Optical activity of neutrinos and antineutrinos

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Using the one-loop helicity amplitudes for low-energy $\nu\gamma\to\nu\gamma$ and $\overline{\nu}\gamma\to\overline{\nu}\gamma$ scattering in the standard model with massless neutrinos, we study the optical activity of a sea of neutrinos and antineutrinos. In particular, we estimate the values of the index of refraction and rotary power of this medium in the absence of dispersion.

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I. INTRODUCTION

The elastic neutrino-photon scattering process, $\nu\gamma\to\nu\gamma$, and the corresponding crossed channels have been the subject of numerous investigations related to astrophysical applications [1–9]. When the center of mass energy $\sqrt{s}$ is much less than the W-boson mass m_W, the amplitudes for these processes have a relatively simple form. In particular, the forward helicity nonflip amplitudes for the elastic processes $\nu\gamma\to\nu\gamma$ and $\overline{\nu}\gamma\to\overline{\nu}\gamma$ with massless neutrinos can be expanded in powers of s/m_W^2 by making use of unitarity [9]. The series are of leading order s^2/m_W^4, and the inclusion of higher powers of s, such as s^3/m_W^6, enables us to use the forward elastic scattering amplitudes for $\nu\gamma\to\nu\gamma$ and $\overline{\nu}\gamma\to\overline{\nu}\gamma$ to study the optical activity of a sea of neutrinos and antineutrinos. This property of a neutrino sea was recognized by Nieves and Pal [10] and studied by Mohanty, Nieves, and Pal [11] in the case where the photons move in a plasma and therefore experience dispersion. Dispersive effects give rise to mass dependent terms of the form k^2/m_e^2 (k^μ denoting the photon momentum and m_e the electron mass), which vanish in the vacuum. Here, we treat the vacuum case where k^μ satisfies $k^2=0$, and find a rather different dependence on m_W and m_e [12].

In the next section, using the results of Ref. [9], we write the forward helicity amplitudes for the elastic scattering $\nu\gamma\to\nu\gamma$ and $\overline{\nu}\gamma\to\overline{\nu}\gamma$. We then present a calculation for the index of refraction and the rotary power for a sea of neutrinos and antineutrinos as a function of the energy of an incident photon, the temperature of the sea, and the neutrino degeneracy parameter $\xi\equiv\mu/T_\nu$, where μ is the chemical potential for neutrinos and T_ν is the temperature of the sea.

II. OPTICAL ACTIVITY

Using Eqs. (24)–(26) of Ref. [9], the helicity non-flip amplitudes $\mathcal{A}_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(s)$ for the forward elastic scattering of a photon with helicity λ and an electron neutrino when $\sqrt{s}\ll m_W$, can be written as

$$\mathcal{A}_{++}^{\nu\gamma\to\nu\gamma}(s)=\frac{-\alpha^2 s^2}{8m_W^4\sin^2\theta_W}h(s), \tag{1}$$

$$\mathcal{A}_{--}^{\nu\gamma\to\nu\gamma}(s)=\frac{-\alpha^2 s^2}{8m_W^4\sin^2\theta_W}h(-s), \tag{2}$$

where

$$h(s)=-4-\frac{16}{3}\ln\left(\frac{m_W^2}{m_e^2}\right)+\frac{m_e^2}{m_W^2}\left[\left(\frac{8}{3}-\frac{8}{3}\frac{s}{m_e^2}\right)\ln\left(\frac{m_W^2}{m_e^2}\right)-10+\frac{64}{9}\frac{s}{m_e^2}\right]. \tag{3}$$

Here, θ_W is the weak mixing angle, α is the fine structure constant, and we have neglected higher powers of m_e^2/m_W^2 and s/m_W^2. Using Eq. (12) of Ref. [9], the helicity non-flip amplitudes, $\mathcal{A}_{\lambda\lambda}^{\overline{\nu}\gamma\to\overline{\nu}\gamma}(s)$, for forward elastic scattering of a photon of helicity λ and an antineutrino, can be expressed as

$$\mathcal{A}_{\lambda\lambda}^{\overline{\nu}\gamma\to\overline{\nu}\gamma}(s)=\mathcal{A}_{-\lambda-\lambda}^{\nu\gamma\to\nu\gamma}(s), \tag{4}$$

which is a consequence of CPT invariance.

