard the coupling to be complex in general, $z\equiv\beta_0 a(Q)/n=|z|\exp(i\phi)$, corresponding to complex momenta Q. Then the Borel integral $\Delta_{\text{BI}}(z)$ can be defined as

$$\Delta_{\text{BI}}(z)=\frac{1}{\beta_0}\int_0^{+\infty\exp(i\phi)}db\,\exp\left(-\frac{b}{nz}\right)B(b)$$
$$(z=|z|e^{i\phi},b=|b|e^{i\phi}). \tag{3}$$

For z near the positive real axis, $z=|z|\pm i\varepsilon$ $[|z|=\beta_0 a(Q)/n>0]$, it is straightforward to show from Eq. (3), with the help of the Cauchy theorem, the following formula:

$$\Delta_{\text{BI}}(z)|_{z=|z|\pm i\varepsilon}$$
$$=\frac{1}{\beta_0}\int_{\pm i\varepsilon}^{+\infty\pm i\varepsilon}db\,\exp\left(-\frac{b}{n|z|}\right)B(b)$$
$$(\varepsilon\to+0). \tag{4}$$

The real part of this is the principal value. As a consequence of the IR singularity (2) at $b\geqslant n$, the integral (3) has a discontinuity (cut) at the positive real axis $z\geqslant 0$; namely, the quantity (4) is not real and its discontinuity shows up in its imaginary part. This can be seen by introducing in the integrand of Eq. (4) the new complex integration variable t via $b=n(1+|z|t)$:

$$\text{Im}\,\Delta_{\text{BI}}(z=|z|\pm i\varepsilon)=\pm\frac{1}{2i\beta_0}\int_{\mathcal{C}_t}db\,\exp\left(-\frac{b}{n|z|}\right)$$
$$\times\frac{C}{(1-b/n)^{1+\nu}}$$
$$=\mp C\frac{n}{\beta_0}e^{-1/|z|}\Gamma(-\nu)\sin(\pi\nu)|z|^{-\nu}. \tag{5}$$

The (Hankel) contour $\mathcal{C}_t$ is depicted in Fig. 1, and the expression (5) is obtained from the known Hankel contour form of the gamma function (see, for example, [8])

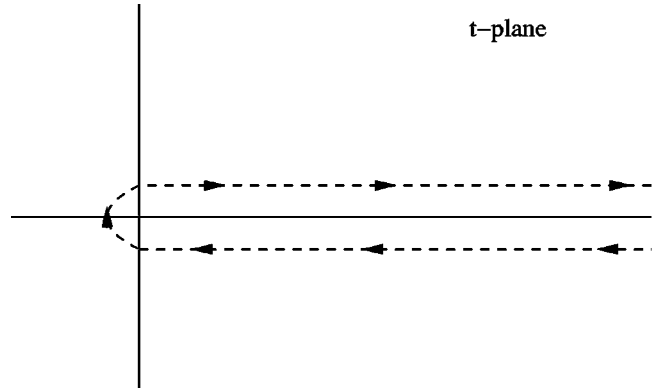

FIG. 1. The path $\mathcal{C}_t$ in the integrals (5) and (6).

$$\Gamma(s)\sin(\pi s)=\frac{\pi}{\Gamma(1-s)}$$
$$=+\frac{1}{2i}\int_{\mathcal{C}_t}dte^{-t}(-t)^{-1+s}\quad(|s|<\infty) \tag{6}$$

in the special cases $s=-\nu,-\nu+1,\ldots$. Because $(-|z|\mp i\varepsilon)=|z|\exp(\mp i\pi)$ and thus $(-|z|\mp i\varepsilon)^{-\nu}=|z|^{-\nu}[\cos(\pi\nu)\pm i\sin(\pi\nu)]$, Eq. (5) leads to

$$\Delta_{\text{BI}}(z)=-C\frac{n}{\beta_0}e^{-1/z}\Gamma(-\nu)(-z)^{-\nu}+\tilde{\Delta}_{\text{BI}}(z), \tag{7}$$

