Scales and hierarchies in warped compactifications and brane worlds

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Warped compactifications with branes provide a new approach to the hierarchy problem and generate a diversity of four-dimensional thresholds. We investigate the relationships between these scales, which fall into two classes. Geometrical scales, such as thresholds for Kaluza-Klein, excited string, and black hole production, are generically determined solely by the spacetime geometry. Dynamical scales, notably the scale of supersymmetry breaking and moduli masses, depend on other details of the model. We illustrate these relationships in a class of solutions of type IIB string theory with imaginary self-dual fluxes. After identifying the geometrical scales and the resulting hierarchy, we determine the gravitino and moduli masses through explicit dimensional reduction, and estimate their value to be near the four-dimensional Planck scale. In the process we obtain expressions for the superpotential and Kähler potential, including the effects of warping. We identify matter living on certain branes to be effectively *sequestered* from the supersymmetry breaking fluxes: specifically, such ''visible sector'' fields receive no tree-level masses from the supersymmetry breaking. However, loop corrections are expected to generate masses, at the phenomenologically viable TeV scale.

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I. INTRODUCTION

Recent years have opened up a new universe of string compactifications. Much of the work done on string phenomenology after the ''first superstring revolution'' of 1984 had focused on traditional Kaluza-Klein compactifications of string theory to four dimensions. However, we now see a great range of extensions of this picture: one may first of all consider more generally *warped compactifications*, and second one may have *brane world* scenarios in which branes—wrapped or otherwise—are present. This leads to a wide new spectrum of possibilities for reproducing four-dimensional Poincaré invariant physics from higher-dimensional string or M theory. Particularly interesting are the resulting geometrical or dynamical mechanisms that allow the string scale to be many orders of magnitude lower than the traditional value $\sim 10^{19}$ GeV—and perhaps even as low as $\mathcal{O}(1\ \text{TeV})$, providing a completely new potential resolution of the hierarchy problem. We still seem only to have scratched the surface in exploring this new universe.

The added complexities of these models imply the possibility of various new phenomena taking place at differing scales. In the case where some of these thresholds are lowered to $\mathcal{O}(1\ \text{TeV})$—or even lower—clearly it is especially interesting to understand what they are, and how they are related to the geometry and fields on the internal manifold. The diversity of possible scales include the natural scale for scalar masses, the apparent and fundamental Planck scales, thresholds for production of Kaluza-Klein states, excited string states or microscopic black holes, and the supersymmetry breaking scale. We will discuss the emergence of these in general warped compactifications or brane worlds that occur in string or M-theory.

Models exhibiting these phenomena include the large extra dimensions scenario of [1] and the warped model of [2]. Although inspired by stringy developments, the original proposals were not directly related to an underlying microscopic theory, but were solutions of effective theories capturing essential ideas. Large extra dimensions were subsequently discussed in the context of string theory in [3], and more complete embeddings of warped scenarios have emerged. As a specific example, [4] provides a string solution that geometrically realizes a hierarchically low fundamental string scale via warping, along the lines of [2]. A warped geometry is created within a Calabi-Yau threefold by fluxes in the spirit of [5–7], with a throat that comes to a smooth end playing the role of an infrared brane, while the Calabi-Yau manifold itself plays the role of an ultraviolet brane by terminating the throat at the top. Since the total space is compact, this picture bears similarities to both [1] and [2]. In these theories, the fluxes have the additional benefit of freezing many geometric moduli of the Calabi-Yau background, as well as the dilaton. The ''Gukov-Vafa-Witten'' (GVW) superpotential [8] that freezes these moduli also can break supersymmetry spontaneously.

In this paper we study the relationship of the various thresholds of physical phenomena in a warped or brane world compactification, both to each other, and to properties of the underlying geometry. Several of these scales rely only on simple properties of the geometry, and very general statements can be made. We refer to these as *geometrical scales.* Some of the scales, particularly involving supersymmetry breaking, are less generic and more model dependent; we

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refer to these as *dynamical scales*. We will illustrate some of this model dependence in the context of the model of [4] and the GVW superpotential.

In outline, we begin with a brief general discussion of warped compactifications and brane worlds. We follow this by discussing the general relationships between geometrical scales: the fundamental and apparent four-dimensional Planck scales, the string scale, and the typical mass scales for brane matter. This is essentially a simple extension of known results. We then discuss the more model-dependent (but still geometrical) question of the thresholds for Kaluza-Klein modes. We next turn to dynamical scales, particularly the supersymmetry breaking scale. Here the observed scales depend sensitively both on the *form* of supersymmetry breaking (e.g., gravity mediated from the moduli sector or a hidden brane sector, or gauge mediated from extended gauge dynamics on the branes), and on the warping in the region where it is localized.

We then give an extensive illustration of our comments in the context of the compactifications of [4]. With a moderate choice of discrete fluxes, these solutions generate a hierarchy between the weak and Planck scales, while at the same time breaking supersymmetry and fixing many of the problematic moduli familiar from traditional Calabi-Yau compactifications. After outlining properties of these solutions, we derive expressions for the gravitino mass and for the potential for moduli. While not essential for the derivation, these can be thought of as arising from a four-dimensional effective supergravity action, and we exhibit the corresponding Kähler and superpotentials, explicitly including the effects of warping. Generically the gravitino and moduli masses are estimated to be large, of order $M_4 \sim 10^{19}$ GeV, an apparent phenomenological disaster. However, as a result of no-scale structure, the tree level masses for scalars living on an IR brane vanish. Moreover, fermion masses also vanish at the tree level [9], producing a close analogue of the *sequestered* scenarios of [10]. To our knowledge this is the first realization of sequestering in a string theory background. The sequestered form persists even incorporating brane back reaction, although it may not survive α' corrections. These "visible sector" masses receive contributions from loops; the warped structure of the solution indicates that these corrections should be of order $\mathcal{O}$ (TeV) for solutions where the hierarchy is indeed generated by warping. Section V is rather long and technical, but the reader interested in a brief overview is directed to a summary in Sec. V F.

