

## Can the mechanism for $\pi_1 \rightarrow \eta\pi, \eta'\pi$ hybrid decays be detected?

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Two mechanisms for the  $\pi_1(J^{PC}=1^{-+})$  hybrid meson decay processes  $\pi_1 \rightarrow \eta\pi, \eta'\pi$  are investigated. These mechanisms are applied to  $\phi \rightarrow \eta\gamma, \eta'\gamma$  and  $J/\psi \rightarrow \eta\gamma, \eta'\gamma$  decays to illustrate the validity of the decay mechanisms and to obtain independent information on the coupling of  $\eta, \eta'$  to quark and gluonic operators. From this information, we find that  $\Gamma(\pi_1 \rightarrow \eta\pi)/\Gamma(\pi_1 \rightarrow \eta'\pi)$  is substantially different in the two decay mechanisms, and hence future experimental measurements of this ratio will provide valuable information for determining the mechanism for these hybrid decays.

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Hybrid meson states, which contain a gluonic degree of freedom in addition to a  $q\bar{q}$  pair, can have exotic  $J^{PC}$  quantum numbers. Experimental evidence for the existence of two such exotic isovector  $\pi_1(1^{-+})$  states is summarized in [1]. The  $\pi_1(1400)$ , formerly known as the  $\hat{\rho}(1405)$  (mass  $1376 \pm 17$  MeV and width  $300 \pm 40$  MeV [1]), was observed by both E852 and Crystal Barrel in very different production processes decaying into  $\eta\pi$  [2,3]. The  $\pi_1(1600)$  (mass  $1596_{-14}^{+25}$  MeV, width  $312_{-24}^{+64}$  MeV [1]) was observed by E852 in  $\rho\pi$  and  $\eta'\pi$  decay channels [2]. The branching ratios for all these channels are not yet measured, but as will be shown below, they can provide important information for subsequently detecting their dominant decay mechanism.

Theoretically, the features of hybrids have been studied with the MIT bag model [4], flux tube models [5], potential models [6], quark-gluon constituent model [7,8], QCD sum rules [9,10], lattice simulations [11], and other methods [12], but the exploration is neither complete nor definitive. In particular, predictions for the hybrid decay widths exhibit some disagreement with the experimental results.

Hybrids can possess exotic  $J^{PC}$  quantum numbers such as  $0^{+-}, 1^{-+}$  and  $2^{+-}$  which are distinct from those of conventional  $q\bar{q}$  mesons. As such, these exotic hybrids have no mixing with other conventional hadrons which provides an advantage in the investigation and detection of these states. The predicted mass of the exotic  $1^{-+}$  hybrid is approximately 2.0 GeV in lattice simulations [11], while in QCD sum rules the resulting mass prediction is 1.4–2.1 GeV [9,10]. The sum-rule predictions are slightly lower than those of the lattice, but are consistent with experiment. However, considering the possible accuracy of the sum-rule and lattice calculations, any apparent deviation between the predicted and observed  $\pi_1$  masses is insufficient to assist in the interpretation of the observed states. Thus it is important to study decay features because they are more sensitive to the nature of the  $\pi_1$  states.

The decay modes and relevant decay widths of the exotic  $1^{-+}$  hybrid have been studied using QCD sum rules for three-point correlation functions, but the scheme employed for  $\eta-\eta'$  mixing has a significant effect on the  $\pi_1 \rightarrow \pi\eta, \pi\eta'$  widths. In the traditional singlet-octet mixing scheme, these widths are found to be small [10], and are similar to those found from selection rules [8,13]. These predictions seem inconsistent with the experimental observations. However, in a different  $\eta-\eta'$  mixing scheme [15,16], the three-point sum-rule analysis results in an enhancement in the  $\pi_1 \rightarrow \eta\pi$  width [14].

Enhancement of the  $\pi_1 \rightarrow \eta\pi$  width in the three-point sum-rule analysis clearly indicates the importance of the composition of the  $\eta, \eta'$  system. However, there are a number of issues that complicate the sum-rule analyses of the  $\pi_1$  decays. For example, there is phenomenological evidence that the  $\eta-\eta'$  system has a gluonic component [17,18], which would clearly have an effect on the sum-rule analyses. Furthermore, the necessary three-point functions have only been calculated at the symmetric Euclidean point which leads to a single Borel transformation instead of the double transformation needed for a full analysis. Finally, the traditional three-point sum-rule method obscures determination of the dominant hadron decay mechanism, indicating the need for further investigation.

