Hadronic τ decay, the renormalization group, analyticity of the polarization operators, and QCD parameters

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The ALEPH data on hadronic τ decay are thoroughly analyzed in the framework of QCD. The perturbative calculations are performed in the (1–4)-loop approximation. The analytical properties of the polarization operators are used in the whole complex q^2 plane. It is shown that the QCD prediction for R_τ agrees with the measured value R_τ not only for conventional $\Lambda_3^{\text{conv}}=(618\pm29)$ MeV but also for $\Lambda_3^{\text{new}}=(1666\pm7)$ MeV. The polarization operator calculated using the renormalization group has a nonphysical cut $[-\Lambda_3^2,0]$. If $\Lambda_3=\Lambda_3^{\text{conv}}$, the contribution of only the physical cut is deficient in the explanation of the ALEPH experiment. If $\Lambda_3=\Lambda_3^{\text{new}}$ the contribution of the nonphysical cut is very small and only the physical cut explains the ALEPH experiment. The new sum rules which follow only from analytical properties of polarization operators are obtained. Based on the sum rules obtained, it is shown that there is an essential disagreement between QCD perturbation theory and the τ-lepton hadronic decay experiment at the conventional value Λ_3. In the evolution upwards to larger energies the matching of $r(q^2)$ at the masses J/ψ, Υ, and $2m_t$ was performed. The obtained value $\alpha_s(-m_z^2)=0.141\pm0.004$ (at $\Lambda_3=\Lambda_3^{\text{new}}$) differs essentially from the conventional value, but the calculation of the values $R(s)=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$, $R_l=\Gamma(Z\to\text{hadrons})/\Gamma(Z\to\text{leptons})$, $\alpha_s(-3\ \text{GeV}^2)$, and $\alpha_s(-2.5\ \text{GeV}^2)$ does not contradict the experiments.

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I. INTRODUCTION

The purpose of this work is to combine the analyticity requirements of QCD polarization operators with the renormalization group. This work is the continuation of Refs. [1–3]. In the Ref. [1], analytical properties of polarization operators were used to improve perturbation theory in QCD. In the Refs. [2,3] the high precision data on hadronic τ decay obtained by the ALEPH [4], OPAL [5], and CLEO [6] Collaborations were analyzed in the framework of QCD. The analyticity requirements of the QCD polarization operators follow from the microcausality and the unitarity; therefore, we have no doubts about them. On the other hand, the calculation according to renormalization group leads to the appearance of nonphysical singularities. So, the one-loop calculation gives a nonphysical pole, while in the calculation in a larger number of loops the pole disappears, but a nonphysical cut appears $[-\Lambda_3^2,0]$. As will be shown, there are only two values of Λ_3, such that theoretical predictions of QCD for $R_{\tau,V+A}$ [formulas (23) and (24)] agree with the experiments [4–6]. These values are the following: one conventional value $\Lambda_3^{\text{conv}}=(618\pm29)$ MeV and the other value of Λ_3 is $\Lambda_3^{\text{new}}=(1666\pm7)$ MeV. Only in these values of Λ_3 are the predictions of QCD consistent with the experiments [4–6]. As far as I know, the value $\Lambda_3=\Lambda_3^{\text{new}}$ was not considered before now.[1] If one simply puts out the nonphysical cut and leaves the conventional value Λ_3^{conv}, then the discrepancy between the theory and experiment will arise. As will be shown, if instead of the conventional value Λ_3^{conv} one chooses the value $\Lambda_3^{\text{new}}=(1565\pm193)$ MeV then only the physical cut contribution is enough to explain the experiment of the hadronic τ decay.[2] It is convenient to introduce the Adler function (11)–(13) instead of the polarization operator. The Adler function is an analytical function of q^2 in the whole complex q^2 plane with a cut along the positive q^2 semiaxis. We will use the renormalization group only for negative q^2, where the value $\alpha_s(q^2)$ is real and positive. For other q^2 the value $\alpha_s(q^2)$ becomes complex and is obtained by analytical continuation.

The plan of the paper is the following. In Sec. II the formulas obtained in paper [3] are transformed to a form suitable for this paper. In Sec. III the values Λ_3 are found such that $R_{\tau,V+A}=3.475\pm0.022$ [formula (24)]. In Sec. IV we obtain new sum rules for polarization operator which follow only from analytical properties of the polarization operator. These sum rules imply that there is an essential discrepancy between perturbation theory in QCD and the experiment in hadronic τ decay at conventional value of Λ_3. The power corrections and instantons cannot eliminate this discrepancy.

Section V suggests the method of resolving these discrepancies. At $\Lambda_3=(1565\pm193)$ MeV the nonphysical cut gives no contribution into $R_{\tau,V+A}$ and the physical cut gives the experimentally observed value $R_{\tau,V+A}$. The previously derived sum rules make no sense if Λ_3 is as large.

In Sec. VI we go over to larger energies. In the matching

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[1]One can see this also from Fig. 2 of Ref. [3]. The line obtained in conventional approach crosses the experimentally allowed strip of $R_{\tau,V+A}$ at two values of Λ_3 equal to Λ_3^{conv} and Λ_3^{new} [in Ref. [3] the parameter $\alpha_s(-m_\tau^2)$ related to Λ_3 was used]. In Ref. [3] the new value Λ_3^{new} was not considered.

[2]The errors here and in the following formulas are due only to the error in the measurement of the value $R_{\tau,V+A}$ [Eq. (24)].

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procedure we require continuity of $r(s)$ [Eq. (12)] [1] at masses J/ψ, Υ, and $2m_t$ when going over from n_f to n_f+1 flavors. The number of flavors on the cut is a good quantum number. At every point off the cut all flavors give a contribution. This follows from the dispersion relation for the Adler function. The continuity requirement off the cut when changing the number of flavors violates the analytical properties of the polarization operator. Section VI presents the results of the calculations in 1–4 loops for estimation of the precision of the calculations. In Secs. VII–IX we compare the theory with experiment. In Sec. VII the prediction of the function $R(s)$ is compared with experiments. The calculated values of the function $R(s)$ are in a very good agreement with the experiment (Tables V and VI) at $2\leqslant\sqrt{s}\leqslant 47.6$ GeV except for the resonance region.

In Sec. VIII we compare the calculated values of $\alpha_s(-3\ \mathrm{GeV}^2)$ and $\alpha_s(-2.5\ \mathrm{GeV}^2)$ with the values $\alpha_s(-3\ \mathrm{GeV}^2)$ and $\alpha_s(-2.5\ \mathrm{GeV}^2)$ obtained from the Gross-Llewellyn-Smith sum rule [7] and the Bjorken sum rule [8]. The results of the calculations are in agreement with the values obtained from the experiment using these sum rules. Section IX presents the calculation of $R_l=\Gamma(Z\to\text{hadrons})/\Gamma(Z\to\text{leptons})$. The obtained value R_l does not contradict the experiment. Section X is devoted to a discussion on the analiticity $\alpha_s(q^2)$. It is shown that the statement that $\alpha_s(0)=4\pi/\beta_0=1.396$ is valid only for one-loop calculations.

