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Factorization and end point singularities in heavy-to-light decays

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We prove a factorization theorem for heavy-to-light form factors. Our result differs in several important ways from previous proposals. A proper separation of scales gives hard kernels that are free of end point singularities. A general procedure is described for including soft effects usually associated with the tail of wave functions in hard exclusive processes. We give an operator formulation of these soft effects using the soft-collinear effective theory, and show that they appear at the same order in the power counting as the hard spectator contribution.

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Exclusive hadronic form factors simplify dramatically at momentum transfers much larger than hadronic scales, $Q^2 \gg \Lambda^2$. Typically, they factor into nonperturbative light cone wave functions $\phi_{a,b}$ for mesons a and b, convoluted with a calculable hard scattering kernel T [1]:

$$F(Q^2)=\frac{f_a f_b}{Q^2}\int dx dy T(x,y,\mu)\phi_a(x,\mu)\phi_b(y,\mu)+ \dots . \tag{1}$$

Here f_i are meson decay constants, the hard scattering kernel $T(x,y)$ is calculated perturbatively in an expansion in α_s, and the ellipsis denotes terms suppressed by additional powers of $1/Q$. For example, the electromagnetic form factor of a pion has $a=b=\pi$ and at $\mu=Q$ [1] $T(x,y) = 8\pi\alpha_s(Q)/(9xy)$. For Eq. (1) to be well defined it is sufficient that $\phi_i(x) \overset{x\to 0}{\sim} x^n$, $\phi_i(x) \overset{x\to 1}{\sim} (1-x)^m$ with any $n,m > 0$. A linear falloff is sometimes assumed, but we will not use this assumption.

Beyond leading order (LO) in $1/Q$ issues arise. There are soft contributions to the form factor, which arise from configurations where a single quark carries most of the meson momentum and leaves $p^\mu \sim \Lambda$ for the remaining constituents [2], and these have been estimated using QCD sum rules. Furthermore, power suppressed hard exchange contributions tend to give contributions diverging as $\int dx/x$. Examples include $1/Q$ corrections to the pion form factor, $1/m_b$ corrections in $B\to\pi\pi$, $K\pi$ decays, and one-gluon exchange for heavy-to-light form factors [3].

The soft-collinear effective theory (SCET) [4–7], reproduces the factorization in Eq. (1) [8], and provides a framework to analyze power corrections based solely on QCD. This theory consists of collinear fields interacting with soft or ultrasoft degrees of freedom. The fields are categorized by the scaling of their momenta: collinear $p_c=(p_c^+,p_c^-,p_c^\perp) = (n\cdot p_c, \bar{n}\cdot p_c, p_c^\perp) \sim Q(\lambda^2,1,\lambda)$, soft $p_s^\mu \sim Q\lambda$ and ultrasoft $p_{us}^\mu \sim Q\lambda^2$, where $n^2=\bar{n}^2=0$, $n\cdot\bar{n}=2$, and $\lambda \ll 1$ is the expansion parameter.

In this paper we show how SCET can be used to understand factorization and soft end point contributions in heavy-to-light form factors for decays such as $B\to\pi\ell\nu$, $B\to K^* e^+e^-$ and $B\to\rho\gamma$, building on [4]. Here the large scales are $Q=\{m_b, E\}$, where the final meson has $E=m_B/2 - q^2/(2m_B)$. Several ideas are developed, which we now summarize.

(1) We prove a factorization formula for heavy-to-light decays involving the LO light-cone wave functions, a jet function, plus a reduced set of non-perturbative matrix elements which obey form factor relations.

(2) Calculable kernels are free of divergences. End point singularities are fake and arise from improperly matching onto $T(x,y)$. They appear in non-factorizable operators and can be parametrized without invoking suppression from Sudakov effects.

(3) A single collinear meson state can be used to categorize all contributions. Soft effects associated with the tail of wave functions are described by matrix elements of operators with a definite power counting. The categories "factorizable" and "non-factorizable" are more accurate than "hard" and "soft" contributions.

(4) There are two perturbative scales in the problem: Q and $\mu_0 \simeq \sqrt{Q\Lambda}$. We separate these scales by matching in two stages, onto a $\mathrm{SCET_I}$ at $\mu=Q$, and onto a $\mathrm{SCET_{II}}$ at $\mu=\mu_0$.

(5) The LO result for heavy-to-light decays comes from power suppressed operators in $\mathrm{SCET_I}$, which match onto LO operators in $\mathrm{SCET_{II}}$.