Hybrids are a many body system containing a quark, antiquark and gluon ( $\bar{q}qg$ ), which complicates the determination of the allowed  $J^{PC}$  values. To get some feel for what is involved, consider these complexities in the MIT bag model. In this model, the gluon in hybrids may be in two different modes ( $TM(1^{--})$  or  $TE(1^{+-})$ ), and the quark-antiquark pairs may also take on different  $J^{PC}$  configurations. As a consequence, there exist many kinds of quantum number combinations. For example, when the quark and antiquark have no relative orbital angular momentum in the pair, the  $J^{PC}$  of this pair may be  $0^{-+}$  or  $1^{--}$ . Therefore, besides the normal  $q\bar{q}$  meson  $J^{PC}$  quantum numbers, hybrids may also have exotic  $J^{PC}$  such as  $0^{+-}, 1^{-+}$  and  $2^{+-}$ . These kinds of exotic states simplify both the theoretical investigation and experimental detection. Among these exotic states, the hy-

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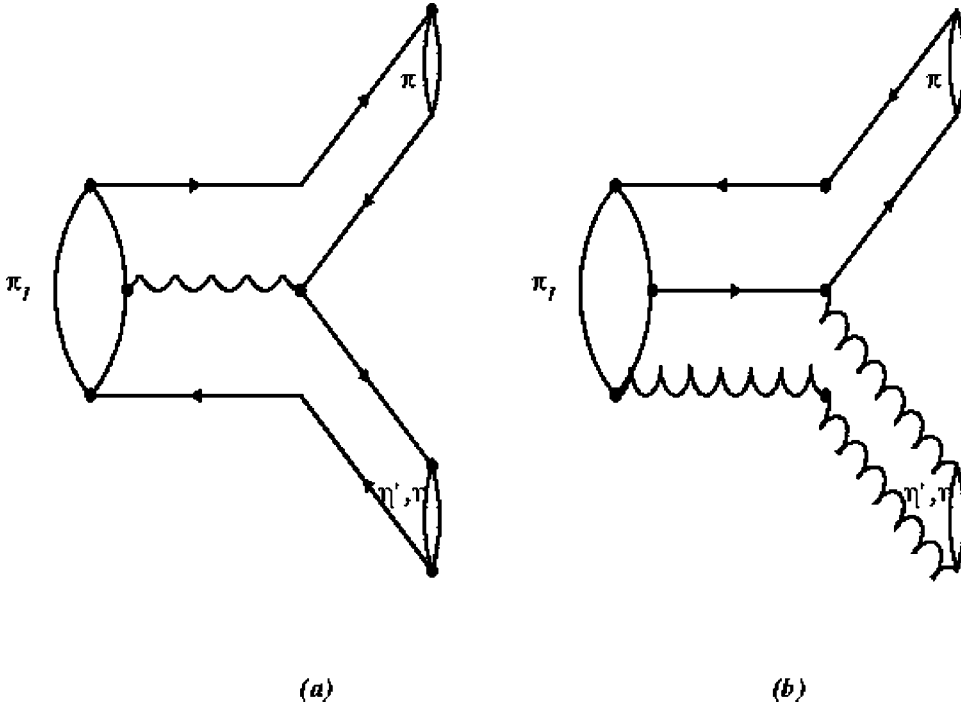


FIG. 1. The decay  $\pi_1 \rightarrow \pi\eta', \eta$  via the  $\bar{q}q$  mechanism is shown in (a), while decay via the  $gg$  mechanism is shown in (b).

brid  $\pi_1$  ( $1^{-+}$ ) is predicted as the lowest-lying state in the hybrid spectrum. Its  $J^{PC}$  construction is:  $1^{--} \times 1^{+-}$ , i.e., the gluon is in transverse electric mode  $TE(1^{+-})$ .

There are a number of models to describe the strong decay of a meson to a two-meson final state through creation of a quark-antiquark pair which then combines with the quark and antiquark of the original meson to form the two-meson final state. However, at a more detailed level, the quark pair creation process can be viewed as either a  $^3S_1(1^{--})$  or  $^3P_0(0^{++})$  intermediate state.

