Final-state phases in $B \to D\pi$, $D^*\pi$, and $D\rho$ decays

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The final-state phases in $\bar{B} \to D\pi$, $D^*\pi$, and $D\rho$ decays appear to follow a pattern similar to those in $D \to \bar{K}\pi$, $\bar{K}^*\pi$, and $\bar{K}\rho$ decays. Each set of processes is characterized by three charge states but only two independent amplitudes, so the amplitudes form triangles in the complex plane. For the first two sets the triangles appear to have nonzero area, while for the $D\rho$ or $\bar{K}\rho$ decays the areas of the triangles are consistent with zero. Following an earlier discussion of this behavior for D decays, a similar analysis is performed for B decays, and the relative phases and magnitudes of contributing amplitudes are determined. The significance of recent results on $\bar{B}^0 \to D^{(*)0}\bar{K}^{(*)0}$ is noted. Open theoretical and experimental questions are indicated.

I. INTRODUCTION

The decays of B mesons (those containing the b quark) are potentially rich sources of information on CP violation. One such manifestation of this phenomenon involves an asymmetry $A(f)$ between the rate for a decay of a B meson to a final state f and the corresponding CP-conjugate process:

$$A(f) \equiv \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}. \quad (1)$$

Such an asymmetry requires there be at least two contributing amplitudes $A_{1,2}$, each characterized by distinct weak phases $\phi_{1,2}$ and strong phases $\delta_{1,2}$. Under CP conjugation, the weak phases change sign but the strong phases do not:

$$A(B \to f) = |A_1| e^{i\phi_1} e^{i\delta_1} + |A_2| e^{i\phi_2} e^{i\delta_2}, \quad (2)$$

$$A(\bar{B} \to \bar{f}) = |A_1| e^{-i\phi_1} e^{i\delta_1} + |A_2| e^{-i\phi_2} e^{i\delta_2}, \quad (3)$$

so that $A(f) \propto \sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2)$. In these decays the observation of a so-called "direct" CP asymmetry thus requires both the weak and the strong phases of the two contributing amplitudes to differ from one another. Thus it is of great importance to understand the patterns of strong final-state phases in as wide as possible a set of decays.

The strong final-state phases in decays of strange particles are appreciable. For example, in $K_{S,L} \to \pi\pi$ the final-state phases in the $I_{\pi\pi} = 0$ and $I_{\pi\pi} = 2$ channels differ from one another by many tens of degrees. Furthermore, they can be measured directly in elastic $\pi\pi$ scattering, and then applied to the decays $K_{S,L} \to \pi\pi$ using Watson's theorem [1]. However, in the decays of charmed and heavier mesons to two-body final states, these states constitute only a small fraction of the available decays, and elastic phase shifts are no longer relevant [2,3].

In the limit of a very heavy decaying quark, certain nonleptonic decays are expected to be characterized by small final-state interactions. These are the ones such as $\bar{B}^0 \to D^{(*)+}\pi^-$ to which the *factorization* hypothesis [4,5] applies: The decay amplitude can be regarded as the product of two color-singlet currents, one associated with the $\bar{B}^0 \to D^{(*)+}$ transition and the other creating the π^- from the vacuum. The large relative momentum of the two final-state particles leaves little time for them to interact with one another before they are safely out of each other's range. This expectation is confirmed in recent analyses of the factorization hypothesis based on QCD [6], though the role of final-state interactions in other nonleptonic heavy quark decays is more open to question (see, e.g., [7]).

Some processes involve *color-suppressed* weak decays, in which the weak current produces a pair of quarks each of which ends up in a different meson. Other processes involve interactions in which the quark and antiquark in the initial meson annihilate with one another or exchange a W boson. While these last processes are expected to be suppressed by a factor of (decay constant)/(heavy meson mass) in the amplitude relative to those to which factorization should apply, they have been found not to exhibit such suppression in charmed meson decays [8]. As a result, charmed particle decays exhibit an interesting pattern of final-state phases, in which there are large relative phases between the amplitudes for the various charge states in $D \to \bar{K}\pi$ and $D \to \bar{K}^*\pi$, but the amplitudes for $D \to \bar{K}\rho$ seem to be relatively real with respect to one another [9–16].

The decays $\bar{B} \to D\pi$, $D^*\pi$, $D\rho$ now have been studied with sufficient accuracy that a similar pattern appears to be emerging. The rates for the three charge states in the first two processes favor relative phases between contributing amplitudes (e.g., those of definite isospin) [19,20], while the $\bar{B} \to D\rho$ rates favor amplitudes which are relatively real [21]. In the present paper we perform an analysis parallel to that for charmed particles in Ref. [8], finding that the source of the final-state phases in the $\bar{B}$ decays under discussion is very similar to that in charm decays, but that the effects are

diminishing as expected with increasing heavy quark mass. We point out open theoretical and experimental problems and indicate what further data would be useful in resolving them.

Now that the color-suppressed decays $\bar{B}^0 \to D^0 \bar{K}^0$ and $\bar{B}^0 \to D^0 \bar{K}^{*0}$ have been observed [22], one can perform a similar analysis for $\bar{B} \to D\bar{K}$ decays. However, in contrast to a recent claim [23], we find that the experimental errors on these Cabibbo-suppressed decay modes are still too large to permit any firm conclusion about relative strong phases. We shall discuss the importance of such modes in reducing ambiguities in the Cabibbo-favored amplitudes.

We review the experimental situation for $\bar{B} \to D\pi$, $D^*\pi$, $D\rho$ in Sec. II, performing a standard isospin analysis and confirming that the isospin amplitudes have a nonzero relative phase for the first two processes but not for the third. We then introduce a description of the decays in terms of topological amplitudes in Sec. III. The implications of the data for these amplitudes are discussed in Sec. IV, while we discuss missing pieces of the puzzle and experimental prospects in Sec. V. We summarize in Sec. VI.

