

Lepton helicity distributions in the polarized Drell-Yan process

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The lepton helicity distributions in the polarized Drell-Yan process at BNL RHIC energy are investigated. For events with a relatively low invariant mass of a lepton pair in which the weak interaction is negligible, only a measurement of lepton helicity can prove the antisymmetric part of the hadronic tensor. Therefore it might be interesting to consider the helicity distributions of leptons to obtain more information on the structure of the nucleon from the polarized Drell-Yan process. We estimate the QCD corrections at the $\mathcal{O}(\alpha_s)$ level to the hadronic tensor including both intermediate γ and Z bosons. We present numerical analyses for different invariant masses and show that the u (\bar{u}) and d (\bar{d}) quarks give different and characteristic contributions to the lepton helicity distributions. We also estimate the lepton helicity asymmetry for the various proton spin configurations.

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I. INTRODUCTION

The measurement of the polarized nucleon structure function $g_1(x, Q^2)$ by the European Muon Collaboration [1] in the late 1980s has opened the door to hadron spin physics. In the last 15 years, great progress has been made both theoretically and experimentally that has considerably improved our knowledge of the spin structure of the nucleon. Through these developments, hadron spin physics has grown up as one of the most active fields attracting considerable attention. Now our interest has spread out to various processes to explore the spin structure of hadrons. In particular, in conjunction with new kinds of experiments, the BNL Relativistic Heavy-Ion Collider (RHIC) spin project, etc., we are now in a position to obtain more information on the structure of hadrons and the dynamics of QCD. The spin dependent quantity is, in general, very sensitive to the structure of interactions among various particles. Therefore, we will be able to study the detailed structure of hadrons based on QCD. We also hope that we can find some clue to new physics beyond the standard model through the new experimental data.

It is now expected that polarized proton-proton collisions (RHIC-spin) at the BNL Relativistic Heavy-Ion Collider [2] will provide sufficient experimental data to unveil the structure of nucleon. Therefore it is important and interesting to investigate various processes that might be measured in RHIC experiments. One of those is the polarized Drell-Yan process [3].

The polarized Drell-Yan process has been studied by many authors for both longitudinally [4–7] and transversely [8–12] polarized cases. The W/Z production from polarized hadrons is also investigated [13–15]. In this article, we discuss the lepton helicity distributions from the polarized Drell-Yan process at the QCD one-loop level. The lepton helicity distributions carry more information on the nucleon structure [16–19] than “inclusive” Drell-Yan observables

such as the invariant mass distribution of lepton pair. Now let us consider, for simplicity, the virtual γ mediated Drell-Yan process. The subprocess cross section $d\hat{\sigma}$ is written in terms of the hadronic and leptonic tensors as

$$d\hat{\sigma} \propto (W_{\mu\nu}^S + W_{\mu\nu}^A)(L^{S\mu\nu} + L^{A\mu\nu}) = W_{\mu\nu}^S L^{S\mu\nu} + W_{\mu\nu}^A L^{A\mu\nu}.$$

The antisymmetric part of the hadronic tensor $W_{\mu\nu}$ contains spin information on the annihilating partons. However, for observables obtained after summing over the helicities of the lepton or integrating out the lepton distributions, this antisymmetric part drops out. Furthermore, the chiral structure of QED and QCD interactions tells us that only particular helicity states are selected for $q-\bar{q}$ annihilation. This observation shows that the polarized and unpolarized Drell-Yan processes are governed by essentially the same dynamics. On the other hand, if we measure the lepton helicity distributions, we can reveal the whole structure of the hadronic tensor. We will show that the u (\bar{u}) and d (\bar{d}) quarks give characteristic contributions to the lepton helicity distributions.

The article is organized as follows. In Sec. II, we reproduce the tree level cross section to define our conventions. We present our calculations for the helicity distributions of lepton at the QCD one-loop level in Sec. III. We adopt the massive gluon scheme to regularize the infrared and mass singularities, avoiding the complexity in the treatment of γ_5 . The scheme dependence will be discussed in Sec. IV and we change the scheme to the modified minimal subtraction (\overline{MS}) scheme to perform the numerical studies. We give the numerical results in Sec. V. Finally, Sec. VI contains the conclusions. The explicit forms for the subprocess cross sections are listed in Appendix A. The invariant mass distribution of lepton pairs in the massive gluon scheme is given in Appendix B, which is used to identify the scheme changing factor.

II. POLARIZED DRELL-YAN PROCESS AT TREE LEVEL

In this section, we reproduce the tree level result for the helicity distributions of leptons in the polarized Drell-Yan process to establish our notation. For the longitudinally polarized Drell-Yan process

$$N_A(P_A, \lambda_A) + N_B(P_B, \lambda_B) \rightarrow l(l, \lambda) + \bar{l}(l', \lambda') + X,$$

with λ ($= \pm$) being the helicity of each particle, we introduce the parton distributions $f_a^A(x), \Delta f_a^A$ by

$$f_a^A(x) = f_a^A(x, +) + f_a^A(x, -),$$

$$\Delta f_a^A(x) = f_a^A(x, +) - f_a^A(x, -),$$

where $f_a^A(x, +/-)$ denotes the distribution of parton type a with positive/negative helicity in nucleon A with positive helicity. Based on the factorization theorem, the hadronic cross section for the helicity distribution of the lepton is given as the convolution of the parton distributions with the hard subprocess cross section $d\hat{\sigma}^{ab}$ as

$$d\sigma(\lambda_A, \lambda_B; \lambda) = \sum_{a,b} \int_{\tau}^1 dx_1 \int_{\tau/x_1}^1 dx_2 \sum_{\lambda_a, \lambda_b} \frac{f_a^A(x_1) + \lambda_a \lambda_A \Delta f_a^A(x_1)}{2} \frac{f_b^B(x_2) + \lambda_b \lambda_B \Delta f_b^B(x_2)}{2} d\hat{\sigma}^{ab}(\lambda_a, \lambda_b; \lambda), \quad (1)$$

where

$$\tau = \frac{Q^2}{S} \equiv \frac{(l+l')^2}{(P_A + P_B)^2}.$$

The helicity dependent cross section $d\hat{\sigma}^{ab}(\lambda_a, \lambda_b; \lambda)$ for the subprocess

$$a(p_a, \lambda_a) + b(p_b, \lambda_b) \rightarrow l(l, \lambda) + \bar{l}(l', \lambda') + X$$

is a function of the partonic invariant variables

$$s = (p_a + p_b)^2 = (x_1 P_A + x_2 P_B)^2 = x_1 x_2 S,$$

$$t = (p_a - l)^2, \quad u = (p_b - l)^2,$$

and

$$Q^2 = q^2 = (l+l')^2 \equiv z s.$$

The tree level polarized cross section for lepton pair production,

$$q(p_q, \lambda_q) + \bar{q}(p_{\bar{q}}, \lambda_{\bar{q}}) \rightarrow l(l, \lambda) + \bar{l}(l', \lambda'),$$