The special role of the constituent gluon in hybrids leads to some different decay possibilities. In the constituent parton picture, two main decay mechanisms, denoted by  $\bar{q}q$  and  $gg$  will be studied in this paper. In the  $\bar{q}q$  process the gluon in the initial hybrid becomes a quark-antiquark pair, while in the  $gg$  process a new gluon is emitted from an original constituent quark and combine with the original constituent gluon into a final meson. For the  $\pi_1 \rightarrow \pi\eta', \eta$  decays, these two decay mechanisms are illustrated in Fig. 1.

It is difficult to accurately describe or calculate these decays directly from QCD because of the fundamentally non-perturbative nature of strong decays. In this paper we show that it is possible to extract the  $\eta, \eta'$  couplings to quark and gluonic currents from existing experimental data on the decays  $J/\psi \rightarrow \gamma\eta', \gamma\eta$  and  $\phi \rightarrow \gamma\eta', \gamma\eta$ , and to apply this information to two mechanisms for  $\pi_1 \rightarrow \pi\eta', \pi\eta$  decays. This approach avoids the issues that have complicated the sum-rule analyses, and allows determination of the dominant hybrid decay mechanism.

### I. GLUONIC COUPLINGS OF $\eta, \eta'$ THROUGH $J/\psi \rightarrow \eta\gamma, \eta'\gamma$ DECAYS

To extract the  $\eta, \eta'$  couplings to gluonic currents, consider the processes  $J/\psi \rightarrow \eta\gamma, \eta'\gamma$ . In these processes the

photon is emitted by one of the  $c$  quarks before their annihilation into lighter quark pairs [17,18]. The resulting ratio of decay rates occurring if the  $J/\psi \rightarrow \eta\gamma, \eta'\gamma$  decay processes occur through  $\bar{c}c \rightarrow gg \rightarrow \eta, \eta'$  is

$$\frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} \simeq \frac{|\langle 0|G\tilde{G}|\eta\rangle|^2}{|\langle 0|G\tilde{G}|\eta'\rangle|^2} \left( \frac{1 - m_\eta^2/m_{J/\psi}^2}{1 - m_{\eta'}^2/m_{J/\psi}^2} \right)^3, \quad (1)$$

where it has been assumed that the  $gg$  pair is sufficiently hard so that the use of the local operator  $G\tilde{G} = 1/2\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}^a G_{\mu\nu}^a$  extracted from the  $gg$  pair is a good approximation. The experimental value of this decay-width ratio is [20]

$$\frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} = 0.200 \pm 0.023. \quad (2)$$

Using the known masses  $m_{J/\psi} = 3.1$  GeV,  $m_\eta = 0.548$  GeV, and  $m_{\eta'} = 0.958$  GeV, the following relation

$$\frac{\langle 0|G\tilde{G}|\eta\rangle}{\langle 0|G\tilde{G}|\eta'\rangle} \simeq 0.404 \pm 0.023 \quad (3)$$

is obtained. The uncertainty given in Eq. (3) is only associated with the experimental value (2).

The agreement between the value (3) and the corresponding sum-rule estimates [16,18] provides support for the assumed  $\bar{c}c \rightarrow gg \rightarrow \eta, \eta'$  mechanism and associated approximations used for these decays. However, the sum-rule analyses are based on specific models of the  $\eta\text{-}\eta'$  system, while the extraction (3) is independent of such considerations and would remain valid with the addition of a gluonic component mixing with  $\eta\text{-}\eta'$ . For this reason we will use (3) in our subsequent analyses.