To calculate the optical activity of a neutrino-antineutrino sea, we consider a photon of helicity λ and energy ω traversing a bath of neutrinos and antineutrinos that are in thermal equilibrium at the temperature T_ν. To give an order of magnitude estimate of the index of refraction n_λ of this sea, we write [13,14]

$$n_\lambda-1=\frac{2\pi}{\omega^2}\int dN_\nu f_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(0)+\frac{2\pi}{\omega^2}\int dN_{\overline{\nu}} f_{\lambda\lambda}^{\overline{\nu}\gamma\to\overline{\nu}\gamma}(0), \tag{5}$$

which is a generalization of Eq. (57) of Ref. [9]. The forward-scattering amplitudes $f_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(0)$ and $f_{\lambda\lambda}^{\overline{\nu}\gamma\to\overline{\nu}\gamma}(0)$, from Eq. (58) of Ref. [9], are

$$f_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(0)=\frac{\omega}{4\pi s}\mathcal{A}_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(s), \tag{6}$$

$$f_{\lambda\lambda}^{\overline{\nu}\gamma\to\overline{\nu}\gamma}(0)=\frac{\omega}{4\pi s}\mathcal{A}_{\lambda\lambda}^{\overline{\nu}\gamma\to\overline{\nu}\gamma}(s), \tag{7}$$

and dN_ν and $dN_{\bar\nu}$ are the usual Fermi–Dirac distributions,

$$dN_\nu = \frac{1}{(2\pi)^3}\frac{d^3\vec{p}_\nu}{e^{(E_\nu-\mu)/T_\nu}+1}, \tag{8}$$

$$dN_{\bar\nu} = \frac{1}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\nu}}{e^{(E_{\bar\nu}+\mu)/T_\nu}+1}. \tag{9}$$

Here, T_ν is the temperature of the neutrino-antineutrino sea, μ is the chemical potential for the neutrinos [15], $\vec{p}_\nu$ and E_ν are the momentum and energy of a neutrino, and $s = 4\omega E_\nu \sin^2(\theta_{\nu\gamma}/2)$, where $\theta_{\nu\gamma}$ is the angle between the incoming photon and the incoming neutrino (with similar definitions for the antineutrino). From Eqs. (5)–(7) we obtain

$$n_\lambda - 1 = \int \frac{dN_\nu}{2\omega s}\mathcal{A}_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(s) + \int \frac{dN_{\bar\nu}}{2\omega s}\mathcal{A}_{\lambda\lambda}^{\bar\nu\gamma\to\bar\nu\gamma}(s). \tag{10}$$

Due to the apparent baryon asymmetry in the universe, it is natural to consider the possibility of the lepton asymmetry. To study this possibility, and for numerical calculations of the index of refraction and the rotary power, it is convenient to introduce the following ratios

$$L_1 \equiv \frac{3}{11}\frac{N_\nu - N_{\bar\nu}}{N_\nu + N_{\bar\nu}}, \tag{11}$$

$$L_2 \equiv \frac{2\pi^2}{11\zeta(3)}\frac{N_\nu - N_{\bar\nu}}{T_\nu^3}, \tag{12}$$

where N_ν and $N_{\bar\nu}$ are the number densities for the neutrino and antineutrino, respectively, and $\zeta(x)$ is the Riemann zeta function. From Eqs. (8), (9), (11), and (12), we find

$$L_1 = \frac{1}{11}\frac{\xi^3+\pi^2\xi}{I(\xi)\xi^3 + 2\ln 2\,\xi^2 + 3\zeta(3)}, \tag{13}$$

$$L_2 = \frac{1}{33\zeta(3)}(\xi^3+\pi^2\xi), \tag{14}$$

where

$$I(\xi) = \int_0^1 (1-x)^2 \frac{e^{\xi x}-1}{e^{\xi x}+1}dx, \tag{15}$$

and $\xi \equiv \mu/T_\nu$ is the neutrino degeneracy parameter. As it is clear from Eqs. (13)–(15), the parameters L_1 and L_2 are solely functions of ξ. These neutrino asymmetry parameters L_1 and L_2 are defined such that for $|\xi| \ll 1$ they coincide with the customary definition of the neutrino asymmetry parameter L_ν [16] for a sea of relic neutrino-antineutrino that decoupled from photons in the early universe

$$L_1 \simeq L_2 \simeq L_\nu \equiv \frac{N_\nu - N_{\bar\nu}}{N_\gamma} \simeq \frac{\pi^2 \xi}{33\zeta(3)}. \tag{16}$$