II. WARPED GEOMETRIES, BRANE WORLDS, AND THE HIERARCHY

In traditional Kaluza-Klein compactifications, the extra dimensions y^m, $m=1,\dots,D-4$, of D-dimensional spacetime (or more generally, in string theory the extra-dimensional conformal field theory) are taken to form a direct product geometry with the visible dimensions x^μ, $\mu=0,1,2,3$:

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n, \tag{1}$$

where g_{mn} is the metric in the extra dimensions. However, we have increasingly realized the potential importance of compactifications in which this geometry is extended to the most general 4D Poincaré-invariant form:

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n. \tag{2}$$

Such a compactification is known as a *warped compactification*, and the function e^{2A} as a *warp factor* [11–14].

A second important extension, following from the "second superstring revolution," is the inclusion of branes. In order to preserve 4D Poincaré invariance, these should be fully extended over the dimensions of observed four-dimensional spacetime. Their configuration in the extra dimensions is more flexible. Simplest is the case of D3-branes, which then are pointlike in the extra dimensions. But more generally, the compact geometry can have non-trivial closed cycles on which some of the dimensions of a Dp-brane, with $p>3$, can wrap.

Within the context of string theory, there are also higher-form antisymmetric tensor fields that can acquire vacuum expectation values (VEVs) in the compact directions, without spoiling Poincaré invariance.

D-branes, fluxes, and warping are of course in general related, since D-branes serve as sources for fluxes, and both D-branes and fluxes may serve as sources of non-trivial warp factors. It is also possible for D-branes and fluxes to transmute into one another.

Our interest is in string or M theory propagating on the spacetime (2). As long as geometrical features are larger than the fundamental Planck length, the dynamics is well described in terms of a low-energy effective action of the form

$$S = \frac{M_D^{D-2}}{(2\pi)^{D-4}}\int d^Dx\sqrt{-g}\,\frac{1}{2}\mathcal{R} + \int d^Dx\sqrt{-g}\mathcal{L}, \tag{3}$$

where M_D is the fundamental Planck mass (in the phenomenologically useful conventions of [15]), $\mathcal{R}$ is the Ricci scalar, and $\mathcal{L}$ is the Lagrangian for other fields and sources, including matter, fluxes, and branes.

We would like to determine the parameters that govern four-dimensional phenomenology, in terms of the parameters of the underlying fundamental theory. In the example of a string compactification of the type II string, for which $D=10$, the string frame Lagrangian takes the form

$$S \propto M_s^8 \int d^{10}x\sqrt{-g}e^{-2\phi}\mathcal{R} + \cdots, \tag{4}$$

up to a numerical constant, where ϕ is the dilaton and the string coupling is $g_s = e^{\langle\phi\rangle}$. The relation between the fundamental string scale and the Planck scale immediately follows:

$$M_{10} = g_s^{-1/4}M_s. \tag{5}$$

The relation between the fundamental and apparent 4-dimensional Planck scales is nearly as simple. Indeed, replace the metric (2) by one including 4D fluctuations

$$ds^2=e^{2A(y)}g_{\mu\nu}dx^\mu dx^\nu+g_{mn}(y)dy^m dy^n, \tag{6}$$

and substitute into the action (4). We find that fluctuations of the 4D metric about internal geometries obeying the equations of motion are governed by an effective action

$$S_4=\frac{M_4^2}{2}\int d^4x\sqrt{-g_4(x)}\mathcal{R}_4, \tag{7}$$

with the four- and D-dimensional Planck masses related by

$$\frac{M_4^2}{M_D^2}=\left(\frac{M_D}{2\pi}\right)^{D-4}\int d^{D-4}y\sqrt{g_{D-4}}e^{2A}\equiv\left(\frac{M_D}{2\pi}\right)^{D-4}V_w. \tag{8}$$

This equation defines the ''warped volume'' V_w.

Next consider mass scales for matter fields. In particular, if fermion masses are generated by a Higgs scalar H, in the absence of a protection mechanism, radiative corrections are expected to generate scalar masses M_H of the order of the cutoff, which here is expected to be $\mathcal{O}(M_D)$, in the Lagrangian $\mathcal{L}$. In the general brane world scenario, where fermion and Higgs fields propagate on a ''standard model'' p-brane with coordinates z, this results in a contribution to the action of the form

$$S_H=-\frac{1}{2}\int d^4x\int d^{p-4}z\sqrt{g_{\rm brane}}[e^{2A}(\nabla_\mu H)^2+M_H^2e^{4A}H^2], \tag{9}$$

where $g_{\rm brane}$ is the induced metric on the brane. From this we find that the Higgs boson mass scale is given in terms of averages of the warp factor over the standard-model brane, by

$$M_0^2=\frac{\int d^{p-4}z\sqrt{g_{\rm brane}}e^{4A}}{\int d^{p-4}z\sqrt{g_{\rm brane}}e^{2A}}M_H^2\sim e^{2A_{\rm SM}}M_D^2 \tag{10}$$

where we denote the average of the warp factor on the standard model (SM) or visible brane by

$$e^{2A_{\rm SM}}=\int d^{p-4}z\sqrt{g_{\rm brane}}e^{2A}. \tag{11}$$

This makes it clear that fields localized in regions where $e^A\ll 1$ have their masses suppressed relative to the fundamental scale M_D; natural TeV scale masses can be generated by the warp factor.

An alternative viewpoint of this mechanism comes from using the Weyl symmetry of the actions (4), (9). Define the new variables

$$g=\lambda^2\bar{g},\quad (M_D,M_0)=(\bar{M}_D,\bar{M}_0)/\lambda,\quad H=\bar{H}/\lambda, \tag{12}$$

with corresponding scalings for other fields and dimensionful parameters. This choice of scale may be used to set the average

$$e^{2\bar{A}_{\rm SM}}=1. \tag{13}$$

In these units [barring a large ratio of the different averages that enter in Eq. (9)], the fundamental Planck scale and the Higgs boson mass are both naturally comparable, and we therefore have a choice.

(1) *Conventional Planck-scale compactification:* Take $\bar{M}_D\sim M_4\sim 10^{19}$ GeV, and find a mechanism, such as supersymmetry, to suppress the Higgs boson mass to a far smaller scale.

(2) TeV-*scale gravity scenario:* Take $\bar{M}_D\sim 1$ TeV, which then requires $V_w\gg 1/\bar{M}_D^{D-4}$.

From the definition (8) of the warped volume and the convention (13) we see that the latter choice results from either large volume or a large warp factor away from the brane, or some combination of the two; in the barred variables, these two effects are on the same footing.