II. EVIDENCE FOR RELATIVE PHASES IN (SOME) $\bar{B}$ DECAYS

We review the isospin decomposition for the decays $\bar{B} \to D\pi$, following closely the corresponding discussion for $D \to \bar{K}\pi$ [13]. Similar decompositions then follow when one of the final-state particles is a vector meson, since in all cases there is a single partial wave in the decay.

The decays of interest are governed by the subprocess $b \to c d \bar{u}$, which has $\Delta I = 1$, $\Delta I_3 = -1$. Since the initial $\bar{B}$ state has $I = 1/2$, the processes are characterized by two amplitudes $A_{1/2}$ and $A_{3/2}$ labeled by the total isospin of the final $D\pi$ state. The amplitudes are given by

$$\mathcal{A}(D^0\pi^-) = A_{3/2},$$

$$\mathcal{A}(D^+\pi^-) = \frac{2}{3}A_{1/2} + \frac{1}{3}A_{3/2},$$

$$\mathcal{A}(D^0\pi^0) = -\frac{\sqrt{2}}{3}A_{1/2} + \frac{\sqrt{2}}{3}A_{3/2}, \quad (4)$$

where we omit the initial particle. They thus satisfy a triangle relation

$$\mathcal{A}(D^0\pi^-) = \mathcal{A}(D^+\pi^-) + \sqrt{2}\mathcal{A}(D^0\pi^0). \quad (5)$$

A nonzero area of the triangle would signify nontrivial final-state phases between the two isospin amplitudes.

Letting Φ_i denote kinematic factors which we shall specify shortly, where the subscript denotes the final state, we can define reduced partial widths with the kinematic factors removed, e.g.,

$$|A_{3/2}|^2 = \tilde{\Gamma}(D^0\pi^-) \equiv \Gamma(D^0\pi^-)/\Phi_{D^0\pi^-}, \quad (6)$$

$$|A_{1/2}|^2 = \frac{3}{2}[\tilde{\Gamma}(D^+\pi^-) + \tilde{\Gamma}(D^0\pi^0)] - \frac{1}{2}\tilde{\Gamma}(D^0\pi^-), \quad (7)$$

and the relative phase $\delta_I = \text{Arg}(A_{3/2}/A_{1/2})$ between isospin amplitudes satisfies

$$\cos\delta_I = \frac{3\tilde{\Gamma}(D^+\pi^-) + \tilde{\Gamma}(D^0\pi^-) - 6\tilde{\Gamma}(D^0\pi^0)}{4|A_{1/2}A_{3/2}|}. \quad (8)$$

The search for relative phases in $\bar{B} \to D\pi$, $D^*\pi$, and $D\rho$ decays goes back at least as far as the unpublished work of Yamamoto [24], in which only upper limits existed at the time for the color-suppressed decays $\bar{B}^0 \to D^0\pi^0$, $D^{*0}\pi^0$ and $D^0\rho^0$. Belle and CLEO reported observation of the first two final states about a year and a half ago. The rate for $\bar{B}^0 \to D^0\pi^0$ was found to be large enough that the triangle of complex amplitudes for $\bar{B}^0 \to D^0\pi^0$, $\bar{B}^0 \to D^+\pi^-$, and $B^- \to D^0\pi^-$ appeared to have nonzero area. More recently, CLEO reported an analysis of a larger data sample of the last two modes [19,20], which strengthens the argument for a nonzero final state phase difference between the $I = 1/2$ and $I = 3/2$ amplitudes. With the new branching ratios $\mathcal{B}(\bar{B}^0 \to D^+\pi^-) = (26.8 \pm 2.9) \times 10^{-4}$, $\mathcal{B}(B^- \to D^0\pi^-) = (49.7 \pm 3.8) \times 10^{-4}$ as well as the Belle-CLEO average $\mathcal{B}(\bar{B}^0 \to D^0\pi^0) = (2.92 \pm 0.45) \times 10^{-4}$, one finds $|A_{3/2}| = (7.70 \pm 0.29) \times 10^{-7}$ GeV, $|A_{1/2}| = (5.30 \pm 0.58) \times 10^{-7}$ GeV, and $\cos\delta_I = 0.86 \pm 0.05$ or $\cos\delta_I < 1$ at 2.8σ.

The same formulae (4)–(8) can be readily applied to both $D^*\pi$ and $D\rho$ decays by replacing the final state mesons with the appropriate ones. It is found that the $D^*\pi$ decays have $|A_{3/2}| = (3.32 \pm 0.14) \times 10^{-7}$, $|A_{1/2}| = (2.50 \pm 0.20) \times 10^{-7}$, and $\cos\delta_I = 0.86 \pm 0.06$ or $\cos\delta_I < 1$ at 2.4σ. Similar conclusions were drawn for isospin amplitudes in $\bar{B} \to D^{(*)}\pi$ in Ref. [25].

$D^*\pi$ decays thus have a phase structure very similar to that of the $D\pi$ decays. However, with the newly reported branching ratio $\mathcal{B}(\bar{B}^0 \to D^0\rho^0) = (2.9 \pm 1.0 \pm 0.4) \times 10^{-4}$ [21], the $D\rho$ decays give $|A_{3/2}| = (5.74 \pm 0.39) \times 10^{-7}$, $|A_{1/2}| = (4.00 \pm 0.76) \times 10^{-7}$, and $\cos\delta_I = 0.99 \pm 0.08$, consistent with a vanishing strong phase.