Our procedure is quite general and similar analyses apply to other exclusive processes. To understand the origin of the end point divergences, we consider the spectator interaction for heavy-to-light decays at $\mathcal{O}(g^2)$ in Fig. 1. Taking $p_{1,2}$ collinear and k,r ultrasoft, the λ expansion of these graphs gives $i\mathcal{A}=g^2\bar{u}_n(p_1)XT^A u_v(p_b)\bar{v}(-r)VT^A v_n(-p_2)/P_g$ with

$$(X\otimes V)^{(a)}=\frac{\Gamma\otimes\bar{n}\!\!\!/}{\bar{n}\cdot p_2}+\frac{\Gamma n\!\!\!/\gamma_\mu^\perp\otimes\gamma_\perp^\mu}{2m_b}+\dots,$$

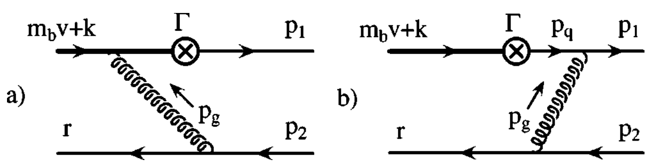

FIG. 1. Tree level QCD graphs for heavy-to-light decays with one perturbative gluon. Note that $p_g^2 \sim Q\Lambda$.

$$(X\otimes V)^{(b)}=\left\{\frac{\not p_1^\perp \gamma_\perp^\mu \bar{n}\cdot p}{P_q\ \ \bar{n}\cdot p_1}+\frac{\gamma_\perp^\mu}{P_q}\left(\frac{n\cdot p\bar{\not n}}{2}+\not r\right)\right\}\Gamma\otimes\gamma_\mu^\perp - \frac{2\bar{n}\cdot p\ \ \Gamma\otimes\not p_2^\perp}{P_q\bar{n}\cdot p_2}+\frac{\bar{n}\cdot p\ \ \not p_1^\perp\gamma_\perp^\mu\Gamma\otimes\gamma_\mu^\perp\not p_2^\perp\bar{\not n}}{2P_q\ \ \bar{n}\cdot p_1\bar{n}\cdot p_2} + \ldots, \qquad (2)$$

where $P_g=\bar{n}\cdot p_2\ n\cdot r$, $P_q=\bar{n}\cdot p n\cdot r+p^2$ and $p=p_1-p_2$. Equation (2) agrees with Ref. [9]. The ellipsis denotes terms $\propto p_\perp$. If one interprets Eq. (2) using Eq. (1) with $a=M$, $b=B$, it is tempting to extract $T(x,y)$ setting $p_i^\perp=0$, $(p_1-p_2)^2=0$, $\bar{n}\cdot p_2=-2xE$, $n\cdot r=y$. However, the result includes terms $\propto 1/x^2$ or $1/y^2$ leading to singular integrals. Note that in full QCD there are no singularities since they are regulated by momenta of order Λ.

Several proposals have been made for dealing with these divergences. One approach regulates these singularities by introducing transverse parton momenta and including Sudakov form factors [10,11]. However, this proposal does not include all the non-perturbative contributions, or deal with the possibility that Sudakov suppression may not be large at $m_b \approx 5$ GeV. In [11] it was shown that at LO these divergences can be reabsorbed into "soft" form factors which satisfy form factor relations [4,12]. However, this analysis was only performed to order α_s. Furthermore, neither a rigorous field theoretic definition for these soft contributions exists, nor does a first principle derivation of their power counting.

To fully understand these issues requires a factorization formula with the generality to account for non-perturbative contributions. In this paper we prove that at leading order in $1/Q$ and all orders in α_s a generic heavy-to-light form factor F can be split into factorizable and non-factorizable components $F=f^{\rm F}(Q)+f^{\rm NF}(Q)$ where

$$f^F(Q)=N_0\int_0^1 dz\int_0^1 dx\int_0^\infty dr_+\ \ T(z,Q,\mu_0) \times J(z,x,r_+,Q,\mu_0,\mu)\phi_M(x,\mu)\phi_B(r_+,\mu), \quad (3)$$