Thus to summarize there are two possible conventions from which to understand the physics of the hierarchy in the context of a TeV-scale gravity model. In the first, the fundamental scale is $M_D\sim 10^{19}$ GeV, and scalar masses are suppressed to a TeV. The second corresponds to a definition of four dimensional energy relative to an observer localized on the brane; for such an observer, the fundamental scale is reached at four-dimensional energies $\bar{M}_D\sim$TeV, and this is also the natural scale for scalar masses. The four-dimensional Planck scale M_4 is enhanced relative to these by the large warp factor in Eq. (8). We will find the barred variables to be convenient for most the the following sections, although we will revert to the unbarred variables for the purposes of calculating masses of bulk fields in Sec. V.

III. GEOMETRICAL SCALES AND THRESHOLDS

In conventional Planck-scale compactifications, many of the new phenomena resulting from the compactification are only accessible in the vicinity of the four-dimensional Planck scale, $M_4\sim 10^{19}$ GeV. One of the reasons for the great interest in warped compactifications is the much greater latitude in the possible scales at which observable phenomena may occur. Many of these scales are determined purely from the *geometry* of the warped compactification, as opposed to other dynamical information. We have just seen two examples: the relationships between the fundamental Planck scale, the apparent four-dimensional Planck scale, and the naturalness scale for scalar masses are determined through relations (8) and (10) and depend only on the warp factor and the geometry of the internal manifold. In this section we will extend this discussion of physical scales and the corresponding thresholds for other physical phenomena.

A. Strings and black holes

The most exciting possibility raised by warped compactification is that, as outlined above, the fundamental Planck scale may be much lower than the apparent four-dimensional Planck scale. This means that we may begin to experimentally access the dynamics of quantum gravity much sooner than previously anticipated.

For example, it is believed that the generic high-energy physics of gravity is the production of black holes. If, as in the preceding section, we work in units where the warping is unity on the IR brane, the fundamental Planck scale $\bar{M}_D$ may be as low as a TeV. Of course, the fundamental Planck scale generically represents the threshold for production of microscopic black holes, so above this energy collisions of particles on the SM brane can produce black holes; this corresponding phenomenology is discussed in [16–18].[1]

In the context of string theory, this threshold may be pushed up to make room for an intermediate regime where string states are produced. This depends on the value of g_s. As we see from Eq. (5), at weak coupling the threshold for string production is below the Planck energy. At the same time, the string length exceeds the Planck length,

$$l_s \sim g_s^{-1/4} l_p . \tag{14}$$

Objects smaller than this will explicitly exhibit behavior characterized by non-local string dynamics, and classical black holes will only begin to exist once their radii exceed this value, at the *correspondence* scale [31]

$$M_c \sim \frac{M_s}{g_s^2}. \tag{15}$$

Between M_s and M_c we expect perturbative string states gradually to become more strongly coupled and morph into black hole states, perhaps with intermediate states best described as ‘‘string balls’’ [32,33].

So, to summarize the results of this subsection, for weakly coupled string theory, we should start seeing perturbative string states at the threshold $g_s^{1/4} M_{10}$; these become more strongly coupled, and evolve into the generic gravitational physics of black holes above the threshold M_s/g_s^2. Some of the phenomenology of the initial perturbative string regime has been discussed in [34].

B. Kaluza-Klein modes

Another generic phenomenon is production of Kaluza-Klein modes. For simplicity we just discuss these in the case of scalar fields, although results for higher-spin fields should be qualitatively similar.

Specifically, consider a D-dimensional scalar field Φ, with action

$$S_\Phi = -\frac{1}{2}\int d^D x \sqrt{-g}[(\nabla\Phi)^2 + M_\Phi^2 \Phi^2]$$
$$= -\frac{1}{2}\int d^4 x \sqrt{-g_4}\int d^6 y \sqrt{g_6} e^{4A}[e^{-2A}\eta^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi + g^{mn}\nabla_m\Phi\nabla_n\Phi + M_\Phi^2\Phi^2]. \tag{16}$$

This gives an equation of motion

$$\Box_4 \Phi + e^{-2A}[\nabla^m(e^{4A}\nabla_m\Phi) - M_\Phi^2 e^{4A}\Phi] = 0. \tag{17}$$

Therefore the masses of Kaluza-Klein states will be given by the eigenvalues of the wave operator

$$e^{-2A}[\nabla^m(e^{4A}\nabla_m Y_i(y)) - M_\Phi^2 e^{4A} Y_i(y)] = -M_i^2 Y_i(y). \tag{18}$$

The size of these masses for Kaluza-Klein modes localized in the vicinity of the SM brane is generically determined by the scales on which the 6D metric and warp factor A vary. For example, in an unwarped compactification, the lightest scale is roughly $1/L$, where L is the size of the largest dimension. In the case of the model of [2], the Kaluza-Klein masses are of size $1/R$, where R is the AdS radius, in other words the scale of variation of the warp factor which in this case is just

$$A = -y/R; \tag{19}$$

a similar result is found for the string solutions of [4] which have an approximately AdS region. Either kind of geometrical scale will typically be larger than the fundamental length scale (otherwise a geometrical description may not apply), so the Kaluza-Klein (KK) masses will typically be below the fundamental scale, even far below as in the extreme case of [1].

Of course, there may be more complicated scenarios where contributions of the warp factor relative to that on the SM brane rescale these masses. Kaluza-Klein (KK) modes localized in a region with warp factor A will have their masses scaled by e^A. For example, Ref. [36] investigates scenarios with multiple throats that are approximately anti–de Sitter; if we consider the KK modes localized in throat j in a vicinity with warp factor A_j, then the corresponding masses will be renormalized by the factor e^{A_j} as seen by an observer on the SM brane. Of course couplings of such modes in a distinct throat to those of the visible sector are expected to be correspondingly suppressed.