The decays $\bar{B} \to D^{(*)}\bar{K}^{(*)}$ are governed by the quark subprocess $b \to c\bar{u}s$, which has $\Delta I = -\Delta I_3 = 1/2$. Combining this interaction with the isospin $I = 1/2$ of the initial $\bar{B}$, one has two amplitudes A_0^{DK} and A_1^{DK} labeled by total isospin. For example, for $\bar{B} \to D\bar{K}$, using the phase convention of Ref. [23],

$$\mathcal{A}(\bar{B}^0 \to D^+K^-) = \frac{1}{2}A_1^{DK} + \frac{1}{2}A_0^{DK},$$

$$\mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0) = \frac{1}{2}A_1^{DK} - \frac{1}{2}A_0^{DK},$$

$$\mathcal{A}(B^- \to D^0K^-) = A_1^{DK}. \quad (9)$$

One thus has the sum rule

$$\mathcal{A}(\bar{B}^0 \to D^+ K^-) + \mathcal{A}(\bar{B}^0 \to D^0 \bar{K}^0) = \mathcal{A}(B^- \to D^0 K^-), \tag{10}$$

with similar sum rules when one final pseudoscalar is replaced by a vector meson. (When both final mesons have spin 1, there are three helicity amplitudes or partial waves; the sum rule holds for each. We shall not consider such decays further here.)

Xing [23] has argued that the observed amplitudes for $D\bar{K}$ and $D\bar{K}^*$ decays favor non-zero relative phases between the isospin amplitudes. We shall see that these amplitudes are consistent with being relatively real at better than 1σ, and will identify the improvements in measurements that are likely to be needed in order to establish a nonzero relative phase.

III. TOPOLOGICAL AMPLITUDES

Meson wave functions are assumed to have the following quark content, with phases chosen so that isospin multiplets contain no relative signs [17,18]:

b-flavored mesons: $\bar{B}^0 = b\bar{d}$, $B^- = -b\bar{u}$, $\bar{B}_s = b\bar{s}$.

charmed mesons: $D^0 = -c\bar{u}$, $D^+ = c\bar{d}$, $D_s^+ = c\bar{s}$, with corresponding phases for vector mesons.

pseudoscalar mesons P: $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, assuming a specific octet-singlet mixing [26,18] in the η and η' wave functions.

vector mesons V: $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $\bar{K}^{*0} = s\bar{d}$, $K^{*-} = -s\bar{u}$, $\phi = s\bar{s}$.

The partial width Γ for a specific two-body decay to PP is expressed in terms of an invariant amplitude $\mathcal{A}$ as

$$\Gamma(B \to PP) = \frac{p^*}{8\pi M^2} |\mathcal{A}|^2, \tag{11}$$

where p^* is the center-of-mass (c.m.) 3-momentum of each final particle, and M is the mass of the decaying particle. The kinematic factor of p^* is appropriate for the S-wave final state. The amplitude $\mathcal{A}$ will thus have dimensions of (energy)$^{-1}$.

For PV decays a P-wave kinematic factor is appropriate instead, and

$$\Gamma(B \to PV) = \frac{(p^*)^3}{8\pi M^2} |\mathcal{A}'|^2. \tag{12}$$

Here $\mathcal{A}'$ is dimensionless. These conventions agree with those of Chau *et al.* [26].

The amplitudes $\mathcal{A}$ are then expressed in terms of topological amplitudes of three types.

Tree amplitudes T: These are associated with the transition $b \to c\bar{d}u$ (favored) or $b \to c\bar{s}u$ (suppressed) in which the light (color-singlet) quark-antiquark pair is incorporated into one meson, while the charmed quark combines with the

spectator antiquark to form the other meson. We denote (favored, suppressed) amplitudes by (unprimed, primed) quantities, respectively.

Color-suppressed amplitudes C: The transition is the same as in the tree amplitudes, namely $b \to c\bar{d}u$ or $b \to c\bar{s}u$, while the charmed quark and the $\bar{u}$ combine into one meson and the light quark and the spectator antiquark combine into the other meson.

Exchange amplitudes E: The b and spectator antiquark exchange a W to become a $c\bar{u}$ pair, which then hadronizes through the creation of a light quark-antiquark pair.

We neglect a fourth type of (annihilation) transition in which a b and a $\bar{u}$ annihilate to form an $s\bar{c}$ or $d\bar{c}$ pair. Such transitions do not contribute in any case to $\bar{B} \to D + X$ decays.

For reference, the relation between isospin amplitudes and topological ones for Cabibbo-favored decays is

$$A_{3/2} = \mathcal{A}(D^0 \pi^-) = -(T + C),$$

$$A_{1/2} = \frac{3}{2}\mathcal{A}(D^+ \pi^-) - \frac{1}{2}\mathcal{A}(D^0 \pi^-)$$

$$= \frac{1}{2}C - T - \frac{3}{2}E, \tag{13}$$

with similar relations for $D^*\pi$ and $D\rho$ decays. The corresponding relation for Cabibbo-suppressed decays is

$$A_1^{DK} = \mathcal{A}(D^0 K^-) = -(T' + C'),$$

$$A_0^{DK} = \mathcal{A}(D^+ K^-) - \mathcal{A}(D^0 \bar{K}^0) = C' - T', \tag{14}$$

with similar relations for D^*K and DK^* decays.

IV. TOPOLOGICAL AMPLITUDES: MAGNITUDES AND PHASES

In Tables I–III we summarize the rates, invariant amplitudes, and their flavor-SU(3) representations for decays of $\bar{B}$ mesons to $D\pi$, $D^*\pi$, and $D\rho$, respectively. Also shown are decays to other final states related by flavor SU(3). Branching ratios and lifetimes are taken from the compilation of Ref. [27] except where indicated otherwise. In particular, we take $\tau(B^-) = (1.674 \pm 0.018) \times 10^{-12}$ s, $\tau(\bar{B}^0) = (1.542 \pm 0.016) \times 10^{-12}$ s. For the $D\pi$ decays we use the updated values quoted in Refs. [19,20]. The branching ratio for $\bar{B}^0 \to D_s^+ K^-$ is based on our average of new values from Belle [28]: $\mathcal{B} = (4.6^{+1.2}_{-1.1} \pm 1.3) \times 10^{-5}$ and BaBar [29]: $\mathcal{B} = (3.2 \pm 1.0 \pm 1.0) \times 10^{-5}$. The branching ratios and limits for $\bar{B}^0 \to D^{(*)0} \bar{K}^{(*)0}$ are based on a recent report by the Belle Collaboration [22].