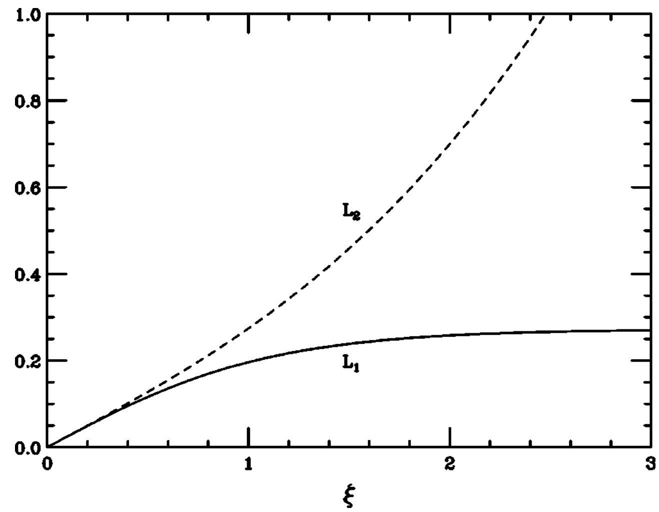

FIG. 1. The asymmetry parameters L_1 and L_2, as functions of the neutrino degeneracy parameter $\xi \equiv \mu/T_\nu$. The relations $L_1(-\xi) = -L_1(\xi)$ and $L_2(-\xi) = -L_2(\xi)$ hold.

Here, N_γ is the photon number density

$$N_\gamma = \frac{1}{(2\pi)^3}\int \frac{2d^3\vec{p}_\gamma}{e^{E_\gamma/T_\gamma}-1} = \frac{2\zeta(3)T_\gamma^3}{\pi^2}, \tag{17}$$

and the relation $T_\nu/T_\gamma = (4/11)^{1/3}$, which holds for the present day temperatures of the relic neutrinos and photons, is used. When ξ is large, L_1 approaches a finite value, while L_2 increases. In Fig. 1 L_1 and L_2 are plotted as functions of ξ. As it is clear from this figure, for $\xi \gtrsim 3$, L_1 is close to its maximum value of 3/11.

After using Eqs. (8) and (9) for dN_ν and $dN_{\bar\nu}$, Eqs. (1)–(4) for the amplitudes $\mathcal{A}_{\lambda\lambda}^{\nu\gamma\to\nu\gamma}(s)$ and $\mathcal{A}_{\lambda\lambda}^{\bar\nu\gamma\to\bar\nu\gamma}(s)$, and performing integrations in the Eq. (10), we obtain

$$n_+ - 1 = \frac{T_\nu^4}{m_W^4}c_0 + \frac{\omega T_\nu^5}{m_W^6}c_1, \tag{18}$$

$$n_- - 1 = \frac{T_\nu^4}{m_W^4}c_0 - \frac{\omega T_\nu^5}{m_W^6}c_1, \tag{19}$$

where (for $\alpha = 1/137$)

$$\begin{aligned} c_0 &= \frac{\alpha^2}{3\pi^2\sin^2\theta_W}\left[\ln\left(\frac{m_W^2}{m_e^2}\right)+\frac{3}{4}\right]\left(\frac{1}{4}\xi^4+\frac{\pi^2}{2}\xi^2+\frac{7\pi^4}{60}\right) \\ &\simeq 1.9\times 10^{-4}\left(\frac{1}{4}\xi^4+\frac{\pi^2}{2}\xi^2+\frac{7\pi^4}{60}\right), \end{aligned} \tag{20}$$

$$\begin{aligned} c_1 &= \frac{4\alpha^2}{9\pi^2\sin^2\theta_W}\left[\ln\left(\frac{m_W^2}{m_e^2}\right)-\frac{8}{3}\right]\left(\frac{1}{5}\xi^5+\frac{2\pi^2}{3}\xi^3+\frac{7\pi^4}{15}\xi\right) \\ &\simeq 2.2\times 10^{-4}\left(\frac{1}{5}\xi^5+\frac{2\pi^2}{3}\xi^3+\frac{7\pi^4}{15}\xi\right). \end{aligned} \tag{21}$$