where $\tilde{\Delta}_{\text{BI}}(z)$ is a function without cuts in the complex z plane since the first expression on the right-hand side absorbs the renormalon-induced cut (5) at $z\geqslant 0$. The first expression, when $z=|z|\pm i\varepsilon=\beta_0 a(Q)/n\pm i\varepsilon$ is at the real positive axis, has the Q dependence $Q^{-2n}\alpha_s(Q)^{\gamma_{2n}/\beta_0}[1+\mathcal{O}(\alpha_s)]$, the same as the corresponding power-suppressed (higher-twist) term $\langle\mathcal{O}_{2n}\rangle^{(Q)}/(Q^2)^n$ in OPE. The imaginary part of this expression, i.e., expression (5), must be identified as the imaginary part of the contribution from the leading IR renormalon (2). The central assumption of the method is that the full first expression on the right-hand side of Eq. (7), which contains the full nonphysical cut (5) along $z>0$ and no cut along $z<0$, represents the nonphysical cut function which is to be eliminated:

$$\Delta_{(\text{cut1})}(z)=-C\frac{n}{\beta_0}e^{-1/z}\Gamma(-\nu)(-z)^{-\nu}$$
$$=C\frac{n}{\beta_0}e^{-1/z}\frac{\pi}{\Gamma(1+\nu)\sin(\pi\nu)}(-z)^{-\nu}. \tag{8}$$

This same cut contribution, for $z=|z|\pm i\varepsilon$ at the positive real axis, can be obtained also by Borel-integrating the nonanalytic part of the Borel transform (2) along its cut $b>n$ above or below the real axis, in analogy with the full Borel-integrated quantity (4). To show this, we use the new real integration variable t such that $b=n(1+zt)$, and therefore $b>n\pm i\varepsilon$ corresponds to $t>0$:

$$\Delta_{\rm BI}^{\rm (nonan)}(z=|z|\pm i\varepsilon;b>n)$$

$$=\frac{1}{\beta_0}\int_{n\pm i\varepsilon}^{\infty\pm i\varepsilon}db\,\exp\left(-\frac{b}{nz}\right)B^{\rm (nonan)}(b) \tag{9}$$

$$=+C\frac{n}{\beta_0}ze^{-1/z}\int_0^{+\infty}dte^{-t}(-z)^{-1-\nu}t^{-1-\nu} \tag{10}$$

$$=-C\frac{n}{\beta_0}e^{-1/z}\Gamma(-\nu)(-z)^{-\nu}. \tag{11}$$

For z away from the real positive axis, the analytic continuation of this expression in z keeps its form (11) unchanged, i.e., precisely the cut function (8). The Borel integration over $t>0$ ($\Leftrightarrow b>n\pm i\varepsilon$) in Eq. (10) converges only when $\mathrm{Re}(\nu)<0$ and gives the result (11). When $\mathrm{Re}(\nu)\geqslant 0$, the result (11) represents the analytic continuation in ν.

The subtraction of the cut-function contribution (8) leads to the final result for the resummed value of the observable at the positive real value $z=|z|=\beta_0 a(Q)/n>0$:

$$\begin{aligned}\Delta[z=\beta_0 a(Q)/n]&=\Delta_{\rm BI}(|z|\pm i\varepsilon)-\Delta_{\rm (cut1)}(|z|\pm i\varepsilon)\\&=\tilde{\Delta}_{\rm BI}[z=\beta_0 a(Q)/n]\\&=[\mathrm{Re}\mp\cot(\pi\nu)\mathrm{Im}]\int_{\pm i\varepsilon}^{\infty\pm i\varepsilon}\frac{db}{\beta_0}\\&\quad\times\exp\left[-\frac{b}{\beta_0 a(Q)}\right]\frac{R(b)}{(1-b/n)^{1+\nu}}.\end{aligned} \tag{12}$$

In the Borel integration here, the exactly known IR renormalon singularity has been factored out explicitly. The function $R(b)=(1-b/n)^{1+\nu}B(b)$ (whose truncated perturbation series up to $\sim b^2$ is known exactly in the case of several QCD observables) has a much weaker singularity at $b=n$ than $B(b)$. In Eq. (12), the equality $\mathrm{Re}\Delta_{\rm (cut1)}(|z|\pm i\varepsilon)=\pm\cot(\pi\nu)\,\mathrm{Im}\Delta_{\rm (cut1)}(|z|\pm i\varepsilon)$ was taken into account, a direct consequence of Eqs. (5), (7), and $(-|z|\mp i\varepsilon)^{-\nu}=|z|^{-\nu}[\cos(\pi\nu)\pm i\sin(\pi\nu)]$. The results (8) and (12) are those arrived at in Ref. [6].