In Table I the amplitudes T, C, and E were described above; in Tables II and III the amplitudes are labeled with subscripts which denote the meson containing the spectator quark: P for pseudoscalar, V for vector [30]. We omit contributions of disconnected diagrams [31,32] in which η and η'

TABLE I. Rates and invariant amplitudes for decays of $\bar{B}$ mesons to $D\pi$ and related modes. Primed amplitudes are related to unprimed amplitudes by a factor of $\lambda f_K/[f_\pi(1-\lambda^2/2)]=0.275$. Except where noted, the branching ratios are quoted from the Particle Data Group [27].

Decay	M (GeV)	Branching ratio (units of 10^{-4})	$p*$ (GeV)	$\|\mathcal{A}\|$ (10^{-7} GeV)	Representation
$B^-\to D^0\pi^-$	5.2790	49.7 ± 3.8 [a]	2.308	7.70 ± 0.29	$-(T+C)$
$\to D^0 K^-$		3.7 ± 0.6	2.281	2.11 ± 0.17	$-(T'+C')$
				7.7 ± 0.6 [b]	$\|T+C\|$
$\bar{B}^0\to D^+\pi^-$	5.2794	26.8 ± 2.9 [a]	2.306	5.89 ± 0.32	$-(T+E)$
$\to D^+K^-$		2.0 ± 0.6	2.279	1.62 ± 0.24	$-T'$
				5.9 ± 0.9 [b]	$\|T\|$
$\to D^0\pi^0$		2.92 ± 0.45 [a]	2.308	1.94 ± 0.15	$(E-C)/\sqrt{2}$
$\to D^0\eta$		$1.4^{+0.6}_{-0.5}$	2.274	1.36 ± 0.27	$(C+E)/\sqrt{3}$
$\to D^0\eta'$		<9.4	2.198	<3.6	$-(C+E)/\sqrt{6}$
$\to D^0\bar{K}^0$		$0.50^{+0.13}_{-0.12}\pm0.06$ [c]	2.280	0.81 ± 0.11	$-C'$
				2.94 ± 0.41 [b]	$\|C\|$
$\to D_s^+K^-$		0.38 ± 0.10 [d]	2.242	0.71 ± 0.10	$-E$

[a]References [19,20].
[b]Value implied by (broken) flavor SU(3).
[c]Reference [22].
[d]Average of [28,29].

exchange no quark lines with the rest of the diagram, and couple through their SU(3)-singlet components.

Tables I–III contain several tests of flavor SU(3). The breaking of this symmetry is incorporated via ratios of decay constants: $f_K/f_\pi=1.22$, $f_{K*}/f_\rho=1.04$. For example, the decay $B^-\to D^0K^-$ is related to $B^-\to D^0\pi^-$ by the U-spin substitution $d\to s$, and so one expects $\mathcal{A}(B^-\to D^0K^-)/\mathcal{A}(B^-\to D^0\pi^-)=(f_K/f_\pi)(\lambda/[1-\lambda^2/2])=0.275$, where $\lambda=0.22$ describes the hierarchy of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [33] and the small form factor difference is ignored throughout the paper.

When one corrects for this factor, the derived values of $|T+C|$ are equal within errors. Similar results hold for the ratio $\mathcal{A}(B^-\to D^{*0}K^-)/\mathcal{A}(B^-\to D^{*0}\pi^-)=(f_K/f_\pi)(\lambda/[1-\lambda^2/2])=0.275$ and $\mathcal{A}(B^-\to D^0K^{*-})/\mathcal{A}(B^-\to D^0\rho^-)=(f_{K*}/f_\rho)(\lambda/[1-\lambda^2/2])=0.235$.

The amplitudes for $\bar{B}^0\to D^+\pi^-$ and $\bar{B}^0\to D^+K^-$ (and similar modes with one final-state pseudoscalar replaced by a vector meson) would be related to one another by U-spin if one neglected the presence of the spectator quark. The spectator quark contributes an additional exchange amplitude, whose magnitude is seen to be small from the decay B^0

TABLE II. Rates and invariant amplitudes for decays of $\bar{B}$ mesons to $D^*\pi$ and related modes. Primed amplitudes are related to unprimed amplitudes by a factor of $\lambda f_K/[f_\pi(1-\lambda^2/2)]=0.275$. The branching ratios are quoted from the Particle Data Group [27].

Decay	M (GeV)	Branching ratio (units of 10^{-4})	$p*$ (GeV)	$\|\mathcal{A}\|$ (10^{-7})	Representation
$B^-\to D^{*0}\pi^-$	5.2790	46 ± 4	2.256	3.32 ± 0.14	$-(T_V+C_P)$
$\to D^{*0}K^-$		3.6 ± 1.0	2.227	0.95 ± 0.13	$-(T'_V+C'_P)$
				3.4 ± 0.5 [a]	$\|T_V+C_P\|$
$\bar{B}^0\to D^{*+}\pi^-$	5.2794	27.6 ± 2.1	2.255	2.68 ± 0.10	$-(T_V+E_P)$
$\to D^{*+}K^-$		2.0 ± 0.5	2.226	0.74 ± 0.09	$-T'_V$
				2.7 ± 0.3 [a]	$\|T_V\|$
$\to D^{*0}\pi^0$		2.5 ± 0.7	2.256	0.81 ± 0.11	$(E_P-C_P)/\sqrt{2}$
$\to D^{*0}\eta$		<2.6	2.220	<0.84	$(C_P+E_P)/\sqrt{3}$
$\to D^{*0}\eta'$		<14	2.141	<2.1	$-(C_P+E_P)/\sqrt{6}$
$\to D^{*0}\bar{K}^0$		<0.66 [b]	2.227	<0.42	$-C'_P$
				<1.54 [a]	$\|C_P\|$
$\to D_s^{*+}K^-$		<0.25	2.185	<0.27	$-E_P$

[a]Value implied by (broken) flavor SU(3).
[b]Reference [22].