is given by

$$d\hat{\sigma}^T = \frac{1}{2N_c^2 S} |M^T|^2 d\Phi_2,$$

where $d\Phi_2$ is the two-particle phase space and N_c ($=3$) is the color factor. The square of the tree amplitude reads

$$|M^T|^2 = \delta_{\lambda_q, -\lambda_{\bar{q}}} \delta_{\lambda, -\lambda'} N_c \left(\frac{4\pi\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 L^{\mu\nu} W_{\mu\nu}^T, \quad (2)$$

where the tree level hadronic and leptonic tensors are defined as

$$W_{\mu\nu}^T = \text{Tr}(\omega_{\lambda_q p_{\bar{q}}} \gamma_{\mu} p_q \gamma_{\nu}), \quad L_{\mu\nu} = \text{Tr}(\omega_{\lambda} l \gamma_{\mu} l' \gamma_{\nu}), \quad (3)$$

and

$$L^{\mu\nu} W_{\mu\nu}^T = 2[t^2 + u^2 + \lambda_q \lambda (u^2 - t^2)].$$

In Eq. (3) and also below, all momenta p under the Tr operator are understood to be \not{p} and $\omega_{\lambda} \equiv (1 + \lambda \gamma_5)/2$. In Eq. (2), α is the QED fine structure constant and the quantity $f^{\lambda_q \lambda}$ depends on the fermion couplings to the photon and Z boson in the following way:

$$f^{\lambda_q \lambda} = -e_q + Q_q^{\lambda_q} Q_l^{\lambda} \frac{Q^2}{Q^2 - M_Z^2 + iM_Z \Gamma_Z}, \quad (4)$$

where e_q is the electric charge of the quark in units of the electron charge e , M_Z is the Z mass, and Γ_Z is the Z width. The helicity labels λ_q, λ of the quark and lepton take \pm . The quark and lepton couplings to the Z boson are

$$Q_{q,l}^- = \frac{1}{\sin \theta_W \cos \theta_W} (T_{q,l}^3 - e_{q,l} \sin^2 \theta_W),$$

$$Q_{q,l}^+ = -e_{q,l} \frac{\sin \theta_W}{\cos \theta_W},$$

where $e_l = -1$, T^3 is the third component of the isospin, and θ_W is the Weinberg angle. In the center of mass (c.m.) frame of annihilating quarks, the cross section becomes

$$\frac{d\hat{\sigma}^T(\lambda_q, \lambda_{\bar{q}}; \lambda)}{dQ^2 d \cos \theta} = \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{\pi}{2N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 \times (1 + \cos^2 \theta + 2\lambda_q \lambda \cos \theta) \delta(1 - z), \quad (5)$$

where θ is the scattering angle of the lepton produced.

In Eq. (5), the third term that depends on the helicities of the quark and lepton comes from the antisymmetric part of the hadronic tensor $W_{\mu\nu}^T$. For the observable after taking the spin sum of the lepton, this antisymmetric part appears in the cross section only through the parity violating Z interaction in Eq. (4). Therefore, for the events with ‘‘small’’ values of Q^2 , the information from the antisymmetric part will be

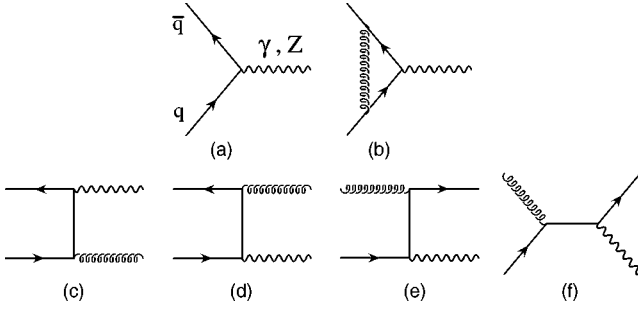


FIG. 1. Parton-level subprocesses contributing to the Drell-Yan process at $\mathcal{O}(\alpha_s)$: (a,b) virtual correction, (c,d) real gluon emission (e,f) quark-gluon Compton.

completely lost. Furthermore, the chiral structure of the standard model interaction forces only particular helicity states to participate in the process as shown in Eq. (5). These observations tell us that for the spin summed over final states and low Q^2 events, the polarized and unpolarized Drell-Yan processes are governed by essentially the same dynamics at least for the $q\bar{q}$ initiated process.

III. QCD ONE-LOOP CALCULATION

The principal result of this section will be the polarized Drell-Yan cross section at the QCD one-loop level with the helicity of the lepton being fixed. At the QCD one-loop level, infrared and mass singularities appear and we regularize them by giving a nonzero mass κ to the gluon [10,20] to avoid the complexities from the treatment of γ_5 and phase space integrals. To perform the numerical analyses by using the known $\overline{\text{MS}}$ parametrization for the parton densities, we have to change the scheme. However, it is well known how to do it [21]. The diagrams to be calculated are given in Fig. 1.

The virtual gluon correction [Figs. 1(a) and 1(b)] to this process yields

$$\frac{d\hat{\sigma}^V}{dQ^2 d\cos\theta} = \frac{d\hat{\sigma}^T}{dQ^2 d\cos\theta} \left(\frac{\alpha_s}{\pi} C_F \right) \times \left[-\frac{1}{2} \ln^2 \frac{Q^2}{\kappa^2} + \frac{3}{2} \ln \frac{Q^2}{\kappa^2} - \frac{7}{4} + \frac{\pi^2}{6} \right], \quad (6)$$

where $\alpha_s = g_s^2/4\pi$ is the strong coupling constant and $C_F = (N_c^2 - 1)/2N_c$ for $\text{SU}(N_c)$ of color.

The amplitude for the real gluon emission [Figs. 1(c) and 1(d)],

$$q(p_q, \lambda_q) + \bar{q}(p_{\bar{q}}, \lambda_{\bar{q}}) \rightarrow l(l, \lambda) + \bar{l}(l', \lambda') + g(k),$$

is given by

$$M^R = \delta_{\lambda_q, -\lambda_{\bar{q}}} \delta_{\lambda, -\lambda'} \left(-\frac{e^2 g_s}{Q^2} \right) f^{\lambda_q \lambda} \bar{u}_{\lambda}(l) \gamma^\mu v_{\lambda'}(l') \bar{v}_{\lambda_{\bar{q}}}(p_{\bar{q}}) \times \left[\gamma_\mu \frac{1}{\not{p}_q - \not{k}} \gamma_\nu + \gamma_\nu \frac{1}{\not{p}_{\bar{q}} - \not{k}} \gamma_\mu \right] T^a u_{\lambda_q}(p_q) \epsilon_a^\nu(k),$$

where $\epsilon_a^\nu(k)$ is the polarization vector of the gluon and T^a is the color matrix. We have defined $q = l + l'$. The expression for the square of the amplitude given below has been summed over the spin and colors of unobserved particles:

$$|M^R|^2 = \delta_{\lambda_q, -\lambda_{\bar{q}}} N_c C_F \left(\frac{4\pi\alpha_s g_s}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 L^{\mu\nu} W_{\mu\nu}^R,$$

where

$$W_{\mu\nu}^R = \frac{2}{\hat{t}^2} [2(p_{\bar{q}} \cdot k) \text{Tr}(\omega_{\lambda_q} k \gamma_\mu p_q \gamma_\nu) - \kappa^2 \text{Tr}(\omega_{\lambda_q} p_{\bar{q}} \gamma_\mu p_q \gamma_\nu)] + \frac{2}{\hat{u}^2} [2(p_q \cdot k) \text{Tr}(\omega_{\lambda_q} p_{\bar{q}} \gamma_\mu k \gamma_\nu) - \kappa^2 \text{Tr}(\omega_{\lambda_q} p_{\bar{q}} \gamma_\mu p_q \gamma_\nu)] + \frac{4}{\hat{t}\hat{u}} [(2p_q \cdot p_{\bar{q}} - 2p_q \cdot k - 2p_{\bar{q}} \cdot k) W_{\mu\nu}^T + (p_{q\mu} + p_{\bar{q}\mu}) k^\alpha W_{\alpha\nu}^T + W_{\mu\alpha}^T k^\alpha (p_{q\nu} + p_{\bar{q}\nu})]. \quad (7)$$

The tree level hadronic and leptonic tensors $W_{\mu\nu}^T, L_{\mu\nu}$ were defined in the previous section and new invariant variables are introduced:

$$\hat{t} = (p_q - q)^2, \quad \hat{u} = (p_{\bar{q}} - q)^2.$$

In Eq. (7), we have dropped terms that do not contribute when $\kappa^2 \rightarrow 0$. It should be noted that the κ^2 terms in the first and second lines cannot be neglected since the phase space integral produces a $1/\kappa^2$ singularity [20].

The cross section is given by

$$d\hat{\sigma}^R = \frac{1}{2N_c^2 s} |M^R|^2 d\Phi_3, \quad (8)$$

where $d\Phi_3$ is the three-particle phase space. To integrate out the irrelevant variables in Eq. (8), we take the c.m. frame of $q\bar{q}$:

$$p_q = \frac{\sqrt{s}}{2} (1, 0, 0, 1), \quad p_{\bar{q}} = \frac{\sqrt{s}}{2} (1, 0, 0, -1).$$

Parametrizing the momenta of the lepton l and lepton pair q by

$$l^\mu = |\mathbf{l}|(1, \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta),$$

$$q^0 = \frac{s + Q^2 - \kappa^2}{2\sqrt{s}}, \quad |\mathbf{q}|^2 = q^{02} - Q^2,$$

$$q^\mu = (q^0, \mathbf{q} \sin \hat{\theta} \cos \hat{\varphi}, \mathbf{q} \sin \hat{\theta} \sin \hat{\varphi}, \mathbf{q} \cos \hat{\theta}),$$

$$|\mathbf{l}| = \frac{Q^2}{2[q^0 - |\mathbf{q}|(\sin \hat{\theta} \sin \theta \cos(\hat{\varphi} - \varphi) + \cos \hat{\theta} \cos \theta)]},$$

where

it is easy to write the phase space element $d\Phi_3$ as

$$d\Phi_3 = \frac{|\mathbf{q}| Q^2 dQ^2 d \cos \hat{\theta} d \hat{\varphi} d \cos \theta d \varphi}{32(2\pi)^5 \sqrt{s} [q^0 - |\mathbf{q}|(\sin \hat{\theta} \sin \theta \cos(\hat{\varphi} - \varphi) + \cos \hat{\theta} \cos \theta)]^2}.$$

After integrating over the angular variables [10] and dropping terms that vanish when $\kappa^2 \rightarrow 0$, we write the inclusive cross section for the polarized lepton in the following form:

$$\frac{d\hat{\phi}^{RT}(\lambda_q, \lambda_{\bar{q}}; \lambda)}{dQ^2 d \cos \theta} = \frac{d\hat{\phi}^{RT}(\lambda_q, \lambda_{\bar{q}}; \lambda; z)}{dQ^2 d \cos \theta} F^R(Q^2, z) + \frac{d\hat{\phi}^{RF}(\lambda_q, \lambda_{\bar{q}}; \lambda)}{dQ^2 d \cos \theta}, \quad (9)$$

where the first term contains the infrared and mass singularities as well as terms associated with them, and

$$\begin{aligned} \frac{d\hat{\phi}^{RT}(\lambda_q, \lambda_{\bar{q}}; \lambda; z)}{dQ^2 d \cos \theta} &= \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{\pi}{2N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 \cdot \frac{1}{2} \left[\frac{8z^2}{[1 + \cos \theta + z(1 - \cos \theta)]^4} \{(1 + \lambda_q \lambda)(1 + \cos \theta)^2 \right. \\ &\quad \left. + (1 - \lambda_q \lambda)(1 - \cos \theta)^2 z^2\} + \frac{8z^2}{[1 - \cos \theta + z(1 + \cos \theta)]^4} \{(1 + \lambda_q \lambda)(1 + \cos \theta)^2 z^2 \right. \\ &\quad \left. + (1 - \lambda_q \lambda)(1 - \cos \theta)^2\} \right] \end{aligned} \quad (10)$$

with

$$\begin{aligned} F^R(Q^2, z) &= \frac{\alpha_s}{\pi} C_F \left[\left(\frac{1}{2} \ln^2 \frac{Q^2}{\kappa^2} - \frac{\pi^2}{6} \right) \delta(1-z) \right. \\ &\quad \left. + \frac{1+z^2}{(1-z)_+} \ln \frac{Q^2}{\kappa^2} + 2(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ \right. \\ &\quad \left. - 2(1+z^2) \frac{\ln z}{1-z} - (1-z) \right]. \end{aligned}$$

The second term in Eq. (9) gives the $\mathcal{O}(\alpha_s)$ finite contribution whose explicit form is listed in Appendix A. It should be noted that the first (second) term in Eq. (10) is just the tree level cross section for the $q\bar{q}$ annihilation with momenta zp_q and $p_{\bar{q}}$ (p_q and $zp_{\bar{q}}$) times z which arises from the difference between the flux normalizations. These terms express the processes with collinear gluon emissions. The function F^R can be understood to be a probability to emit a collinear gluon with momentum $(1-z)p_q$ or $(1-z)p_{\bar{q}}$ and does not depend on the helicities of quarks because of the helicity conservation of QCD interaction.