The renormalon power coefficient $\nu=(nc_1-\gamma_{2n})/\beta_0$ should be regarded as a general parameter which can take on, in principle, any (real) value. For example, when varying the number of effectively active quark flavors n_f continuously, ν changes continuously. Yet another example is given by the large-β_0 approximation, when $c_1\to 0$ and $\nu\to 0$. The residue C of the renormalon in general does not vanish in the large-β_0 limit.

The cut function (8), as a function of general ν and when the coupling parameter is near the positive real axis $z_\pm=|z|\pm i\varepsilon=\beta_0 a(Q)/n\pm i\varepsilon$, can be rewritten in the following form:

$$\begin{aligned}&\Delta_{\rm (cut1)}(z=|z|\pm i\varepsilon)\\&\quad=-C\frac{n}{\beta_0}e^{-1/|z|}\frac{[-\pi\cot(\pi\nu)\mp i\pi]}{\Gamma(1+\nu)}|z|^{-\nu}.\end{aligned} \tag{13}$$

The central assumption of the method, i.e., the subtraction of the cut function (8) from the Borel-integrated value, appears to be plausible and natural, especially because the cut function (8) [$\Leftrightarrow$ (13)] has a simple form, and because it represents precisely the contribution of the Borel integration of the nonanalytic part of the Borel transform (2) along the cut $b\geqslant n$ parallel to the real axis, as explained in Eqs. (9)–(11). However, there are at least two problems with this method when we impose on it the plausible condition that it should work for any (real) value of the power coefficient ν.

(1) When ν is a non-negative integer ($\nu=k$; $k=0,1,2,\ldots$), the cut function (13) is infinite, because $\cot(\pi\nu)$ diverges there. This would not present a problem if the residue C of the renormalon disappeared, making this cut function finite. This appears to be unlikely, as argued above; in particular, for the Adler function in the large-β_0 approximation ($n=2,c_1=0,\gamma_4=0$) this would mean the disappearance of the leading IR renormalon. Note that, in contrast to $\cot(\pi\nu)$, the factor $1/\Gamma(1+\nu)$ in Eq. (13) is an analytic function in the entire complex ν plane.

(2) When ν is a negative integer $\nu=-1,-2,\ldots$, the IR renormalon singularity (2) with the cut disappears (even when $C\neq 0$), and the Borel transform is analytic. This means that $\Delta_{\rm (cut1)}(z)$ must be zero. However, according to Eq. (13) $\Delta_{\rm (cut1)}(z)\neq 0$—note that $\pi\cot(\pi\nu)/\Gamma(1+\nu)=(-1)^k(k-1)=0$ when $\nu=-k=-1,-2,\ldots$. Again, it is the poles of $\cot(\pi\nu)$ that cause the problems, but this time at $\nu=-1,-2,\ldots$.

If we take the viewpoint that the method should be taken as the starting point nonetheless, because of the mentioned plausibility and naturalness of the choice of Eq. (8), we should definitely modify it so that the aforementioned two problematic points are eliminated. This can be done in a natural way, by inspecting again the expression (13). The two problematic aspects arose because of the poles of the term $\cot(\pi\nu)$, at $\nu=0,\pm1,\pm2$, etc. The function $f(\nu)=\cot(\pi\nu)$ should thus be regularized in order to eliminate both problems. The apparently most natural regularization is obtained by subtracting simple pole functions with the coefficients equal to the residues of $f(\nu)$ at those poles. This gives

$$-\pi\cot(\pi\nu)\mapsto-\pi\cot(\pi\nu)+\frac{1}{\nu}+\sum_{k=1}^{\infty}\left(\frac{1}{\nu-k}+\frac{1}{\nu+k}\right) \tag{14}$$

$$=-\pi\cot(\pi\nu)+\frac{1}{\nu}+2\nu\sum_{k=1}^{\infty}\frac{1}{\nu^2-k^2}\equiv 0. \tag{15}$$