## II. QUARK COUPLINGS OF $\eta, \eta'$ THROUGH $\phi \rightarrow \eta\gamma, \eta'\gamma$ DECAYS

The  $\phi \rightarrow \eta\gamma, \eta'\gamma$  decays must occur through a different mechanism than the  $J/\psi$  decays, since a  $\bar{s}s \rightarrow gg \rightarrow \eta, \eta'$  process similar to the  $J/\psi$  decays considered earlier would lead to

$$\frac{\Gamma(\phi \rightarrow \eta'\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)} \simeq \frac{|\langle 0|G\tilde{G}|\eta'\rangle|^2}{|\langle 0|G\tilde{G}|\eta\rangle|^2} \left( \frac{1-m_{\eta'}^2/m_\phi^2}{1-m_\eta^2/m_\phi^2} \right)^3 \simeq 2.8 \times 10^{-2}, \quad (4)$$

where the mass  $m_\phi = 1.02$  GeV and (3) has been used. This value is an order of magnitude larger than the known experimental value [21]

$$\frac{\Gamma(\phi \rightarrow \eta'\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)} = 4.7 \pm 0.47 \pm 0.31 \times 10^{-3}, \quad (5)$$

indicating that the  $\bar{s}s \rightarrow gg \rightarrow \eta, \eta'$  process does not properly describe the  $\phi \rightarrow \eta\gamma, \eta'\gamma$  processes. Since a high energy scale is needed to approximate the  $gg$  mechanism as the coupling of a gluonic operator to mesons, it is not surprising that the  $gg$  mechanism cannot be applied to  $\phi$  radiative decays to  $\eta, \eta'$  [19].

Alternatively, consider the  $\bar{s}s \rightarrow \bar{q}q \rightarrow \eta, \eta'$  mechanism, which could include the direct process  $\bar{s}s \rightarrow \eta, \eta'$ . Although the detailed decay mechanism is unknown, this process can be modeled through  $\eta, \eta'$  couplings to quark currents. Considering the nature of the the initial  $\phi$  state and final  $\eta, \eta'$  states, the appropriate quark current is of the form  $\bar{q}i\gamma_5q$  indicating a general flavor structure. Note that a coupling to an axial vector current would contain a (dominant) anomaly term which would then lead to the gluonic current couplings as ruled out in the above analysis. The resulting ratio of decay rates in this general  $\bar{s}s \rightarrow \bar{q}q \rightarrow \eta, \eta'$  mechanism is

$$\frac{\Gamma(\phi \rightarrow \eta'\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)} \simeq \frac{|\langle 0|\bar{q}i\gamma_5q|\eta'\rangle|^2}{|\langle 0|\bar{q}i\gamma_5q|\eta\rangle|^2} \left( \frac{1-m_{\eta'}^2/m_\phi^2}{1-m_\eta^2/m_\phi^2} \right)^3. \quad (6)$$

Using Eq. (5), we obtain

$$\frac{|\langle 0|\bar{q}i\gamma_5q|\eta'\rangle}{|\langle 0|\bar{q}i\gamma_5q|\eta\rangle|} \simeq 0.984 \pm 0.082, \quad (7)$$

where the uncertainty only reflects the experimental value (5).

To disentangle the actual flavor structure occurring in the estimate (7), we turn to the theoretical estimates

$$\frac{|\langle 0|\bar{s}i\gamma_5s|\eta\rangle}{|\langle 0|\bar{s}i\gamma_5s|\eta'\rangle|} \simeq 0.76 \pm 0.10 \quad (8)$$

$$\frac{|\langle 0|\bar{n}i\gamma_5n|\eta\rangle}{|\langle 0|\bar{n}i\gamma_5n|\eta'\rangle|} \simeq 2.5 \quad (9)$$

as obtained in a recent QCD sum-rule analysis [16] (see also [15] for other estimates of this quantity), where  $n$  denotes the non-strange  $u, d$  quarks in the  $SU(2)$  limit. Comparison of Eqs. (7), (8) and (9) indicates that the direct  $\bar{s}s \rightarrow \eta\eta'$  process is predominant in  $\phi \rightarrow \eta\gamma, \eta'\gamma$ , and the agreement between Eqs. (7) and (8) validates the approximations used to obtain Eq. (6). In particular, this agreement indicates that the coupling of the final states to operators ignored in obtaining Eq. (6) must be small enough such that the pseudoscalar current dominates the process.

It is important to note that the numerical value (9) is almost unchanged between  $\eta, \eta'$  mixing schemes, while Eq. (8) shows some scheme dependence. Thus we can consider our result (7) as a mixing-scheme independent extraction of the ratio of the couplings to strange pseudoscalar currents, and can safely use the theoretical value for nonstrange currents, obviating the absence of experimental data that could be used to extract the nonstrange ratio.

