$(g-2)_\mu$ anomaly, Higgs bosons, and heavy neutrinos

G. G. Boyarkina and O. M. Boyarkin*

Byelorussian State Pedagogical University, Soviet Street 18, Minsk, 220050, Belarus

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Within the model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group and having a bidoublet and two triplets of the Higgs fields (left-right model), the Higgs sector impact on the value of the muon anomalous magnetic moment (AMM) is considered. The contributions coming from the doubly charged Higgs bosons, the singly charged Higgs bosons, and the lightest neutral Higgs boson are taken into account. The obtained value of the muon AMM is a function of the Higgs boson masses and the Higgs boson coupling constants (CC's). We express the largest part of the CC's as a function of the heavy neutrino sector parameters. We show that at the particular parameter values the model under study could explain the BNL 2000 result.

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I. INTRODUCTION

The measurement of the spin magnetic dipole moment of particles has a rich history as a harbinger of impressive progress in quantum theory. Thus, registration of the anomalous values of the nucleon magnetic moments was a powerful argument benefiting the π-mesonic theory of nuclear forces formulated by Yukawa. The determination of the anomalous magnetic moment (AMM) of the electron has played an important role in the development of modern quantum electrodynamics and renormalization theory. It appeared to be reasonable that the ongoing muon $(g-2)_\mu$ measurement E821 at Brookhaven National Laboratory (BNL) would be a sensitive test for the results of the standard model electroweak corrections. However, the measurements of the muon AMM indicate a deviation δa_μ from the theoretical value predicted by the standard model (SM). Since the E821 data have been thoroughly collected and studied over many years, it is most unlikely that this discrepancy also could be explained as a mere statistical fluctuation, as several earlier deviations from the SM turned out to be. Attention is drawn to the fact of the extremely small variation of the muon AMM central value in all the BNL results presented up to now. This circumstance could be a weighty argument in favor of the trustworthiness of the E821 experiment. While it is often argued that the SM should be augmented by new physics at higher energy scales because of some unanswered fundamental questions, the $(g-2)_\mu$ anomaly with such phenomena as the neutrino oscillations [1], 3σ departure of $\sin^2\theta_W$ from the SM predictions measured in deep inelastic neutrino-nucleon scattering [2], and the observation of the neutrinoless double beta decay [3] may already serve as a new physics signal at the weak scale.

Suggestions already made in the literature for explaining δa_μ include supersymmetry [4], additional gauge bosons [5], anomalous gauge boson couplings [6], leptoquarks [7], extra dimensions [8], muon substructure [9], exotic flavor-changing interactions [10], exotic vectorlike fermions [11], possible nonperturbative effects at the 1 TeV order [12], the violation of CPT and Lorentz invariance [13], and so on.

Some explanations of the E821 experiment turn out to be excluded by the current experimental data. To cite some examples: The possibility of muon substructure can be immediately ruled out since the necessary compositeness scale of the muon should already have been seen in processes involving highly energetic muons at the CERN e^+e^- collider LEP, DESY ep collider HERA, and the Fermilab Tevatron.

For the anomalous W-boson dipole magnetic moment

$$\mu_W = \frac{e}{2m_W}(1+\kappa_\gamma)$$

the additional one-loop contribution to a_μ is given by the expression

$$a_\mu(\kappa_\gamma) \approx \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \ln\left(\frac{\Lambda^2}{m_W^2}\right)(\kappa_\gamma - 1),$$

where Λ is the high momentum cutoff required to give a finite result. For $\Lambda \approx 1$ TeV, in order to obtain accord between theory and observation, one should demand

$$\delta\kappa_\gamma \equiv \kappa_\gamma - 1 \approx 0.4.$$

However, such a big value of $\delta\kappa_\gamma$ is already eliminated by the $e^+e^- \to W^+W^-$ data at LEP II, which give [14]

$$\delta\kappa_\gamma = 0.08 \pm 0.17.$$

In this manner, at the moment the $(g-2)_\mu$ anomaly plays the role of an Occam's razor for the existing SM extensions.

The purpose of this work is to investigate the $(g-2)_\mu$ anomaly within the left-right model (LRM) based on the gauge group $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$. One-loop contributions to a_μ from extra gauge bosons have been calculated in Ref. [15]. However, the contribution coming from the Z_2 gauge boson is negative, while in order to accommodate the discrepancy between the BNL results and the LRM prediction the mass value of the W_2 gauge boson should lie around 100 GeV, which is clearly ruled out by direct searches and precision measurements [14]. In the LRM the Higgs bosons

*Email address: boyarkin@bspu.unibel.by

may also appear to be candidates for particles generating significant contributions to the muon AMM.

The paper is organized as follows. In the next section we discuss the BNL results and the SM theoretical predictions. In Sec. III we consider the structure of both the Higgs boson and the lepton sectors in the LRM. There we establish the connection between the Higgs boson coupling constants (CC's) and the neutrino oscillation parameters. In Sec. IV we discuss the current constraints on the LRM parameters. Section V is devoted to computation of the Higgs boson contributions to the muon AMM. In Sec. VI we compare the theoretical and experimental values of a_μ and find the bounds on the Higgs boson sector parameters which provide in their turn information on the masses and the mixing angles of the heavy neutrinos. Section VII is devoted to analysis of the results obtained.

II. THE BNL RESULTS AND THE SM THEORETICAL PREDICTIONS

The first BNL result based on the data taken through 1997 was [16]

$$a_\mu^{expt} = (116\,592\,500 \pm 1500) \times 10^{-11}\mu_0 \quad (\text{BNL 1997}), \tag{1}$$

where μ_0 is the muon magnetic moment predicted by the Dirac theory. The 1998 and 1999 runs had much higher statistics and gave results with increased precision:

$$a_\mu^{expt} = (116\,591\,910 \pm 590) \times 10^{-11}\mu_0 \quad (\text{BNL 1998 [17]}), \tag{2}$$

$$a_\mu^{expt} = (116\,592\,020 \pm 160) \times 10^{-11}\mu_0 \quad (\text{BNL 1999 [18]}). \tag{3}$$

The BNL 1998 and BNL 1999 results averaged with older measurements made at CERN [19] gave the following value of the muon AMM:

$$a_\mu^{expt} = (116\,592\,023 \pm 151) \times 10^{-11}\mu_0 . \tag{4}$$

In the SM the expression for the muon AMM can be presented as the sum

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}, \tag{5}$$

in which $a_\mu^{QED} = 11\,658\,470.57(0.29) \times 10^{-10}\mu_0$ (see [20] and references therein) and $a_\mu^{EW} = 15.2(0.4) \times 10^{-10}$ (see [21] and references therein).

The term a_μ^{had} arises from virtual hadronic contributions to the photon propagator in fourth $a_\mu^{had}(VP1)$ and sixth orders, where the latter includes hadronic vacuum polarization $a_\mu^{had}(VP2)$ and light-by-light scattering $a_\mu^{had}(LbyL)$. The dominant contribution to a_μ^{had} as well as one of the largest ambiguities in its value come from $a_\mu^{had}(VP1)$. $a_\mu^{had}(VP1)$ was derived in Ref. [22] from the e^+e^- hadronic cross section and the hadronic τ decay data:

$$a_\mu^{had}(VP1) = 6924(62) \times 10^{-11}\mu_0. \tag{6}$$

Evolution of the three-loop hadron vacuum polarization contribution $a_\mu^{had}(VP2)$ has given the result [23]

$$a_\mu^{had}(VP2) = -100(6) \times 10^{-11}\mu_0 . \tag{7}$$

It is important to keep in mind that all the estimations of the $LbyL$ scattering contribution $a_\mu^{had}(LbyL)$ made so far are model dependent. The calculations are based on the chiral perturbation or extended Nambu–Jona-Lasinio model. Also, vector meson dominance is assumed and the phenomenological parametrization of the pion form factor $\pi\gamma^*\gamma^*$ is introduced in order to regularize the divergence. The previous average value for $a_\mu^{had}(LbL)$ is given by [24,25]

$$a_\mu^{had}(LbyL) = -85(25) \times 10^{-11}\mu_0 . \tag{8}$$

With this value of the $LbyL$ hadronic correction the total SM prediction of a_μ^{SM} was

$$a_\mu^{SM} = 116\,591\,597(67) \times 10^{-11}\mu_0 . \tag{9}$$

Comparing Eq. (9) with the experimental average in Eq. (4) one finds

$$\delta a_\mu \equiv a_\mu^{expt} - a_\mu^{SM} = 426(165) \times 10^{-11}\mu_0 . \tag{10}$$

Equation (10) means that there is a 2.6σ deviation between experiment and the SM prediction.