We see that the function $\cot(\pi\nu)$ is identical to the sum of the corresponding simple pole functions. Only some of the meromorphic functions have this remarkable property. The procedure (14),(15) therefore eliminates the real part of the cut function (13) at the real axis $z=|z|\pm i\varepsilon$; the modified expression there is purely imaginary. The final result of the modified method is then just the principal value (PV) of the Borel integral, when $a(Q)>0$,

$$\Delta[z=\beta_0 a(Q)/n] = \mathrm{Re}\,\Delta_{\mathrm{BI}}(z\pm i\varepsilon) = \mathrm{Re}\int_{\pm i\varepsilon}^{\infty\pm i\varepsilon}\frac{db}{\beta_0}\exp\left[-\frac{b}{\beta_0 a(Q)}\right] \times\frac{R(b)}{(1-b/n)^{1+\nu}}, \tag{16}$$

and the new cut function can be written for a general complex z as

$$\Delta_{\mathrm{cut}}^{(\mathrm{PV})}(z) = +C\frac{n}{\beta_0}e^{-1/z}\frac{\pi}{\Gamma(1+\nu)\sin(\pi\nu)} \times[(-z)^{-\nu}-\cos(\pi\nu)z^{-\nu}]. \tag{17}$$

It can be checked explicitly that this function is finite for any finite complex ν, including $\nu=0,\pm1,\pm2,\ldots$. It is an analytic function of z outside the real axis, with the cut along the real axis. The subtraction of the poles of $\cot(\pi\nu)$ (15) introduced in Eq. (17) terms proportional to $z^{-\nu}$. The consistency of the results for $\nu=0,\pm1,\ldots$ thus introduced an additional cut along the negative real axis.

In general, any regularization in ν of the singular $\cot(\pi\nu)$ factor in Eq. (13), not just the simplest regularization (14), would give an acceptable result. The factor $1/\Gamma(1+\nu)$ is already nonsingular (even analytic) for all ν. The factor $\exp(-1/|z|)|z|^{-\nu}$ in Eq. (13) cannot be modified because it reflects the correct Q dependence of the aforementioned higher-twist term $\langle\mathcal{O}_{2n}\rangle^{(Q)}/(Q^2)^n$ in OPE. The imaginary part in Eq. (13) cannot be modified because it is needed to make the amplitude $\Delta(z)\equiv\Delta_{\mathrm{BI}}(z)-\Delta_{\mathrm{cut}}(z)$ real for real positive z. Therefore, the most general ν regularization of the cut function (13) for $z=|z|\pm i\varepsilon$ is represented by a simple substitution $\cot(\pi\nu)\mapsto g(\nu)$, where $g(\nu)$ is an arbitrary nonsingular (possibly analytic) function of ν which is real for real ν:

$$\Delta_{\mathrm{cut}}(z=|z|\pm i\varepsilon) = -C\frac{n}{\beta_0}e^{-1/|z|}\frac{[-\pi g(\nu)\mp i\pi]}{\Gamma(1+\nu)}|z|^{-\nu}. \tag{18}$$

This cut function then has the following form in the complex coupling plane:

$$\Delta_{\mathrm{cut}}(z) = +C\frac{n}{\beta_0}e^{-1/z}\frac{\pi}{\Gamma(1+\nu)\sin(\pi\nu)}\{(-z)^{-\nu}-[\cos(\pi\nu) -g(\nu)\sin(\pi\nu)]z^{-\nu}\}. \tag{19}$$

Again, as in Eq. (17), we see that the ν regularization introduces an additional cut along the negative axis $z<0$. The amplitude $\Delta\equiv\Delta_{\mathrm{BI}}-\Delta_{\mathrm{cut}}$ is now represented by the principal value and a higher-twist term proportional to $g(\nu)$, where $g(\nu)$ is nonsingular in ν. We stress that any such choice is physically acceptable, the principal value choice being distinguished in this context only by the mathematical simplicity of the corresponding ν regularization (14),(15).