$$f^{\rm NF}(Q)=C_k(Q,\mu)\ \ \zeta_k(Q\Lambda,\mu), \quad (4)$$

and $N_0=f_B f_\pi m_B/(4E^2)$. The hard coefficients C_k and T can be calculated in an expansion in $\alpha_s(Q)$, the jet function J is dominated by momenta $p^2 \simeq Q\Lambda$ and calculable perturbatively in $\alpha_s(\sqrt{Q\Lambda})$. The functions ϕ_M and ϕ_B are standard non-perturbative light-cone wave functions, cf. [9,13], where our ϕ_B denotes ϕ_B^+ or ϕ_B^-. Only ϕ_B^+ appears if J is calculated at tree level. End point singularities only arise in matrix elements which determine the soft, non-perturbative form factors $\zeta_k(Q,\mu)$, leaving the convolution integrals in the factorizable terms finite. There are three soft form factors $\zeta_k(Q,\mu)$; one for pseudoscalar, and two for vector mesons. We show that terms proportional to ϕ_B^- can be absorbed into a redefinition of $\zeta_k(E,\mu)$ at any order in perturbation theory.

Our expression for the heavy-to-light form factors differs in several important ways from previous proposals. In the approach of Ref. [10] possible non-perturbative soft contributions are dropped with the *ex post facto* assumption that they are negligible. Furthermore, their perturbative pieces contain singular terms in the hard kernels, which are then regulated by resummations [14]. In Ref. [9] both soft and non-singular hard contributions were included. Unlike [10] the soft pieces were found to dominate, due to the fact that the hard terms were suppressed by $\alpha_s(\sqrt{Q\Lambda})$. However, their soft and hard definitions do not clearly avoid double counting. Furthermore, in order to show that these two contributions are the same order in $1/m_b$ it was necessary to use assumptions about the scaling of the tails of the meson wave functions.

In contrast, in our work the f^F and f^{NF} pieces appear from matrix elements of distinct operators with the same states, avoiding any possibility of double counting. Furthermore, singular hard scattering kernels do not appear. We prove a factorization theorem to all orders in perturbation theory. Our result differs from the proposed formula in Ref. [9] because it involves both a hard kernel T and a jet function J, which separate the scales Q and $\sqrt{Q\Lambda}$. This separation is necessary if one wants to distinguish factors of $\alpha_s(Q)$ from $\alpha_s(\sqrt{Q\Lambda})$, or more accurately resum large logarithms between these scales. Our result differs from Ref. [10] in that f^{NF} contains non-perturbative matrix elements of operators with $D_\perp$'s which do not appear in [10], and our f^F does not involve $k_\perp$ convolutions. The operator definitions for the various pieces in the factorization formula in Eqs. (3), (4) allow us to rigorously power count the two types of contributions in a model independent way. Finally, the form of our result appears to indicate that the soft and hard contributions may actually be comparable in size, since the $\alpha_s(\sqrt{Q\Lambda})$ suppression in J could be compensated for by a similar factor in ζ_k. A complete answer to this question requires a full resummation of the double Sudakov logarithms, which are known [15] to appear in both our factorizable and non-factorizable contributions. This is left to future work.

To begin, we need a definition of the non-perturbative hadronic states. They can be defined by any interpolating field which has the right quantum numbers and significant overlap with the physical state. For the B we pick the standard heavy quark effective theory (HQET) state $|B_v\rangle$ [16], while for the light meson M we pick a state $|M_n\rangle$ whose interpolating field is built out of two collinear quarks, and involves all interactions in the LO collinear Lagrangian. Thus, the B/M states are generated by soft/collinear fields in $\rm SCET_{II}$ which have $p_s^2 \sim p_c^2 \sim \Lambda^2$. Time-ordered products account for corrections to these states. We do *not* define $|M_n\rangle$ with collinear quarks in $\rm SCET_I$ since here the off-shellness is still large.

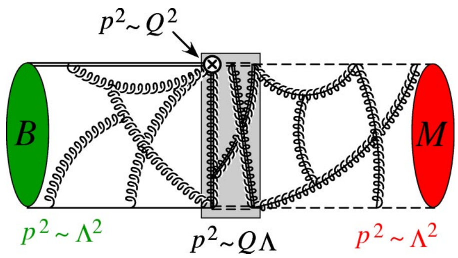

FIG. 2. Levels of factorization. The gray area corresponds to gluons in $\mathrm{SCET_I}$ which are integrated out in $\mathrm{SCET_{II}}$.