Recently, the theoretical prediction for $a_\mu^{had}(LbyL)$ has undergone a significant revision because of a change in sign. The overestimations have given the following values for $a_\mu^{had}(LbyL)$:

$$a_\mu^{had}(LbyL) = \begin{cases} 83(12) \times 10^{-11}\mu_0 & [26], \\ 89(15) \times 10^{-11}\mu_0 & [27], \\ 83(32) \times 10^{-11}\mu_0 & [28]. \end{cases} \tag{11}$$

Taking the average of these new results one finds

$$a_\mu^{SM} = 116\,591\,770(70) \times 10^{-11}\mu_0 . \tag{12}$$

Using Eqs. (4) and (12) one obtains

$$\delta a_\mu = 260(160) \times 10^{-11}\mu_0 . \tag{13}$$

Thus the deviation value has dropped from 2.6σ up to 1.6σ.

On July 30, 2002 the Muon $g-2$ Collaboration announced a new result based on the μ^+ data collected in the year 2000 [29]:

$$a_\mu^{expt} = 116\,592\,040(70)(50) \times 10^{-11}\mu_0 \quad (\text{BNL 2000}). \tag{14}$$

The uncertainty of BNL 2000 is almost two times smaller than in BNL 1999 and only two times larger than the final aim of the E821 experiment. With this new result the present world average experimental value is

$$a_\mu^{expt} = 116\,592\,030(80) \times 10^{-11}\mu_0 . \tag{15}$$

Improved calculations of $a_\mu^{had}(VP1)$ have been presented recently [30,31]. These are data-driven analyses using the most recent data from the e^+e^- hadronic cross section observed at the CMD-2, BES, and SND experiments [32]. Their precision $\sim 58\times10^{-11}$ is now even smaller than those in Eq. (6). Further on for $a_\mu^{had}(VP1)$ we shall use the result of Ref. [30], where the experimental input is based only on the e^+e^- data:

$$a_\mu^{had}(VP1)=6889(58)\times10^{-11}\mu_0. \tag{16}$$

For estimation of $a_\mu^{had}(LbyL)$ we invoke the new result obtained in [33]:

$$a_\mu^{had}(LbyL)=80(40)\times10^{-11}\mu_0. \tag{17}$$

Then with the help of Eqs. (7), (16), and (17) the full hadronic contributions are given by

$$a_\mu^{had}=6869(71)\times10^{-11}\mu_0. \tag{18}$$

This leads us to the SM prediction

$$a_\mu^{SM}=116\,591\,726.7(70.9)\times10^{-11}\mu_0. \tag{19}$$

So, at present the deviation between experimental data and the SM prediction has reached the value

$$\delta a_\mu=303.3(106.9)\times10^{-11}\mu_0, \tag{20}$$

that is, the deviation is roughly about 3σ.

If the deviation of Eq. (20) can be attributed to the effects of physics beyond the SM, then at 95% C.L., $\delta a_\mu/\mu_0$ must lie in the range

$$93.8\times10^{-11}\leqslant\frac{\delta a_\mu}{\mu_0}\leqslant512.8\times10^{-11}. \tag{21}$$

This contribution is positive, and has the same order as the electroweak corrections to a_μ, namely, $\sim G_F m_\mu^2/(4\pi^2\sqrt{2})$.

III. THE LEFT-RIGHT MODEL

In the SM, the Higgs boson contribution to a_μ is negligible because $\overline{\mu}\mu h$ coupling is extremely small, namely, $\sim m_\mu/v$, where v is the vacuum expectation value (VEV) being equal to 246 GeV. In the LRM the Higgs boson sector is much richer than that in the SM. It includes four doubly charged scalars $\Delta_{1,2}^{(\pm\pm)}$, four singly charged scalars $h^{(\pm)}$ and $\tilde{\delta}^{(\pm)}$, four neutral scalars S_i $(i=1,2,3,4)$, and two neutral pseudoscalars $P_{1,2}$. The current experimental data allow some of these Higgs bosons to have masses around the electroweak scale and couplings of at least electroweak strength. It is well to bear in mind that among the extensions of the SM the LRM is of special interest because its Higgs boson sector contains elements belonging to other models very popular nowadays. The presence of the bidoublet in the LRM causes the existence of the same physical Higgs bosons as in the two Higgs doublet modification of the SM (THDM) [34] and in the MSSM [35]. Owing to the availability of the triplets the LRM has the Higgs bosons that are present in the model based on the $SU(3)_L\times U(1)_N$ gauge group [36].

One more fascinating property of the LRM resides in the fact that the LRM belongs among the models in which the Higgs boson coupling constants determining the interaction of the Higgs bosons both with leptons and with gauge bosons are connected to the neutrino oscillation parameters (NOP's). Therefore, in such models the bounds obtained on the Higgs boson sector parameters could be extended to the bounds on the NOP's.

In the LRM the choice of the Yukawa potential has an influence on the form of the Lagrangian describing the Higgs boson interactions both with fermions and with gauge bosons. The most general Yukawa potential $\mathcal{L}_Y^g$ has been proposed in [37]. In spite of the fact that $\mathcal{L}_Y^g$ has a very complicated form, the diagonalization of the charged Higgs boson mass matrix presents no special problems. However, for the neutral Higgs boson mass matrix M_n this procedure can only be realized when some simplifications in $\mathcal{L}_Y^g$ have been done [38]. For example, the matrix M_n can be diagonalized in the following conditions (we use the same notation as in Ref. [37]):

$$\alpha_1=\frac{2\alpha_2k_2}{k_1},\quad \alpha_3=\frac{2\alpha_2k_-^2}{k_1k_2},\quad \beta_1=\frac{2\beta_3k_2}{k_1}, \tag{22}$$

where $\alpha_{1,2,3}$ and $\beta_{1,2}$ are the constants entering into the Yukawa potential, k_1,k_2 are the VEV's of the neutral components of the Higgs bidoublet, and $k_\pm=\sqrt{k_1^2\pm k_2^2}$ $(k_+=174\ \text{GeV})$.

Of all the Higgs bosons, the $\Delta_{1,2}^{(\pm\pm)}$, $h^{(\pm)}$, $\tilde{\delta}^{(\pm)}$, and S_1 bosons have been of our main interest here because the current data allow their masses to lie on the electroweak scale (recall that the S_1 boson is the analogue of the SM Higgs boson). Assuming the conditions (22) to be satisfied one obtains the squared masses of these particles:

$$m_h^2=\alpha(v_R^2+k_0^2)+\frac{\beta_1^2k_+^4k_0^2}{k_-^4(\alpha+\rho_1-\rho_3/2)}, \tag{23}$$

$$m_{\tilde{\delta}}^2=(\rho_3/2-\rho_1)v_R^2-\frac{\beta_1^2k_+^4k_0^2}{k_-^4(\alpha+\rho_1-\rho_3/2)}, \tag{24}$$

$$m_{\Delta_1}^2=\frac{\alpha_3k_-^2+4\rho_2v_R^2}{2}+\frac{k_-^4(\beta_3k_+^2+\beta_1k_1k_2)^2}{2k_1^4(4\rho_2+\rho_3-2\rho_1)v_R^2}, \tag{25}$$

$$m_{\Delta_2}^2=\frac{\alpha_3k_-^2-(2\rho_1-\rho_3)v_R^2}{2}-\frac{k_-^4(\beta_3k_+^2+\beta_1k_1k_2)^2}{2k_1^4(4\rho_2+\rho_3-2\rho_1)v_R^2}, \tag{26}$$

$$m_{S_1}^2=2\lambda_1 k_+^2+8k_1^2k_2^2(2\lambda_2+\lambda_3)/k_+^2-8\lambda_4 k_1 k_2 + \frac{4k_1k_2k_-^4[2(2\lambda_2+\lambda_3)k_1k_2/k_+^2-\lambda_4]^2}{\alpha_2 v_R^2 k_+^2}, \quad (27)$$

where

$$\alpha=\frac{\alpha_3 k_+^2}{2k_-^2}=\frac{\alpha_3(1+\tan^2\beta)}{2(\tan^2\beta-1)},$$

$$\beta_0=\frac{\beta_1 k_+^2}{k_-^2}=\frac{\beta_1(1+\tan^2\beta)}{(\tan^2\beta-1)},$$

$$k_0=\frac{k_-^2}{\sqrt{2}k_+}=\frac{k_+(\tan^2\beta-1)}{\sqrt{2}(1+\tan^2\beta)},$$

$\rho_{1,3}$ are the constants entering into the Yukawa potential, $\tan\beta=k_1/k_2$, and v_R is the VEV of the neutral component of the right-handed Higgs triplet, $v_R\gg\max(k_1,k_2)$. From the relations (24) and (26) it follows that the masses of the $\tilde{\delta}^{(\pm)}$ and $\Delta_2^{(\pm\pm)}$ bosons are very close to each other. From expressions (23)–(26) likewise, it is obvious that in order that the $h^{(\pm)}$, $\tilde{\delta}^{(\pm)}$, and $\Delta_{1,2}^{(\pm\pm)}$ boson masses are on the electroweak scale the constants α_3, ρ_2, and $(\rho_3/2-\rho_1)$ should have the order of $\sim 10^{-2}$.