C. Summary of geometric thresholds

To summarize the results of this section, in a TeV-scale gravity scenario with hierarchy generated by warping, the sequence of thresholds is as follows. (This summary is given in the ‘‘brane-based’’ conventions outlined in the preceding section.) The lowest energy threshold is generically that for Kaluza-Klein states,

$$M_{KK} \sim \frac{e^{A_{KK}}}{R}, \tag{20}$$

[1]It has long been believed that collisions above the Planck energy should create black holes. An early concrete statement is Thorne's hoop conjecture [19], and such processes were further studied in [20] and [21–23]. Reference [24] pointed out the relevance of such black hole formation within the TeV-scale gravity models of [1], and discussed some aspects of the phenomenology. Other aspects of black holes in these models were discussed in [25], and their evaporation in [26]. The experimental relevance of black hole formation in warped scenarios was pointed out in [16]. A general argument for classical black hole formation at high energies appears in [27]. For reviews, see [28–30].

where A_{KK} is the warp factor in the region where the state is localized, and R is a characteristic proper geometrical scale. Next, in the case of a string scenario, and at least for moderately weak string coupling, comes the threshold for producing string states:

$$M_S \sim g_s^{1/4} M_{10}. \tag{21}$$

For the possibly more realistic case of strong string coupling, this is degenerate with the fundamental Planck scale, which as measured by observers on the standard model brane is $\bar{M}_D \sim$ TeV; this is the approximate threshold for producing black holes. Scalar masses are also naturally of this size:

$$M_0 \sim \bar{M}_D. \tag{22}$$

The four-dimensional Planck scale lies far beyond, at

$$\frac{M_4^2}{\bar{M}_D^2} = \left(\frac{\bar{M}_D}{2\pi}\right)^{D-4} V_w. \tag{23}$$

IV. DYNAMICAL SCALES; SUPERSYMMETRY BREAKING

Certain physical thresholds are determined by more detailed dynamical information than that contained in the metric; these are the dynamical scales. An obvious example is that of supersymmetry breaking mass scales: the mass of the gravitino, and of superpartners. Moreover, generic Calabi-Yau compactifications suffer from a plethora of moduli, but these typically also get masses upon supersymmetry breaking. Details of these scales depend sensitively on the dynamics; we will exhibit the mechanism of flux-generated masses in the next section.

There are two broad classes of relevant supersymmetry breaking mechanisms, *gauge mediated* and *gravity mediated*, and in particular the latter appears to offer the possibility of a large range of scales.

In gauge-mediated supersymmetry breaking, we imagine that in addition to the standard model dynamics, the infrared branes produce other dynamics that breaks supersymmetry and is conveyed to the standard model fields via a gauge theory messenger. Such mechanisms have been widely studied; for a review and references see [35]. It should be noted that while many of their features are not necessarily modified by virtue of the warped setting, a TeV-scale gravity scenario does apparently put one strong constraint on allowed scenarios since the highest allowed scale in the gauge theory near the SM brane is the TeV scale. This is problematic in view of the need for SUSY breaking scales of order 100 TeV to avoid flavor problems. We will not explore further aspects of these scenarios in this paper.

In gravity-mediated scenarios, it appears that there can be a much richer interplay between the supersymmetry-breaking dynamics and the warping. For example, first consider supersymmetry breaking produced by gauge dynamics on other IR branes that are only coupled to standard model fields via gravity. In this case, if the hidden-sector supersymmetry breaking scale is Λ, we expect that the splittings in the standard model sector are given by

$$m_{3/2} \sim \frac{\Lambda^2}{M_p} \tag{24}$$

where here $M_p \sim \bar{M}_D \sim 1$ TeV if the branes are separated on scales small as compared to the curvature scales or radii of the extra dimensions, and $M_p \sim M_4$ if the branes are separated on larger scales. For $\Lambda \sim$ TeV this can produce the correct splittings if the effective gravitational mediation scale is $\bar{M}_D$. However, this produces splittings that are far too low, $\mathcal{O}(10^{-4})$ eV, if the mediation scale goes like M_4.

Different scales may, moreover, be generated depending on the *location* of the supersymmetry breaking in the extra dimensions; we expect a general relationship

$$\Lambda \sim e^{A_{SUSY}} \Lambda_{SUSY} \tag{25}$$

where Λ_{SUSY} is the *proper* scale for supersymmetry (SUSY) breaking (as measured by a higher-dimensional observer in the supersymmetry breaking region) and $e^{A_{SUSY}}$ is the corresponding warp factor of that region. For example, one may consider supersymmetry breaking on some branes that have been raised some distance up an AdS throat relative to the standard model branes—although a critical question is how to stabilize such branes. Alternatively, as mentioned previously, one may generically have warped compactifications with more than one region with strong warp factor; standard model branes could be in one region and the supersymmetry breaking sector in another. A large relative warp factor between the two regions can generate a large variation in the supersymmetry breaking scale.[2] Of course one expects that the proper scale of supersymmetry breaking is bounded by the fundamental scale, $\Lambda_{SUSY} \lesssim < \bar{M}_D$. But the relative factor in Eq. (25) can easily produce a sufficiently large gravitino mass,

$$m_{3/2} \sim \frac{e^{2A_{SUSY}} \Lambda_{SUSY}^2}{M_4}. \tag{26}$$

Indeed, the gravitino mass can also in practice be too *high*. For example, SUSY breaking in the vicinity of the UV brane could produce a scale

$$m_{3/2} \sim M_4; \tag{27}$$

we will see a similar phenomenon in models which produce SUSY breaking through flux in the next section. However, there is one other interesting caveat: supersymmetry breaking does not always generate tree-level masses for superpartners. This may for example happen if the Kähler and superpotentials of the visible and hidden sectors completely separate. Such a mechanism was proposed in the ‘‘sequestered’’ scenario of [10]. In this case the splittings will be

[2]Reference [36] proposed a different mechanism, *tunneling mediation*, for supersymmetry breaking in such scenarios, although for a large range of parameters gravity mediation dominates.

produced by loop corrections. If the gravitino has a mass given by Eq. (27), it is effectively removed from the theory on scales smaller than M_4. One might think this leads to loop corrections of order M_4 to scalar masses in the visible sector, but note that when one computes the divergent diagrams that give such masses, the cutoff should actually be the fundamental Planck scale $\overline{M}_D$. The important point is that as seen from the perspective of an observer on the standard model brane, she lives in a theory that is not supersymmetric, but in which the fundamental scale and cutoff is $\overline{M}_D\sim$ TeV. Quantum corrections should thus produce scalar masses of TeV size. Similar, though less general, observations were made in [37].

V. A STRING THEORY EXAMPLE: HIERARCHIES FROM FLUXES

A concrete realization of many of these ideas is provided by the warped compactification solutions described in [4]. These exhibit some of the basic ideas of the two-brane scenario of [2] in a known microscopic theory, namely type IIB string theory. They also have other appealing features, as they improve on a standard phenomenological difficulty of string theory by stabilizing many of the moduli fields. Supersymmetry is generically broken, but both the cosmological constant and, as we discuss below, masses for ''visible sector'' fields living on a brane are zero at tree level; this can be related to a ''pseudo Bogomol'nyi-Prasad-Sommerfield (BPS)'' condition on the branes, which we describe shortly.