TABLE III. Rates and invariant amplitudes for decays of $\bar{B}$ mesons to $D\rho$ and related modes. Primed amplitudes are related to unprimed amplitudes by a factor of $\lambda f_{K*}/[f_\rho(1-\lambda^2/2)]=0.235$. Except where noted, the branching ratios are quoted from the Particle Data Group [27].

Decay	M (GeV)	Branching ratio (units of 10^{-4})	p^* (GeV)	$\|\mathcal{A}\|$ (10^{-7})	Representation
$B^- \to D^0\rho^-$	5.2790	134 ± 18	2.238	5.74 ± 0.39	$-(T_P+C_V)$
$\to D^0 K^{*-}$		6.1 ± 2.3	2.213	1.25 ± 0.23	$-(T_P'+C_V')$
				5.3 ± 1.0 [a]	$\|T_P+C_V\|$
$\bar{B}^0 \to D^+\rho^-$	5.2794	78 ± 14	2.236	4.57 ± 0.41	$-(T_P+E_V)$
$\to D^+ K^{*-}$		3.7 ± 1.8	2.211	1.01 ± 0.25	$-T_P'$
				4.3 ± 1.0 [a]	$\|T_P\|$
$\to D^0\rho^0$		$2.9\pm1.0\pm0.4$ [b]	2.238	0.88 ± 0.16	$(E_V-C_V)/\sqrt{2}$
$\to D^0\omega$		1.8 ± 0.6	2.235	0.69 ± 0.12	$-(C_V+E_V)/\sqrt{2}$
$\to D^0\bar{K}^{*0}$		$0.48^{+0.11}_{-0.10}\pm0.05$ [c]	2.212	0.36 ± 0.04	$-C_V'$
				1.55 ± 0.19 [a]	$\|C_V\|$
$\to D_s^+ K^{*-}$		<9.9	2.172	<1.7	$-E_V$

[a]Value implied by (broken) flavor SU(3).
[b]Reference [21].
[c]Reference [22].

$\to D_s^+ K^-$ in Table I and the upper limit on $B^0 \to D_s^{*+} K^-$ in Table II. Thus, for example, one cannot tell the difference between $|T+E|$ extracted from $\bar{B}^0 \to D^+\pi^-$ and $|T|$ extracted from $\bar{B}^0 \to D^+ K^-$. A similar conclusion applies to $|T_V+E_P|$ versus $|T_V|$ in Table II and $|T_P+E_V|$ versus $|T_P|$ in Table III.

We now discuss the amplitude triangles for each set of processes. It is interesting to determine the individual magnitudes and phases of the contributing topological amplitudes T, C, and E, as was done for charmed particle decays [8]. This can be important for understanding the systematics of B decays involving two amplitudes with different weak *and* strong phases, for which direct *CP* violation can be observed. In the present case, of course, it is only the strong phases which may differ from one another. The decays $\bar{B} \to D\pi$ and those related to it by flavor SU(3) permit one to map out the amplitudes (up to a discrete ambiguity), while those involving one vector meson in the final state are missing key information which one hopes will be provided by BaBar, Belle, or hadron colliders.

A. $\bar{B} \to D\pi$ and related decays

The amplitudes $A(B^- \to D^0\pi^-) = -(T+C)$, $A(\bar{B}^0 \to D^+\pi^-) = -(T+E)$, and $\sqrt{2}A(\bar{B}^0 \to D^0\pi^0) = E-C$ form a triangle in the complex plane, as shown in Fig. 1. Here we have arbitrarily taken $T+C$ to be real and positive. The favored area of the triangle is non-zero, as our earlier discussion of isospin amplitudes also implies.

The decay $\bar{B}^0 \to D_s^+ K^-$ [29] provides a value of $|E|$, whose central value is used to draw a circle of radius $|E|$ around the intersection of the two sides $T+E$ and $C-E$. The decay $\bar{B}^0 \to D^0\eta$ provides a value of $|C+E|$. Using the relation $(C-E)/2+E=(C+E)/2$ for complex amplitudes, we draw a circle of radius $|C+E|/2$ about the midpoint of

the side $C-E$. The intersections O and O' of the two circles then denote the allowed phases of E. One can now identify the amplitudes T and C corresponding to each of these solutions.

In principle the value of $|T|$, provided through broken flavor SU(3) by the decay $\bar{B}^0 \to D^+ K^-$, could help to resolve the discrete ambiguity. In the solution shown in Fig. 1, we have $|T| \simeq 5.6$ (here and in the following analysis, we express topological amplitudes in units of 10^{-7} GeV for PP modes and 10^{-7} for PV modes), while in the solution in which T points to O', one has $|T| \simeq 6.6$. The error on $|T|=5.9\pm0.9$ from $\bar{B}^0 \to D^+ K^-$ is at least a factor of three too large to permit any distinction between the two solutions.

Similarly, the value $|C|=2.94\pm0.41$, obtained via flavor SU(3) from the recently reported decay [22] $\bar{B}^0 \to D^0\bar{K}^0$, can be compared with that implied by Fig. 1, which is the same for the two discrete solutions. The measurements in Table I provide $|C\pm E|$ and $|E|$. One can then solve to find $|C| = 2.46\pm0.25$, consistent with the above value.