The Lagrangians that are required for our purposes are given by the expressions

$$\mathcal{L}_{\gamma\Delta\Delta}=2ie\{[\partial_\mu\Delta_1^{(--)*}(x)]\Delta_1^{(--)}(x)-\Delta_1^{(--)*}(x) \times[\partial_\mu\Delta_1^{(--)}(x)]\}+(1\to 2), \quad (28)$$

$$\mathcal{L}_l^{dc}=-\sum_{a,b}\frac{f_{ab}}{2}[\overline{l_a^c}(x)(1+\gamma_5)l_b(x)c_{\theta_d} -\overline{l_a^c}(x)(1-\gamma_5)l_b(x)s_{\theta_d}]\Delta_1^{(--)*}(x) +\left(1\to 2,\theta_d\to\theta_d-\frac{\pi}{2}\right)+\text{conj.}, \quad (29)$$

$$\mathcal{L}_{W_1\gamma h}=\frac{g_R e m_{W_1}(1-\tan^2\beta)(\alpha-\rho_3/2+\rho_1+1)s_\xi}{g_L(1+\tan^2\beta)} \times h^{(-)*}(x)W_{1\mu}(x)A_\mu(x)+\text{conj.}, \quad (30)$$

$$\mathcal{L}_{W_2\gamma h}=\mathcal{L}_{W_1\gamma h}(s_\xi\to c_\xi), \quad (31)$$

$$\mathcal{L}_{W_1\gamma\tilde{\delta}}=g_R e g_L^{-1}\beta_1 m_{W_1}s_\xi\tilde{\delta}^{(-)*}(x)W_{1\mu}(x)A_\mu(x)+\text{conj.}, \quad (32)$$

$$\mathcal{L}_{W_2\gamma\tilde{\delta}}=\mathcal{L}_{W_1\gamma\tilde{\delta}}(s_\xi\to c_\xi), \quad (33)$$

$$\mathcal{L}_l^{sc}=\sum_{a,b}\left\{\left[\frac{h'_{ab}k_2-h_{ab}k_1}{2k_+}\overline{\nu_a}(x)(1-\gamma_5)l_b(x) -\frac{h_{ab}k_2-h'_{ab}k_1}{2k_+}\overline{N_a}(x)(1+\gamma_5)l_b(x)\right]h^{(-)*}(x) +\frac{f_{ab}}{\sqrt{2}}\left[\overline{l_a^c}(x)(1+\gamma_5)\nu_b(x) \times\left(\frac{\beta_0 k_0^2}{(\alpha+\rho_1-\rho_3/2)v_R^2}h^{(-)*}(x)-\tilde{\delta}^{(-)*}(x)\right) +\overline{l_a^c}(x)(1-\gamma_5)N_b(x)\left(\frac{k_0}{v_R}h^{(-)*}(x) +\frac{\beta_0 k_0}{(\alpha+\rho_1-\rho_3/2)v_R}\tilde{\delta}^{(-)*}(x)\right)\right]+\text{conj.}\right\}, \quad (34)$$

$$\mathcal{L}^n=-\frac{1}{\sqrt{2}k_+}\left\{\sum_{a,b}\overline{l}_a(x)l_b(x)[(h_{ab}k_1+h'_{ab}k_2)s_{\theta_0} +(h'_{ab}k_1-h_{ab}k_2)c_{\theta_0}]\right\}S_1(x), \quad (35)$$

where the superscript c denotes the charge conjugation operation, $c_{\theta_d}=\cos\theta_d$, $s_{\theta_d}=\sin\theta_d$, θ_d is the mixing angle of the doubly charged Higgs bosons ($\tan\theta_d\sim k_+^2/v_R^2$), f_{ab} is the Yukawa triplet coupling constant, g_R is the gauge coupling of the $SU(2)_R$ subgroup (further on we shall speculate that $g_L=g_R$), $N_a(x)$ describes the heavy neutrino with flavor a, ξ is the mixing angle of the charged gauge bosons, and the angle θ_0 is determined by the Yukawa potential parameters and the VEV's:

$$\tan 2\theta_0=\frac{4k_1k_2k_-^2[-2(2\lambda_2+\lambda_3)k_1k_2+\lambda_4k_+^2]}{k_1k_2[(4\lambda_2+2\lambda_3)(k_-^4-4k_1^2k_2^2)-k_+^2(2\lambda_1k_+^2-8\lambda_4k_1k_2)]+\alpha_2v_R^2k_+^4}. \quad (36)$$

The impact of the Yukawa potential choice upon the physical results can easily be seen by the example of the Lagrangian (35). Really, when in the condition (22) the change $k_2 \to -k_2$ is carried out; then instead of Eq. (35) one obtains

$$\mathcal{L}^n = -\frac{1}{\sqrt{2}k_+}\left\{\sum_a m_a \overline{l}_a(x) l_a(x) c_{\theta_0} + \sum_{a,b} \overline{l}_a(x) l_b(x)(h_{ab}k_1 - h'_{ab}k_2) s_{\theta_0}\right\} S_1(x). \quad (37)$$

Since $\tan\beta_0 \sim k_-^2/v_R^2$ and for the muon $m_\mu/k_+ \sim 6\times 10^{-3}$, then, as an example, the cross section of the electron-muon recharge,

$$e^-\mu^+ \to e^+\mu^-,$$

never could have the resonance peak connected with the S_1 boson when the Lagrangian (37) is used [38], while the existence of such a peak could be quite possible when one works with the Lagrangian (35) [39].

Let us express the CC's in terms of the lepton sector parameters. It in turn will also allow us to measure the CC's in the processes including only the leptons. We shall find the connection between the Higgs boson sector parameters and the lepton sector parameters restricted ourselves to the two-flavor approximation. For this purpose we need the neutrino mass matrix $\mathcal{M}$. Once one chooses the basis $\Psi^T = (\nu_{aL}^T, N_{aR}^T, \nu_{bL}^T, N_{bR}^T)$, the $\mathcal{M}$ takes the form

$$\mathcal{M} = \begin{pmatrix} f_{aa}v_L & m_D^a & f_{ab}v_L & M_D \\ m_D^a & f_{aa}v_R & M_D & f_{ab}v_R \\ f_{ab}v_L & M_D & f_{bb}v_L & m_D^b \\ M_D & f_{ab}v_R & m_D^b & f_{bb}v_R \end{pmatrix}, \quad (38)$$

where v_L is the VEV of the neutral component of the left-handed Higgs triplet $[v_L \ll (\max(k_1,k_2)]$ and

$$m_D^a = h_{aa}k_1 + h'_{aa}k_2, \quad (39)$$

$$M_D = h_{ab}k_1 + h'_{ab}k_2. \quad (40)$$

In their turn the elements of the matrix $\mathcal{M}$ are connected with the neutrino oscillations parameters [38,40]

$$m_D^a = c_{\varphi_a}s_{\varphi_a}(-m_1 c_{\theta_\nu}^2 - m_3 s_{\theta_\nu}^2 + m_2 c_{\theta_N}^2 + m_4 s_{\theta_N}^2),$$

$$m_D^b = m_D^a(\varphi_a \to \varphi_b, \theta_{\nu,N} \to \theta_{\nu,N} + \pi/2), \quad (41)$$

$$M_D = c_{\varphi_a}s_{\varphi_b}c_{\theta_\nu}s_{\theta_\nu}(m_1 - m_3) + s_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_4 - m_2), \quad (42)$$

$$f_{ab}v_R = s_{\varphi_a}s_{\varphi_b}c_{\theta_\nu}s_{\theta_\nu}(m_3 - m_1) + c_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_4 - m_2), \quad (43)$$

$$f_{aa}v_R = (s_{\varphi_a}c_{\theta_\nu})^2 m_1 + (c_{\varphi_a}c_{\theta_N})^2 m_2 + (s_{\varphi_a}s_{\theta_\nu})^2 m_3 + (c_{\varphi_a}s_{\theta_N})^2 m_4, \quad (44)$$

$$f_{bb}v_R = f_{aa}v_R(\varphi_a \to \varphi_b + \pi/2, \theta_N \to \theta_N + \pi/2),$$

$$f_{ll'}v_L = f_{ll'}v_R\left(\varphi_{l,l'} \to \varphi_{l,l'} + \frac{\pi}{2}\right), \quad l,l' = a,b, \quad (45)$$

where φ_a is the mixing angle in the a generation between the light neutrino and the heavy neutrino (recall that the ν_a neutrino enters into the left-handed lepton doublet

$$\begin{pmatrix} \nu_a \\ l_a \end{pmatrix}_L,$$

while the N_a neutrino enters the right-handed lepton doublet

$$\begin{pmatrix} N_a \\ l_a \end{pmatrix}_R,$$

respectively), θ_ν (θ_N) is the mixing angle between the ν_{aL} neutrino and the ν_{bL} neutrino (N_{aR} and N_{bR}), $c_{\varphi_a} = \cos\varphi_a$, $s_{\varphi_a} = \sin\varphi_a$ and so on. As $m_\nu \ll m_N$, then with the help of Eqs. (43) and (45) it is possible to find a relationship for an estimation of the mixing angles between light and heavy neutrinos. Further on, we shall assume that mixing takes place between the μ and τ generations ($a=\mu, b=\tau$) only. Then for the mixing angles we obtain

$$\sin 2\varphi_\mu \approx \frac{f_{\mu\mu}\sqrt{v_R v_L}}{c_{\theta_N}^2 m_{N_\mu} + s_{\theta_N}^2 m_{N_\tau}}, \quad (46)$$

$$\sin 2\varphi_\tau \approx \frac{f_{\tau\tau}\sqrt{v_R v_L}}{s_{\theta_N}^2 m_{N_\mu} + c_{\theta_N}^2 m_{N_\tau}}. \quad (47)$$