By adding the tree level Eq. (5) and virtual Eq. (6) contributions to Eq. (9), the double logarithmic infrared singularities cancel out and the cross section from the $q\bar{q}$ initial states becomes

$$\begin{aligned} \frac{d\hat{\sigma}^{T+V+R}}{dQ^2 d \cos \theta} &= \frac{d\hat{\sigma}^{RT}}{dQ^2 d \cos \theta} \left[\left(1 - \frac{7}{4} \frac{\alpha_s}{\pi} C_F \right) \delta(1-z) \right. \\ &\quad \left. + \frac{\alpha_s}{\pi} \left(P_{qq}(z) \ln \frac{Q^2}{\kappa^2} + C_F \left\{ 2(1+z^2) \right. \right. \right. \\ &\quad \left. \left. \left. \times \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2(1+z^2) \frac{\ln z}{1-z} - (1-z) \right\} \right) \right] \\ &\quad \left. + \frac{d\hat{\sigma}^{RF}}{dQ^2 d \cos \theta}, \right] \quad (11) \end{aligned}$$

where

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right),$$

which is the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) one-loop splitting functions P_{qq} . To obtain the above result, we have used the fact that

$$\frac{d\hat{\sigma}^{RT}(\lambda_q, \lambda_{\bar{q}}; \lambda; z)}{dQ^2 d \cos \theta} \delta(1-z) = \frac{d\hat{\sigma}^T(\lambda_q, \lambda_{\bar{q}}; \lambda)}{dQ^2 d \cos \theta}.$$

Finally, the contribution from the quark-gluon Compton process [Figs. 1(e) and 1(f)],

$$q(p_q, \lambda_q) + g(k, h) \rightarrow l(l, \lambda) + \bar{l}(l', \lambda') + q(p'_q, \lambda'_q),$$

where h is the helicity of gluon, can be calculated in the same way as before. To avoid singularities in the physical region, the gluon mass is taken to be $-\kappa^2$ [20] and the spin projection for incoming gluons reads

$$\epsilon_\alpha(k, h) \epsilon_\beta^*(k, h) = \frac{1}{2} \left[-g_{\alpha\beta} + ih \epsilon_{\alpha\beta\gamma\delta} \frac{k^\gamma p_q^\delta}{k \cdot p_q} \right].$$

The amplitude is

$$M^C = \delta_{\lambda_q, \lambda'_q} \delta_{\lambda, -\lambda'} \left(-\frac{e^2 g_s}{Q^2} \right) f^{\lambda_q \lambda} \bar{u}_\lambda(l) \gamma^\mu v_{\lambda'}(l') \bar{u}_{\lambda'_q}(p'_q) \\ \times \left[\gamma_\mu \frac{1}{\not{p}_q + \not{k}} \gamma_\alpha + \gamma_\alpha \frac{1}{\not{p}_q - \not{k}} \gamma_\mu \right] T^a u_{\lambda_q}(p_q) \epsilon_a^\alpha(k, h),$$

and its square becomes

$$|M^C|^2 = N_c C_F \left(\frac{4\pi\alpha_s}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 L^{\mu\nu} W_{\mu\nu}^C,$$

where

$$W_{\mu\nu}^C = \frac{1}{s} (1 + \lambda_q h) \left(1 + \frac{\hat{u}}{\hat{t}} \right) \text{Tr}(\omega_{\lambda_q} p'_q \gamma_\mu k \gamma_\nu) + \left\{ \frac{2\hat{u}}{s\hat{t}} + \frac{1}{s} (1 + \lambda_q h) + \frac{1}{\hat{t}} (1 - \lambda_q h) \right\} \text{Tr}(\omega_{\lambda_q} p'_q \gamma_\mu p_q \gamma_\nu) \\ - \frac{1}{\hat{t}} (1 - \lambda_q h) \left(1 + \frac{\hat{u}}{s} \right) \text{Tr}(\omega_{\lambda_q} k \gamma_\mu p_q \gamma_\nu) + \frac{4}{\hat{t}} (1 + \lambda_q h) p'_{q\mu} p'_{q\nu} + \frac{4}{s} (1 - \lambda_q h) p_{q\mu} p_{q\nu} \\ + \frac{\kappa^2}{\hat{t}^2} \left(1 - \lambda_q h \frac{2Q^2 - s}{s} \right) \text{Tr}(\omega_{\lambda_q} (p'_q - k) \gamma_\mu p_q \gamma_\nu)$$

with

$$s = (p_q + k)^2, \quad \hat{t} = (p_q - q)^2, \quad \hat{u} = (k - q)^2.$$

From the cross section formula for this process,

$$d\hat{\sigma}^C = \frac{1}{2N_c(N_c^2 - 1)s} |M^C|^2 d\Phi_3,$$

we obtain the polarized lepton distribution and write it in the same form as Eq. (9):

$$\frac{d\hat{\sigma}^C(\lambda_q, h; \lambda)}{dQ^2 d \cos \theta} = \frac{d\hat{\sigma}^{CT}(\lambda_q; \lambda; z)}{dQ^2 d \cos \theta} F^C(Q^2, z, \lambda_q, h) \\ + \frac{d\hat{\sigma}^{CF}(\lambda_q, h; \lambda)}{dQ^2 d \cos \theta}. \quad (12)$$

The first term, in this case, is given by

$$\frac{d\hat{\sigma}^{CT}(\lambda_q; \lambda; z)}{dQ^2 d \cos \theta} = \frac{\pi}{2N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 \\ \times \frac{8z^2}{[1 - \cos \theta + z(1 + \cos \theta)]^4} \{ (1 + \lambda_q \lambda) \\ \times (1 + \cos \theta)^2 z^2 + (1 - \lambda_q \lambda)(1 - \cos \theta)^2 \}, \quad (13)$$

and

$$F^C(Q^2, z, \lambda_q, h) = \left(\frac{\alpha_s}{2\pi} \right) \left[P_{qg}^C(z; \lambda_q, h) \left(\ln \frac{Q^2}{\kappa^2} + \ln \frac{1-z}{z^2} \right) \right. \\ \left. - \frac{1}{4} [1 + \lambda_q h(1 - 2z)] \right].$$

The quantity P_{qg}^C is related to the unpolarized and polarized DGLAP splitting functions $P_{gq}(z)$ and $\Delta P_{qg}(z)$ in the following way:

$$P_{qg}^C(z; \lambda_q, h) = \frac{1}{2} [P_{qg}(z) - \lambda_q h \Delta P_{qg}(z)] \\ = \frac{(1 + \lambda_q h)(1 - z)^2 + (1 - \lambda_q h)z^2}{4}.$$

Equation (13) again expresses the tree level cross section for the $q\bar{q}$ annihilation with momenta p_q and zk where the \bar{q} is emitted collinearly from the initial gluon. In contrast to the $q\bar{q}$ annihilation case, the function F^C does depend on the helicities of the initial quark and gluon since the emitted antiquark should have the helicity $\lambda_{\bar{q}} = -\lambda_q$ to annihilate into electroweak gauge bosons. The remaining finite contribution $d\hat{\sigma}^{CF}$ can be found in Appendix A.

The cross section for the $\bar{q}g$ subprocess can be obtained by changing (λ_q, h) to $(-\lambda_q, -h)$ and $\mu \leftrightarrow \nu$ in $W_{\mu\nu}^C$ in the above formulas.