II. INITIAL FORMULAS

In this section the formulas obtained in paper [3] are transformed to a form suitable for this paper. We will consider three-loop approximation thoroughly. Polarization operators of hadronic currents are defined by the formula

$$\Pi^J_{\mu\nu}(q)=i\int e^{iqx}\langle 0|TJ_\mu(x)J^+_\nu(0)|0\rangle d^4x$$
$$=(q_\mu q_\nu-g_{\mu\nu}q^2)\Pi^{(1)}_J(q^2)+q_\mu q_\nu\Pi^{(0)}(q^2), \quad (1)$$

where

$$J=V,A;\quad V_\mu=\overline{u}\gamma_\mu d,\quad A_\mu=\overline{u}\gamma_\mu\gamma_5 d.$$

Imaginary parts $\Pi^{(1)}_J,\Pi^{(0)}_J$ are connected with the measured, the so called spectral functions $v_1(s),a_1(s),a_0(s)$, by the formulas

$$v_1(s)/a_1(s)=2\pi\,\mathrm{Im}\,\Pi^{(1)}_{V/A}(s+i0),$$
$$a_0(s)=2\pi\,\mathrm{Im}\,\Pi^{(0)}_A(s+i0). \quad (2)$$

Functions $\Pi^{(1)}_{V/A}$ are analytical functions of q^2 with the cuts $[4m_\pi^2,\infty]$ for $\Pi^{(1)}_V$ and $[9m_\pi^2,\infty]$ for $\Pi^{(1)}_A,a_0(s)=2\pi^2f_\pi^2\delta(s-m_\pi^2),f_\pi=130.7$ MeV.

To get QCD predictions let us use the renormalization group equation in three-loop approximation [9,10]

$$q^2\frac{\partial a}{\partial q^2}=-\beta_0^{(n_f)}\,a^2(1+b_1^{(n_f)}a+b_2^{(n_f)}\,a^2), \quad (3)$$

where

$$a(q^2)=\frac{\alpha_s(q^2)}{4\pi},\quad \beta_0^{(n_f)}=11-\frac{2}{3}n_f,\quad \beta_1^{(n_f)}=51-\frac{19}{3}n_f,$$

$$\beta_2^{(n_f)}=2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2,\quad b_1^{(n_f)}=\frac{2\beta_1^{(n_f)}}{\beta_0^{(n_f)}},$$

$$b_2^{(n_f)}=\frac{\beta_2^{(n_f)}}{2\beta_0^{(n_f)}}. \quad (4)$$

Here n_f is the number of flavors.

Let us consider for the moment $n_f=3$ and omit the mark n_f. Find singularities of $a(q^2)$. Integrate Eq. (3) [3]:

$$\beta_0\ln\frac{q^2}{\mu^2}=-\int_{a(\mu^2)}^{a(q^2)}\frac{da}{a^2(1+b_1a+b_2a^2)}. \quad (5)$$

Denote the value q^2 at which $a(q^2)=\infty$ as $-\Lambda_3^2$.[3] Then we get, instead of Eq. (5),

$$\beta_0\ln\left(\frac{-q^2}{\Lambda_3^2}\right)=\int_{a(q^2)}^{\infty}\frac{da}{a^2(1+b_1a+b_2a^2)}\equiv f(a). \quad (6)$$

According to the known value $a(-m_\tau^2)$ Λ_3^2 is determined by the formula

$$\Lambda_3^2=m_\tau^2e^{-f[a(-m_\tau^2)]/\beta_0}. \quad (7)$$

The integral in formula (6) is taken and the answer is written as

$$f(a)=\frac{1}{a}+b_1\ln a-\frac{1}{\sqrt{b_1^2-4b_2}}\times\left[\frac{1}{x_1^2}\ln(a-x_1)-\frac{1}{x^2}\ln(a-x_2)\right], \quad (8a)$$

where

$$x_{1,2}=\frac{-b_1\pm\sqrt{b_1^2-4b_2}}{2b_2}. \quad (8b)$$

At $\alpha_s(-m_\tau^2)=0.355$ [3] we obtain $\Lambda_3^2=0.394\ \mathrm{GeV}^2$.

The expansion of the function $f(a)$ in the Taylor series at large a over $1/a$ is of the form

$$f(a)=\frac{1}{3b_2}\frac{1}{a^3}-\frac{b_1}{4b_2^2}\frac{1}{a^4}+\frac{(b_1^2-b_2)}{5b_2^3}\frac{1}{a^5}+0\left(\frac{1}{a^6}\right). \quad (9)$$

[3]This is the definition of Λ_3.

It follows from Eqs. (6)–(9) that the singularity $a(q^2)$ at $q^2 \to -\Lambda_3^2$ has the form [3]

$$a(q^2) = \left(-\frac{2\Lambda_3^2}{3\beta_2(q^2+\Lambda_3^2)} \right)^{1/3}. \tag{10}$$

Since for massless quarks the contributions of V and A coincide, we will omit in all formulas the mark J. Introduce the Adler function

$$D(q^2) = -q^2 \frac{d\Pi(q^2)}{dq^2}. \tag{11}$$

It is convenient to write for three flavors

$$D(q^2) = 3[1+d(q^2)], \quad R(q^2) = 3[1+r(q^2)],$$
$$\Pi(q^2) = 3[-\ln(-q^2/\mu^2) + p(q^2)]$$
$$r(q^2) = \frac{1}{\pi}\mathrm{Im}\, p(q^2) = \frac{1}{2\pi i}[p(q^2+i0) - p(q^2-i0)]. \tag{12}$$

In three-loop approximation for modified numerical subtraction ($\overline{\mathrm{MS}}$) renormalization scheme function $d(q^2)$ for negative q^2 is written as [11]

$$d(q^2) = 4a(q^2)[1 + 4d_1^{(n_f)} a(q^2) + 16 d_2^{(n_f)} a(q^2)^2], \tag{13}$$

where

$$d_1^{(n_f)} = 1.9857 - 0.1153 n_f,$$
$$d_2^{(n_f)} = 18.244 - 4.216 n_f + 0.086 n_f^2. \tag{14}$$

Hereafter we will follow [3]

$$d(q^2) = -q^2 \frac{dp(q^2)}{dq^2}, \tag{15}$$

$$p(q^2) = -\int \frac{d(q^2)}{q^2} dq^2. \tag{16}$$

Using formula (3),

$$\frac{dq^2}{q^2} = -\frac{da}{\beta_0 a^2 (1 + b_1 a + b_2 a^2)} \tag{17}$$

we get for the function $p(q^2)$ the expression

$$p(q^2) = \frac{1}{\beta_0} \int \frac{d(a)da}{a^2(1+b_1 a + b_2 a^2)}$$
$$= \frac{1}{\beta_0 b_2} \int \frac{d(a)da}{a^2(a-x_1)(a-x_2)}. \tag{18}$$

After taking the integral (18) we get

$$p(a) = \frac{4}{\beta_0}\ln a + \frac{4}{\beta_0\sqrt{b_1^2 - 4b_2}} \left\{ \left(\frac{1}{x_1} + 4d_1 + 16 d_2 x_1 \right) \times \ln(a - x_1) - \left(\frac{1}{x_2} + 4d_1 + 16 d_2 x_2 \right) \ln(a - x_2) \right\}, \tag{19}$$

x_1, x_2 are determined by formula (8b).