The estimation of v_L can be done with the help of the quantity

$$\rho = \frac{m_Z^2 c_W^2}{m_W^2}.$$

In the LRM the quantity ρ is defined by the relation [41]

$$\rho=\frac{1+4x}{1+2x}, \tag{48}$$

where

$$x=\left(\frac{v_L}{k_+}\right)^2.$$

As the experiment for today yields

$$\rho=1.0107\pm 0.0006,$$

then the value of v_L can reach 13 GeV.

Taking into consideration both the definition of m_D^a [Eqs. (39),(41)] and the formulas for the charged lepton masses

$$m_{l_a}=h_{aa}k_2+h'_{aa}k_1, \tag{49}$$

it is not difficult to obtain

$$\begin{aligned}\alpha_{\bar{l}_a\nu_a h}&=\frac{h'_{aa}k_2-h_{aa}k_1}{2k_+}\\&=\frac{1+\tan^2\beta}{2k_+(1-\tan^2\beta)}\left(\frac{2m_{l_a}\tan\beta}{1+\tan^2\beta}+m_D^a\right)\\&\approx\frac{1+\tan^2\beta}{2k_+(1-\tan^2\beta)}\left[\frac{2m_{l_a}\tan\beta}{1+\tan^2\beta}\right.\\&\quad\left.+c_{\varphi_a}s_{\varphi_a}(m_2c^2_{\theta_N}+m_4s^2_{\theta_N})\right].\end{aligned} \tag{50}$$

The analogous mathematics for $\alpha_{\bar{l}_aN_ah}$ leads to the expression

$$\begin{aligned}\alpha_{\bar{l}_aN_ah}&=\frac{h'_{aa}k_1-h_{aa}k_2}{2k_+}\\&=\frac{1+\tan^2\beta}{2k_+(1-\tan^2\beta)}\left(\frac{2m_D^a\tan\beta}{1+\tan^2\beta}-m_{l_a}\right)\\&\approx\frac{1+\tan^2\beta}{2k_+(1-\tan^2\beta)}\\&\quad\times\left[\frac{2c_{\varphi_a}s_{\varphi_a}(m_2c^2_{\theta_N}+m_4s^2_{\theta_N})\tan\beta}{1+\tan^2\beta}-m_{l_a}\right].\end{aligned} \tag{51}$$

It is pertinent to note that there is a connection between the coupling constants $\alpha_{\bar{l}_aN_bh}$ and $\alpha_{\bar{l}_al_bS_1}$:

$$\alpha_{\bar{l}_aN_bh}\approx\frac{\alpha_{\bar{l}_al_bS_1}}{\sqrt{2}}. \tag{52}$$

The next step is the determination of the nondiagonal CC's. To suppress the mixing in the charged lepton sector (between l_a and l_b) it is necessary to demand

$$h_{ab}k_2+h'_{ab}k_1=0. \tag{53}$$

Then, with regard to the definitions of the quantity M_D [Eqs. (40) and (42)] one obtains

$$\alpha_{\bar{l}_a\nu_bh}=-\frac{M_D}{2k_+}\approx-\frac{s_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_4-m_2)}{2k_+}, \tag{54}$$

$$\begin{aligned}\alpha_{\bar{l}_aN_bh}&=-\frac{M_D\tan\beta}{k_+(1+\tan^2\beta)}\\&\approx-\frac{s_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_4-m_2)\tan\beta}{k_+(1+\tan^2\beta)}.\end{aligned} \tag{55}$$

IV. THE EXISTING CONSTRAINTS ON THE LRM PARAMETERS

We shall start with a discussion about the constraints on the Higgs boson masses. The present limit on the singly charged Higgs boson mass has been obtained within the THDM by investigation of the reaction

$$e^+e^-\to H^+H^-. \tag{56}$$

The lowest value for the mass of the charged Higgs boson, independent of its branching ratio, is currently 78.6 GeV [14]. It is evident that this limit may be broken for singly charged Higgs bosons of the LRM $\tilde{\delta}^{(\pm)}$ and $h^{(\pm)}$. Really, in the THDM the charged Higgs boson interacts with the quarks at the tree level while in the LRM such interaction exists for the $h^{(\pm)}$ boson only. Furthermore, in both models the coupling constants of the charged Higgs bosons with the Z boson are not equal to each other:

$$\frac{(g_{HHZ})_{2HDM}}{(g_{\tilde{\delta}\tilde{\delta}Z})_{LRM}}=\frac{\cot 2\theta_W}{g'\cos^{-1}\theta_W(\alpha+\rho_1-\rho_3/2)(g'^{1}\sin\theta_W\cos\Phi+g_R^{-1}\sin\Phi)}, \tag{57}$$

$$\frac{(g_{HHZ})_{2HDM}}{(g_{hhZ})_{LRM}}=-\frac{\cot 2\theta_W}{\cos^{-1}\theta_W(\sin^{-1}\theta_W\cos 2\theta_W\cos\Phi+g_Rg'^{-1}\sin\Phi)/2}. \tag{58}$$

However, since an analysis of the process (56) from the LRM point of view is absent up to now, we shall assume that the lower bound on the masses of the singly charged Higgs bosons of the LRM is 78.6 GeV too.

As regards the doubly charged Higgs boson mass, the situation is somewhat more simple. The $\Delta_{1,2}^{(\pm\pm)}$ bosons are typical representatives of the LRM. For this reason experiments aimed at their appearance are analyzed from the LRM point of view only. The current lower bounds on their masses obtained by the OPAL Collaboration at 95% C.L. [42] are 98.5 GeV.

We also should discuss the implementation of the lower bound 115 GeV on the mass of the lightest neutral Higgs boson (LNH) in the SM extensions. This bound has been obtained on LEP II by investigation of the reaction

$$e^+e^-\to Z^*\to Zh \tag{59}$$

from the SM point of view [43]. The reaction (59) is analyzed for the four Zh decay channels

$$Zh\to q\bar{q}q'\bar{q}',q\bar{q}\nu\bar{\nu},q\bar{q}l_a\bar{l}_a(l_a=e,\mu),\tau^+\tau^-q\bar{q},$$

where the final state $h\to q\bar{q}$ includes both the quark-antiquark and the gluon-gluon pairs. For the THDM's, as an example, the substantial deviations from the SM are present both in the cross section $\sigma_{e^+e^-\to Zh}$ and in the decay widths Γ_h. Since in these models the coupling constant of the neutral CP-even h boson (analogue of the SM Higgs boson) with the Z boson has the form

$$g_{ZZh}=\frac{g_L m_Z\sin(\beta-\alpha)}{c_{\theta_W}} \tag{60}$$

where $\sin(\beta-\alpha)\sim 0$, then $(\sigma_{e^+e^-\to Zh})_{THDM}$ is much less relative to the SM value. On the other hand, as the relation takes place

$$\frac{(g_{f\bar{f}h})_{THDM}}{(g_{f\bar{f}h})_{SM}}\approx\tan\beta,\qquad f=b,\tau,$$

the h-boson decay widths through quarks and leptons have greater values than those in the SM. It is evident that analysis of the reaction (59) leads to different mass values for the SM Higgs boson and the LNH boson of the THDM. Actually, when $|\sin(\beta-\alpha)|\leqslant 0.06$ the LEP data result in $m_h\sim 10$ GeV at 98% C.L. [44].