Equations (3), (4) separate the contributions from hard momenta ($p^2 \sim Q^2$), jet momenta ($p^2 \sim Q\Lambda$), and nonperturbative momenta ($p^2 \sim \Lambda^2$), as illustrated in Fig. 2. In Fig. 1 it is the gluon that connects to the spectator which scales like a jet momentum. To separate these scales we match QCD onto an intermediate effective theory $\mathrm{SCET_I}$, valid for $\sqrt{Q\Lambda} < \mu < Q$, which contains collinear particles with off-shellness $p_c^2 \sim Q\Lambda$ and a power counting in $\lambda = \sqrt{\Lambda/Q}$. Since the collinear particles in $\mathrm{SCET_I}$ satisfy $p_c^2 \sim Q\Lambda$ this theory does not describe the complete $B \to M$ process in QCD. A second step of matching is required onto $\mathrm{SCET_{II}}$, containing collinear particles with off-shellness $p_c^2 \sim \Lambda^2$ and power counting in $\lambda' = \lambda^2 = \Lambda/Q$. Wilson coefficients in $\mathrm{SCET_I}$ determine T, C of Eq. (3), while those in $\mathrm{SCET_{II}}$ determine J. The ultrasoft fields in $\mathrm{SCET_I}$ are identical to soft fields in $\mathrm{SCET_{II}}$. This two-step procedure provides a simple and more general method of determining the $\mathrm{SCET_{II}}$ soft-collinear operators compared to the procedure in Ref. [7].

$\mathrm{SCET_I}$ is defined by its Lagrangian and heavy-to-light currents. The terms in the expansion of the collinear Lagrangian we require are $\mathcal{L}_c = \mathcal{L}_c^{(0)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\xi\xi}^{(1)} + \mathcal{L}_{cg}^{(1)} + \mathcal{L}_{\xi q}^{(2a)} + \mathcal{L}_{\xi q}^{(2b)}$. The superscript denotes the order in λ that these terms contribute in the power counting [17]. The LO action for collinear quarks and gluons is [4,7]

$$\mathcal{L}_c^{(0)} = \bar\xi_n \left[in\cdot D + i\not{D}_\perp^c \frac{1}{i\bar n\cdot D_c} i\not{D}_\perp^c \right] \frac{\not{\bar n}}{2} \xi_n + \mathcal{L}_{cg}^{(0)}, \qquad (5)$$

with $i\bar n\cdot D_c = \bar{\mathcal{P}} + g\bar n\cdot A_c$, $iD_c^\perp = \mathcal{P}^\perp + gA_c^\perp$, $in\cdot D = in\cdot\partial + gn\cdot A_{us} + gn\cdot A_c$. The gluon action $\mathcal{L}_{cg}^{(0)}$ can be found in Ref. [7]. For the subleading action we find [18,19,15]

$$\mathcal{L}_{\xi\xi}^{(1)} = \bar\xi_n i\not{D}_\perp^{us} \frac{1}{i\bar n\cdot D_c} i\not{D}_\perp^c \frac{\not{\bar n}}{2} \xi_n + \text{H.c.},$$

$$\mathcal{L}_{cg}^{(1)} = \frac{2}{g^2} \mathrm{tr}\{[i\mathcal{D}^\mu, iD_c^{\perp\nu}][i\mathcal{D}_\mu, iD_{us\ \nu}^\perp]\} + \text{g.f.}, \qquad (6)$$

with $\mathcal{D}^\mu = n^\mu \bar n\cdot D_c/2 + D_c^{\perp\mu} + \bar n^\mu n\cdot D/2$ and g.f. denotes gauge fixing terms. In our proof the mixed collinear-ultrasoft Lagrangian $\mathcal{L}_{\xi q}$ will play a crucial role and was first considered in [20]. Using the label operator formalism [6] we obtain the gauge invariant QCD result:

$$\mathcal{L}_{\xi q}^{(1)} = ig\bar\xi_n \frac{1}{i\bar n\cdot D_c} \not{B}_\perp^c W q_{us} + \text{H.c.},$$

$$\mathcal{L}_{\xi q}^{(2a)} = ig\bar\xi_n \frac{1}{i\bar n\cdot D_c} \not{M} W q_{us} + \text{H.c.}, \qquad (7)$$

$$\mathcal{L}_{\xi q}^{(2b)} = ig\bar\xi_n \frac{\not{\bar n}}{2} i\not{D}_\perp{}^c \frac{1}{(i\bar n\cdot D_c)^2} \not{B}_\perp^c W q_{us} + \text{H.c.},$$

where $ig\not{B}_\perp^c = [i\bar n\cdot D^c, i\not{D}_\perp^c]$ and $ig\not{M} = [i\bar n\cdot D^c, i\not{D}^{us} + (\not{\bar n}/2) gn\cdot A^c]$. A possible four quark operator $(\bar\xi_n W T^A \not{\bar n} W^\dagger \xi_n) 1/\bar{\mathcal{P}}^2 (\bar\xi_n W T^A \not{\bar n} q_{us})$ has been eliminated using the collinear gluon equations of motion. Finally the $\mathrm{SCET_I}$ currents we will need are [4,21,18,20,15]