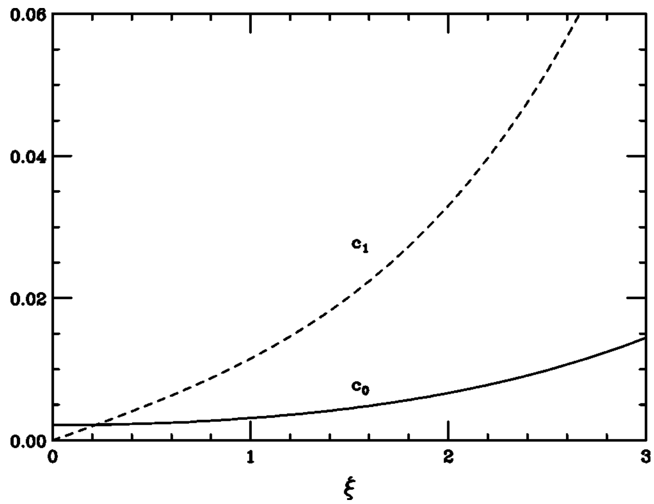

FIG. 2. The coefficients c_0 and c_1 as functions of the neutrino degeneracy parameter $\xi \equiv \mu/T_\nu$. The relations $c_0(-\xi)=c_0(\xi)$ and $c_1(-\xi)=-c_1(\xi)$ hold.

Notice that the ξ-dependent parts of the L_2, c_0, and c_1 are related as

$$\frac{1}{4}\frac{\partial}{\partial\xi}\left(\frac{1}{5}\xi^5+\frac{2\pi^2}{3}\xi^3+\frac{7\pi^4}{15}\xi\right)=\frac{1}{4}\xi^4+\frac{\pi^2}{2}\xi^2+\frac{7\pi^4}{60}, \tag{22}$$

$$\frac{\partial}{\partial\xi}\left(\frac{1}{4}\xi^4+\frac{\pi^2}{2}\xi^2+\frac{7\pi^4}{60}\right)=\xi^3+\pi^2\xi. \tag{23}$$

In the Figs. 2–4 the coefficients c_0 and c_1 are plotted as functions of ξ, L_1, and L_2, respectively.

The range of the validity of Eqs. (18)–(21), as far as energy is concerned, is related to that of Eqs. (1)–(3), which is $s=4\omega E_\nu \sin^2(\theta_{\nu\gamma}/2) \ll m_W^2$. In Eq. (10), if we change the upper limits of the integrations on E_ν and $E_{\bar{\nu}}^-$ from the infinity to fT_ν, the contributions of these integrals to c_0 and c_1 in

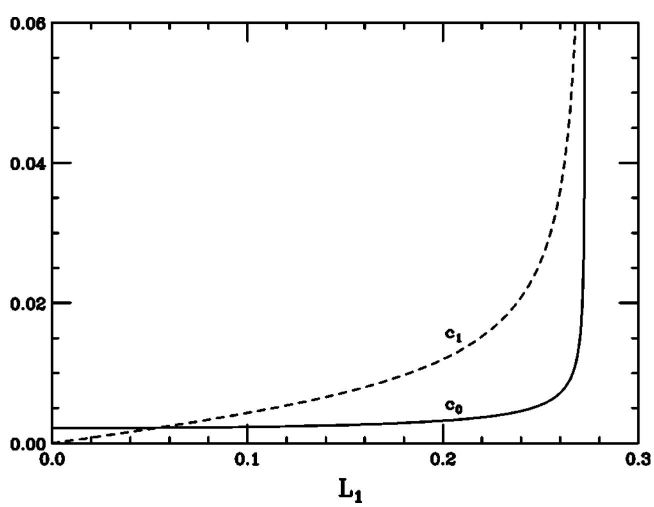

FIG. 3. The coefficients c_0 and c_1 as functions of the asymmetry parameter L_1. The relations $c_0(-L_1)=c_0(L_1)$ and $c_1(-L_1)=-c_1(L_1)$ hold.