The only reason that the LRM cross section $\sigma_{e^+e^-\to ZS_1}$ could not coincide with that of the SM is the coupling of the S_1 boson with the Z_1 boson:

$$g_{Z_1Z_1S_1}=\frac{g_L s_W^2 m_{Z_1}[c_{\theta_0}(1-\tan^2\beta)-2\,s_{\theta_0}\tan\beta)]}{2\,c_W(\tan^2\beta+1)}\times(2g_Rg'^{-1}c_\Phi\,s_\Phi\,s_W^{-1}-c_\Phi^2\,s_W^{-2}-g_R^2g'^{-2}\,s_\Phi^2), \tag{61}$$

where Φ is the mixing angle of Z_1 and Z_2 bosons ($\Phi\approx 10^{-2}$–10^{-3}) and g' is the gauge constant of subgroup $U(1)_{B-L}$. When

$$\Phi=k_2=0$$

then $g_{Z_1Z_1S_1}$ converts to the constant describing the interaction between the Higgs boson and the Z boson in the SM. Since the symmetric LRM reproduces the SM under the following values of g_R and g':

$$g_L=g_R=e\,s_W^{-1},\quad g'=e\sqrt{c_W^2-s_W^2}, \tag{62}$$

then the quantity $g_Rg'^{-1}$ may differ moderately from unity. Therefore, the major contributor to the deviation $g_{Z_1Z_1S_1}$ from its SM value is the factor

$$\Delta g=\frac{[c_{\theta_0}(1-\tan^2\beta)-2\,s_{\theta_0}\tan\beta)]}{(\tan^2\beta+1)}. \tag{63}$$

From the expression (36) it follows that the value of the angle θ_0 is basically determined by the parameter α_2 which appears in the Higgs potential. When $\alpha_2\sim 10^{-2}$ then the angle θ_0 may reach the value $\pi/4$. In this condition the S_1 boson could remain light as usual but the S_2 boson ceases to be superheavy:

$$m_{S_2}^2=\frac{\alpha_2 v_R^2k_+^2}{k_1k_2}-\frac{4k_1k_2k_-^4[2(2\lambda_2+\lambda_3)k_1k_2/k_+^2-\lambda_4]^2}{\alpha_2 v_R^2k_+^2}. \tag{64}$$

Recall that the demand

$$m_{S_2}\geqslant 10\ \ \text{TeV} \tag{65}$$

is caused by the necessity to suppress at the tree level the flavor-changing neutral currents (FCNC's) in the Lagrangian

$$\begin{aligned}\mathcal{L}_q^n=&-\frac{1}{\sqrt{2}k_+}\sum_{a,b}\bar{u}_a\Bigg\{\Bigg[m_{u_a}\bigg(c_{\theta_0}+\frac{2k_1k_2}{k_-^2}s_{\theta_0}\bigg)S_1-m_{u_a}\\&\times\bigg(s_{\theta_0}-\frac{2k_1k_2}{k_-^2}c_{\theta_0}\bigg)S_2-im_{d_a}\gamma_5P_1\Bigg]\delta_{ab}\\&+\frac{k_+^2}{k_-^2}(\mathcal{K}\mathcal{M}_d\mathcal{K}^*)_{ab}(S_1\,s_{\theta_0}+S_2\,c_{\theta_0})\Bigg\}u_b\\&+(u_a\to d_a,m_{u_a}\leftrightarrow m_{d_a},\gamma_5\to-\gamma_5),\end{aligned} \tag{66}$$

where $\mathcal{K}$ is the Cabibbo-Kobayashi-Maskawa matrix and $\mathcal{M}_d$ is the diagonal mass matrix for the down quarks. The absence of the FCNC's in its turn allows one to describe properly the $\bar{K}^0\leftrightarrow K^0$ transitions. However, as shown in [38] the successful building of the LRM demands the redefinition of the traditional Yukawa Lagrangian for quarks. The expression (66) must be replaced by

$$\mathcal{L}_q^n = -\frac{1}{\sqrt{2}k_+}\sum_a \overline{u}_a\left\{ m_{u_a}\left[\left(c_{\theta_0}-\frac{k_1}{k_2}s_{\theta_0}\right)S_1 - \left(s_{\theta_0}+\frac{k_1}{k_2}c_{\theta_0}\right)S_2\right] + \frac{i m_{u_a}k_1}{k_2}\gamma_5 P_1\right\} u_a + (u_a \to d_a, \theta_0 \to -\theta_0). \tag{67}$$

Since the Lagrangian (67) does not induce any FCNC's, the inequality (65) breaks down. Then from Eq. (63) it follows that with increase of the angle θ_0 the deviation Δg from unity could be large enough.

From the form of the Lagrangian (67) it is evident that the decay widths of the S_1 boson into quarks and gluons may significantly differ from those of the SM. Since the coupling of the S_1 boson with the τ lepton is determined by Eq. (52) then the value $\Gamma_{S_1\to\tau^+\tau^-}$ also might not coincide with the corresponding value in the SM. Thus, it is apparent that the LNH boson mass lower bound in the LRM may not agree with that in the SM. However, since up to now any work containing an analysis of the process (59) from the point of view of the LRM is absent, we shall also take the value 115 GeV as the low bound on the S_1-boson mass.

We are coming now to the question concerning the values of the coupling constants $\alpha_{L_aL_bH_i}$ $(L_a = l_a, \nu_a, N_a)$. At present information about $\alpha_{L_aL_bH_i}$ follows from looking for deviations from the SM predictions. It is usually reported in terms of the upper limits for quantities of the type $\alpha_{L_aL_bH_i}/m_{H_i}$ or, which is more frequent, for quantities of the type

$$\sum_i C_i \epsilon_i^{aba'b'} = \sum_i C_i \frac{(\alpha_{L_aL_bH_i}\alpha_{L_{a'}L_{b'}H_i})^2}{m_{H_i}^4},$$

where C_i are the constants. As a rule the determination of the upper bound for only one quantity $\alpha_{L_aL_bH_i}/m_{H_i}$ is a very involved task (see [45] for a review).

As a possible way forward in this context one might use the connection between the CC's $\alpha_{\overline{L}_aL_bH_i}$ and the lepton sector parameters. From the expressions (50)–(52), (54) and (55) it is obvious that the values of $\alpha_{\overline{L}_aL_bH_i}$ are basically defined by the oscillation parameters of the heavy neutrinos. Nowadays information concerning the heavy neutrino sector is very poor too. All we have is the upper bound for the heavy electron neutrino mass resulting from experiments aimed at finding the neutrinoless double β decay

$$m_{N_e} > (63\ \ \mathrm{GeV})\left(\frac{1.6\ \ \mathrm{TeV}}{m_{W_2}}\right)^4. \tag{68}$$

In the existing situation we can choose only the minimal set of heavy neutrino sector parameters and other parameters are expressed through it with the help of Eqs. (51)–(55). Further on, as the minimal set we shall take m_{N_μ}, m_{N_τ}, and θ_N.

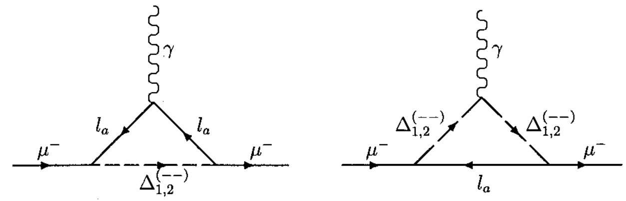

FIG. 1. One-loop diagrams contributing to the muon AMM due to the doubly charged Higgs bosons $\Delta_{1,2}^{(--)}$. The wavy line represents the electromagnetic field.

V. HIGGS BOSON CORRECTIONS TO A_μ

We now proceed to calculations of the contribution to the muon AMM caused by the Higgs bosons of the LRM. The diagrams corresponding to the exchange of doubly charged Higgs bosons are shown in Fig. 1. They give the following corrections to the muon AMM value:

$$\frac{\delta a_\mu^\Delta}{\mu_0} = \frac{1}{8\pi^2}\left(4f_{\mu e}^2\sum_{i=1}^2 I_e^{\Delta_i} + f_{\mu\mu}^2\sum_{i=1}^2 I_\mu^{\Delta_i} + 4f_{\mu\tau}^2\sum_{i=1}^2 I_\tau^{\Delta_i}\right), \tag{69}$$

where

$$I_{l_a}^{\Delta_i} = \int_0^1\left(\frac{2m_\mu^2(z^2-z^3)}{m_\mu^2(z^2-z)+m_{\Delta_i}^2 z + m_{l_a}^2(1-z)} + \frac{m_\mu^2(z^2-z^3)}{m_\mu^2(z^2-z)+m_{\Delta_i}^2(1-z)+m_{l_a}^2 z}\right)dz,$$

and $I_{l_a}^{\Delta_i} > 0$.

