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Measuring γ in $B^{\pm}\to K^{\pm}(KK^{*})_D$ decays

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We develop a method to measure the Cabibbo-Kobayashi-Maskawa angle γ without hadronic uncertainties from the analysis of $B^{\pm}\to K^{\pm}D^0$ and $K^{\pm}\bar{D}^0$ followed by singly Cabibbo-suppressed D decays to non-CP eigenstates, such as $K^{\pm}K^{*\mp}$. This method utilizes the interference between $b\to c\bar{u}s$ and $b\to u\bar{c}s$ decays, and we point out several attractive features of it. All the modes that need to be measured for this method are accessible in the present data.

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Some of the theoretically cleanest determinations of the weak phase γ rely on $B\to DK$ and related decays [1–4] (for definitions, see [3,4]). The original idea of Gronau and Wyler (GW) [2] was to measure two decay rates arising from $b\to c\bar{u}s$ and $b\to u\bar{c}s$ amplitudes. By measuring the rate of a third decay that involves the interference between these two amplitudes, one can gain sensitivity to their relative phase, which is γ. Since all the quarks which appear in $B\to DK$ decays have distinct flavors, the theoretical uncertainty arises only from higher order weak interaction effects (including, possibly, $D-\bar{D}$ mixing, which we discuss below). However, there are no penguin contributions to these decays.

A practical difficulty of the GW method is that the amplitude ratio $A(B^-\to \bar{D}^0K^-)/A(B^-\to D^0K^-)$ is expected to be small. As a result, the measurement of $|A(B^-\to \bar{D}^0K^-)|$ using hadronic D decays is hampered by a significant contribution from the decay $B^-\to D^0K^-$, followed by a doubly Cabibbo-suppressed decay of the D^0. To avoid this problem, Atwood, Dunietz, and Soni (ADS) [5] proposed to study final states where Cabibbo-allowed and doubly Cabibbo-suppressed D decays interfere. Several other variants of the GW method have been proposed [6,7]. An important point is that most of the methods require the measurements of very small rates, which have yet to be observed. (One exception is Ref. [7], where relatively large rates are expected.)

In this Rapid Communication we propose to use singly Cabibbo-suppressed D decays to final states that are not CP eigenstates, and have sizable rates in both D^0 and $\bar{D}^0$ decay. The use of such final states has been mentioned in Ref. [5]; here we develop the details and point out the advantages. The simplest examples are the final states $K^{*+}K^-$ and $K^{*-}K^+$. In the formalism below, we assume that all relevant decays are dominated by the standard model amplitudes.[1] We define

$$A_B\equiv A(B^-\to D^0K^-),\quad \bar{A}_B\equiv A(B^-\to \bar{D}^0K^-). \tag{1}$$

We shall further denote

$$\frac{\bar{A}_B}{A_B}=r_Be^{i(\delta_B-\gamma)},\quad r_B\equiv\left|\frac{\bar{A}_B}{A_B}\right|, \tag{2}$$

where δ_B is the relative strong phase between $\bar{A}_B$ and A_B, and we have neglected the deviation of the weak phase of $\bar{A}_B/A_B$ from $-\gamma$, which is suppressed by four powers of the Cabibbo angle ($\lambda^4\simeq 2\times 10^{-3}$). Then the ratio of the CP conjugate decay amplitudes is given by

$$\frac{A(B^+\to D^0K^+)}{A(B^+\to \bar{D}^0K^+)}=r_Be^{i(\delta_B+\gamma)}. \tag{3}$$

We denote the following D decay amplitudes:

$$A_D\equiv A(D^0\to K^{*+}K^-),\quad \bar{A}_D\equiv A(\bar{D}^0\to K^{*+}K^-), \tag{4}$$

and their ratio

$$\frac{\bar{A}_D}{A_D}=r_De^{i\delta_D},\quad r_D\equiv\left|\frac{\bar{A}_D}{A_D}\right|. \tag{5}$$

Here we neglected the $c\to u$ penguin contribution compared to the $c\to s\bar{s}u$ tree diagram, which is a very good approximation. Then the ratio of the CP conjugate amplitudes is

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[1] New physics would have to compete with tree-level $b\to u\bar{c}s$ or $c\to s\bar{s}u$ decays, or give rise to $D-\bar{D}$ mixing near the present experimental limits to influence our determination of γ.

$$\frac{A(D^0\to K^{*-}K^+)}{A(\bar{D}^0\to K^{*-}K^+)}=r_D e^{i\delta_D}. \tag{6}$$