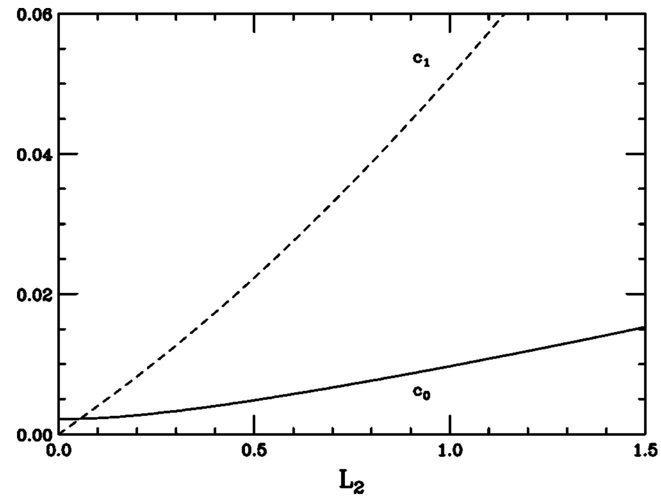

FIG. 4. The coefficients c_0 and c_1 as functions of the asymmetry parameter L_2. The relations $c_0(-L_2)=c_0(L_2)$ and $c_1(-L_2)=-c_1(L_2)$ hold.

Eqs. (18) and (19) change, at most, by 9% and 18%, respectively, if we use $f=\sqrt{7^2+\xi^2}$ (for $f=\sqrt{8^2+\xi^2}$, the corresponding changes are at most 5% and 10%). Here, we set the following criterion:

$$4\omega f T_\nu \ll m_W^2, \tag{24}$$

which for $f=\sqrt{7^2+\xi^2}$ is

$$\omega T_\nu \sqrt{7^2+\xi^2} \ll 2\times 10^{16}\ \text{GeV K}, \tag{25}$$

where ω is the photon energy in GeV, and T_ν is the neutrino temperature in Kelvin.

From Eqs. (18) and (19), we have

$$n_+ - n_- = 2\frac{\omega T_\nu^5}{m_W^6}c_1 \simeq 3.5\times 10^{-77} c_1 \omega T_\nu^5, \tag{26}$$

and the following approximate relation:

$$n_+ - 1 \simeq n_- - 1 \simeq \frac{T_\nu^4}{m_W^4}c_0 \simeq 1.3\times 10^{-60} c_0 T_\nu^4. \tag{27}$$

Equation (27) implies that the leading term of the index of refraction is independent of the helicity and the energy of the incident photon, as long as Eq. (25) is satisfied.

When linearly polarized light propagates through a medium that has different indices of refraction for positive and negative helicities ($n_+ \neq n_-$), the plane of polarization of the light rotates by an angle ϕ, which is [17]

$$\phi = \frac{\pi}{\lambda_\gamma}(n_+ - n_-)l = \frac{\omega}{2}(n_+ - n_-)l, \tag{28}$$

where ω and $\lambda_\gamma = 2\pi/\omega$ are the energy and wavelength of the photon and l is the distance traveled by photons in the

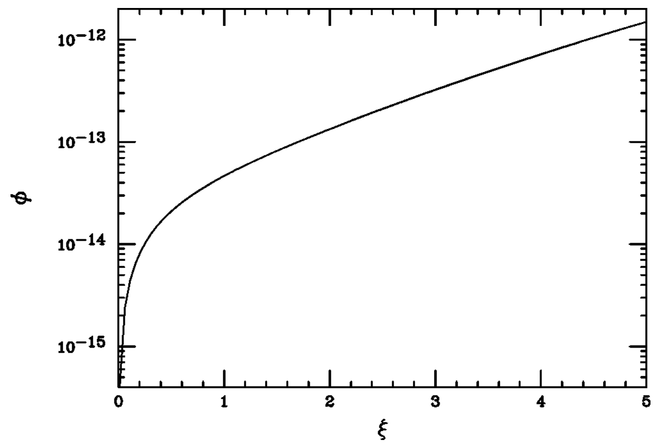

FIG. 5. The polarization rotation angle ϕ is shown as a function of ξ for $\omega = 10^{11}$ GeV. The curve scales as ω^2.