The solution shown has some similarity to that for $D \to \bar{K}\pi$ [8]. In that solution, the amplitudes T, C, and E all had distinctly different phases. Denoting δ_{AB} as the angle of
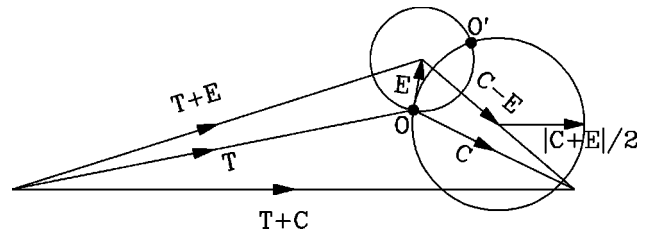

FIG. 1. Amplitude triangle for $\bar{B} \to D\pi$ and related decays. The amplitude E points from either O or O' to the center of the small circle. The amplitudes T and C are shown only for the first of these two solutions.
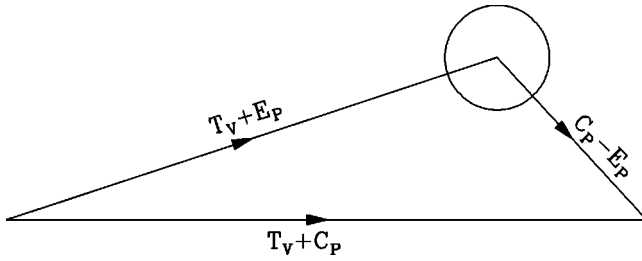
FIG. 2. Amplitude triangle for $\bar{B} \to D^* \pi$ and related decays. The amplitude E_P points from anywhere inside the small circle to the intersection of the lines $T_V + E_P$ and $T_V + C_P$.

rotation from the amplitude B to A, then we have as the central values $\delta_{CT} \simeq -38°$ and $\delta_{ET} \simeq 69°$. The other solution (not shown) has T and E relatively real, both with a large phase relative to C. Explicitly, we obtain $\delta_{CT} \simeq -73°$ and $\delta_{ET} \simeq 180°$. In comparison, the phases in the corresponding D decays are $\delta_{CT} \simeq -151°$ and $\delta_{ET} \simeq 115°$ [8]. The sign flip in Re(C/T) (negative for charm, positive for b-flavored) also occurs in a simplified analysis in which C/T is taken to be real and the effects of the amplitude E are not taken into account. In such a case the sign flip is merely a consequence of $\Gamma(D^+ \to \bar{K}^0 \pi^+) < \Gamma(D^0 \to K^- \pi^+)$, $\Gamma(B^- \to D^0 \pi^-) > \Gamma(\bar{B}^0 \to D^+ \pi^-)$.

It is interesting to compare the ratios $|C/T|$ and $|E/T|$ with those found for $D \to \bar{K}\pi$, where our favored solution [8] had $|C/T| \simeq 0.8$ and $|E/T| \simeq 0.6$. Here, the solution shown in Fig. 1 has $|C/T| \simeq 0.4$ and $|E/T| \simeq 0.1$. The amplitude E is indeed vanishing faster than the others as the heavy quark mass m_Q increases, in accord with expectations for heavy-quark systems, but as some power between m_Q^{-1} and m_Q^{-2}.

We comment briefly on the claim by Xing [23] that the pattern of branching ratios in $\bar{B} \to D\bar{K}$ decays implies nonzero final-state phases between isospin amplitudes. This is certainly true for central values. However, the sum rule (10) when written for amplitudes which are relatively real reads

$$(2.43 \pm 0.27) \times 10^{-7} \text{ GeV} = (2.11 \pm 0.17) \times 10^{-7} \text{ GeV} \tag{15}$$

using the amplitudes in Table I. This is satisfied at the 1σ level. The errors are dominated by those for the color-favored processes; reduction by a factor of 2 would help considerably.

B. $\bar{B} \to D^* \pi$ and related decays

The triangle formed by the amplitudes $A(B^- \to D^{*0} \pi^-) = -(T_V + C_P)$, $A(\bar{B}^0 \to D^{*+} \pi^-) = -(T_V + E_P)$, and $\sqrt{2} A(\bar{B}^0 \to D^{*0} \pi^0) = E_P - C_P$ is shown in Fig. 2. We have taken $T_V + C_P$ to be real and positive. Nonzero area again is favored.

Here the situation is much less satisfactory than for $\bar{B} \to D\pi$. We have only an upper bound on $|E_P|$ based on the non-observation of $\bar{B}^0 \to D_s^{*+} K^-$. The decay $\bar{B}^0 \to D^{*+} K^-$ tells us that $|T_V| = 2.7 \pm 0.3$, whereas on the basis of $|E_P| \lesssim 0.27$ and $|T_V + E_P| = 2.68 \pm 0.10$ (Table II) we could have
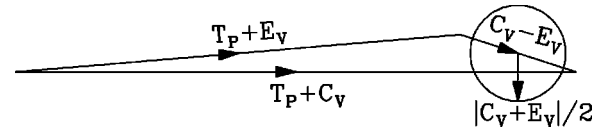
any value of $|T_V|$ between 2.2 and 3.1. We also have no information on $|C_P + E_P|$, only a poor upper bound from $\bar{B}^0 \to D^{*0} \eta$ and a much worse one from $\bar{B}^0 \to D^{*0} \eta'$. If the pattern in Fig. 2 is anything like that for the corresponding charm decays $D \to \bar{K}^* \pi$, it should resemble that for the solution displayed in Fig. 1. Improved information on $|C_P|$, obtainable via flavor SU(3) from the decay $\bar{B}^0 \to D^{*0} \bar{K}^0$ for which only an upper limit is quoted [22], would also be helpful. This process also would be useful in implementing the suggestion of Xing [23] to search for relative final-state phases between isospin amplitudes in $\bar{B} \to D^* \bar{K}$.

C. $\bar{B} \to D\rho$ and related decays

FIG. 3. Amplitude triangle for $\bar{B} \to D\rho$ and related decays.