The polarization operator is an analytical function with the cut $[0,\infty]$. The polarization operator calculated in three-loop approximation has the physical cut $0 < q^2 < \infty$ and nonphysical one $-\Lambda_3^2 < q^2 < 0$. The contribution of the physical cut in the value $R_{\tau,V+A}^{S=0}$ is equal to

$$R_{\tau,V+A}^{\mathrm{QCD}}|_{\text{phys. cut}} = 6|V_{ud}|^2 S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right) \times [1 + r(s)]ds + \Delta R_\tau^{(0)}, \tag{20}$$

where $|V_{ud}| = 0.9735 \pm 0.0008$ is the Cabibbo-Kobayashi-Maskawa matrix element [12], $S_{EW} = 1.0194 \pm 0.040$ is the contribution of electroweak corrections [13].

$$\Delta R_\tau^{(0)} = -24\pi^2 \frac{f_\pi^2 m_\pi^2}{m_\tau^2} = -0.008 \tag{21}$$

is a small correction from the pion pole [3]. The nonphysical cut contribution is equal to

$$R_{\tau,V+A}^{(\mathrm{QCD})}|_{\text{nonphys. cut}} = 6|V_{ud}|^2 S_{EW} \int_{-\Lambda_3^2}^{0} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left(1 + 2\frac{s}{m_\tau^2}\right)[1 + r(s)]ds. \tag{22}$$

III. FINDING OF THE VALUE Λ_3

The value measured in the experiment is

$$R_{\tau,V+A} = \frac{B(\tau \to \nu_\tau + \text{hadrons}, S=0)}{B(\tau \to e^- \overline{\nu_e} \nu_\tau)}$$
$$= 6|V_{ud}|^2 S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \times [v_1(s) + a_1(s) + a_0(s)] - 2\frac{s}{m_\tau^2} a_0(s) \right]. \tag{23}$$

For the value $R_{\tau,V+A}$, the ALEPH [4], OPAL [5] and CLEO [6] Collaborations had obtained

$$R_{\tau,V+A} = 3.475 \pm 0.022. \tag{24}$$

Here new value of $R_{\tau,S}$ [14,15] is taken into account,

$$R_{\tau,S} = 0.161 \pm 0.007. \tag{25}$$

The convenient way to calculate the R_τ in QCD is to transform the integral in the complex s plane [16–19] around the circle $|s|=m_\tau^2$ and thus getting a satisfactory agreement with the experiment:

$$R_{\tau,V+A}=6\pi i|V_{ud}|^2 S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+2\frac{s}{m_\tau^2}\right)\times\left(1+2\frac{s}{m_\tau^2}\right)\Pi(s)+\Delta R_\tau^{(0)}. \qquad (26)$$

There are two values of Λ_3, such that $R_{\tau,V+A}=3.475\pm 0.022$ [Eq. (24)]. These values are: conventional value

$$\Lambda_3^{\text{conv}}=618\pm 29 \text{ MeV} \qquad (27)$$

and the alternative new value

$$\Lambda_3^{\text{new}}=1666\pm 7 \text{ MeV}. \qquad (28)$$

We can calculate $R_{\tau,V+A}^{\text{QCD}}$ also with the help of formulas (20) and (22). At $\Lambda_3=618\pm 29$ MeV,

$$R_{\tau,V+A}^{\text{QCD}}|_{\text{phys. cut}}=3.305\pm 0.008 \qquad (29)$$

and

$$R_{\tau,V+A}^{\text{QCD}}|_{\text{nonphys. cut}}=0.162\pm 0.015. \qquad (30)$$

The sum of integrals on the physical and nonphysical cuts is equal to the integral over the circle (it follows from the Cauchy theorem) and coincides with the measured value $R_{\tau,V+A}$ (24). The contribution of only one physical cut is insufficient to explain the experiment.

If $\Lambda_3=1666\pm 7$ MeV,

$$R_{\tau,V+A}^{\text{QCD}}|_{\text{phys. cut}}=3.480\pm 0.0007 \qquad (31)$$

and

$$R_{\tau,V+A}^{\text{QCD}}|_{\text{nonphys. cut}}=0.0129\pm 0.0024. \qquad (32)$$

If $\Lambda_3=\Lambda_3^{\text{conv.}}$, the nonphysical cut must be taken into account to avoid a discrepancy with the experiment. If $\Lambda_3=\Lambda_3^{\text{new}}$, there are two possibilities. It is possible to omit the contribution of the nonphysical cut and to satisfy the requirements of microcausality and unitarity. Alternatively, the contribution of the nonphysical cut is taken into account and the requirement of microcausality and unitarity will be satisfied only in a future comprehensive theory.

In Refs. [4] $R_{\tau,V}$ and $R_{\tau,A}$ are measured separately:

$$R_{\tau,V}=1.775\pm 0.017, \qquad (33)$$

$$R_{\tau,A}=1.717\pm 0.018. \qquad (34)$$

The values $R_{\tau,V}$ and $R_{\tau,A}$ have been corrected taking into account papers [14,15].

In QCD one should have for massless u and d quarks

$$R_{\tau,V}^{(\text{QCD})}=R_{\tau,A}^{(\text{QCD})}. \qquad (35)$$

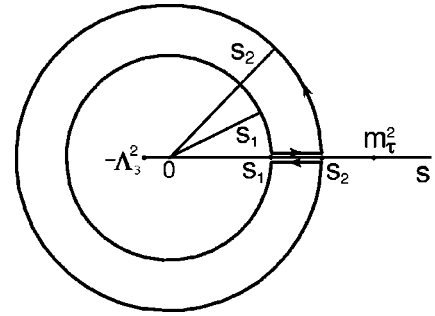

FIG. 1. Contour integral.

The results of the experiments (33), and (34) contradict formula (35). This contradiction was resolved in paper [2].

IV. NEW SUM RULES FOR POLARIZATION OPERATORS

To derive the sum rule, let us consider the integral over closed contour from the function $W(s)\Pi(s)$, where $\Pi(s)$ is one of the functions $\Pi_V^{(1)}(s)$, $\Pi_A^{(1)}(s)$, $\Pi_V^{(1)}(s)+\Pi_A^{(1)}(s)$, $\Pi_V^{(1)}(s)-\Pi_A^{(1)}(s)$, and $W(s)$ is the weight analytical function, which will be chosen later. As a contour, we choose the one which contains the upper and lower edges of the cut from s_1 to s_2 and of two circles with radii s_1 and s_2 (see Fig. 1). Let us choose the values $s_1=0.6,0.8,\ldots,2$ GeV^2 and the values $s_2=s_1+0.2$, $s_1+0.4,\ldots,3$ GeV^2. The integral considered through the Cauchy theorem is zero. It is does not contain the contribution of power corrections[4] and nonphysical cut. As a weight function we choose

$$W(s)=(s-s_1)(s_2-s). \qquad (36)$$

The sum of integrals over cut edges is $2i\int_{s_1}^{s_2}W(s)\,\text{Im}\,\Pi(s)ds$. This sum is equal to the sum of integrals with inverse sign over the circles, for which owing to that the weight function vanishes at the points s_1 and s_2, one may take $\Pi^{(\text{QCD})}(s)$ instead of the true value $\Pi(s)$.