From these results, the unpolarized cross sections are easily obtained. As a check, we have reproduced the results of Ref. [20] by neglecting the Z boson contribution.

IV. FACTORIZATION AND SCHEME CHANGE

Before proceeding to numerical studies, we must factorize the mass singularities into the parton densities in the $\overline{\text{MS}}$ scheme since their two-loop evolution, which should be combined with the one-loop corrections for the hard part, is given in this scheme.

The renormalized parton densities at the factorization scale μ_F^2 which are relevant to this work are written as

$$q(x, \mu_F^2) = \int_x^1 \frac{dy}{y} \left(q(y) \left\{ \delta \left(1 - \frac{x}{y} \right) + \frac{\alpha_s}{2\pi} \left[P_{qq} \left(\frac{x}{y} \right) \ln \frac{\mu_F^2}{\kappa^2} + C_{qq} \left(\frac{x}{y} \right) \right] \right\} + g(y) \frac{\alpha_s}{2\pi} \left[P_{qg} \left(\frac{x}{y} \right) \ln \frac{\mu_F^2}{\kappa^2} + C_{qg} \left(\frac{x}{y} \right) \right] \right),$$

$$\Delta q(x, \mu_F^2) = \int_x^1 \frac{dy}{y} \left(\Delta q(y) \left\{ \delta \left(1 - \frac{x}{y} \right) + \frac{\alpha_s}{2\pi} \left[P_{qq} \left(\frac{x}{y} \right) \ln \frac{\mu_F^2}{\kappa^2} + C_{qq} \left(\frac{x}{y} \right) \right] \right\} + \Delta g(y) \frac{\alpha_s}{2\pi} \left[\Delta P_{qg} \left(\frac{x}{y} \right) \ln \frac{\mu_F^2}{\kappa^2} + \Delta C_{qg} \left(\frac{x}{y} \right) \right] \right),$$

where C and ΔC are finite functions depending on the scheme of factorization. Since the mass singularity is of universal nature and not affected by the lepton distributions, we can find the relation between different regularization schemes by comparing the results calculated in the corresponding schemes for the simpler process. We have calculated the invariant mass distribution of the lepton pair in our scheme and the results are shown in Appendix B. The same quantity has been calculated in the dimensional regulariza-

tion scheme in Refs. [5], [7]. Note that the result depends also on the prescriptions of γ_5 in this scheme. We change our massive gluon scheme to the one adopted in Ref. [7] which can be combined with the known two-loop evolution equations for the parton densities in Refs. [22], [23]. We find

$$C_{qq}^{\kappa^2 \rightarrow \overline{\text{MS}}}(z) = -C_F \left[(1+z^2) \frac{\ln z}{1-z} + 2(1-z) + \left(\frac{\pi^2}{3} - \frac{9}{4} \right) \delta(1-z) \right],$$

$$C_{qg}^{\kappa^2 \rightarrow \overline{\text{MS}}}(z) = - \left(\frac{(1-z)^2 + z^2}{2} \ln[z(1-z)] + 1 \right),$$

and

$$\Delta C_{qg}^{\kappa^2 \rightarrow \overline{\text{MS}}}(z) = - \left(\frac{2z-1}{2} \ln[z(1-z)] + \frac{1}{2} \right).$$

We must, therefore, factorize these finite terms together with the singular terms from the subprocess cross sections.

The final results for the hard part at $\mathcal{O}(\alpha_s)$ level in the $\overline{\text{MS}}$ factorization scheme read for $q\bar{g}$ annihilation from Eq. (11)

$$\frac{d\hat{\sigma}^{q\bar{q}}(\mu_F^2)}{dQ^2 d \cos \theta} = \frac{d\hat{\sigma}^{RT}}{dQ^2 d \cos \theta} \left(\left\{ 1 + \frac{\alpha_s}{\pi} C_F \left(\frac{\pi^2}{3} - 4 \right) \right\} \delta(1-z) + \frac{\alpha_s}{\pi} \left[P_{qq}(z) \ln \frac{Q^2}{\mu_F^2} + C_F \left\{ 2(1+z^2) \times \left(\frac{\ln(1-z)}{1-z} \right)_+ - (1+z^2) \frac{\ln z}{1-z} + (1-z) \right\} \right] \right) + \frac{d\hat{\sigma}^{RF}}{dQ^2 d \cos \theta}, \quad (14)$$

and for the qg Compton process from Eq. (12)

$$\frac{d\hat{\sigma}^{qg}(\mu_F^2)}{dQ^2 d \cos \theta} = \frac{d\hat{\sigma}^{CT}}{dQ^2 d \cos \theta} \frac{\alpha_s}{2\pi} \left[P_{qg}^C(z; \lambda_q, h) \left\{ \ln \frac{Q^2}{\mu_F^2} + \ln \frac{(1-z)^2}{z} \right\} + \frac{1}{4} [1 - 2\lambda_q h(1-z)] \right] + \frac{d\hat{\sigma}^{CF}}{dQ^2 d \cos \theta}. \quad (15)$$

V. NUMERICAL RESULTS

We are now ready to predict the helicity distributions of leptons in proton-proton annihilation. By denoting the scattering angle of the lepton in the c.m. frame of protons with momenta P_A and P_B by θ^* , the relation between $\cos \theta$ in the c.m. frame of partons with momenta $x_1 P_A$ and $x_2 P_B$ and $\cos \theta^*$ reads

$$\cos \theta = \frac{x_2(1 + \cos \theta^*) - x_1(1 - \cos \theta^*)}{x_2(1 + \cos \theta^*) + x_1(1 - \cos \theta^*)}.$$

The hadronic cross section for the helicity distribution of lepton is finally given by inserting Eqs. (14), (15) in terms of θ^* into Eq. (1) and changing the parton densities into the μ_F^2 dependent ones. We use the rapidity y_l instead of the scattering angle $\cos \theta^*$ of the lepton in this section, which is defined by

$$y_l = \frac{1}{2} \ln \frac{1 + \cos \theta^*}{1 - \cos \theta^*}.$$

We fix the total energy S of the proton-proton system to be $\sqrt{S} = 500$ GeV, which is the planned highest energy at the RHIC. Although our formulas can be applied to the process for arbitrary Q , we have chosen two values $Q = M_Z$ (the Z boson pole) and $Q = 10$ GeV as typical examples. For the strong coupling constant, we use the standard two-loop form with Λ_{QCD} which is set to the value used in the parton densities. The following values for other parameters of the standard model are used:

$$\alpha = \frac{1}{128}, \quad \sin^2 \theta_w = 0.2315,$$

$$M_Z = 91.187 \text{ GeV}, \quad \Gamma_Z = 2.5 \text{ GeV}.$$

Denoting the helicities of initial protons by $P_{A,B}(\pm)$, there are three cases to be analyzed corresponding to $P_A(+)$ $P_B(-)$ ($-$), $P_A(+)$ $P_B(+)$, and $P_A(-)$ $P_B(-)$ configurations.