$$J^{(0)} = C_\Gamma(\omega_1)(\bar\xi_n W)_{\omega_1} \Gamma h_v,$$

$$J^{(1a)} = B_\Gamma^a(\omega_1+\omega_2)(\bar\xi_n W)_{\omega_1}(W^\dagger i\overleftarrow{D}_{c\alpha}^\perp W)_{\omega_2} \frac{\Gamma_a^\alpha}{\bar{\mathcal{P}}^\dagger} h_v, \qquad (8)$$

$$J^{(1b)} = B_\Gamma^b(\omega_1,\omega_2)(\bar\xi_n W)_{\omega_1}(W^\dagger i\vec{D}_{c\alpha}^\perp W)_{\omega_2} \frac{\Gamma_b^\alpha}{m_b} h_v,$$

where we sum over ω_1, ω_2. Here $J^{(1a,1b)}$ correspond to the $J_i^{(1a,1b)}$ of [15] with $\Gamma^a, \Gamma^b \to \Upsilon_i, \Theta_i$, and are the most general allowed operators at any order in α_s, taking $v_\perp = 0$.

In matching onto $\mathrm{SCET_{II}}$ we need two collinear quarks to give non-zero overlap with $|M_n\rangle$, so we only need operators with two collinear quarks in $\mathrm{SCET_I}$. For the graphs in $\mathrm{SCET_I}$ it is necessary to have a $\mathcal{L}_{\xi q}^{(n)}$ interaction to turn the ultrasoft spectator in the B into a collinear quark. This is the generic reason that the form factors in the range of q^2 considered here, $q^2 \lesssim 10\ \mathrm{GeV}^2$, are suppressed relative to their size near q^2_{max}. More than one $\mathcal{L}_{\xi q}^{(n)}$ insertion is forbidden at this order. The relevant time-ordered products are

$$T_0^{\mathrm{F}} = T[J^{(0)}, i\mathcal{L}_{\xi q}^{(1)}] \equiv \int d^4x\, T[J^{(0)}(0) i\mathcal{L}_{\xi q}^{(1)}(x)] \qquad (9)$$

as well as

$$T_1^{\mathrm{F}} = T[J^{(1a)}, i\mathcal{L}_{\xi q}^{(1)}], \quad T_2^{\mathrm{F}} = T[J^{(1b)}, i\mathcal{L}_{\xi q}^{(1)}],$$

$$T_3^{\mathrm{F}} = T[J^{(0)}, i\mathcal{L}_{\xi q}^{(2b)}], \quad T_4^{\mathrm{NF}} = T[J^{(0)}, i\mathcal{L}_{\xi q}^{(2a)}], \qquad (10)$$

$$T_5^{\mathrm{NF}} = T[J^{(0)}, i\mathcal{L}_{\xi\xi}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}], \quad T_6^{\mathrm{NF}} = T[J^{(0)}, i\mathcal{L}_{cg}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}].$$

The time-ordered product T_0^{F} is enhanced by one power of λ in $\mathrm{SCET_I}$ compared to the other terms; however, its matching onto $\mathrm{SCET_{II}}$ does not give rise to enhanced contributions to form factors. Higher order T's do not contribute at the order we are working.

To prove the factorization formula given in Eqs. (3), (4), we decouple the collinear-ultrasoft interaction in the LO Lagrangian $\mathcal{L}_c^{(0)}$ using the field redefinitions [7]

$$\xi_n^{(0)}=Y^\dagger\xi_n\,,\quad A_n^{(0)}=Y^\dagger A_n Y,$$
$$Y(x)=P\exp\left(ig\int_{-\infty\,\mathrm{sign}\,\bar{\mathcal{P}}}^{0}\mathrm{d}s\,n\cdot A_{us}(ns+x)\right). \tag{11}$$

While this introduces a factor of $Y^\dagger$ into the leading current, it only appears in the combination $\mathcal{H}_v=[Y^\dagger h_v]$

$$J^{(0)}=C_\Gamma(\omega_1)(\bar\xi_n^{(0)}W^{(0)})_{\omega_1}\Gamma\mathcal{H}_v\,. \tag{12}$$