Making use of the analytical properties of $\Pi^{(\text{QCD})}(s)$, let us transform the sum of integrals over circles into the integral from $\text{Im}\,\Pi^{(\text{QCD})}(s)$ over the cut from s_1 to s_2. Finally, we obtain the following sum rule:

$$\int_{s_1}^{s_2}W(s)\text{Im}\,\Pi(s)ds=\int_{s_1}^{s_2}W(s)\text{Im}\,\Pi^{(\text{QCD})}(s)ds. \qquad (37)$$

To compare QCD predictions with experiment, let us introduce the notations

$$U_B=\int_{s_1}^{s_2}W(s)\text{Im}\,\Pi_B(s)ds\Big/\int_{s_1}^{s_2}W(s)\text{Im}\,\Pi_B^{(\text{QCD})}(s)ds, \qquad (38)$$

[4]We ignored the α_s corrections to the condensates. The condensates without α_s corrections are the poles off the contour of integration.

TABLE I. Comparison of the sum rules (37) with the ALEPH experiment. U_B is given by Eq. (38). $\mathrm{Im}\,\Pi_B(s)$ is obtained from the ALEPH experimental data. $\mathrm{Im}\,\Pi_B^{(\mathrm{QCD})}(s)$ is calculated by three-loop approximation of QCD at $\Lambda_3=618$ MeV. s_1, s_2 are given in GeV^2.

s_2	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	
U_V	0.552	0.489	0.479	0.498	0.534	0.592	0.668	0.745	0.816	0.881	
U_A	1.169	1.398	1.502	1.514	1.465	1.389	1.309	1.229	1.161	1.107	$s_1=0.8$
U_{V+A}	0.861	0.942	0.988	1.003	0.995	0.985	0.982	0.981	0.982	0.984	
s_2	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	
u_V	0.438	0.425	0.451	0.494	0.546	0.62	0.71	0.796	0.871	0.939	
u_A	1.491	1.629	1.649	1.594	1.493	1.383	1.279	1.889	1.114	1.059	$s_1=1$
u_{V+A}	0.967	1.024	1.047	1.039	1.015	0.995	0.998	0.986	0.986	0.988	
s_2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3		
u_V	0.429	0.477	0.531	0.592	0.677	0.778	0.869	0. 945	1.011		
u_A	1.712	1.668	1.563	1.427	1.299	1.188	1.098	1.027	0.978		$s_1=1.2$
u_{V+A}	1.065	1.069	1.042	1.003	0.98	0.976	0.976	0.979	0.983		
s_2	1.6	1.8	2	2.2	2.4	2.6	2.8	3			
u_V	0.533	0.586	0.65	0.749	0.862	0.956	1.028	1.091			
u_A	1.601	1.456	1.302	1.172	1.067	0.986	0.927	0.891			$s_1=1.4$
u_{V+A}	1.063	1.015	0.968	0.951	0.956	0.963	0.970	0.978			
s_2	1.8	2	2.2	2.4	2.6	2.8	3				
u_V	0.642	0.715	0.837	0.962	1.053	1.118	1.173				
u_A	1.299	1.146	1.032	0.944	0.88	0.833	0.816				$s_1=1.6$
u_{V+A}	0.964	0.922	0.924	0.944	0.959	0.97	0.98				
s_2	2	2.2	2.4.	2.6	2.8	3					
u_V	0.788	0.956	1.086	1.161	1.209	1.252					
u_A	1.006	0.918	0.847	0.799	0.77	0.765					$s_1=1.8$
u_{V+A}	0.884	0.923	0.958	0.974	0.983	0.922					
s_2	2.2	2.4	2.6	2.8	3						
u_V	1.133	1.217	1.255	1.281	1.312						
u_A	0.833	0.778	0.744	0.728	0.738						$s_1=2$
u_{V+A}	0.972	0.993	0.996	0.998	1.005						

where $B=V,A,V+A$. The results of the calculations of U_B are given in Table I.

It is seen from Table I that QCD predictions agree with experiment for $V+A$ and disagree with experiments for V and A separately. The results do not change if one takes the weight function of the form $(s-s_1)^n(s_2-s)^n$, $n=2,3,\ldots,10$.

Let us consider $V-A$. In this case $\mathrm{Im}(\Pi_V^{(\mathrm{QCD})}(s)-\Pi_A^{(\mathrm{QCD})}(s))=0$, while $\mathrm{Im}[\Pi_V(s)-\Pi_A(s)]\neq 0$. Try to eliminate this disagreement with the help of instantons.

The instanton contribution into $\Pi_V^{(1)}(s)-\Pi_A^{(1)}(s)$ in the model considered in [3] is given by formula (39) [3]:

$$\Pi_{V,inst}^{(1)}(s)-\Pi_{A,inst}^{(1)}(s) = \int_0^\infty d\rho\, n(\rho)\left[-\frac{4}{s^2}-\frac{4\rho^2}{s}\; K_1^2(\rho\sqrt{-s})\right], \tag{39}$$

K_1 is the Macdonald function. Introduce the notation

$$L_{Exp}=\int_{s_1}^{s_2} W(s)\,\mathrm{Im}[\Pi_V^{(1)}(s)-\Pi_A^{(1)}(s)]ds, \tag{40}$$

$$L_{inst}=\int_{s_1}^{s_2} W(s)\,\mathrm{Im}[\Pi_{V,inst}^{(1)}(s)-\Pi_{A,inst}^{(1)}(s)]ds. \tag{41}$$

The results of L_{Exp} and L_{inst} calculations are given in Table II for $n(\rho)=n_0\delta(\rho-\rho_0)$ $\rho_0=1.7\ \mathrm{GeV}^{-1}$, $n_0=1.5\times 10^{-3}\ \mathrm{GeV}^4$.

It is seen from Table II that instantons (in the model under consideration) cannot eliminate the disagreement between QCD theory and experiment.

V. NEW QCD PARAMETERS AND ELIMINATION OF CONTRADICTIONS

In my opinion, the only possible way to resolve the discrepancy which follows from the sum rules (37) is to change the conventional value $\Lambda_3^{\mathrm{conv}}\sim 600$ MeV by $\Lambda_3^{\mathrm{new}}\sim 1600$ MeV. Because s_1 must be larger than Λ_3^2

TABLE II. $s_1=0.8\ \mathrm{GeV}^2$.

s_2/GeV^2	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
L_{Exp}	−0.000424	−0.022	−0.107	−0.286	−0.554	−0.878	−1.198	−1.442	−1.572	−1.563	−1.402
L_{inst}	0.000144	0.00089	0.00234	0.00427	0.00634	0.00815	0.00933	0.00955	0.00853	0.00619	0.00209
L_{Exp}/L_{inst}	−2.907	−24.147	−45.8	−66.82	−87.34	−107.7	−128.7	−151.1	−184.2	−256.7	−670.7

$\sim 2.5\ \mathrm{GeV}^2$, the sum rules (38) become meaningless. At $\Lambda_3^{\mathrm{new}}\sim 1600$ MeV we have a possibility to fulfill the requirement of microcausality and unitarity to omit the nonphysical cut.

The contribution of the physical cut

$$R_{\tau,V+A}^{\mathrm{QCD},S=0}=6|V_{ud}|^2 S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \times\left(1+\frac{2s}{m_\tau^2}\right)[1+r(s)]ds =3.483\pm 0.022 \tag{42}$$

agrees with experiment (24) at

$$\Lambda_3=\Lambda_3^{\mathrm{new}}=(1565\pm 193)\ \ \mathrm{MeV}. \tag{43}$$

The value Λ_3^{new} in Eq. (43) differs from Λ_3^{new} in Eq. (28) since in Eq. (28) we take into account the contribution of the nonphysical cut. It is my belief, that the nonphysical cut must be absent. In spite of that the nonphysical cut is a consequence of three-loop in *p* QCD, if we are able to eliminate this drawback, we must do it. But at $\Lambda_3\sim 600$ MeV we cannot do it while at $\Lambda_3\sim 1600$ MeV we can. In what follows we put $\Lambda_3=(1565\pm 193)$ MeV and omit the nonphysical cut contribution.