Specifically, quantized three-form fluxes are introduced inside a compact six-dimensional manifold, warping a region of the space into an approximately AdS throat. The throat is terminated smoothly at the infrared end by a geometry that is an appropriate analogue of the Klebanov-Strassler solution [7], while the unwarped region of the manifold plays the role of an ultraviolet brane, much as in [5].

Mobile branes that fill the non-compact directions are generically required to be present. Some of these branes are taken to reside in the throat region, where the warping induces a hierarchy of scales for the ''visible sector'' fields on these branes. In principle one would like these to be the standard model fields, which could perhaps be realized by placing an additional singularity (or more generally brane intersections) at the base of the throat, but we will for the moment content ourselves with the simpler case of the $U(N)$ spectrum of D3-branes at a generic point.

A. Solutions and geometric scales

We begin by describing these solutions in more detail. The bosonic low-energy action for type IIB supergravity in the Einstein frame can be written (we use the conventions of [4])

$$S^b_{IIB}=\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{\mathcal{R}-\frac{\partial_M\tau\partial^M\overline{\tau}}{2(\mathrm{Im}\,\tau)^2}-\frac{G_{(3)}\cdot\overline{G}_{(3)}}{12\,\mathrm{Im}\,\tau}-\frac{\tilde{F}^2_{(5)}}{4\cdot 5!}\right\}-\frac{1}{8i\kappa_{10}^2}\int\frac{C_{(4)}\wedge G_{(3)}\wedge\overline{G}_{(3)}}{\mathrm{Im}\,\tau} \tag{28}$$

where we have

$$G_{(3)}\equiv F_{(3)}-\tau H_{(3)},\quad \tau\equiv C_{(0)}+ie^{-\phi}, \tag{29}$$
$$F_{(3)}=dC_{(2)},\quad H_{(3)}=dB_{(2)},$$
$$\tilde{F}_{(5)}=dC_{(4)}-\frac{1}{2}C_{(2)}\wedge H_{(3)}+\frac{1}{2}B_{(2)}\wedge F_{(3)}.$$

Here $\mathcal{R}$ is the Ricci scalar, ϕ is the dilaton, $C_{(0)}$ is the Ramond-Ramond (RR) scalar, $B_{(2)}$ and $C_{(2)}$ are the Neveu-Schwarz–Neveu-Schwarz (NSNS) and RR 2-form potentials, and $C_{(4)}$ is the RR 4-form potential. The five-form field strength $\tilde{F}_{(5)}$ is self-dual:

$$\tilde{F}_{(5)}={}^*\tilde{F}_{(5)}, \tag{30}$$

which does not follow from the action (28); rather, Eq. (28) is understood to produce the correct equations of motion when supplemented by Eq. (30). Dimensionally reducing the action in a background 5-form field must be done with care, as we discuss in Sec. V C.

It is familiar that a four-dimensional, $\mathcal{N}=2$ supersymmetric solution may be obtained from a type II theory by considering a background geometry of the form $R^4\times\mathcal{M}$, where $\mathcal{M}$ is a Calabi-Yau threefold. However, there is a much wider class of warped compactifications preserving the Poincaré symmetry. The general Poincaré invariant configuration allows the axion-dilaton scalar to vary over the compact manifold,

$$\tau=\tau(y), \tag{31}$$

and allows components of the three- and five-form fluxes in the compact directions:

$$G_{(3)}=\frac{1}{3!}G_{mnp}(y)dy^m dy^n dy^p, \tag{32}$$

$$\tilde{F}_{(5)}=\partial_m\alpha(y)(1+*)dy^m dx^0 dx^1 dx^2 dx^3. \tag{33}$$

The expression for the five-form is manifestly consistent with self-duality (30), and is the most general form consistent with the Bianchi identity. Poincaré invariance also allows D3-branes, which will be pointlike in the extra dimensions, 5-branes wrapped on two-cycles, D7-branes wrapped on four-cycles, and D9-branes. The metric in general takes the warped form

$$ds^2=G_{MN}dx^M dx^N=e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu+g_{mn}(y)dy^m dy^n. \tag{34}$$

With a typical configuration of branes and fluxes, g_{mn} is no longer Calabi-Yau.

Reference [4] considers a very general class of string solutions that are obtained by making an additional assumption, and this class will be the focus of our description for the rest of the paper. The assumption is that localized sources such as branes and orientifold planes must satisfy a BPS-like condition relating their stress-energy to their D3-brane charge:

$$\frac{1}{4}(T_m^m - T_\mu^\mu)^{loc} \geqslant T_3 \rho_3 . \tag{35}$$

Here ρ_3 is the D3-brane charge density of the localized sources, and the constant T_3 is the D3-brane tension. This ''pseudo-BPS'' condition roughly states that negative-tension sources (which are of course allowed in string theory, as for example orientifold planes) cannot be too strong.

Under these added assumptions, [4] finds the general solutions in terms of an underlying Calabi-Yau geometry (or more generally, in the case with 7-branes, an *F*-theory background). The warp factor and five-form are related by

$$e^{4A} = \alpha, \tag{36}$$

the internal metric is *conformal* to a Calabi-Yau (or *F*-theory base) metric $\tilde{g}$,

$$g_{mn} = e^{-2A}\tilde{g}_{mn} , \tag{37}$$

the flux must be imaginary self-dual (ISD) in the compact dimensions,

$$*_6 G_{(3)} = i G_{(3)} , \tag{38}$$

where $*_6$ denotes the six dimensional Hodge dual, and finally, the BPS-like condition (35) is in fact saturated for all sources.