We have organized this section as follows. First, we show the numerical results for the helicity distributions of leptons with respect to y_l . Second, we investigate the factorization scale and the parton parametrization dependence of the cross section.

A. The lepton helicity distribution

Here we use the $\overline{\text{MS}}$ parametrization of the parton densities in Refs. [24], [25] and take the factorization scale μ_F and the renormalization scale μ_R to be $\mu_F = \mu_R = Q$. The scale dependence will be discussed later.

We give in Fig. 2 the results at $Q = M_Z$ for both distributions of leptons with negative and positive helicities in the three $P_A(+)$ $P_B(-)$, $P_A(+)$ $P_B(+)$, and $P_A(-)$ $P_B(-)$ configurations. The unit of the cross section (vertical axis) is pb/GeV. The dotted line is the total contribution. The solid line is the contribution from the u quark, namely, $u\bar{u}$ and ug ($\bar{u}g$) annihilation subprocesses, whereas the dot-dashed line is from the d -quark sector: we write simply d for the d and s quarks. The long dashed (dashed) line is the contribution from the tree level $u\bar{u}$ ($d\bar{d}$). We plot in Fig. 3 similar results for $Q = 10$ GeV. Some comments are in order for these results. First, the main effect of QCD corrections is just an enhancement of the tree level cross section: the K factor is 1.5–1.8. It does not change the shape of the lepton distributions significantly from the tree level one. Second, we see that the u and d quarks give different and characteristic con-

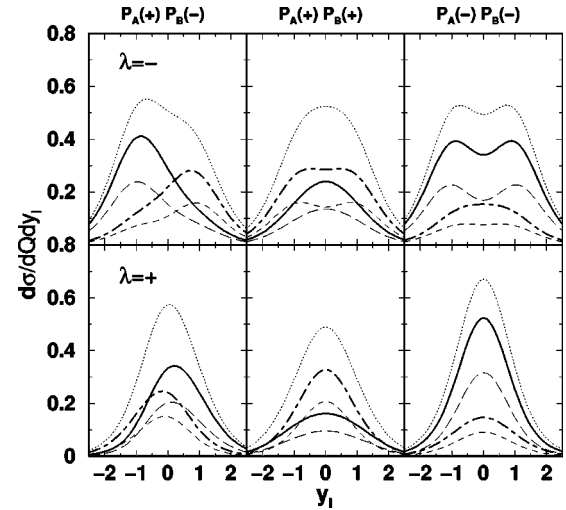


FIG. 2. The cross sections for leptons with negative and positive helicities in the three configurations for the spins of protons with respect to y_l for $Q = M_Z$.

tributions to the helicity distributions for the lepton. In particular, the case of the lepton distribution in the $P_A(+)$ $P_B(-)$ configuration for $Q = M_Z$ deserves some explanation. Our numerical results show that, in the negative rapidity region, leptons with negative helicities are mainly produced and these come from u -quark annihilation. On the other hand, in the positive rapidity region, leptons with both helicities are produced; however, the leptons with negative helicities come mainly from the d -quark annihilation subprocesses. These features can be understood intuitively by observing the following aspects. (1) The polarized quark distributions in the polarized proton $P(+)$ roughly satisfy, for the relevant scale $Q^2 = M_Z^2$ and the momentum fractions of partons $x_1 x_2 = M_Z^2/S$,

$$u(\uparrow) \gg d(\downarrow) \sim u(\downarrow) \gg d(\uparrow) \gg \bar{u}(\uparrow, \downarrow) \sim \bar{d}(\uparrow, \downarrow),$$

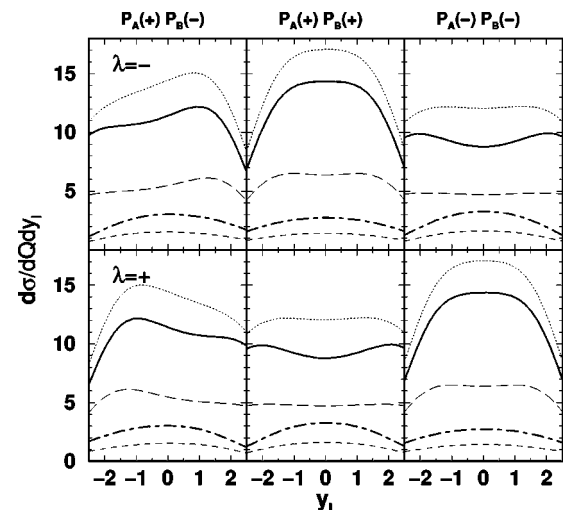


FIG. 3. The cross sections for leptons with negative and positive helicities in the three configurations for the spins of protons with respect to y_l for $Q = 10$ GeV.

where \uparrow and \downarrow denote the quark's spin parallel and antiparallel to the parent proton's spin. This relation implies the dominant subprocesses in the $P_A(+)P_B(-)$ configuration to be (i) $u_A(\uparrow)\bar{u}_B$, (ii) $\bar{u}_A u_B(\uparrow)$, (iii) $d_A(\downarrow)\bar{d}_B$, and (iv) $\bar{d}_A d_B(\downarrow)$. (2) From the angular momentum conservation, the spin of the Z boson produced is aligned with the $P_A (P_B)$ direction for $u_A\bar{u}_B$ and $\bar{u}_A u_B$ ($d_A\bar{d}_B$ and $\bar{d}_A d_B$) annihilations and the negative (positive) helicity lepton from the Z decay has a higher probability of production in the opposite (same) direction as the Z boson's spin. (3) The third point to be noted is that the $V-A$ coupling is larger than the $V+A$ coupling for the quark- Z boson interaction. This suggests that among the four subprocesses in (1), (ii) and (iii) eventually dominate the process. (4) Finally, since the momentum fractions of quarks are bigger than those of antiquarks, the distributions of negative helicity leptons from (ii) and (iii) are Lorentz boosted to the negative and positive rapidity regions, respectively. The fact above (3) also explains that the d (u) quark contribution is larger than that of the u (d) quark for the $P_A(+)P_B(+)$ [$P_A(-)P_B(-)$] configuration. For these configurations, the lepton distribution is symmetric around $y_l=0$, as it should be.

At the lower value of Q ($Q=10$ GeV), the Z -boson contribution can be safely neglected and the electromagnetic interaction becomes dominant. In this case, the situation is not so simple as the previous one ($Q=M_Z$). At first sight, only the point (3) above should be dropped. If so, the subprocesses (i) and (ii) dominate the process, which leads to similar distributions to those for the $Q=M_Z$ case. However, the distribution takes a somewhat different shape for the following reasons. For $Q=10$ GeV, partons with very small momentum fractions participate in the process ($x_1 x_2=10^2/S$). Therefore, for the "dominant" processes for the $Q=M_Z$ case, (i) and (ii), the Lorentz boost effect is fairly large and the Jacobian factor $J=\sin^2\theta^*$ apparently diminishes the cross section in the y_l distributions. Furthermore, at this scale, all subprocesses seem to contribute equally to the cross section, and the partons with small x have less information on the spin of the parent proton. One can see, nonetheless, the contribution from the antisymmetric part of the hadronic tensor, which causes the asymmetric distributions of leptons for the $P_A(+)P_B(-)$ configuration.