medium. To estimate the specific rotary power, ϕ/l, for a sea of neutrinos and antineutrinos, we use Eqs. (26) and (28) to obtain

$$\frac{\phi}{l} = \frac{\omega^2 T_\nu^5}{m_W^6} c_1 \simeq 8.9 \times 10^{-62} c_1 \omega^2 T_\nu^5 \text{ rad/m}, \tag{29}$$

where, again, ω is in GeV and T_ν is in Kelvin. A positive angle of rotation, $\phi > 0$, that is $n_+ > n_-$, corresponds to a clockwise rotation (dextrorotation) of the plane of polarization of the linearly polarized incident photons, as viewed by an observer that is detecting the forward-scattered light. Thus, the optical activity of a neutrino–antineutrino sea with $N_\nu > N_{\bar{\nu}}$ is that of a dextrorotary medium. In addition, it is clear from Eq. (29) that the rotary power, ϕ/l, varies as $1/\lambda_\gamma^2$, which is the same as that of quartz and most transparent substances for visible light.

To get a rough estimate of rotation angle ϕ for linearly polarized photons propagating through the relic neutrino and antineutrino sea (with $\xi = 0.01$), we use Eq. (29) with $l = ct$, $c = 3 \times 10^8$ m/s, $t \sim 15 \times 10^9$ yr, $T_\nu \sim 2$ K, and $\omega \sim 10^{20}$ eV, and we find

$$\phi \sim 4 \times 10^{-16} \text{ rad}, \tag{30}$$

which is exceedingly small. The dependence of ϕ on ξ is shown in Fig. 5. It should be noted that recent studies which include neutrino oscillation effects in the derivation of cosmological bounds on ξ conclude that $|\xi| \lesssim 0.1$ [18–20].

III. CONCLUSIONS

In conclusion, we note that using Eqs. (12), (14), (21), and (29), for the neutrino degeneracy parameter $|\xi| \ll 1$, the specific rotary power, ϕ/l, for a sea of neutrinos and antineutrinos is

$$\frac{\phi}{l} = \frac{112\pi G_F \alpha}{45\sqrt{2}} \left[\ln\left(\frac{m_W^2}{m_e^2}\right) - \frac{8}{3} \right] \frac{\omega^2 T_\nu^2}{m_W^4} (N_\nu - N_{\bar{\nu}}). \tag{31}$$

This equation differs in several respects from the corresponding result in Ref. [11],

$$\frac{\phi}{l} = \frac{G_F \alpha}{3\sqrt{2}\pi} (N_\nu - N_{\bar{\nu}}) \frac{\omega_P^2}{m_e^2}, \tag{32}$$

where, in the non-relativistic limit, the plasma frequency ω_P is related to the electron number density n_e as,

$$\omega_P^2 = \frac{4\pi\alpha n_e}{m_e}. \tag{33}$$

From Eqs. (31) and (32), it is clear that the mass scales for the case of photon dispersion versus no dispersion are very different and this favors the dispersive regime. The size of specific rotary power in the dispersive case is independent of the frequency of the propagating photons and, apart from a dependence on the lepton asymmetry parameter $L_\nu \propto N_\nu - N_{\bar{\nu}}$ shared by both expressions, is controlled by the electron number density n_e. In addition, the non-dispersive case depends on frequency and temperature. These differences clearly indicate that Eqs. (29) and (32) represent complementary limits in the treatment of the optical activity of a neutrino-antineutrino sea.

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that connects the index of refraction to the forward-scattering amplitude, for a medium that is at $T=0$ temperature. Here, we make a non-trivial assumption that the introduction of finite temperature in the medium, will not spoil the coherent condition that was necessary to derive the Lorentz relation. For a similar approach, see P. Langacker and J. Liu, Phys. Rev. D **46**, 4140 (1992).

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