The singly charged Higgs bosons also influence the value of the AMM. The relevant diagrams are depicted in Fig. 2. For the diagrams that contain the loops with the $W_1^\pm$ and $h^{(\pm)}$ bosons the following relation holds:

$$\frac{M_{W_1N_\mu h}}{M_{W_1\nu_\mu h}} = s_\xi,$$

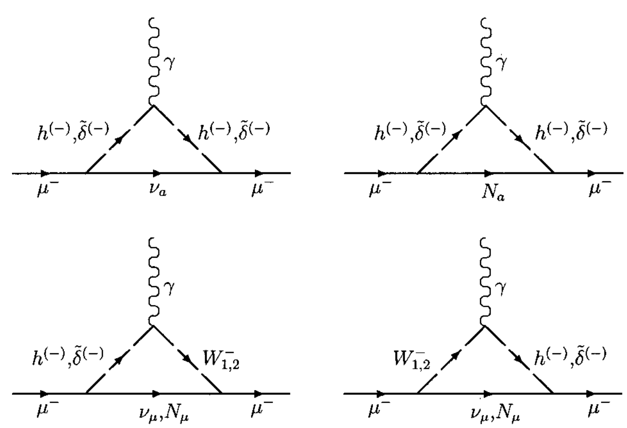

FIG. 2. One-loop diagrams contributing to the muon AMM due to the singly charged Higgs bosons $\tilde{\delta}^{(-)}$ and $h^{(-)}$.

where $M_{W_1\nu_\mu h}$ ($M_{W_1 N_\mu h}$) are the matrix elements appropriate to the diagrams with the exchange of a light (heavy) neutrino. As the mixing angle of the charged gauge bosons is very small, $|\xi|\approx 10^{-2}$–10^{-5} [14], one can neglect the contributions coming from the diagrams with a virtual heavy neutrino. Taking into account the analogous relations

$$\frac{M_{W_2\nu_\mu h}}{M_{W_2 N_\mu h}}=s_\xi,\quad \frac{M_{W_1 N_\mu \tilde{\delta}}}{M_{W_1\nu_\mu\tilde{\delta}}}=s_\xi,\quad \frac{M_{W_2\nu_\mu\tilde{\delta}}}{M_{W_2 N_\mu\tilde{\delta}}}=s_\xi,$$

we find that the dominant contributions to the muon AMM from the diagrams shown in Fig. 2 are

$$\frac{\delta a_\mu^{(hh)}}{\mu_0}=\frac{1}{8\pi^2}\sum_{a=e,\mu,\tau}(\alpha^2_{\mu N_a h}I^{hh}_{N_a}+\alpha^2_{\mu\nu_a h}I^{hh}_{\nu_a}), \tag{70}$$

$$\frac{\delta a_\mu^{(\tilde{\delta}\tilde{\delta})}}{\mu_0}=\frac{1}{8\pi^2}\sum_{a=e,\mu,\tau}(\alpha^2_{\mu N_a\tilde{\delta}}I^{\tilde{\delta}\tilde{\delta}}_{N_a}+\alpha^2_{\mu\nu_a\tilde{\delta}}I^{\tilde{\delta}\tilde{\delta}}_{\nu_a}), \tag{71}$$

$$\frac{\delta a_\mu^{(W_1 h)}}{\mu_0}=\frac{(\alpha-\rho_3/2+\rho_1+1)(1-\tan^2\beta)s_\xi m_{W_1}}{16\sqrt{2}\pi^2(1+\tan^2\beta)}\times\alpha_{\mu\nu_\mu h}I^{W_1 h}, \tag{72}$$

$$\frac{\delta a_\mu^{(W_2 h)}}{\mu_0}=\frac{(\alpha-\rho_3/2+\rho_1+1)(1-\tan^2\beta)c_\xi m_{W_1}}{16\sqrt{2}\pi^2(1+\tan^2\beta)}\times\alpha_{\mu N_\mu h}I^{W_2 h}, \tag{73}$$

$$\frac{\delta a_\mu^{(W_1\tilde{\delta})}}{\mu_0}=\frac{\beta_1(1-\tan^2\beta)s_\xi m_{W_1}\alpha_{\mu\nu_\mu\tilde{\delta}}}{16\sqrt{2}\pi^2(1+\tan^2\beta)}I^{W_1\tilde{\delta}}, \tag{74}$$

$$\frac{\delta a_\mu^{(W_2\tilde{\delta})}}{\mu_0}=\frac{\beta_1(1-\tan^2\beta)c_\xi m_{W_1}\alpha_{\mu N_\mu\tilde{\delta}}}{16\sqrt{2}\pi^2(1+\tan^2\beta)}I^{W_2\tilde{\delta}}, \tag{75}$$

where

$$\alpha_{l_a\nu_b h}=\frac{h'_{ab}k_2-h_{ab}k_1}{2k_+},$$

$$\alpha_{l_a N_b h}=\frac{h'_{ab}k_1-h_{ab}k_2}{2k_+},$$

$$\alpha_{l_a\nu_b\tilde{\delta}}=\frac{f_{ab}}{\sqrt{2}},$$

$$\alpha_{l_a N_b\tilde{\delta}}=\frac{f_{ab}\beta_0 k_0}{\sqrt{2}(\alpha+\rho_1-\rho_3/2)v_R}=\frac{f_{ab}\beta_1 k_+}{2(\alpha+\rho_1-\rho_3/2)v_R},$$

$$I^{hh}_i=\int_0^1\frac{m_\mu^2(z^3-z^2)dz}{m_\mu^2z^2+(m_h^2-m_i^2-m_\mu^2)z+m_i^2}$$

,

$$i=\nu_a,N_a,$$

$$I^{\tilde{\delta}\tilde{\delta}}_i=I^{hh}_i(m_h\to m_{\tilde{\delta}}),\quad I^{hh}_i<0,$$

$$I^{W_1 h}=\frac{m_\mu}{m^2_{W_1}-m_h^2}\left\{\ln\left(\frac{m^2_{W_1}}{m_h^2}\right)-\int_0^1\frac{z^2[m_\mu^2(2z-1)+m^2_{W_1}-m^2_{\nu_\mu}]dz}{m_\mu^2z^2+(m^2_{W_1}-m^2_{\nu_\mu}-m^2_\mu)z+m^2_{\nu_\mu}}+(m_{W_1}\to m_h)\right\},$$

$$I^{W_2 h}=I^{W_1 h}(m_{W_1}\to m_{W_2},m_{\nu_\mu}\to m_{N_\mu}),$$

$$I^{W_k\tilde{\delta}}=I^{W_k h}(m_h\to m_{\tilde{\delta}}),\quad I^{W_k h}>0.$$

The contribution of the neutral Higgs boson S_1 to the muon AMM value is due to the diagram shown in Fig. 3 and is given by

$$\frac{\delta a_\mu^{(S_1)}}{\mu_0}=\frac{1}{8\pi^2}\sum_a\alpha^2_{\mu l_a S_1}I^{S_1}_{l_a}, \tag{76}$$

where

$$\alpha_{l_a l_b S_1}=-\frac{1}{\sqrt{2}k_+}[(h_{ab}k_1+h'_{ab}k_2)s_{\theta_0}+(h'_{ab}k_1-h_{ab}k_2)c_{\theta_0}],$$

$$I^{S_1}_{l_a}=\int_0^1\frac{[m_\mu^2(z^2-z^3)+m^2_{l_a}z^2]dz}{m_\mu^2(z^2-z)+m^2_{S_1}(1-z)+m^2_{l_a}z},\quad I^{S_1}_{l_a}>0.$$

The total correction value to the muon AMM motivated by the Higgs bosons, $\delta a_\mu/\mu_0$, is defined by the sum of the expressions (69)–(76).

VI. ANALYSIS OF THE RESULTS

Now we apply $\delta a_\mu/\mu_0$, obtained in Sec. V, to constrain the parameters of the LRM. Let us determine some key moments in our strategy for calculation of the CC's. To evaluate the VEV of the right-handed Higgs triplet v_R we invoke the relation [38]

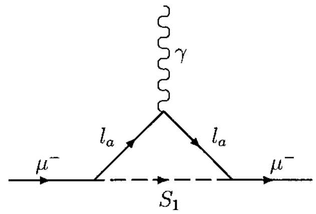

FIG. 3. One-loop diagrams contributing to the muon AMM due to the lightest neutral Higgs boson S_1.

$$v_R=\sqrt{\frac{m_{W_2}^2-m_{W_1}^2}{g_L^2(1+\tan^2 2\xi)}}. \tag{77}$$

The current bounds on the W_2 gauge boson mass and the mixing angle ξ are varied within a broad range in relation to what kind of reactions and what assumptions have been used in the analysis [14]. For example, the lower bound on m_{W_2} being equal to 484 GeV is obtained from investigation of the polarized muon decay under the assumption $\xi=0$. The analysis of the process $b\to s\gamma$ leads to the constraints

$$-0.01\leqslant\xi\leqslant 0.003. \tag{78}$$

Having specified $m_{W_2}=0.8$ TeV and $\xi=10^{-2}$ we evaluate v_R. Then, setting the values of m_{N_μ}, m_{N_τ}, θ_N, and v_L we can present the quantities φ_μ, φ_τ, $f_{\mu\tau}$, m_D^μ, and M_D as functions of $f_{\mu\mu}$.