We also estimate the lepton helicity asymmetry A_l , which is defined by

$$A_l \equiv \frac{d\sigma(\lambda=-1) - d\sigma(\lambda=+1)}{d\sigma(\lambda=-1) + d\sigma(\lambda=+1)}.$$

This asymmetry is plotted in Fig. 4 for the three spin configurations for protons. The upper three graphs show the asymmetry for the $Q=M_Z$ case and the lower three for the $Q=10$ GeV case. The solid line is the $\mathcal{O}(\alpha_s)$ result and the long dashed line is the tree level one. In these results, we recognize that there are non-negligible effects from the QCD higher order corrections other than just the enhancement effects of tree level results. This asymmetry amounts to around 20–30% in the central rapidity region for $Q=M_Z$. The asymmetry for $Q=10$ GeV is much smaller than that for Q

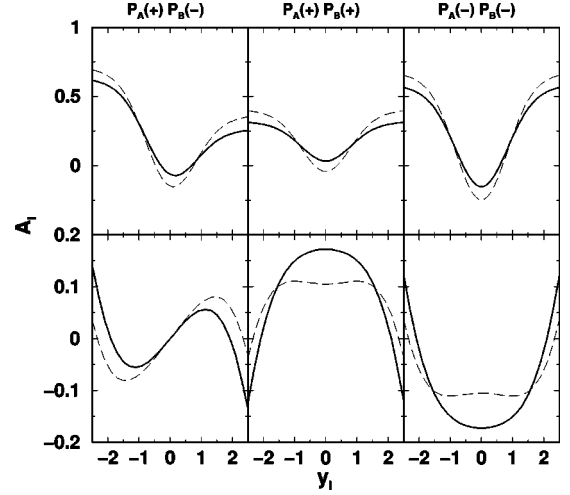


FIG. 4. The helicity asymmetry in the three configurations for the spins of protons with respect to y_l .

$=M_Z$. This is consistent with the previous observation that partons with small x that do not have enough spin information about the parent proton contribute to the process at smaller Q .

B. The scale and parametrization dependence

Physical quantities such as the cross sections or asymmetries are obviously independent of the renormalization and the factorization scales. However, the truncation of the perturbative series at a fixed order induces a dependence on these scales which leads to uncertainty in the theoretical predictions. We study the scale dependence of the cross section and asymmetry by comparing the results obtained with three different choices for the scale: $\mu_F^2=2Q^2$, Q^2 , and $0.5Q^2$. We also mention the dependence on the various parton parametrizations in this subsection.

We give the results only for the case of the $P_A(+)P_B(-)$ configuration with $Q=M_Z$ since the results for other spin configurations of protons and the value of Q are found to be essentially the same. The scale dependence of the lepton distributions (the upper two graphs) as well as the asymmetry (the lower left graph) are shown in Fig. 5, where the dashed line corresponds to $\mu_F^2=2Q^2$, the solid line to $\mu_F^2=Q^2$ and the dotted line to $\mu_F^2=0.5Q^2$. In this calculation, we use the parton densities in Ref. [24]. One can see that the variation is less than 10% in the central rapidity region.

For the parton parametrization dependence, we calculate the asymmetry with the three sets in Ref. [26] since the difference appears more clearly in the asymmetry than in the cross sections. The results are shown in Fig. 5 (the lower right graph). The solid line, dashed line, and dotted line correspond to the Gehrman-Stirling set A (GS-A), GS-B, and GS-C, respectively. The dependence on the parton parametrizations is similar to the scale dependence in size and not so large. The reason will be that the process is dominated by the quark and antiquark annihilation in the RHIC energy region and the gluon initiated Compton subprocess, in which the ambiguity of the gluon distribution appears, gives a tiny correction to the cross section.

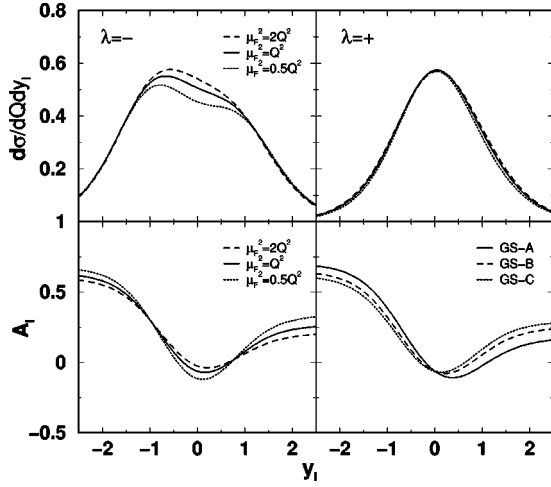


FIG. 5. The scale and the parton parametrization dependence.

VI. CONCLUSION

We have presented a complete calculation at the $\mathcal{O}(\alpha_s)$ order in QCD of the lepton helicity distributions in the polarized Drell-Yan process. We have numerically analyzed the cross section at $Q = 10$ GeV and $Q = M_Z$ and pointed out that the u (\bar{u}) and d (\bar{d}) quarks give different and characteristic contributions to the lepton helicity distributions which deserve some theoretical interest. The QCD corrections mainly enhance the tree level cross sections and this fact can explain qualitatively the lepton helicity distributions from the various proton spin configurations. We have defined and estimated the lepton helicity asymmetry which amounts to around 20–30 %.

1. The real gluon emission process

$$\begin{aligned}
 \frac{d\hat{\sigma}^{RF}(\lambda_q, \lambda_{\bar{q}}, \lambda)}{dQ^2 d\cos\theta} &= \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{\pi}{2N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 \left(\frac{\alpha_s \pi}{C_F} \right) \\
 &\times \left[4z^2 \frac{1+z^2}{1-z} \left\{ \frac{(1+\lambda_q \lambda)(1+c)^2 z^2 + (1-\lambda_q \lambda)(1-c)^2}{[1-c+z(1+c)]^4} \ln \left(\frac{[1-c+z(1+c)]^2}{4z} \right) \right. \right. \\
 &+ \left. \frac{(1+\lambda_q \lambda)(1+c)^2 + (1-\lambda_q \lambda)z^2(1-c)^2}{[1+c+z(1-c)]^4} \ln \left(\frac{[1+c+z(1-c)]^2}{4z} \right) \right\} - 2z(1-z) \\
 &+ \frac{4z(1-z)(1+z^2)\{(1+z)^2 - c^2(1+6z+z^2)\}}{[1+c+z(1-c)]^2 [1-c+z(1+c)]^2} + \frac{8\lambda_q \lambda z^2(1-z)c}{[1+c+z(1-c)]^3 [1-c+z(1+c)]^3} \{(1+z)^2(7-4z \\
 &+ 7z^2) - 2c^2(3+14z+10z^2+14z^3+3z^4) - c^4(1-z)^2(1+z^2)\} + \frac{2z(1-z)}{3[1+c+z(1-c)]^4 [1-c+z(1+c)]^4} \\
 &\times \{ -(1+z)^4(1+14z-18z^2+14z^3+z^4) + 2c^2(1+z)^2(1+4z+239z^2-56z^3+239z^4+4z^5+z^6) + 8c^4 z(3 \\
 &- 45z-151z^2-118z^3-151z^4-45z^5+3z^6) - 2c^6(1-z)^2(1+8z+71z^2+48z^3+71z^4+8z^5+z^6) + c^8(1 \\
 &- z)^4(1-2z+6z^2-2z^3+z^4) \}.
 \end{aligned}$$