The triangle formed by the amplitudes $A(B^- \to D^0 \rho^-) = -(T_P + C_V)$, $A(\bar{B}^0 \to D^+ \rho^-) = -(T_P + E_V)$, and $\sqrt{2} A(\bar{B}^0 \to D^0 \rho^0) = E_V - C_V$, shown in Fig. 3, has a much smaller area than either of the previous two, and is consistent with the same phase for each of the three amplitudes. Here we have taken $T_P + C_V$ to be real and positive.

Additional information is available from the decay $\bar{B}^0 \to D^0 \omega$, which provides a value of $|C_V + E_V|$. As in Fig. 1, we draw a circle of radius $|C_V + E_V|/2$ from the midpoint of the line $C_V - E_V$. The solution points O and O' would correspond to the intersection of this circle with one of radius $|E_V|$ whose center is the intersection of the lines $T_P + E_V$ and $C_V - E_V$. We would need an improved upper bound on $\bar{B}^0 \to D_s^+ K^{*-}$ in order to draw a useful version of this last circle.

As in the two previous cases, an estimate of the tree amplitude ($|T_P|$ in this case) would also be helpful. From the decay $\bar{B}^0 \to D^0 K^{*-}$ we find $|T_P| = 4.3 \pm 1.0$, to be compared with $|T_P + E_V| = 4.57 \pm 0.41$. Obviously no conclusion can be drawn at present about the relative phase of T_P and E_V.

Flavor SU(3) can be applied to the decay $\bar{B}^0 \to D^0 \bar{K}^{*0}$, yielding the magnitude $|C_V| = 1.55 \pm 0.19$ quoted in Table III. An independent upper limit on this quantity can be obtained by combining information on $|C_V + E_V|$ from $\bar{B}^0 \to D^0 \rho^0$ and $|C_V - E_V|$ from $\bar{B}^0 \to D^0 \omega$ to obtain $(|C_V|^2 + |E_V|^2)^{1/2} = 1.12 \pm 0.15$. There is clearly not very much room for $|E_V|$ at the upper limit quoted in Table III, and even the hint ($<2\sigma$) of an inconsistency. The most incisive test would probably be direct observation of the decay $\bar{B}^0 \to D_s^+ K^{*-}$, providing a value of $|E_V|$.

As in the case of $\bar{B} \to D\bar{K}$, the decays $\bar{B} \to D\bar{K}^*$ in principle provide information on relative strong final-state phases [23]. Here the sum rule (10), if written for relatively real amplitudes, would read

$$(1.38 \pm 0.25) \times 10^{-7} = (1.25 \pm 0.23) \times 10^{-7} \tag{16}$$

using the amplitudes in Table III. This is satisfied at considerably better than 1σ. Again, the error is dominated by that in the color-favored processes.

If the pattern of $\bar{B} \to D\rho$ decays is similar to that for the corresponding $D \to \bar{K}\rho$ decays, the triangle in Fig. 3 would assume its squashed shape as a result of a negative largely imaginary contribution of E_V, so that the relative phase of T_P and C_V would be nontrivial. In the case of $\bar{B} \to D^*\pi$ decays (Fig. 2), by contrast, the contribution of E_P would be largely imaginary and positive, leading to a triangle with greater area as in the case of $\bar{B} \to D\pi$ (Fig. 1). This pattern is what occurs in charm decays [8]. We now discuss what measurements might determine if a similar picture applies to $\bar{B}$ decays.

V. MISSING PIECES OF THE PUZZLE

We began by asking the question of whether the pattern of T, C, and E amplitudes in the decays $\bar{B} \to D\pi$, $D^*\pi$, and $D\rho$ bore any relation to that in $D \to \bar{K}\pi$, $\bar{K}^*\pi$, and $\bar{K}\rho$. We see that there is some resemblance of the two cases in that the decay amplitudes for the first two processes appear to have nontrivial relative phases which could well be absent for each of the third processes. However, we are frustrated in our quest for the topological amplitudes by the fragmentary nature of the data. We believe this situation could well improve in the near future.

One key element responsible for the pattern in charm decays is the flip of the sign of the exchange amplitude when one interchanges which particle in the final state is a pseudoscalar and which is a vector. Thus, for charm, we found that the relation $E_V = -E_P$, which could be justified if the exchange amplitude really proceeded through a quark-antiquark state [34], was responsible for the very different pattern of amplitudes in $D \to \bar{K}^*\pi$ and $D \to \bar{K}\rho$. We do not yet have enough information to draw such a conclusion for $\bar{B} \to D^*\pi$ and $\bar{B} \to D\rho$ decays.

In Table IV we summarize some useful decays that would help to sort out the question of the topological amplitudes. We quote in each case the present error and a desirable error in $\mathcal{B}$, in units of 10^{-5}. We see that improvements by factors of three in branching ratios or roughly a tenfold increase in the data sample would permit a fairly clear pattern to emerge. There may be some shortcuts to this procedure which would be less demanding in data. We now briefly justify each of the entries in Table IV.

It would be helpful to have the error on $|T|$ from the decay $\bar{B}^0 \to D^+K^-$ small enough that one could tell the difference between $|T+E|$ and $|T|$ at least for the case of maximal constructive or destructive $T-E$ interference. Thus, we ask for the error on $|T|$ to be less than 1/3 the value of $|E|$, or $\Delta|T| \le 0.2$. (Our convention for units was mentioned in Sec. IV A.) This is a factor of 4.5 increase in present accuracy, both in $\mathcal{A}$ and in branching ratio. We also require $\Delta|T+C| \le 0.2$ as obtained from $B^- \to D^0\pi^-$ and, therefore, the error on the branching ratio should be reduced by roughly a factor of 1.5. We ask for similar errors in $|T+C|$

TABLE IV. $\bar{B}$ decays which would provide useful information on topological amplitudes for $\bar{B} \to D\pi$, $D^*\pi$, and $D\rho$.