A. Scenario with contributions from $\Delta_2^{(--)}$ boson

We assume that the dominant contribution comes from the $\Delta_2^{(--)}$ boson. A negligible value of the corrections from the S_1, $\Delta_1^{(--)}$, $\tilde{\delta}^{(-)}$, and $h^{(-)}$ bosons could be caused both by the large values of their masses and by the small values of their coupling constants. It is natural to require that $f_{\mu\mu}$ should be less than 1. Then analysis shows that the interval of the $\Delta_2^{(--)}$-boson mass at which the satisfaction of the BNL 2000 results is possible critically depends on the value of $f_{\mu\tau}$ (note that the value of $f_{\mu\tau}$ very weakly depends on the angles φ_μ and φ_τ and is basically determined by the difference of the heavy neutrino masses). When one sets v_L equal to 1.7 GeV then the $\Delta_2^{(--)}$-boson mass would reach the greatest value $(m_{\Delta_2})_{max}$ at $f_{\mu\tau}\approx 0.15$. For this case in the m_{Δ_2} vs $f_{\mu\mu}$ parameter space two contour lines marked 93.8 and 512.8 corresponding to 95% C.L. limits for the contribution of new physics to $\delta a_\mu/\mu_0$ are exhibited in Fig. 4. The range of the Higgs boson sector parameters allowed by the BNL 2000 result lies between the contours 93.8 and 512.8.

When $f_{\mu\tau}>0.15$ and $m_{\Delta_2}>140$ GeV the value of $f_{\mu\mu}$ becomes more than 1 for the upper bound of $\delta a_\mu/\mu_0$ in Eq. (21). A decrease of $f_{\mu\tau}$ results in a reduction of $(m_{\Delta_2})_{max}$. At fixed $f_{\mu\tau}$ the reduction of v_L practically has no effect on the final result. However, the increase of v_L by an order of magnitude gives rise to a significant growth of $f_{\mu\mu}$. For example, when $v_L=10$ GeV and $m_{\Delta_2}=100$ GeV the value of $f_{\mu\mu}$ lies in the interval $(0.318, 0.743)$.

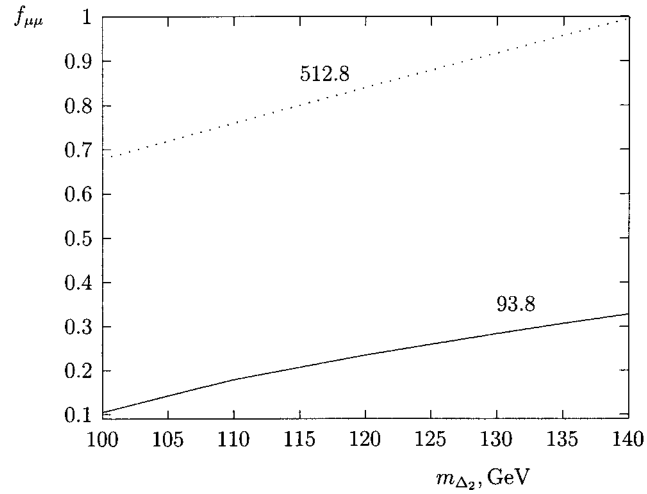

FIG. 4. Contours of the one-loop contribution from the $\Delta_2^{(--)}$ boson to the muon AMM.

B. Scenario with contributions from $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons

Inasmuch as the masses of the $\Delta_1^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons are close to each other then the following possibility should be considered: the observable value of the muon AMM is caused by $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons. Notice that the quantities $\delta a_\mu^{(W_{1,2}\tilde{\delta})}$ change sign as one passes from the region $\tan\beta<1$ to the region $\tan\beta>1$. We shall restrict our consideration to a specific case, namely, when the quantities $\beta_1\alpha_{\mu\nu_\mu\tilde{\delta}}$ and $\beta_1\alpha_{\mu N_\mu\tilde{\delta}}$ are positive (recall that $I^{(W_{1,2}\tilde{\delta})}>0$).

In numerical calculations we shall use the following parameter values:

$$m_{N_\mu}=110 \text{ GeV},\quad m_{N_\tau}=125 \text{ GeV},\quad v_L=0.17 \text{ GeV},$$

$$\tan\beta=0.8,\quad \alpha-\rho_3/2+\rho_1=1,\quad \beta_1=1,\quad \theta_N=0.78, \tag{79}$$

and shall assume a hierarchy of the Higgs boson masses

$$m_{\Delta_1}=1.1 m_{\Delta_2},\quad m_{\tilde{\delta}}=1.05 m_{\Delta_2}.$$

In the m_{Δ_2} vs $f_{\mu\mu}$ parameter space two contour lines marked 93.8 and 512.8 are shown in Fig. 5. At the chosen values of the heavy neutrino masses the quantity $f_{\mu\tau}$ is approximately equal to 0.01.

With an increase of the heavy neutrino masses, but provided that

$$m_{N_\tau}-m_{N_\mu}=\text{const},$$

the function $f_{\mu\mu}(m_{\Delta_2})$ grows faster. However, this rise becomes essential only when the heavy neutrino masses are approximately changed by a order of magnitude. So, for example, choosing $m_{N_\mu}=900$ and $m_{N_\tau}=915$ GeV, we obtain that at $m_{\Delta_2}=200$ GeV the value of $f_{\mu\mu}$ lies in the interval $(0.092, 0.444)$. In this case $f_{\mu\tau}$ is approximately equal to

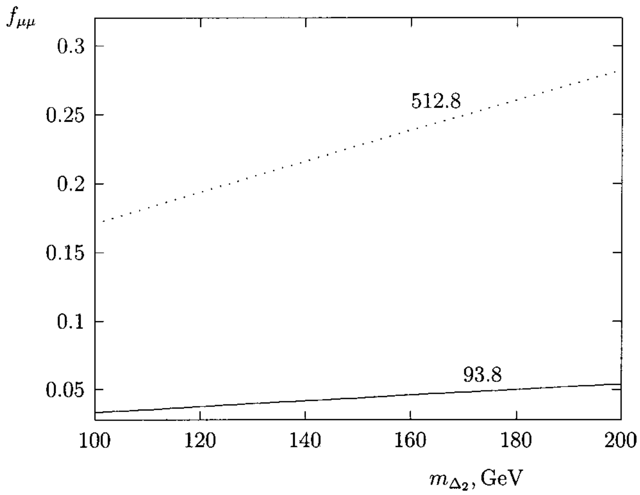

FIG. 5. Contours of the one-loop contribution from the $\Delta_{1,2}^{(--)}$ and the $\tilde{\delta}^{(-)}$ boson to the muon AMM.

0.006. On the other hand, with an increase of $m_{N_\tau}-m_{N_\mu}$ the rate of growth of the function $f_{\mu\mu}(m_{\Delta_2})$ goes down. For example, when we set

$$m_{N_\mu}=900 \text{ GeV}, \quad m_{N_\tau}=1100 \text{ GeV},$$

and leave all the remaining parameters without change, the value of $f_{\mu\mu}$ will lie in the interval $(0.029,0.235)$ when m_{Δ_2} is equal to 100 GeV. The reduction of $\tan\beta$ results in decreasing $f_{\mu\mu}$ as a function of m_{Δ_2}. For example, in the case $m_{N_\mu}=110$ and $m_{N_\tau}=125$ GeV (all the remaining parameter values are unchanged) when $\tan\beta$ has been set to 0.3 we have

$$f_{\mu\mu}\in(0.009,0.049) \quad \text{when} \quad m_{\Delta_2}=100 \text{ GeV}$$

and

$$f_{\mu\mu}\in(0.014,0.079) \quad \text{when} \quad m_{\Delta_2}=200 \text{ GeV}.$$

The results obtained are practically unchanged on increasing v_L up to its maximum value.

C. Scenario with contributions from S_1 and $h^{(-)}$ bosons

At present we assume that the muon AMM value can be explained by the S_1-boson contribution only. To suppress the contributions coming from the remaining Higgs bosons it is enough to assume that

$$\alpha\sim 1, \quad \rho_2\sim 1, \quad \rho_3/2-\rho_1\sim 1 \tag{80}$$

(it will make the $h^{(-)}$, $\Delta_{1,2}^{(--)}$, and $\tilde{\delta}^{(-)}$ bosons superheavy). The contours 93.8 and 512.8 in the m_{S_1} vs $\tan\beta$ parameter space are represented in Fig. 6. In numerical calculations the following parameters values have been used:

$$f_{\mu\mu}=0.04, \quad m_{N_\mu}=110 \text{ GeV}, \quad m_{N_\tau}=125 \text{ GeV},$$

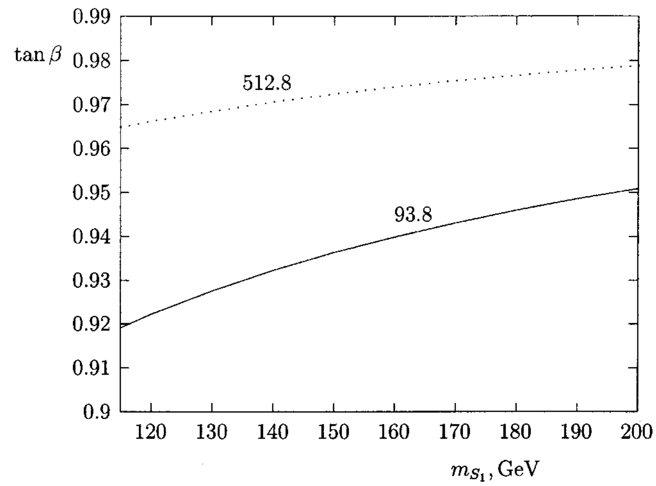

FIG. 6. Contours of the one-loop contribution from the S_1 boson to the muon AMM.

$$v_L=1.7 \text{ GeV}.$$

Increasing (decreasing) v_L results in the reduction (the enhancement) of the allowed values of $\tan\beta$. For example, when v_L is equal to 10 GeV we obtain

$$\tan\beta\in(0.804,0.913) \quad \text{at} \quad m_{S_1}=115 \text{ GeV}$$

and

$$\tan\beta\in(0.88,0.947) \quad \text{at} \quad m_{S_1}=200 \text{ GeV}.$$

Increasing the heavy neutrino masses and the values of $f_{\mu\mu}$ does not cause an appreciable change of the results obtained.