The presence of localized sources is not an option, but is forced on us by flux conservation. Because the $H_{(3)}$ and $F_{(3)}$ fluxes participate in the 5-form Bianchi identity,

$$d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} , \tag{39}$$

together they produce a source of D3-brane charge. Additional sinks of D3-brane flux must then be introduced on the compact manifold to cancel this charge. Two options were discussed in Ref. [4]: one may quotient the space by a discrete symmetry so as to introduce orientifold 3-planes, or one may add 7-branes wrapped on four-cycles, both of which carry a D3-brane charge (in the latter case the charge is induced by the curvature of the four-cycle). The 7-branes require a non-Ricci-flat unwarped geometry as well as a varying axion-dilaton τ, all of which is summarized as an *F*-theory compactification on a Calabi-Yau fourfold *X*. The total charge that must vanish is then

$$Q_{D3} = N_{D3} - \tfrac{1}{4} N_{O3} - \frac{\chi(X)}{24} + \frac{1}{2\kappa_{10}^2 T_3} \int_{\mathcal{M}} H_{(3)} \wedge F_{(3)} = 0. \tag{40}$$

$\chi(X)$ is the Euler number of *X*, and N_{D3} and N_{O3} denote the numbers of D3-branes and O3-planes, respectively. Notice that with a general choice of fluxes, satisfying this constraint *requires* the presence of some number of explicit D3-branes, on which gauge dynamics may live. To avoid the complications of the *F*-theory examples, we will often keep the orientifold case in mind, but it should be remembered that both are possible.

The underlying Calabi-Yau (CY) manifold in general has a large collection of both Kähler and complex structure moduli, and this is typically a problem for string phenomenology. However, for given quantized fluxes, the ISD condition (38) fixes many of these moduli [4]. This condition can be reexpressed in terms of the Dolbault cohomology of the CY manifold, as permitting only a primitive (2,1) form (i.e., a $G_{ij\bar{k}}$ satisfying $g^{j\bar{k}}G_{ij\bar{k}} = 0$) and a (0,3) form. The former preserves $\mathcal{N}=1$ supersymmetry, while the latter breaks all SUSY. Generically, one expects both types to be present in a given compact background, and so SUSY is generally broken. These models are found classically to be no-scale models [38,39], and in particular the cosmological constant vanishes despite supersymmetry breaking.

The 3-form fluxes must satisfy quantization conditions with respect to the 3-cycles on $\mathcal{M}$; if C_I form a homology basis for three cycles,

$$\int_{C_I} F_{(3)} = (2\pi)^2 \alpha' M_I , \quad \int_{C_I} H_{(3)} = -(2\pi)^2 \alpha' K_I . \tag{41}$$

Consequently they are fixed and do not fluctuate. A particularly interesting case, which we will bear in mind as an example, arises if we work in the vicinity of a conifold point in the Calabi-Yau moduli space. Call the degenerating cycle *A* and its dual cycle *B*, and suppose we have turned on a flux configuration with

$$\int_A F_3 = (2\pi)^2 \alpha' M, \quad \int_B H_3 = -(2\pi)^2 \alpha' K. \tag{42}$$

As [4] found, this generates an approximately AdS region, locally resembling the Klebanov-Strassler geometry [7].

These particular solutions exemplify the features of warped compactifications that we have discussed in earlier sections. The most fundamental is the warping that arises in the AdS-like region. The fluxes (42) produce a relative warp factor

$$e^{A_{min}} \sim \exp(-2\pi K/3Mg_s) \tag{43}$$

between the unwarped region and the bottom of the throat.

Since the gravitational potential is minimized at the bottom of the throat, and the configuration is not truly BPS, a reasonable hypothesis is that a potential for the position of the branes is generated at loop level and has a minimum when they are at the bottom of the throat. (We will return to related comments when we discuss generating masses for brane matter.) Fields living on branes at the bottom of the throat will perceive a hierarchy of scales between the apparent M_4 and the fundamental Planck scale $\bar{M}_D$; realistic values of $M_4 \sim 10^{19}$ GeV, $\bar{M}_D \sim 1$ TeV may be generated through Eqs. (8),(43) with quite modest values for the flux quanta.

The rest of the discussion of geometrical scales of Sec. III also directly applies. If the fundamental Planck scale has been lowered to $\mathcal{O}$ (TeV), black holes may of course be produced above this threshold on the SM brane. Likewise, string states may be produced, at comparable or lower thresholds depending on the value of the string coupling [thus to agree with phenomenological bounds, weakly coupled models should instead have M_s set to $\mathcal{O}$ (TeV) or higher]. Furthermore, the lightest Kaluza-Klein modes will have masses given by the approximate geometrical scales at the bottom of the throat; from [7] we find

$$E_{KK} \sim \frac{\overline{M}_s}{g_s M}. \tag{44}$$

This exhausts the discussion of the geometrical scales.

An important question is to determine the corresponding dynamical scales, in particular the scale of supersymmetry breaking, the magnitude of the resulting splittings in supermultiplets in the visible sector, and the masses of the moduli fields. We turn to this task in the coming sections. We calculate the mass of the gravitino broken by (0,3) flux as a measure of supersymmetry breaking, as well as determining the potential for the moduli.[3] We also comment on how supersymmetry breaking is not communicated to the visible sector fields at the tree level, a phenomenon analogous to the *sequestered* scenarios of [10]. In the process, we develop expressions for the Kähler and superpotentials for such warped compactifications, which heretofore have not been calculated with the warping taken into account.

B. The gravitino

In the absence of (0,3) flux, $\mathcal{N}=1$ supersymmetry is preserved in four dimensions. Correspondingly there is a massless gravitino. When SUSY is broken by the flux, the mass $m_{3/2}$ of this gravitino is a useful measure of the breaking. We shall begin by computing this quantity by dimensional reduction of the 10D theory, and in the process relate this to the expressions for the superpotential and Kähler potential including the effects of warping.

The equations of motion for the IIB fermions are given in Appendix B. We find it convenient to work in terms of an action from which these equations can be derived, and to determine the gravitino mass it is sufficient to consider the gravitino squared terms:

$$\frac{1}{\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left\{ i\bar{\Psi}_M\Gamma^{MNP}\left(D_N\Psi_P-\frac{i}{2}Q_N\Psi_P-R_P\Psi_N\right)\right.$$
$$\left.-\left[\frac{i}{2}\bar{\Psi}_M\Gamma^{MNP}S_P\Psi_N^*+\text{H.c.}\right]\right\}, \tag{45}$$

where Ψ_M is Weyl but not Majorana, Q_N is a composite connection composed of derivatives of τ (see [41]), and the supercovariantizations are[4]

$$R_M \equiv -\frac{i}{16\cdot 5!}(\Gamma^{M_1\cdots M_5}\tilde{F}_{M_1\cdots M_5})\Gamma_M,$$
$$S_M \equiv \frac{1}{96(\text{Im}\,\tau)^{1/2}}(\Gamma_M{}^{NPQ}G_{NPQ}-9\Gamma^{NP}G_{MNP}). \tag{46}$$

The supersymmetry variation of the gravitino is

$$\delta\Psi_M=\left(D_M-\frac{i}{2}Q_M\right)\varepsilon+R_M\varepsilon+S_M\varepsilon^*, \tag{47}$$

where the supersymmetry parameter ε is a 10D Weyl spinor field.