We have also studied the dependence of the cross section and the asymmetry on the scale and parton parametrization. Since the $q\bar{q}$ subprocess is dominant in the RHIC energy region, we will not be able, unfortunately, to find a clear difference between various parton parametrizations which have big ambiguities in the gluon distributions.

From the experimental point of view, it seems rather difficult to measure the helicity of the produced muon and/or electron in the Drell-Yan process. However, if we can observe the τ lepton produced from the Drell-Yan process and its decay, we will be able to compare the experimental data and theoretical prediction presented in this paper. We believe that our calculations and numerical studies may provide information complementary to the existing results for the invariant mass and rapidity or x_F distributions of lepton pair in the polarized proton-proton collisions.

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APPENDIX A: THE CROSS SECTION FORMULAS

The explicit forms for the finite contributions to the cross sections are listed in this appendix. We use the abbreviation $c \equiv \cos\theta$.

2. The quark gluon Compton process

$$\begin{aligned} \frac{d\hat{\sigma}^{CF}(\lambda_q, h; \lambda)}{dQ^2 d \cos \theta} &= \frac{\pi}{2N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 \left(\frac{\alpha_s}{\pi} \right) \times \left[4z^2 P_{qg}^C(z; \lambda_q, h) \frac{(1+\lambda_q \lambda)(1+c)^2 z^2 + (1-\lambda_q \lambda)(1-c)^2}{[1-c+z(1+c)]^4} \right. \\ &\times \ln \left(\frac{[1-c+z(1+c)]^2}{4z} \right) + \frac{2z(1-z)[1-z^2-c(1+6z+z^2)]}{[1+c+z(1-c)][1-c+z(1+c)]^3} \times \left\{ [(1-\lambda_q \lambda)(1-c) + (1+\lambda_q \lambda)z(1 \right. \\ &+ c)] P_{qg}^C(z; \lambda_q, h) - \frac{z}{2} \{ (1-\lambda_q h)z(1-c) + (1+\lambda_q h)[1+c+z(1-c)] \} \left. \right\} + \frac{1}{4} (1+\lambda_q \lambda)(1-\lambda_q h)z^2(1-z \\ &- c \ln z) + \left. \frac{2z(1-z^2)c P_{qg}^C(z; \lambda_q, h)}{[1+c+z(1-c)][1-c+z(1+c)]} + G_S + \lambda_q \lambda G_A \right], \end{aligned}$$

where

$$\begin{aligned} G_S &= \frac{z(1-z)}{12[1+c+z(1-c)][1-c+z(1+c)]^4} \times \{ (1-z^2)[(1-\lambda_q h)(1+5z+21z^2-7z^3-2z^4) - 2(1+\lambda_q h)(1+2z-42z^2 \\ &+ 2z^3+z^4)] - c[(1-\lambda_q h)(4+16z+61z^2+98z^3-34z^4+2z^5-3z^6) - 4(1+\lambda_q h)(1-3z^2-92z^3-3z^4+z^6)] + c^2[(1 \\ &-\lambda_q h)(5+17z+65z^2+208z^3+13z^4-17z^5-3z^6) - 72(1+\lambda_q h)z^2(1-z^2)] - 2c^3[(1-\lambda_q h)z(4+13z+38z^2+16z^3 \\ &- 2z^4+3z^5) + 2(1+\lambda_q h)(1-z)^2(1-2z-2z^3+z^4)] - c^4(1-z^2)[(1-\lambda_q h)(5-7z+7z^2+13z^3) - 2(1+\lambda_q h)(1 \\ &- z)^2(1-4z+z^2)] + c^5(1-\lambda_q h)(1-z)^2(4+3z^2+4z^3+3z^4) - c^6(1-\lambda_q h)(1+z)(1-z)^3(1-z+z^2) \}, \\ G_A &= \frac{z(1-z)}{4[1+c+z(1-c)][1-c+z(1+c)]^3} \{ (1-z^2)(1+4z+z^2)[-(1-\lambda_q h)z + 2(1+\lambda_q h)(1-z)] + 2c[(1-\lambda_q h)z(1+2z \\ &+ 4z^2+z^4) - 2(1+\lambda_q h)(1+3z^2-3z^3-z^5)] + 2c^2(1-z)[(1-\lambda_q h)z^2(1-2z-z^2) + (1+\lambda_q h)(1-z^4)] - 2c^3z(1-z) \\ &\times [(1-\lambda_q h)(1+z+3z^2+z^3) + 2(1+\lambda_q h)(1-z)^2] + c^4(1-\lambda_q h)z(1-z)^2(1+z^2) \}. \end{aligned}$$

APPENDIX B: THE INVARIANT MASS DISTRIBUTIONS

Here we present the invariant mass distribution of the lepton pair in the massive gluon scheme before factorizing the mass singularity. Comparing these expressions with those in the dimensional regularization, we can find the relation between the two regularization schemes.

1. The quark and antiquark annihilation process

$$\begin{aligned} \frac{d\hat{\sigma}^{T+V+R}}{dQ^2} &= \delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{4\pi}{3N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 z \left(\left(1 - \frac{7}{4} \frac{\alpha_s}{\pi} C_F \right) \delta(1-z) + \frac{\alpha_s}{\pi} \left[P_{qq}(z) \ln \frac{Q^2}{\kappa^2} + C_F \left\{ 2(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right) \right. \right. \right. \\ &\left. \left. \left. - 2(1+z^2) \frac{\ln z}{1-z} - 2(1-z) \right\} \right] \right). \end{aligned}$$

2. The quark gluon Compton process

$$\frac{d\hat{\sigma}^C}{dQ^2} = \frac{4\pi}{3N_c} \left(\frac{\alpha}{Q^2} \right)^2 |f^{\lambda_q \lambda}|^2 z \left(\frac{\alpha_s}{2\pi} \right) \left[P_{qg}^C(z; \lambda_q, h) \left(\ln \frac{Q^2}{\kappa^2} + \ln \frac{1-z}{z^2} \right) - \frac{1}{8} (1-2z+3z^2 + \lambda_q h \{ 3-2z-3z^2 \}) \right].$$

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