VI. TRANSITIONS TO A LARGER NUMBER OF FLAVORS

Let us introduce the notation

$$f_{n_f}(a)=\frac{1}{a}+b_1^{(n_f)}\ln a-\frac{1}{\sqrt{b_1^{(n_f)^2}-4b_2^{(n_f)}}}\left[\frac{1}{x_1^{(n_f)^2}}\ln(a-x_1^{(n_f)}) -\frac{1}{x_2^{(n_f)^2}}\ln(a-x_2^{(n_f)})\right],\quad n_f\leqslant 5, \tag{44}$$

$$f_{n_f}(a)=\frac{1}{a}+b_1^{(n_f)}\ln a-\frac{1}{\sqrt{b^{(n_f)^2}-4b_2^{(n_f)}}}\left[\frac{1}{x_1^{(n_f)^2}}\ln(a-x_1^{(n_f)}) -\frac{1}{x^{(n_f)^2}}\ln(x_2^{(n_f)}-a)\right],\quad n_f=6, \tag{45}$$

where[5]

$$x_{1,2}^{(n_f)}=\frac{-b_1^{(n_f)}\pm\sqrt{b_1^{(n_f)^2}-4b_2^{(n_f)}}}{2b_2^{(n_f)}}, \tag{46}$$

$$x_1^{(3)}=-0.0497+0.107i,\quad x_2^{(3)}=x_1^{(3)*},$$

$$x_1^{(4)}=-0.0632+0.1296i,\quad x_2^{(4)}=x_1^{(4)*},$$

$$x_1^{(5)}=-0.107+0.176i,\quad x_2^{(5)}=x_1^{(5)*},$$

$$x_1^{(6)}=-0.213,\quad x_2^{(6)}=1.013.$$

The value $a(q^2)$ is found by numerical solution of the equation

$$\beta_0^{(n_f)}\ln\left(-\frac{q^2+i0}{\Lambda_f^2}\right)=f_{n_f}(a),\quad q^2>0,\quad n_f=3,4,5,6. \tag{47}$$

The function $r(s)$ can be obtained with the help of Eqs. (19) and (12).

In the evolution upwards to larger energies the matching of $r(q^2)$ at the masses J/ψ, Υ, and $2m_t$ is performed.

There are three alternatives.

(1) The nonphysical cut is absent $\Lambda_3^{\mathrm{new}}=(1565\pm 193)$ MeV. The Adler function $d(q^2)$ may be written in the form

$$d(q^2)=d^{(3)}(q^2)+d^{(4)}(q^2)+d^{(5)}(q^2)+d^{(6)}(q^2), \tag{48}$$

where

$$d^{(3)}(q^2)=-q^2\int_0^{m_\psi^2}\frac{r_3(q'^2)dq'^2}{(q'^2-q^2)^2} \tag{49}$$

is the contribution of the part of the cut with three flavors into the Adler function. Similarly,

$$d^{(4)}(q^2)=-q^2\int_{m_\psi^2}^{m_\Upsilon^2}\frac{r_4(q'^2)dq'^2}{(q'^2-q^2)^2} \tag{50}$$

is the contribution of the part of the cut with four flavors into Adler function;

[5]The sign of the argument in the third logarithm is changed for that to remain on the physical sheet.

TABLE III. The calculation of $\alpha_s(q^2)$ at different q^2 in approximation of 1–4 loops. The matching of $r(q^2)$ at the masses of J/ψ, Υ, and $2m_t$ is performed. The contribution of nonphysical cut is omitted.

	Approximation One loop	Two loops	Three loops	Four loops
Λ_3/MeV	618 ± 59	1192 ± 136	1565 ± 193	1862 ± 230
Λ_4/MeV	508 ± 51	956 ± 113	1257 ± 158	1503 ± 189
Λ_5/MeV	377 ± 40	678 ± 86	872 ± 119	1164 ± 145
Λ_6/MeV	191 ± 22	301 ± 42	312 ± 47	202 ± 31
$\alpha_s(0)$	1.396	0.895 ± 0.001	0.749 ± 0.001	0.749 ± 0.001
$\alpha_s(-m_\tau^2)$	0.469 ± 0.018	0.387 ± 0.015	0.379 ± 0.013	0.379 ± 0.013
$\alpha_s(m_\tau^2+i0)$	0.356 ± 0.019 $+(0.306\pm0.017)i$	0.355 ± 0.014 $+(0.270\pm0.014)i$	0.365 ± 0.023 $+(0.264\pm0.013)i$	0.363 ± 0.013 $+(0.263\pm0.013)i$
$\alpha_s(-m_\psi^2)$	0.377 ± 0.014	0.332 ± 0.013	0.322 ± 0.009	0.326 ± 0.012
$\alpha_s(m_\psi^2-0+i0)$	0.276 ± 0.010 $+(0.216\pm0.013)i$	0.265 ± 0.003 $+(0.200\pm0.014)i$	0.272 ± 0.015 $\pm(0.201\pm0.014)i$	0.273 ± 0.016 $\pm(0.202\pm0.014)i$
$\alpha(m_\psi^2+0+i0)$	0.291 ± 0.012 $+(0.206\pm0.013)i$	0.284 ± 0.002 $+(0.193\pm0.014)i$	0.293 ± 0.017 $+(0.199\pm0.015)i$	0.294 ± 0.018 $+(0.199\pm0.015)i$
$\alpha_s(-m_\Upsilon^2)$	0.254 ± 0.008	0.234 ± 0.009	0.237 ± 0.009	0.237 ± 0.009
$\alpha_s(m_\Upsilon^2-0+i0)$	0.206 ± 0.005 $+(0.107\pm006)i$	0.192 ± 0.004 $+(0.100\pm007)i$	0.194 ± 0.006 $+(0.106\pm0.008)i$	0.194 ± 0.007 $+(0.104\pm0.007)i$
$\alpha_s(m_\Upsilon^2+0+i0)$	0.211 ± 0.006 $+(0.100\pm0.005)i$	0.199 ± 0.005 $+(0.093\pm0.007)i$	0.203 ± 0.007 $+(0.098\pm0.007)i$	0.204 ± 0.008 $+(0.099\pm0.007)i$
$\alpha_s(-m_z^2)$	0.142 ± 0.003	0.137 ± 0.002	0.142 ± 0.004	0.141 ± 0.004
$\alpha_s(m_z^2+i0)$	0.141 ± 0.006 $+(0.039\pm0.002)i$	0.129 ± 0.003 $+(0.036\pm0.002)i$	0.131 ± 0.003 $+(0.037\pm0.002)i$	0.131 ± 0.003 $+(0.038\pm0.002)i$
$r(m_z^2)$	0.0462 ± 0.0009	0.0458 ± 0.0011	0.0464 ± 0.0012	0.0465 ± 0.0012
R_l	20.856 ± 0.017	20.848 ± 0.021	20.860 ± 0.023	20.861 ± 0.023

$$d^{(5)}(q^2)=-q^2\int_{m_\Upsilon^2}^{4m_t^2}\frac{r_5(q'^2)dq'^2}{(q'^2-q^2)^2} \tag{51}$$

is the contribution of the part of the cut with five flavors into the Adler function; and

$$d^{(6)}(q^2)=-q^2\int_{4m_t^2}^{\infty}\frac{r_6(q'^2)dq'^2}{(q'^2-q^2)^2} \tag{52}$$

is the contribution of the part of the cut with six flavors into the Adler function.