We must first identify the 4D $\mathcal{N}=1$ gravitino as a particular component of the 10D field. In a warped background satisfying Eqs. (33),(34),(36) without 3-form fluxes, the preserved 4D supersymmetries are associated with Killing spinors (for more detail, see [42][5]):

$$\varepsilon=\zeta(x)\otimes e^{A(y)/2}\chi(y), \quad \tilde{D}_m\chi=0, \tag{48}$$

where we use the tilde to denote the CY metric.[6] We normalize the covariantly constant spinor on the unwarped compact space χ as $\chi^\dagger\chi=1$.

Knowing the preserved supersymmetry, we can easily determine the associated gravitino as the SUSY partner of the 4D graviton. The supersymmetry variation of the 4D metric $g_{\mu\nu}$ is

$$\delta g_{\mu\nu}\propto\bar{\zeta}\gamma_\mu\psi_\nu+\bar{\zeta}\gamma_\nu\psi_\mu, \tag{49}$$

and its 10D counterpart is analogous. One then finds the 4D gravitino ψ_μ embedded in the 10D gravitino as

$$\Psi_\mu=\psi_\mu\otimes e^{A/2}\chi. \tag{50}$$

It is straightforward to see that under dimensional reduction, the Einstein and Rarita-Schwinger terms for the 4D metric $g_{\mu\nu}$ and gravitino ψ_μ match the standard form:

$$S=\frac{1}{\kappa_4^2}\int d^4x\sqrt{-g_4}\{\tfrac{1}{2}\mathcal{R}_4+i\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho\}, \tag{51}$$

with the 4D gravitational constant κ_4 given in terms of the 10D gravitational constant κ_{10} and the warped volume V_w:

[3] Related work involving partial SUSY breaking in the unwarped case appeared in [40].

[4] The G field picks up an additional τ-dependent phase in transforming from the conventions of [41], which we absorb into a redefinition of Ψ.

[5] The five-form in [42] is related to our $\tilde{F}_5$ by $F_{GP}=-\tilde{F}_5$.

[6] In the case of an F-theory compactification, χ is covariantly constant with respect to $\tilde{D}_m-(i/2)Q_m$.

$$\frac{1}{\kappa_4^2}=\frac{V_w}{\kappa_{10}^2}, \quad V_w\equiv\int d^6y\sqrt{\tilde{g}_6}e^{-4A}. \tag{52}$$

The 4D gravitino ψ_μ is massless as long as supersymmetry is preserved. The S_P term in the action vanishes, and a possible mass contribution from the R_P term is canceled by the term in the spin connection containing a derivative of A.

However, in the presence of 3-form fluxes, supersymmetry is generically broken and the gravitino ψ_μ acquires a mass. For a pseudo-BPS solution, the 5-form/warp factor relation (36) persists and the R_P and spin connection terms continue to cancel. The mass term for ψ_μ is then generated solely from the S_P term in the 10D action. Its reduction is straightforward, and one obtains

$$\frac{1}{\kappa_{10}^2}\int d^4x\sqrt{-g_4}\frac{1}{(\mathrm{Im}\,\rho)^{3/2}}\Bigg\{(\overline{\psi}_\mu\gamma^{\mu\nu}\psi_\nu^*)\times\left(\frac{i}{48}\int d^6y\sqrt{\tilde{g}_6}\frac{1}{(\mathrm{Im}\,\tau)^{1/2}}\chi^\dagger\tilde{\gamma}^{mnp}\chi^*G_{mnp}\right)+\mathrm{H.c.}\Bigg\}, \tag{53}$$

where in the above we included the Kähler modulus ρ controlling the overall scale of the compact directions; we discuss ρ in the next subsection. This is the proper form for a gravitino mass term,[7] with

$$m_{3/2}=\frac{1}{(\mathrm{Im}\,\rho)^{3/2}(\mathrm{Im}\,\tau)^{1/2}V_w}\left(\frac{1}{24}\int d^6y\sqrt{\tilde{g}_6}\chi^\dagger\tilde{\gamma}^{mnp}\chi^*G_{mnp}\right). \tag{54}$$

Taking a complex basis $i,j,k,\overline{\imath,\jmath,k}$ for the Calabi-Yau metric, we may define the covariantly constant spinor to be the "lowest weight" for the Clifford algebra: $\gamma^{\bar{\imath}}\chi=0$. One then sees immediately that only the (0,3) piece of $G_{(3)}$ contributes to the gravitino mass, as expected.

Given our normalization for χ, we have the relation

$$\chi^\dagger\tilde{\gamma}^{\overline{\imath\jmath k}}\chi^*=\epsilon^{\overline{\imath\jmath k}}=\frac{\Omega^{\overline{\imath\jmath k}}}{||\Omega||}, \tag{55}$$

up to an undetermined phase, where Ω_{ijk} is the holomorphic 3-form of the Calabi-Yau manifold and $3!||\Omega||^2=\Omega_{ijk}\overline{\Omega}^{ijk}$. Using

$$||\Omega||^2V_w=||\Omega||^2\int d^6y\sqrt{\tilde{g}_6}e^{-4A}, \tag{56}$$

$$=\int e^{-4A}\Omega\wedge\overline{\Omega} \quad \equiv \quad \omega_w,$$

we then obtain

$$m_{3/2}=(\mathrm{Im}\,\rho)^{-3/2}(V_w\omega_w)^{-1/2}(\mathrm{Im}\,\tau)^{-1/2}\left(\frac{1}{4}\int \Omega\wedge G\right). \tag{57}$$

In the absence of $G_{(0,3)}$ flux, $m_{3/2}\to 0$ and 4D $\mathcal{N}=1$ supersymmetry is restored. This suggests that the supersymmetry breaking can be captured in an $\mathcal{N}=1$ language, where a gravitino mass can be expressed in terms of the Kähler potential $\mathcal{K}$ and superpotential W as

$$m_{3/2}\propto\kappa_4^2e^{\mathcal{K}/2}W. \tag{58}$$

After discussing the moduli in the next subsection, we will present values for the Kähler and superpotentials, and demonstrate that Eq. (57) can be written in the form (58). We will estimate the value of $m_{3/2}$ in Sec. V D.