With these definitions, the four amplitudes we are interested in are given by

$$\begin{aligned}
A[B^-\to K^-(K^{*+}K^-)_D]&=|A_BA_D|[1+r_Br_De^{i(\delta_B+\delta_D-\gamma)}],\\
A[B^-\to K^-(K^{*-}K^+)_D]&=|A_BA_D|e^{i\delta_D}\\
&\quad\times[r_D+r_Be^{i(\delta_B-\delta_D-\gamma)}],\\
A[B^+\to K^+(K^{*-}K^+)_D]&=|A_BA_D|[1+r_Br_De^{i(\delta_B+\delta_D+\gamma)}],\\
A[B^+\to K^+(K^{*+}K^-)_D]&=|A_BA_D|e^{i\delta_D}\\
&\quad\times[r_D+r_Be^{i(\delta_B-\delta_D+\gamma)}].
\end{aligned} \tag{7}$$

Of the unknowns in these equations, $|A_D|$ and r_D have been measured in D decays [8], and $|A_B|$ was measured from the $B^-\to D^0K^-$ rate (and its conjugate) by reconstructing the D^0 in flavor-specific decays [9–11]. (While in practice, measuring $|A_B|$ involves identifying the D^0 through its hadronic decay, which is not a pure flavor tag; this induces a negligible error.) For any given integrated luminosity in the future, the errors in the measurements of $|A_D|$, r_D, and $|A_B|$ will induce a smaller error in the measurement of γ than the statistical error of measuring the decay rates corresponding to Eqs. (7).

This brings us to the key point: by measuring the rates of the four decays in Eqs. (7), one has four measurements for the remaining four unknowns: r_B, δ_B, δ_D, and γ.

A simple analytic solution for γ can only be obtained by neglecting the terms proportional to r_B^2 in the four decay rates corresponding to the amplitudes of Eqs. (7). In this approximation, we obtain

$$\cos^2\gamma=\frac{(R_1+R_3-2)^2-(R_2+R_4-2r_D^2)^2}{4[(R_1-1)(R_3-1)-(R_2-r_D^2)(R_4-r_D^2)]}, \tag{8}$$

where

$$R_1=\left|\frac{A[B^-\to K^-(K^{*+}K^-)_D]}{A_BA_D}\right|^2, \tag{9}$$

and similarly R_{2-4} are the squares of the "reduced amplitudes" corresponding to lines 2–4 in Eqs. (7). Equation (8) illustrates that our method is sensitive to γ, even neglecting terms in the branching ratios proportional to r_B^2. Although r_B is expected to be small, r_B^2 should of course not be neglected when the experimental analysis is carried out numerically. On the other hand, r_D is expected to be of order unity, and our method works best if r_D and $1-r_D$ are both of order unity. This expectation is supported by the data, $r_D=0.73\pm0.21$ [8] (the real uncertainty of r_D may already be smaller; here we assumed that the errors of the measured $D^0\to K^{*+}K^-$ and $D^0\to K^{*-}K^+$ rates are uncorrelated).

In principle, our method requires the analysis of only one type of final state with different charge assignments (such as $K^{*\pm}K^{\mp}$). In practice, the sensitivity can be improved by considering several B and D decays of the type considered so far. When a different B decay mode is used, for example, $B^-\to K^{*-}(K^{*+}K^-)_D$, four more measurements can be done, but only two new parameters are introduced, r_B' and δ_B'. (Here we assumed, as before, that $|A_B'|$ of this B decay channel is measured.) When a different D decay mode is used, for example, $B^-\to K^-(\rho^+\pi^-)_D$, only one new parameter is introduced, δ_D'. (Here we assumed again that $|A_D'|$ and r_D' of this D decay channel are measured.) An especially interesting additional D decay mode is to CP eigenstates. In this case, two extra measurements are possible, but no new parameters are added, since $r_D'=1$ and $\delta_D'=0$ or π.

Next we discuss how the sensitivity to γ changes in some limiting cases. If $r_D=1$ then Eqs. (7) become degenerate if either $\delta_D=0$ or $\delta_B=0$, and γ can no longer be extracted from this mode alone. If $r_D\neq1$, then our method is sensitive to γ independent of δ_D and δ_B. However, if $\delta_D=0$ or $\delta_B=0$ then the sensitivity to γ comes only from terms in the branching ratios proportional to r_B^2. In this case Eqs. (7) are not degenerate, but both the numerator and the denominator of Eq. (8), obtained neglecting r_B^2 terms, vanish due to $|\cos(\delta_D+\delta_B)|=|\cos(\delta_D-\delta_B)|$. This indicates that the error in the determination of γ may become large if either of the strong phases is small. This potential difficulty may be eliminated using several decay modes, as long as there are two sizable and different strong phases. For example, even if δ_B is small, the sensitivity of our analysis to γ can still be large and not rely on the r_B^2 terms in the decay rates if we use two D decay modes with sizable strong phases.