The number of flavors for $r(q^2)$ on the cut is a certain number in contrast to the number of flavors at the point of the complex plane q^2 off the cut. Let us consider $q^2=-m_Z^2$ and find $\alpha_s(-m_Z^2)$.

Return to formula (13).The coefficients d_1 and d_2 in Eq. (13) are defined for a certain number of flavors.

Introduce

$$d_1^{(av)}(-m_Z^2)=[d_1^{(3)}d^{(3)}(-m_Z^2)+d_1^{(4)}d^{(4)}(-m_Z^2)+d_1^{(5)}d^{(5)}\times(-m_Z^2)+d_1^{(6)}d^{(6)}(-m_Z^2)]/d(-m_Z^2), \tag{53}$$

$$d_2^{(av)}(-m_Z^2)=[d_2^{(3)}d^{(3)}(-m_Z^2)+d_2^{(4)}d^{(4)}(-m_Z^2)+d_2^{(5)}d^{(5)}\times(-m_Z^2)+d_2^{(6)}d^{(6)}(-m_Z^2)]/d(-m_Z^2). \tag{54}$$

The values $d^{(n_f)}(-m_Z^2)$ have been calculated.

$$d^{(3)}(-m_Z^2)=0.000169,\quad d^{(4)}(-m_Z^2)=0.000823,$$

$$d^{(5)}(-m_Z^2)=0.0432,\quad d^{(6)}(-m_Z^2)=0.00218,$$

$$d(-m_Z^2)=0.0464. \tag{55}$$

Formula (13) is replaced by

$$d(-m_Z^2)=4a(-m_Z^2)[1+4d_1^{(av)}(-m_Z^2)a(-m_Z^2)+16d_2^{(av)}(-m_Z^2)a^2(-m_Z^2)]. \tag{56}$$

Equation (49) can be solved for $a(-m_Z^2)$,

$$\alpha_s(-m_Z^2)=4\pi a(-m_Z^2)=0.142\pm0.004. \tag{57}$$

The value $\alpha_s(m_Z^2+i0)$ is evaluated from Eqs. (6) and (8),

TABLE IV. The calculation of $\alpha_s(q^2)$ at different q^2 in (1–4)-loop approximation. The matching of $r(q^2)$ at the masses of J/ψ, Υ mesons, and of $2m_\tau$ is performed. The contribution of nonphysical cut is taken into account.

	Approximation			
	One loop	Two loops	Three loops	Four loops
Λ_3/MeV	370 ± 19	539 ± 25	618 ± 29	720 ± 33
Λ_4/MeV	296 ± 16	416 ± 21	475 ± 24	557 ± 28
Λ_5/MeV	211 ± 13	275 ± 15	301 ± 16	359 ± 20
Λ_6/MeV	102 ± 7	111 ± 7	96 ± 5	61 ± 4
$\alpha_s(-m_\tau^2)$	0.445 ± 0.015	0.371 ± 0.012	0.354 ± 0.010	0.375 ± 0.012
$\alpha_s(m_\tau^2+i0)$	$0.222+(0.222\pm007)i$	$0.19+(0.174\pm0.006)i$	0.186 ± 0.001 $+(0.162\pm0.005)i$	0.184 ± 0.001 $+(0.161\pm0.005)i$
$\alpha_s(-m_\psi^2)$	0.33 ± 0.01	0.277 ± 0.006	0.266 ± 0.006	0.273 ± 0.006
$\alpha_s(m_\psi^2-0+i0)$	0.212 ± 0.002 $+(0157\pm0.005)i$	0.182 ± 0.002 $+(0.125\pm0.004)i$	0.177 ± 0.001 $+(0.116\pm0.004)i$	0.177 ± 0.001 $+(0.116\pm0.003)i$
$\alpha_s(m_\psi^2+0+i0)$	0.222 ± 0.002 $+(0.148\pm0.005)i$	0.192 ± 0.002 $+(0.118\pm0.004)i$	0.187 ± 0.002 $+(0.112\pm0.003)i$	0.187 ± 0.002 $\pm(0.111\pm0.003)i$
$\alpha_s(-m_\Upsilon^2)$	0.180 ± 0.003	0.189 ± 0.003	0.184 ± 0.003	0.185 ± 0.003
$\alpha_s(m_\Upsilon^2-0+i0)$	0.180 ± 0.002 $+(0.082\pm0.002)i$	0.157 ± 0.002 $+(0.066\pm0.002)i$	0.154 ± 0.002 $+(0.063\pm0.002)i$	0.154 ± 0.002 $\pm(0.063\pm0.002)i$
$\alpha_s(m_\Upsilon^2+0+i0)$	0.184 ± 0.002 $+(0.076\pm0.002)i$	0.161 ± 0.002 $+(0.061\pm0.002)i$	0.159 ± 0.002 $+(0.059\pm0.001)i$	0.159 ± 0.002 $+(0.059\pm0.002)i$
$\alpha_s(-m_z^2)$	0.135 ± 0.001	0.120 ± 0.001	0.118 ± 0.001	0.118 ± 0.001
$\alpha_s(m_z^2+i0)$	0.186 ± 0.002 $+(0.032\pm0.001)i$	0.113 ± 0.001 $+(0.0274\pm0.0005)i$	0.112 ± 0.001 $+(0.0267\pm0.0004)i$	0.111 ± 0.001 $+(0.0266\pm0.004)i$
$r(m_z^2)$	0.0420 ± 0.0004	0.0395 ± 0.0003	0.0389 ± 0.0003	0.0388 ± 0.0003
R_l	20.772 ± 0.008	20.721 ± 0.007	20.710 ± 0.007	20.708 ± 0.007

$$\alpha_s(m_Z^2+i0)=0.131\pm0.003+(0.037\pm0.002)i, \quad |\alpha_s(m_Z^2+i0)|=0.136\pm0.004. \tag{58}$$

In a similar way $\alpha_s(q^2)$ can be calculated at arbitrary q^2. The values of α_s at the interesting points are given in Table III.

(2) The nonphysical cut is taken into account, $\Lambda_3^{\rm new}=(1666\pm7)$ MeV. In this case the lower limit of the integral in Eq. (49) is equal to $-\Lambda_3^2$ and in the three-loop approximation $\Lambda_4=1591$ MeV, $\Lambda_5=791$ MeV, $\Lambda_6=280$ MeV. Errors in this case are very small. The results of the calculations in the three-loop approximation are the following:

$$\alpha_s(-m_\tau^2)=1.575, \quad \alpha_s(m_\tau^2+i0)=0.142+0.272i, \quad \alpha_s(-m_\psi^2)=0.484,$$
$$\alpha_s(m_\psi^2-0+i0)=0.179+0.209i, \quad \alpha_s(m_\psi^2+0+0i)=0.223+229i, \quad \alpha_s(-m_\Upsilon^2)=0.253,$$
$$\alpha_s(m_\Upsilon^2-0+i0)=0.202+0.110i, \quad \alpha_s(m_\Upsilon^2+0+0i)=0.189+0.093i,$$
$$\alpha_s(-m_z^2)=0.139, \quad \alpha_s(-m_z^2+i0)=0.129+0.036i, \quad r(m_z^2)=0.0457, \quad R_l=20.845. \tag{59}$$

(3) The nonphysical cut is taken into account, $\Lambda_3^{\rm conv}=(618\pm29)$ MeV. The results of the calculations in (1–4)-loop approximation are given in Table IV. Unlike conventional matching procedure at negative q^2, the matching of $r(q^2)$ on the masses of J/Ψ, Υ mesons, and of $2m_t$ is performed. The value $\alpha_s(-m_z^2)$ is practically independent of the matching procedure. I believe that the first alternative is the best.