As a final demonstration of the consistency of our analysis, we return to the  $J/\psi$  decays under the assumption of a  $\bar{c}c \rightarrow \bar{q}q \rightarrow \eta, \eta'$  mechanism, resulting in

$$\frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} \simeq \frac{|\langle 0|\bar{q}i\gamma_5q|\eta'\rangle|^2}{|\langle 0|\bar{q}i\gamma_5q|\eta\rangle|^2} \left( \frac{1-m_\eta^2/m_{J/\psi}^2}{1-m_{\eta'}^2/m_{J/\psi}^2} \right)^3 \geq 1.19, \quad (10)$$

where Eq. (7) has been used to obtain a lower bound. This is clearly an inadequate description of the decay process because of its disagreement with the experimental value (2), illustrating that different mechanisms and the resulting coupling of  $\eta, \eta'$  to different operators are occurring in each case. Indeed, if a common decay mechanism existed in the cases considered so far, then the matrix elements of the relevant operator would cancel in the following double ratio resulting in

$$\frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} \frac{\Gamma(\phi \rightarrow \eta\gamma)}{\Gamma(\phi \rightarrow \eta'\gamma)} \simeq \frac{(1-m_\eta^2/m_{J/\psi}^2)^3 (1-m_{\eta'}^2/m_\phi^2)^3}{(1-m_{\eta'}^2/m_{J/\psi}^2)^3 (1-m_\eta^2/m_\phi^2)^3} = 5.59 \times 10^{-3} \quad (11)$$

where masses have been inserted to obtain the numerical value. By comparison, the experimental value of the double ratio ratio obtained from Eqs. (2) and (5) is

$$\frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} \frac{\Gamma(\phi \rightarrow \eta\gamma)}{\Gamma(\phi \rightarrow \eta'\gamma)} = 0.2 \times 4.7 \times 10^{-3} = 9.4 \times 10^{-4}, \quad (12)$$

demonstrating that the scales associated with the  $\phi$  and  $J/\psi$  decays must be described by couplings to different operators, with the  $J/\psi$  decays best described by coupling to gluonic operators through the  $\bar{c}c \rightarrow \bar{g}g \rightarrow \eta, \eta'$  mechanism, while the

$\phi$  decays are best described by a coupling to quark operators through the  $\bar{s}s \rightarrow \eta, \eta'$  mechanism.

### III. HYBRID $\pi_1 \rightarrow \eta\pi, \eta'\pi$ DECAY MECHANISMS

The details of the  $q\bar{q}g$  hybrid decay mechanisms represented in Fig. 1 are unknown. However, the quark pair (one created from the initial constituent gluon in the hybrid) can be modeled through a quark current, and the gluon pair (one emitted from an initial constituent quark) through a gluonic current. Thus, for  $\pi_1 \rightarrow \eta\pi, \eta'\pi$  decays, the  $gg$  mechanism can be analyzed through the  $\eta, \eta'$  couplings to the pseudo-scalar gluonic current, and the  $\bar{q}q$  mechanism through a pseudoscalar quark current. Although it is possible to anticipate the dominance of the  $gg$  mechanism in the  $J/\psi$  decays and the  $\bar{q}q$  mechanism in the  $\phi$  decays because of the energy scales associated with the decay processes, the special role of the constituent gluon in the hybrid makes it difficult to make a theoretical prediction of which mechanism is dominant in hybrid decays. However, experimental data combined with the phenomenological analysis given below provides a means for distinguishing between these mechanisms.

If  $\pi_1 \rightarrow \eta\pi, \eta'\pi$  decays are dominated by the  $gg \rightarrow \eta, \eta'$  mechanism illustrated in diagram (b) of Fig. 1, then we would find

$$\begin{aligned} \frac{\Gamma(\pi_1 \rightarrow \eta\pi)}{\Gamma(\pi_1 \rightarrow \eta'\pi)} &\approx \frac{|\langle 0 | G\tilde{G} | \eta \rangle|^2}{|\langle 0 | G\tilde{G} | \eta' \rangle|^2} \left( \frac{1 - m_\eta^2/m_{\pi_1}^2}{1 - m_{\eta'}^2/m_{\pi_1}^2} \right)^3 \\ &= 0.425 \pm 0.048, \end{aligned} \quad (13)$$

where  $m_{\pi_1} = 1.6$  GeV has been used along with Eq. (3) for the  $\eta, \eta'$  gluonic couplings. If the hybrid mass is reduced to  $m_{\pi_1} = 1.4$  GeV, the central value of this ratio increases to 0.659.