The situation is similar in $\mathcal{L}_{\xi q}^{(1)}$ and $\mathcal{L}_{\xi q}^{(2b)}$, where ultrasoft fields or interactions now only appear in the combination $\mathcal{Q}=[Y^\dagger q_{us}]$. On the other hand, we have

$$\mathcal{L}_{\xi\xi}^{(1)}=\bar\xi_n^{(0)}[Y^\dagger i\not{D}_{us}^{\perp}Y]\frac{1}{i\bar n\cdot D_c^{(0)}}i\not{D}_{c,\perp}^{(0)}\frac{\not{\bar n}}{2}\xi_n^{(0)}+\mathrm{H.c.},$$
$$\mathcal{L}_{\xi q}^{(2a)}=ig\bar\xi_n^{(0)}\frac{1}{i\bar n\cdot D_c^{(0)}}\,[Y^\dagger\not{M}Y]W^{(0)}\mathcal{Q}+\mathrm{H.c.}. \tag{13}$$

Thus, the time-ordered products fall into two categories: "factorizable," $T^{\mathrm{F}}_{\{0,1,2,3\}}$, in which the ultrasoft interactions all occur in $\mathcal{H}_v$ and $\mathcal{Q}$, and "nonfactorizable," $T^{\mathrm{NF}}_{\{4,5,6\}}$, with an additional $[Y^\dagger D^\mu_{us}Y]$ or $[Y^\dagger\not{M}\,Y]$. It can be clearly seen that there is no double counting when the soft and hard contributions are defined this way. The matching onto $\mathrm{SCET_{II}}$ for these two cases is discussed separately.

For the factorizable terms $T_i^F=T[J_i^F,i\mathcal{L}_i^{\mathrm{F}}]$ each J^{F} and $\mathcal{L}^{\mathrm{F}}$ splits into collinear and ultrasoft parts in $\mathrm{SCET_I}$, $J^{\mathrm{F}}=T'(\omega_j)\bar{\mathcal{J}}_{\omega_j}\Gamma\mathcal{H}_v$, $\mathcal{L}^{\mathrm{F}}=\bar{\mathcal{Q}}\,\mathcal{J}+$H.c., where $\mathcal{J}$'s denote products of collinear fields. To factorize these time-ordered products we follow Ref. [7]. From momentum conservation we have $\omega_1+\omega_2\to\bar n\cdot p_M$ of meson M, so we suppress this dependence and let $\bar\omega=\omega_1-\omega_2$. With this notation we can write

$$T_i^F=T_i'(\bar\omega)\int d^4x\,T[\bar{\mathcal{J}}_{\bar\omega}(0)\Gamma\mathcal{H}(0)\ \bar{\mathcal{Q}}(x)\mathcal{J}(x)]$$
$$=T_i(\bar\omega)\int d^4x\,T[\ \bar{\mathcal{J}}_{\bar\omega}\,(0)\Gamma_c\mathcal{J}(x)]T[\bar{\mathcal{Q}}(x)\Gamma_s\mathcal{H}(0)], \tag{14}$$

where $T_i'(\bar\omega)$ is $\{C_\Gamma(\bar n\cdot p_M),B_\Gamma^a(\bar n\cdot p_M),B_\Gamma^b(\bar n\cdot p_M,\bar\omega),C_\Gamma(\bar n\cdot p_M)\}$. In the second line we performed a Fierz transformation on the color and spin indices, absorbing prefactors to give $T(\bar\omega)$, and dropping a $T^A\otimes T^A$ which gives no contribution in $\mathrm{SCET_{II}}$. We now lower the off-shellness of the external collinear particles to $p_c^2\sim\Lambda^2$. The T_i^F run exactly like their J_i^F currents. Since we have explicitly kept the ultrasoft part of the momentum of collinear particles, matching onto $\mathrm{SCET_{II}}$ amounts to setting $p_\perp^c=n\cdot p^c=0$ on external lines and expanding the T_i^F's. Matching at $\mu_0\simeq\sqrt{Q\Lambda}$ the ultrasoft fields become soft (e.g. $Y\to S$), and the collinear T-product matches onto a bilinear collinear quark operator in $\mathrm{SCET_{II}}$,

$$T[\bar{\mathcal{J}}_{\bar\omega}(0)\mathcal{J}(x)]=\delta(x^+)\,\delta^2(x^\perp)\int d\bar\eta\int dk^+e^{(i/2)k^+x^-}$$
$$\times J(\bar\omega,\bar\eta,k^+)[\bar\xi_n^{\mathrm{II}}W\Gamma_c\delta(\bar\eta-\bar{\mathcal{P}}_+)W^\dagger\xi_n^{\mathrm{II}}]. \tag{15}$$