The main idea behind the result Eqs. (8),(12), as stressed in Ref. [6], was that the cut function which is to be subtracted from a Borel-resummed QCD amplitude with an IR renormalon has a cut only along the positive axis in the coupling plane $z\equiv a(Q)$. In the two exactly solvable non-QCD examples presented in Ref. [6] this idea was shown to hold. Here we showed that the requirement of finiteness with respect to the IR renormalon power parameter ν in QCD amplitudes, and their consistency in the absence of the IR renormalon, imply that the cut function must contain, in addition to the cut along the positive z axis, a cut along the negative z axis also. This, in turn, implies that the amplitude $\Delta(z)\equiv\Delta_{\mathrm{BI}}(z)-\Delta_{\mathrm{cut}}(z)$, while having no cut on the $z>0$ axis in accordance with the unitarity and causality conditions [9], does have a cut along the $z<0$ axis (Landau region) as a consequence of the IR renormalon.[2] This somewhat counterintuitive conclusion was also obtained in Refs. [4,5] by substantially different approaches, where the principal value prescription was adopted. If the considered amplitude has ultraviolet (UV) renormalons, it should have a cut along $z<0$ even if it has no IR renormalons. If the coupling $a(Q)$ is regularized so that it is finite for all $Q^2\geqslant0$ (see, e.g., [12–16]), i.e., in contrast to the PQCD $a(Q)$ it has no Landau singularities, the conclusions about the analyticity of the considered amplitude in the complex coupling plane probably change singnificantly.

The IR renormalons also play an important role in the resummations using modified Borel transforms where the entire integrand in the Borel integration is renormalization scale (RS) invariant. Such transforms were introduced by Grunberg [17] on the basis of a larger class of transforms proposed in Ref. [18] in a somewhat different context. Such RS-invariant Borel transform resummations were applied in Refs. [19,20], by either evaluating the principal value of the Borel integral [19] or adding to the principal value the higher-twist OPE terms [20]. The discussed method of Ref. [6], for the ordinary Borel transforms (3), can be adapted to the method of RS-invariant Borel transforms. The problems (divergences) appearing in this case are similar to those discussed here, but algebraically more complicated. It is not clear whether in such cases an analogous regularization procedure as the one presented here would lead naturally to the principal value of the RS-invariant Borel resummation.

In the present work, the method of subtracting a power-suppressed term from the principal value of the Borel integral for QCD amplitudes with IR renormalons, recently proposed in Ref. [6] and applied in Refs. [7], was scrutinized. It

[2]According to Ref. [9], the physical singularities in confined theories are generated by the physical hadron states. The point $z=0$ is an essential singularity [10,11]. The confinement is not seen by PQCD; thus the cut along $z<0$ (Landau region) is not physical but must appear in quantities involving PQCD coupling $a(Q)$.

was pointed out that the result becomes physically untenable for specific values of the renormalon power coefficient ν, as a consequence of the divergences of the term $[\cot(\pi\nu)]$ appearing in the result. When these divergences are removed in apparently the most natural way, the power-suppressed term of the method disappears and the modified result becomes the principal value. Any removal of the aforementioned divergences results in an IR-renormalon-induced cut along the negative axis (Landau region) in the coupling plane. These conclusions suggest, among other things, that the most natural PQCD Borel integration of a QCD observable remains the principal value. The additional power-suppressed (higher-twist, higher-dimensional) terms cannot be inferred from PQCD (+renormalon) methods in any natural way. Such additional (OPE) terms involve vacuum expectation values of higher-twist operators and can theoretically be obtained or estimated only by genuinely nonperturbative methods. Phenomenologically, such additional OPE terms can be determined by fitting them to the corresponding experimental data. However, in such a procedure, it is important to keep for the leading-twist term in OPE a specific resummed PQCD expression, most naturally the principal value of the Borel integral, i.e., Eq. (16). On the other hand, if the leading-twist term is taken to be a truncated perturbation series (TPS), the strength of the higher-twist terms will sometimes dramatically change when the order of the TPS is changed [21,22].

This work was supported by FONDECYT (Chile) Grant No. 1010094.

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