For the  $\bar{q}q \rightarrow \eta, \eta'$  mechanism illustrated in diagram (a) of Fig. 1 we find

$$\frac{\Gamma(\pi_1 \rightarrow \eta\pi)}{\Gamma(\pi_1 \rightarrow \eta'\pi)} \approx \frac{|\langle 0 | \bar{n}i\gamma_5 n | \eta \rangle|^2}{|\langle 0 | \bar{n}i\gamma_5 n | \eta' \rangle|^2} \left( \frac{1 - m_\eta^2/m_{\pi_1}^2}{1 - m_{\eta'}^2/m_{\pi_1}^2} \right)^3 \quad (14)$$

where  $m_{\pi_1} = 1.6$  GeV has been used, and the isovector nature of the  $\pi_1$  and  $\pi$  necessitates the nonstrange quark operators in the  $SU(2)$  limit. If the theoretical value (9) is used to obtain an approximate value of the non-strange quark operators we obtain

$$\frac{\Gamma(\pi_1 \rightarrow \eta\pi)}{\Gamma(\pi_1 \rightarrow \eta'\pi)} \approx 16, \quad (15)$$

which is clearly distinct from the  $gg$  value (13). Even the lower bound obtained from Eq. (7)

$$\frac{\Gamma(\pi_1 \rightarrow \eta\pi)}{\Gamma(\pi_1 \rightarrow \eta'\pi)} > 2.5, \quad (16)$$

is sufficient to distinguish between the  $gg$  and  $\bar{q}q$  processes. Reducing the hybrid mass to  $m_{\pi_1} = 1.4$  GeV increases the numerical values in Eqs. (15) and (16) by approximately 55%.

In conclusion, experimental information for the decay processes  $J/\psi \rightarrow \eta\gamma, \eta'\gamma$  and  $\phi \rightarrow \eta\gamma, \eta'\gamma$  has been used to demonstrate that these decays occur through different decay mechanisms, allowing the extraction of  $\eta, \eta'$  couplings to gluonic and quark operators. These extractions are consistent with those of QCD sum rules, but have the advantage that they are independent of  $\eta$ - $\eta'$  mixing details. The overall consistency of these couplings substantiates the models and approximations used to study these decays.

Under the assumption of a hybrid nature of the  $\pi_1$  states, the extracted couplings of  $\eta, \eta'$  to the gluonic and quark operators are applied to estimating the decay-width ratio  $\Gamma(\pi_1 \rightarrow \eta\pi)/\Gamma(\pi_1 \rightarrow \eta'\pi)$  through the  $\bar{q}q$  and  $gg$  mechanisms illustrated in Fig. 1. This ratio is substantially different in the two mechanisms, and hence future branching-ratio measurements should identify the dominant decay mechanism, facilitating more detailed theoretical work. However, we note that a sum-rule analysis in the quark scheme for  $\eta$ - $\eta'$  mixing clearly predicts  $\Gamma(\pi_1 \rightarrow \eta\pi)/\Gamma(\pi_1 \rightarrow \eta'\pi) > 1$  [14], suggesting dominance of the  $\bar{q}q$  mechanism. Although effects of final-state interactions and of using a local operator for the  $\bar{q}q$  and  $gg$  pairs have been ignored in these processes, it seems unlikely that they will be large enough to alter the qualitative result  $\Gamma(\pi_1 \rightarrow \eta\pi) < \Gamma(\pi_1 \rightarrow \eta'\pi)$  in the  $gg$  mechanism and  $\Gamma(\pi_1 \rightarrow \eta\pi) > \Gamma(\pi_1 \rightarrow \eta'\pi)$  in the  $\bar{q}q$  mechanism. Conversely,  $\pi_1$  branching ratios that lie outside the extremes associated with Eqs. (13) and (15) may be difficult to accommodate from a theoretical perspective.

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