The jet function $J(\bar\omega,\bar\eta,k^+)$ is the Wilson coefficient for this matching step. Inserting this in Eq. (14),

$$T_i^F=\int d\bar\omega\ d\bar\eta\ dk^+T(\bar\omega)J(\bar\omega,\bar\eta,k^+)\mathcal{O}(\bar\eta,k^+),$$
$$\mathcal{O}(\bar\eta,k^+)=[\bar\xi_n^{\mathrm{II}}W\delta(\bar\eta-\bar{\mathcal{P}}_+)\Gamma_cW^\dagger\xi_n^{\mathrm{II}}]$$
$$\times[\bar q^sS\Gamma_s\delta(\mathcal{P}_+-k^+)S^\dagger h_v^s], \tag{16}$$

where $\mathcal{O}(\bar\eta,k^+)$ is the full operator in $\mathrm{SCET_{II}}$. Now taking the $\mathrm{SCET_{II}}$ matrix element gives

$$\langle M_n|\mathcal{O}(\bar\eta,k^+)|B_v\rangle=Nf_Mf_B\phi_M(x)\phi_B^+(k^+), \tag{17}$$

where N is a normalization factor and $x=\bar\eta/(4E)+1/2$. Combining Eqs. (16) and (17) reproduces Eq. (3).

For the non-factorizable operators T_i^{NF}, it is not possible to write the matrix elements as in f^F. Instead when matched onto $\mathrm{SCET_{II}}$ these terms give f^{NF} in Eq. (4) and should be understood to *define* the soft nonperturbative effects for the form factors. It remains to show that they satisfy the form factor relations [4,12]. Since the relevant time-ordered products only contain the current J_0, the argument is the same as in [4]: any Dirac structure in heavy-to-light currents can be reduced to only three, $\bar\xi_nWh_v$, $\bar\xi_nW\gamma^5h_v$ and $\bar\xi_nW\gamma_\perp^\mu h_v$. These three operators contribute only to $B\to P$, $B\to V_{||}$ and $B\to V_\perp$, respectively, where P, ($V_{||}$, $V_\perp$) denote pseudoscalar (longitudinally, transversely) polarized vector mesons. For J_0 this is true even in arbitrary time-ordered products with Lagrangian insertions, since Lagrangians are parity even Lorentz scalars. The f^F term breaks these relations, but is calculable. At higher order in λ non-factorizable contributions will also break these relations, since subleading currents appear in time-ordered products with non-factorizable Lagrangian insertions.

The matrix elements of $T_{1,2}^F$ contain only ϕ_B^+ to all orders in α_s since inserting a projector next to ξ_n in $\mathcal{L}_{\xi q}^{(1)}$, the q_{us} appears as $\bar q_{us}\not{n}\not{\bar n}$ in the Fierzed operators. On the other hand, T_3^F [which may contribute at $\mathcal{O}(\alpha_s^2)$] has only $\bar q_{us}\not{\bar n}$ and so is proportional to ϕ_B^-. However, T_3^F's matrix element involves J_0 and therefore satisfies the same symmetry relations as the nonfactorizable matrix elements in f^{NF} [15]. Therefore it can be absorbed into a redefinition of the ζ_k^M's to all orders in perturbation theory.

The last step is to understand the power counting of the two contributions in Eqs. (3), (4). When we expand to match onto $\mathrm{SCET_{II}}$ the new operators and coefficients scale with $1/Q$ in the same way as those in $\mathrm{SCET_I}$, up to a global $1/Q$ from switching from the ξ_n^{I} to ξ_n^{II} fields. The one exception is T_0^F, since it is odd in the number of $D_c^\perp$ derivatives and this

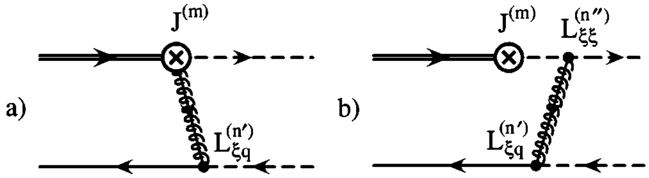

FIG. 3. Tree level graphs in $\mathrm{SCET_I}$. The graphs in (a) are from $T_{1,2,4}$, while those in (b) are from $T_{0,1,3,4,5,6}$.

extra $\perp$ gets suppressed by at least one power of λ. Therefore, T_i^F and T_i^{NF} contribute at the same order in $1/Q$ to the form factors. We find a generic form factor to scale as $(\Lambda/Q)^{3/2}$, which is Λ^2/Q^2 suppressed compared to the scaling in m_b near $q^2_{\max}$ derived from HQET [16].