VII. COMPARISON OF THE CALCULATED VALUES $R_T(s)$ WITH THE MEASURED VALUES $R_E(s)$

The value $R_T(s)$ is calculated by the formula

$$R_T(s)=3\sum e_q^2[1+r(s)]. \tag{60}$$

The results of the calculations of $R(s)$ in three-loop approximation and of their comparison with experiments are given in Table V for $2\leqslant\sqrt{s}\leqslant 4.8$ GeV and for $12\leqslant\sqrt{s}\leqslant 46.6$ GeV in Table VI. The calculated values of the function $R(s)$ are in excellent agreement with the experiment except for the resonance region $3.7\leqslant\sqrt{s}\leqslant 4.4$ GeV. But the accuracy of measurements of $R(s)$ is insufficient to define the value $r(s)$ with a good accuracy [20].

VIII. COMPARISON WITH $\alpha_s(s)$ OBTAINED FROM THE SUM RULES

Perturbative corrections to two measurements, namely, the Gross-Llewellyn-Smith sum rule [7] for the deep inelastic neutrino scattering and the Bjorken sum rule [8] for polarized structure functions, have been determined:

$$\alpha_s(-3\ \ \mathrm{GeV}^2)=0.28\pm 0.035(\mathrm{stat})\pm 0.050(\mathrm{sys}) \tag{61}$$

[12,25] and

$$\alpha_s(-2.5\ \ \mathrm{GeV}^2)=0.375^{+0.062}_{-0.081} \tag{62}$$

[12,26–28]. The results of the calculations give

$$\alpha_s(-3\ \ \mathrm{GeV}^2)=0.381\pm 0.013, \tag{63}$$

$$\alpha_s(-2.5\ \ \mathrm{GeV}^2)=0.39\pm 0.013. \tag{64}$$

IX. CALCULATION OF R_l

The value $R_l=\Gamma(Z\to\mathrm{hadrons})/\Gamma(Z\to\mathrm{leptons})$ is parametrized by the latest version of ZFITTER [29]:

$$R_l=19.934\left[1+1.045\left(\frac{\alpha_s}{\pi}\right)+0.94\left(\frac{\alpha_s}{\pi}\right)^2-15\left(\frac{\alpha_s}{\pi}\right)^3\right]. \tag{65}$$

This is the conventional method. In this parametrization

$$M_H=300\ \ \mathrm{GeV},\quad M_t=174.1\ \ \mathrm{GeV},\quad 0.1<\alpha_s<0.13.$$

In the method under investigation Eq. (65) should be changed by the formula

$$R_l=19.934[1+r(m_Z^2)]. \tag{66}$$

The calculated values of $r(m_z^2)$ and of R_l are presented in Tables III and IV in (1–4)-loop approximation. The value R_l can be compared with the measurement [12].

$$\Gamma(Z\to\mathrm{hadrons})=(1744.4\pm 2)\ \ \mathrm{MeV}, \tag{67}$$

$$\Gamma(Z\to\mathrm{leptons})=(83.984\pm 0.086)\ \ \mathrm{MeV}, \tag{68}$$

and

$$R_l=20.771\pm 0.045. \tag{69}$$

The value R_l (69) does not contradict R_l from Tables III and IV.

TABLE V. Comparison of the calculated values $R_T(s)$ with the measured values R_E [21].

E_{cm} (GeV)	R_T	R_E	E_{cm} (GeV)	R_T	R_E
2.000	2.29	2.18.$\pm$0.07$\pm$0.18	4.110	3.69	3.92$\pm$0.16$\pm$0.19
2.200	2.28	2.38$\pm$0.07$\pm$0.17	4.120	3.69	4.11$\pm$0.24$\pm$0.23
2.400	2.27	2.38$\pm$0.07$\pm$0.14	4.130	3.69	3.99$\pm$0.15$\pm$0.17
2.500	2.27	2.39$\pm$0.08$\pm$0.15	4.140	3.69	3.83$\pm$0.15$\pm$0.18
2.600	2.26	2.38$\pm$0.06$\pm$0.15	4.150	3.69	4.21$\pm$0.18$\pm$0.19
2.700	2.26	2.30$\pm$0.07$\pm$0.13	4.160	3.69	4.12$\pm$0.15$\pm$0.16
2.800	2.25	2.27$\pm$0.06$\pm$0.14	4.170	3.69	4.12$\pm$0.15$\pm$0.19
2.900	2.25	2.22$\pm$0.07$\pm$0.13	4.180	3.69	4.18$\pm$0.17$\pm$0.18
3.000	2.25	2.21$\pm$0.05$\pm$0.11	4.190	3.69	4.01$\pm$0.14$\pm$0.14
3.700	3.71	2.23$\pm$0.08$\pm$0.08	4.200	3.69	3.87$\pm$0.16$\pm$0.16
3.730	3.71	2.10$\pm$0.08$\pm$0.14	4.210	3.69	3.20$\pm$0.16$\pm$0.17
3.750	3.71	2.47$\pm$0.09$\pm$0.12	4.220	3.69	3.62$\pm$0.15$\pm$0.20
3.760	3.71	2.77$\pm$0.11$\pm$0.13	4.230	3.69	3.21$\pm$0.13$\pm$0.15
3.764	3.71	3.29$\pm$0.27$\pm$0.29	4.240	3.69	3.24$\pm$0.12$\pm$0.15
3.768	3.71	3.80$\pm$0.33$\pm$0.25	4.245	3.69	2.97$\pm$0.11$\pm$0.14
3.770	3.71	3.55$\pm$0.14$\pm$0.19	4.250	3.69	2.71$\pm$0.12$\pm$0.13
3.772	3.71	3.12$\pm$0.24$\pm$0.23	4.255	3.69	2.88$\pm$0.11$\pm$0.14
3.776	3.71	3.26$\pm$0.26$\pm$0.19	4.260	3.69	2.97$\pm$0.11$\pm$0.14
3.780	3.71	3.28$\pm$0.12$\pm$0.12	4.265	3.69	3.04$\pm$0.13$\pm$0.14
3.790	3.71	2.62$\pm$0.11$\pm$0.10	4.270	3.69	3.26$\pm$0.12$\pm$0.17
3.810	3.71	2.38$\pm$0.10$\pm$0.12	4.280	3.69	3.08$\pm$0.12$\pm$0.15
3.850	3.70	2.47$\pm$0.11$\pm$0.13	4.300	3.69	3.11$\pm$0.12$\pm$0.12
3.890	3.70	2.64$\pm$0.11$\pm$0.15	4.320	3.69	2.96$\pm$0.12$\pm$0.14
3.930	3.70	3.18$\pm$0.14$\pm$0.17	4.340	3.69	3.27$\pm$0.15$\pm$0.18
3.940	3.70	2.94$\pm$0.13$\pm$0.19	4.350	3.69	3.49$\pm$0.14$\pm$0.14
3.950	3.70	2.97$\pm$0.13$\pm$0.17	4.360	3.68	3.47$\pm$0.13$\pm$0.18
3.960	3.70	2.79$\pm$0.12$\pm$0.17	4.380	3.68	3.50$\pm$0.15$\pm$0.17
3.970	3.70	3.29$\pm$0.13$\pm$0.13	4.390	3.68	3.48$\pm$0.16$\pm$0.16
3.980	3.70	3.13$\pm$0.14$\pm$0.16	4.400	3.68	3.91$\pm$0.16$\pm$0.19
3.990	3.70	3.06$\pm$0.15$\pm$0.18	4.410	3.68	3.79$\pm$0.15$\pm$0.20
4.000	3.70	3.16$\pm$0.14$\pm$0.15	4.420	3.68	3.68$\pm$0.14$\pm$0.17
4.010	3.70	3.53$\pm$0.16$\pm$0.20	4.430	3.68	4.02$\pm$0.16$\pm$0.20
4.020	3.70	4.43$\pm$0.16$\pm$0.21	4.440	3.68	3.85$\pm$0.17$\pm$0.17
4.027	3.70	4.58$\pm$0.18$\pm$0.21	4.450	3.68	3.75$\pm$0.15$\pm$0.17
4.030	3.70	4.58$\pm$0.20$\pm$0.23	4.460	3.68	3.66$\pm$0.17$\pm$0.16
4.033	3.70	4.32$\pm$0.17$\pm$0.22	4.480	3.68	3.54$\pm$0.17$\pm$0.18
4.040	3.70	4.40$\pm$0.17$\pm$0.19	4.500	3.68	3.49$\pm$0.14$\pm$0.15
4.050	3.70	4.23$\pm$0.17$\pm$0.22	4.520	3.68	3.25$\pm$0.13$\pm$0.15
4.060	3.70	4.65$\pm$0.19$\pm$0.19	4.540	3.68	3.23$\pm$0.14$\pm$0.18
4.070	3.70	4.14$\pm$0.20$\pm$0.19	4.560	3.68	3.62$\pm$0.13$\pm$0.16
4.080	3.70	4.24$\pm$0.21$\pm$0.18	4.60	3.68	3.31$\pm$0.11$\pm$0.16
4.090	3.69	4.06$\pm$0.17$\pm$0.18	4.80	3.67	3.66$\pm$0.14$\pm$0.19
4.100	3.69	3.97$\pm$0.16$\pm$0.18			