In the case when the muon AMM is due to the contributions from S_1 and h bosons the contours 93.8 and 512.8 in the m_{S_1} vs $\tan\beta$ parameter space are represented in Fig. 7. The choice of the model parameters is as follows:

$$m_{N_\mu}=110 \text{ GeV}, \quad m_{N_\tau}=125 \text{ GeV},$$

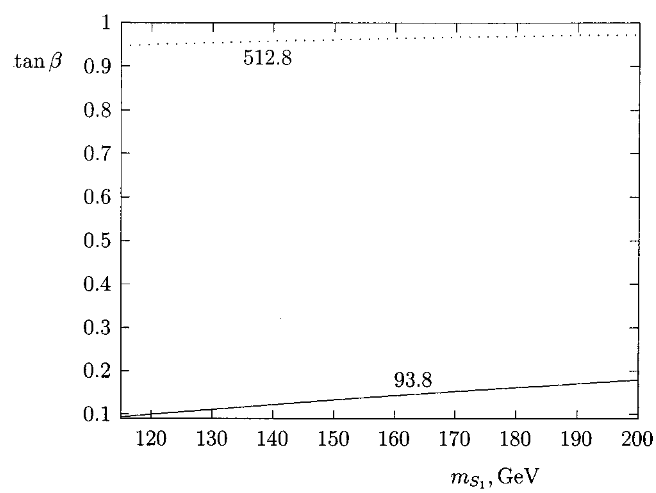

FIG. 7. Contours of the one-loop contribution from the S_1 and $h^{(-)}$ bosons to the muon AMM.

$$v_L = 1.7 \ \text{GeV}, \quad f_{\mu\mu} = 0.04 \tag{81}$$

With decrease of v_L the value of $\tan\beta$ comes closer and closer to 1, while an increase of v_L results in the removal of the values of $\tan\beta$ from 1 to 0. An increase of the heavy neutrino masses does not in practice influence the behavior of the contours shown in Fig. 7. On increase of $f_{\mu\mu}$ there is a reduction of the allowed values of $\tan\beta$ with growth of m_{S_1} values. For example, when $f_{\mu\mu}=0.1$ (all the remaining parameters are unchanged), then at $m_{S_1}=165$ GeV the values of $\tan\beta$ lie within the interval (0.0015, 0.4542) and at $m_{S_1}=200$ GeV they lie within the interval (0.0211, 0.5562).

VII. CONCLUSIONS

We have considered the Higgs boson sector of the LRM as a source of the muon AMM value observed at BNL. The contributions from the interactions of the doubly charged Higgs bosons ($\Delta_{1,2}^{(--)}$), the singly charged ($h^{(-)}$ and $\tilde{\delta}^{(-)}$), and the lightest neutral (S_1) Higgs bosons both with leptons and with gauge bosons were taken into the account. The value of the muon AMM found represents a function of the Higgs boson masses and the Higgs boson coupling constants (CC's). For the majority of the SM extensions the information about the Higgs boson masses is at the level of knowledge of the lower borders only. The situation with the CC's is even more pessimistic. The experimental data derived up to now do not allow one to obtain the constraints on all the CC's. We managed to show that most of the CC's are functions of the neutrino oscillation parameters. From this it turned out that the values of these CC's are in practice not sensitive to the masses and the mixing angles in the light neutrino sector. They are mainly defined by the values of the heavy neutrino masses and by the mixing angles between the light and the heavy neutrinos. It should be particularly emphasized that this property is common for all the models with the "seesaw" mechanism, i.e., for the models with one or more heavy neutrinos. Having expressed the CC's through the heavy neutrino sector parameters we have expanded the range of experiments in which the CC's can be measured. Now information about the CC's can also be obtained by investigation of processes with heavy neutrinos. It is important to keep in mind that these processes might not include the Higgs bosons at all.

To explain the observed value of the muon AMM by contributions from either S_1 and $h^{(-)}$ bosons or from both of them, it is necessary to assume that $\tan\beta$ is close to 1. To put this another way, coincidence with the BNL 2000 result will take place at quasidegeneracy of the bidoublet VEV's ($k_1 \approx k_2$), i.e., at the fine-tuning of the bidoublet VEV's. As in this case the borders obtained on the Higgs boson parameters weakly depend on the neutrino sector parameters, the recovery of some information concerning the masses and the mixing angles of the neutrinos will be rather difficult. However, the reverse side of this history is the fine capability for detecting of S_1 and of $h^{(-)}$ bosons. It appears that when $\tan\beta$ is close to 1 then the values of the CC's for $h^{(-)}$ and S_1 bosons are such that these bosons can be observed as resonance peaks in the whole series of processes. For example, the S_1 boson could be observed as resonance splashes in the cross sections of the reactions

$$\mu^+\mu^- \to \mu^+\mu^-, \tau^+\tau^-, \tag{82}$$

$$\mu^+ e^- \to \mu^- e^+, \tag{83}$$

which have almost no background. The reactions (82) and (83) may be investigated right now, because the energy of the muon beams used in the current experiments is rather high. The Spin Muon Collaboration at CERN has been working with muon beams having energy 190 GeV [46] and the Fermilab experiments investigating the muon-proton interaction have been using muons with energies of 470 GeV [47]. The reactions (82) and (83) can also be investigated at the muon colliders (MC's) which are now in design. For detection of the $h^{(-)}$ bosons one could employ the reactions

$$e^-\nu_e \to W_1^- Z_1, \tag{84}$$

$$e^-\nu_e \to \mu^-\nu_\mu, \tag{85}$$

which have s-channel diagrams with the exchange of the $h^{(-)}$ boson [38]. Ultrahigh energy cosmic neutrinos could be used for studying these two reactions at such neutrino telescopes as BAIKAL NT-200, NESTOR, and AMANDA.

However, one important point to remember is that the fine-tuning of the parameters always corresponds to an extremely rare expedient is used in nature. For this reason a variant with wider range of the parameters ensuring agreement between theory and experiment is preferable. The situation with the dominating contribution to the muon AMM from $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons is just such a case. However, in this case we are still far from a final definition of the heavy neutrino parameters. Having established the values of $f_{\mu\mu}$, v_L, and v_R, we shall obtain only two equations for the definition of the quantities φ_μ, θ_N, m_{N_μ}, and m_{N_τ}, which is obviously not enough. Certainly it is possible to select the conventional way too. Of the five parameters of the heavy neutrino sector ($\varphi_\mu, \varphi_\tau, \theta_N, m_{N_\mu}, m_{N_\tau}$), we may fix four parameters and vary only one, say, m_{N_μ}. With this approach, instead of the contours shown in Fig. 5, we shall have contours constructed in m_{N_μ} vs m_{Δ_2} parameter space, which will not introduce anything essentially new to our analysis. The most important point here is something else, namely, when the contribution to the muon AMM is truly caused by the $\Delta_1^{(-)}$, $\tilde{\delta}^{(-)}$, and/or $\Delta_2^{(-)}$ bosons, a further way of defining the parameters of the heavy neutrinos without their direct observation is evident. For example, we could investigate the reactions

$$\mu^-\mu^- \to \mu^-\mu^-, \tag{86}$$

$$\mu^-\mu^- \to \mu^-\tau^-, \tag{87}$$

$$\mu^-\mu- \to \tau^-\tau^-, \tag{88}$$

which may be observed at MC's. All these reactions go through the *s* channels with exchanges of the $\Delta_{1,2}^{(--)}$ bosons. Therefore, their cross sections have two resonance peaks related to the Higgs bosons. Detecting the reaction (86) will allow us to determine $f_{\mu\mu}$, while investigation of the reactions (87) and (88) will yield information about $f_{\mu\tau}$ and $f_{\tau\tau}$, respectively. Then the use of Eqs. (41)–(45) will allow one to define the regions in which the values of the heavy neutrino masses and the mixing angles are constrained.

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