We finally show that the end point singularities encountered in Eq. (2) do not occur in f^F in the second step of matching. The contributions of the time-ordered products at $\mathcal{O}(g^2)$ are shown in Fig. 3; we find $i\mathcal{A}_i = g^2\overline{u}_n X_i T^A h_v(p_b)\overline{v}(-r)V_i T^A v_n/P_g$ with

$$X_0\otimes V_0 = X_3\otimes V_3 = X_6\otimes V_6 = 0, \quad X_1\otimes V_1 = \frac{\gamma_\perp^\mu \bar{n\!\!\!/}\,\Gamma\otimes\gamma_\mu^\perp}{2\ \bar{n}\cdot p},$$

$$X_2\otimes V_2 = \frac{\Gamma n\!\!\!/\,\gamma_\perp^\mu\otimes\gamma_\mu^\perp}{2m_b}, \quad X_4\otimes V_4 = \frac{1}{\bar{n}\cdot p_2}\Gamma\otimes\left[\bar{n\!\!\!/} - \frac{2p\!\!\!/^{us}_{2\perp}}{n\cdot r}\right], \tag{18}$$

$$X_5\otimes V_5 = \left[\frac{\gamma_\perp^\mu r\!\!\!/_\perp}{\bar{n}\cdot p\, n\cdot r} + \frac{p\!\!\!/^{us}_{1\perp}\gamma_\perp^\mu}{\bar{n}\cdot p_1\ \ n\cdot r}\right]\Gamma\otimes\gamma_\mu^\perp .$$

The $1/x^2$, $1/r_+^2$ singularities only exist in the non-factorizable T_4 and T_5, while the factorizable $T_{1,2}^F$ give non-singular jet functions. This is not surprising, since in full QCD all end point singularities are regulated by Λ. Thus, if (ultra)soft operators are properly included to account for this region of momenta end point singularities will not arise.

As an example, for the form factor f_+ at leading order in $1/Q$ and all orders in α_s we find

$$\begin{aligned} f_+ = N_0\int dx dz dr_+ \Bigg[&\frac{2E-m_B}{m_B}T_a(\mu_0)J_a(z,x,r_+,\mu_0,\mu) \\ &+ \frac{2E}{m_b}T_b(z,\mu_0)J_b(z,x,r_+,\mu_0,\mu)\Bigg]\phi_M(x,\mu) \\ &\times\phi_B^+(r_+,\mu) + C(Q,\mu)\zeta(Q\Lambda,\mu), \end{aligned} \tag{19}$$

where $N_0 = f_B f_\pi m_B/(4E^2)$ and the Q dependence of $T_{a,b}$ and $J_{a,b}$ is implicit. Here $T_{\{a,b\}}$ are the Wilson coefficients of the currents $J^{(1a,1b)}$, the jet functions $J_{a,b}$ are computed from the $T_{1,2}^F$ time ordered products, and we have reabsorbed possible ϕ_B^- contributions from T_3^F into ζ. For the jet functions at order α_s we find

$$J_a = J_b = \frac{\pi C_F}{N_c}\ \frac{\alpha_s(\mu_0)}{x r_+}\delta(x-z). \tag{20}$$

At tree level the coefficients satisfy $C = T_a = T_b = 1$ and using Eq. (20) the first term in Eq. (19) then agrees with the non-singular hard contribution in [11]. This simple approximation misses double logarithms in $T_{a,b}(\mu_0)$ which may be larger than the single logarithms resummed in the $\alpha_s(\mu_0)$ for $\mu_0 \simeq \sqrt{Q\Lambda}$. The one loop expression for $C(Q,\mu)$ can be found in Eqs. (33), (60) of [4]. The non-perturbative matrix element $\zeta(E)$ is the reduced soft form factor describing decays to pseudoscalar mesons.

In this paper we proved a factorization formula for heavy-to-light decays, including spectator effects. The factorizable pieces are finite and determined by one-dimensional convolutions. The nonfactorizable pieces include non-perturbative gluon effects and satisfy form factor relations. They are not determined by the $k_\perp$-dependent light-cone meson wave functions, which is different from the conclusion in [2]. Our leading order analysis needed the currents $J^{(1a,1b)}$, unlike the analysis in Refs. [18,20] where these currents first enter at subleading order.

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