X. ON ANALYTICITY OF $\alpha_s(q^2)$

In Ref. [30] the renormalization group was combined with the analyticity of $\alpha_s(q^2)$. It was assumed that $\alpha_s(q^2)$ is an analytic function of q^2 in the whole complex q^2 plane with a cut along the positive q^2 semiaxis. In particular, it was obtained that $\alpha_s(0)$ is universal and independent of the number of loops, Λ_3 and

TABLE VI. Comparison of the calculated values $R_T(s)$ with the measured values R_E [22–24].

s/Gev	R_T	R_E	$\sqrt{s}$/GeV	R_T	R_E
		Ref. [22]			
14.0	3.92	4.10±0.11±0.11	29.93	3.88	3.55±0.40±0.11
22.0	3.89	3.86±0.12±0.11	30.38	3.87	3.85±0.19±0.12
33.8	3.87	3.74±0.10±0.10	31.29	3.87	3.83±0.28±0.11
38.3	3.86	3.89±0.10±0.09	33.89	3.87	4.16±0.10±0.12
41.5	3.86	4.03±0.17±0.10	34.50	3.87	3.93±0.20±0.12
43.5	3.86	3.97±0.08±0.09	35.01	3.87	3.93±0.10±0.12
44.2	3.86	4.01±0.10±0.08	34.45	3.87	3.93±0.18±0.12
46.0	3.86	4.09±0.21±0.10	36.38	3.87	3.71±0.21±0.11
46.6	3.86	4.20±0.36±0.10	40.32	3.86	4.05±0.19±0.14
		Ref. [23]	41.18	3.86	4.21±0.22±0.14
29	3.88	3.96±0.09	42.55	3.86	4.20±0.22±0.14
		Ref [24]	43.53	3.86	4.00±0.20±0.14
12	3.93	3.45±0.27±0.13	44.41	3.86	3.98±0.20±0.14
14.04	3.92	3.94±0.14±0.14	45.59	3.86	4.40±0.22±0.15
22	3.89	4.11±0.13±0.12	46.47	3.86	4.04±0.24±0.14
25.01	3.88	4.24±0.29±0.13			
27.66	3.88	3.85±0.48±0.12			

$$\alpha_s(0)=4\pi/\beta_0^{(3)}=1.396. \tag{70}$$

The analogous formula had also been obtained in paper [1]. As is seen from Table III, Eq. (70) is valid only in one-loop approximation.

If $\alpha_s(q^2)$ has correct analyticity, the Adler function will have correct analyticity too, but a reverse statement is invalid. If the Adler function has correct analiticity, function $a(q^2)$ may have, generally speaking, additional singularities. This statement will be evident, if one considers Eq. (13) as an equation relative to $a(q^2)$. If only Eq. (13) is solved by expansion in $a(q^2)$, analyticity $a(q^2)$ will be the same as the Adler function. However, using of $d(q^2)$ expansion in $a(q^2)$ at small q^2 seems to be doubtful.

XI. CONCLUSION

In conclusion, let us formulate the main results of this paper.

(1) There are two and only two values of Λ_3, at which $R_{\tau,V+A}=3.475\pm0.022$, one conventional value $\Lambda_3^{\text{conv}}=(618\pm29)$ MeV [Λ_3 is defined by Eq. (7)] and the other, found in this paper, $\Lambda_3^{\text{new}}=(1666\pm7)$ MeV.

(2) The renormalization group calculation leads to the appearance of a nonphysical cut in the Adler function. In complete theory, where everything is taken into account, the nonphysical cut must be absent. The question arises, if it is possible to neglect the nonphysical cut at the present situation with the theory. At conventional value $\Lambda_3=\Lambda_3^{\text{conv}}$ it is impossible. The new value, $\Lambda_3=\Lambda_3^{\text{new}}$, is more preferable than $\Lambda_3=\Lambda_3^{\text{conv}}$, since at $\Lambda_3=\Lambda_3^{\text{new}}$ the contribution of the nonphysical cut into $R_{\tau,V+A}$ is practically absent.

(3) At $\Lambda_3=\Lambda_3^{\text{conv}}$ there is an essential disagreement between the ALEPH experiment and the new obtained sum rules, which follow only from analytical properties of the polarization operator. This disagreement disappears if $\Lambda_3=\Lambda_3^{\text{new}}$.

(4) In (1–4)-loop approximation all calculations are made exactly, without $\pi/\ln(Q^2/\Lambda^2)$ expansion.

(5) At $\Lambda_3^{\text{new}}=(1565\pm193)$ MeV the nonphysical cut may be omitted, the polarization operator has correct analytical properties, and $R_{\tau,V+A}=3.475\pm0.022$. But in this case QCD parameters essentially differ from conventional values.

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