Throughout this analysis we assumed that $|A_D|$ and r_D are known from D decays, but δ_D is not. In the near future, it will be possible to measure δ_D at a charm factory using CP-tagged D decays [13,14], simplifying our γ measurement. With some model dependence, δ_D may also be measured at the B factories using a Dalitz plot analysis of the D decay [15]. In this case, one typically assumes that the variation of δ_D over the Dalitz plot can be accounted for by using phases of Breit-Wigner resonances. With enough events, the validity of this assumption can be checked in the analysis. Both the charm and the B factory measurements of r_D and δ_D can be carried out as a function of the Dalitz plot variables.

Measurements of γ that depend only on one strong phase, δ, are in general subject to an eightfold discrete ambiguity, due to invariance of the observables under the three symmetry operations [12]

$$\begin{aligned}
S_{\rm ex}&:\quad \gamma\to\delta,\quad \delta\to\gamma,\\
S_{\rm sign}&:\quad \gamma\to-\gamma,\quad \delta\to-\delta,\\
S_\pi&:\quad \gamma\to\gamma+\pi,\quad \delta\to\delta+\pi.
\end{aligned} \tag{10}$$

In the modes we propose, the variation of δ_D across the D decay Dalitz plot will be largely determined by the Breit-Wigner shape of the dominant resonance (such as the K^*). As a result, the theoretical expressions for the decay rates are no longer invariant under $S_{\rm ex}$ and $S_{\rm sign}$, and the only ambiguity that is relevant for our method is the twofold S_π ambiguity [7].

To compare the sensitivity to γ of the different methods, we assume that the only small parameters are λ and r_B. The latter has been estimated, assuming factorization as

$$r_B \sim \left|\frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*}\right| \frac{1}{N_C} \sim \lambda/2, \tag{11}$$

where $N_C=3$ is the number of colors. The accuracy of this estimate is expected to depend on the specific hadronic mode. Thus, for example, the numerical values of r_B in $B \to K(K^*K)_D$ and in $B\to K^*(K^*K)_D$ are expected to be different. Since the uncertainty of this estimate of r_B is large, it is not yet known whether r_B is closer to λ or to λ^2. The statistical significance of a CP asymmetry measurement scales roughly as the smallest amplitude that is needed in order to generate the asymmetry. Thus, to compare the methods we need to identify the smallest such amplitude in each of them. Compared to A_B, the smallest amplitude in the ADS method is of order min(λ^2,r_B), while in the GW and in our methods it is of order $r_B\lambda$. This simple argument suggests that if $r_B<\lambda$ then the ADS method has the largest sensitivity while if $r_B>\lambda$ then it is one of the others. There are many additional factors and experimental differences that will influence this comparison when the measurements are actually carried out. For example, since in our case measurements of doubly Cabibbo-suppressed D decays are not needed, the induced experimental error from D decay rates is expected to be the smallest. We conclude that the sensitivity of these methods are comparable and depend on yet unknown hadronic amplitudes and experimental details, and so all should be pursued.

So far we have neglected $D-\bar{D}$ mixing in our analysis. Its effects on the GW and ADS methods were studied in Ref. [13], and it is straightforward to generalize it to our case. We find that the leading effect on our determination of γ is generically of order x_D/r_B and y_D/r_B, where $x_D=\Delta m_D/\Gamma_D$ and $y_D=\Delta\Gamma_D/2\Gamma_D$. Since the present experimental bounds on x_D and y_D are at the few percent level, and even the standard model values may not be much smaller [16], $D-\bar{D}$ mixing gives rise to a theoretical error of order 10%. This is the largest theoretical uncertainty in our method at present, but it will be reduced as experiments yield tighter bounds on or measurements of x_D and y_D. (The leading sensitivity to $D-\bar{D}$ mixing in the GW method is similar to our case, while in the ADS method there are potentially even larger effects of order x_D/λ^2 and y_D/λ^2.)

We note that while there are experimental advantages to using the resonant D decay final states K^*K or $\rho\pi$, in general the full three-body Dalitz plot may be used to perform this analysis. (Higher multiplicity final states may also be used, although they will suffer from low reconstruction efficiencies.) Our method should also work well in some regions of the Dalitz plots of Cabibbo-allowed decay modes, such as $K_S\pi^+\pi^-$ or $K_SK^+K^-$. However, the regions where r_D and $1-r_D$ are both of order unity are expected to be relatively small, and so the advantage of the high branching fractions of these decays is not fully realized.

In conclusion, we proposed a variant of the GW method to measure γ. It requires measurement of only Cabibbo-allowed and singly Cabibbo-suppressed D decays in color-allowed B decays. Because it involves only large decay rates it may be carried out with current data sets. The branching fraction and reconstruction efficiency of the decay $B^{\pm}\to K^{\pm}(KK^*)_D$ are similar to that of $B^{\pm}\to K^{\pm}(K^+K^-)_D$, which has already been observed [11]. This might provide the first measurement of γ that is free of hadronic uncertainties.

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