Amplitude	Decay	Present error in $\mathcal{B} \times 10^5$	Desirable error in $\mathcal{B} \times 10^5$
$|T|$	$\bar{B}^0 \to D^+K^-$	6	1.3
$|T+C|$	$B^- \to D^0\pi^-$	38	26
$|T+C|$	$B^- \to D^0K^-$	6	2
$|C|$	$\bar{B}^0 \to D^0\bar{K}^0$	1.4	0.7
$|C+E|$	$\bar{B}^0 \to D^0\eta$	6	2
$|T_V|$	$\bar{B}^0 \to D^{*+}K^-$	5	1.5
$|C_P|$	$\bar{B}^0 \to D^{*0}\bar{K}^0$	<6.6 [a]	0.7
$|C_P+E_P|$	$\bar{B}^0 \to D^{*0}\eta$	<26 [a]	2
$|E_P|$	$\bar{B}^0 \to D_s^{*+}K^-$	<2.5 [a]	1
$|T_P|$	$\bar{B}^0 \to D^+K^{*-}$	18	2
$|T_P+C_V|$	$B^- \to D^0\rho^-$	180	45
$|T_P+C_V|$	$B^- \to D^0K^{*-}$	23	2
$|E_V|$	$\bar{B}^0 \to D_s^+K^{*-}$	<99 [a]	1

[a]Present 90% C.L. upper limit on branching ratio.

and $|C|$ as obtained via flavor SU(3) from $B^- \to D^0K^-$ and $\bar{B}^0 \to D^0\bar{K}^0$, respectively, and in $|C+E|$ obtained from $\bar{B}^0 \to D^0\eta$. Demanding that $\Delta|T+C| = \Delta|C| = \Delta|C+E| \le 0.2$ we find that the corresponding branching ratios should be specified to an error of $\pm(2, 0.7, 2) \times 10^{-5}$. The proposed errors on $|T|$, $|T+C|$, and $|C|$ also would allow one to draw a useful conclusion about the relative strong phases of isospin amplitudes in $\bar{B} \to D\bar{K}$.

If $\bar{B}^0 \to D^{*+}K^-$ is to provide a useful value of $|T_V|$ to compare with $|T_V+E_P|$ from $\bar{B}^0 \to D^+\pi^-$, we need $\Delta|T_V|$ to be no more than 1/3 of $|E_P|$, or at most 0.1. This, again, represents a threefold improvement in the error on the branching ratio. The color-suppressed decay $\bar{B}^0 \to D^{*0}\bar{K}^0$ would provide a useful value of $|C_P|$ via flavor SU(3) if measured with an error similar to that desired for $\bar{B}^0 \to D^0\bar{K}^0$, or $\Delta\mathcal{B} = 0.7 \times 10^{-5}$.

The decay $\bar{B}^0 \to D^{*0}\eta$ is very poorly measured, corresponding only to a rather weak upper bound. An error on its branching ratio comparable to that for $\bar{B}^0 \to D^0\eta$ would be highly desirable.

Evidence for the exchange amplitude E_P at something approaching the 3σ level would be useful for the present program. Thus, if the branching ratio for $\bar{B}^0 \to D_s^{*+}K^-$ is close to its present upper limit of 2.5×10^{-5}, an error of no more than 1×10^{-5} would be desirable.

The likelihood that $E_V = -E_P$ sets the scale of useful errors in decays related to $\bar{B} \to D\rho$. One would like errors in $|T_P|$ (from $\bar{B}^0 \to D^+K^{*-}$) and $|E_V|$ (from $\bar{B}^0 \to D_s^+K^{*-}$) no larger than 0.1, leading to the rather stringent demands in Table IV. A similar error requirement on $|T_P+C_V|$ as obtained directly from $B^- \to D^0\rho^-$ means only a fourfold improvement in the branching ratio. It may be worth searching for alternative strategies to constrain these amplitudes. The

proposed error on $B^- \to D^0 K^{*-}$, when combined with other proposed measurements, will be more than sufficient to check the relative phases of isospin amplitudes in $\bar{B} \to D\bar{K}^*$.

Effects of the exchange amplitudes E, E_P, and E_V (indeed, also of C, C_P, and C_V) may be generated by rescattering effects [8,25,35,36], as has been emphasized in reports of the decay $\bar{B}^0 \to D_s^+ K^-$ [28,29]. In this case we may not be able to justify the assumption [34] $E_V = -E_P$. However, this relation does appear consistent with charm decays [8], and for the moment with b-flavored decays as well.

VI. SUMMARY AND DISCUSSION

We have compared the decays $\bar{B} \to D\pi$, $D^*\pi$, and $D\rho$ with the corresponding charmed particle decays $D \to \bar{K}\pi, \bar{K}^*\pi$, and $\bar{K}\rho$. In the first two of each set, there appear to be nontrivial final-state phases between the decay amplitudes, while in the third case in each set, the decay amplitudes appear to be relatively real.

In the case of charm decays, we traced the apparent relative reality of $D \to \bar{K}\rho$ amplitudes to an accidental cancellation of nontrivial final-state phases among the tree (T), color-suppressed (C), and exchange (E) amplitudes. Our analysis of $\bar{B}$ decays suggests that while the exchange amplitude is diminishing in importance, with $|E/T| \simeq 0.1$ for $\bar{B} \to D\pi$ as compared with about 0.6 for $D \to \bar{K}\pi$, it still can play a significant role in contributing to the observed final-state phases, at least for $\bar{B} \to D\pi$. We have identified a number of measurements which could determine whether the apparently different shapes of the amplitude triangles for $\bar{B} \to D^*\pi$ (Fig. 2) and $\bar{B} \to D\rho$ (Fig. 3) are due to a simple sign flip of the exchange amplitude, as occurs for charm. We have also indicated improvements in accuracy likely to be needed to identify nonzero final-state phases between amplitudes in $\bar{B} \to D\bar{K}$ and related decays.

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