C. The moduli

Ordinary Calabi-Yau compactifications possess a large number of moduli, massless fields corresponding to the deformations of the compact manifold consistent with the Calabi-Yau condition, as well as the axion-dilaton. Since our solutions have an underlying Calabi-Yau space, in the absence of fluxes such moduli would also be present there. Specifically, the corresponding light fields are the complex structure moduli $z^\alpha(x)$, the Kähler moduli $\rho^i(x)$, and the axion-dilaton $\tau(x)$. However, an advantage of the pseudo-BPS warped compactifications, beyond their original motivation of solving the hierarchy problem, is that many of the moduli are fixed by the fluxes, including the dilaton. This was understood in [4]; one explanation follows from the assumption of a superpotential of the Gukov-Vafa-Witten form [8],

$$W=\frac{a}{\kappa_4^8}\int_{\mathcal{M}}\Omega\wedge G \tag{59}$$

(where a is a convention-dependent numerical constant) which is believed to arise in a wide variety of compactifications of string or M theory with fluxes turned on threading calibrated submanifolds. The flux is fixed, and the moduli (in this case the complex structure and axion-dilaton) adjust to minimize F terms arising from Eq. (59).

In order to give a more complete treatment of these moduli, in this section we turn to the problem of working out their 4D effective action and in particular their potential. The Appendix of [4] began the process of explicitly demonstrating this action, by working out the kinetic terms and potential, together with their connection with the superpotential (59), in the limit where warping can be neglected. The purpose of the present section is to give a more complete derivation, and in particular to find the effective action and Kähler and superpotentials in the presence of non-trivial warping. This means not just including the warp factor in the terms studied in [4], but also incorporating the contributions from the Einstein and five-form terms, which vanished there. We proceed by fixing the fluxes—in accord with the quantization condition (41)—and investigating the action for slowly varying fields $z^\alpha(x)$, $\rho^i(x)$, and $\tau(x)$.

[7]The bilinear $\overline{\psi}_\mu\gamma^{\mu\nu}\psi_\nu^*$ may seem unfamiliar if one is used to 4D gravitini written in Majorana form, but it is the correct expression for a Weyl gravitino, which arises naturally from our reduction.

First we calculate the moduli kinetic terms including the warping, and derive the corresponding warped Kähler potential. The geometrical moduli fields arise in the metric as

$$ds^2=e^{2A(y)}e^{-6u(x)}g_{\mu\nu}dx^\mu dx^\nu+e^{-2A(y)}e^{2u(x)}(\tilde{g}_{mn}(y)+T^I(x)\,\delta g_{Imn}(y))dy^m dy^n, \tag{60}$$

where the δg_I are traceless, $\tilde{g}^{mn}\delta g_{Imn}=0$, so that fluctuations of e^{2u} scale the total volume, while the T^I, which in principle include both the remaining Kähler structure moduli and the complex structure moduli, are volume preserving at linear order. The factor e^{-6u} on the 4D part must be introduced to decouple $u(x)$ from the 4D graviton.

The kinetic terms for the moduli fields are found by extracting the quadratic order terms in an expansion of the Einstein-Hilbert term in the Lagrangian (28) using the decomposition (60). These are calculated to be

$$\begin{aligned}S_{mod}&=\frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g_4}\,d^6y\sqrt{\tilde{g}_6}\,e^{-4A}\left\{-\frac{6}{4}e^{-8u}(\partial_\mu e^{4u})^2-\frac{1}{4}\partial_\mu T^I\partial^\mu T^J\delta g_{Imn}\delta g_J^{\tilde{m}\tilde{n}}\right\}\\&=\frac{1}{\kappa_4^2}\int d^4x\sqrt{-g_4}\left\{-3\frac{\partial_\mu\bar{\rho}\partial^\mu\rho}{|\rho-\bar{\rho}|^2}-\frac{1}{8V_w}\partial_\mu T^I\partial^\mu T^J\int d^6y\sqrt{\tilde{g}_6}\,e^{-4A}\delta g_{Imn}\delta g_J^{\tilde{m}\tilde{n}}\right\},\end{aligned} \tag{61}$$

where we have defined the complex field ρ such that $\mathrm{Im}\,\rho=e^{4u}$; the real part is a form field that was discussed in [4]. The moduli space metric for the remaining fields is seen to be a suitably warped version of the Weil-Petersson metric. From Eq. (28), one easily calculates the 4D dilaton kinetic term to be

$$S_{dil}=\frac{1}{\kappa_4^2}\int d^4x\sqrt{-g_4}\left\{-\frac{\partial_\mu\bar{\tau}\partial^\mu\tau}{|\tau-\bar{\tau}|^2}\right\}. \tag{62}$$

The kinetic terms (61),(62) are consistent with the Kähler potential

$$\mathcal{K}=-3\log[-i(\rho-\bar{\rho})]-\log\left(-\frac{i}{\kappa_4^6}\int d^6y\,e^{-4A}\sqrt{\hat{g}_6}\right)-\log\left(-\frac{i}{\kappa_4^6}\int e^{-4A}\Omega\wedge\bar{\Omega}\right)-\log[-i(\tau-\bar{\tau})], \tag{63}$$

where the volume piece is computed using the metric

$$\hat{g}(x,y)=\tilde{g}_{mn}(y)+T^I(x)\,\delta g^I_{mn}(y) \tag{64}$$

with the overall scale piece removed, as in Eq. (60). Notice that $\mathcal{N}=1$ supersymmetry will match the real T^I fields corresponding to Kähler moduli with a set of p-form modes into complex pairs, but these p-form fields do not appear in the Kähler potential.

The Kähler potential (63) has a form quite similar to that which arises in the unwarped case, with a correction due to the warp factor inserted to the volume integrals. The coefficient of the ρ term identifies this Kähler potential as being of the no-scale form, as noticed in [4]. As a result the tree-level cosmological constant will vanish despite supersymmetry breaking. The result (63) is valid only to leading order in α'; some next-to-leading-order results were examined in [43] (neglecting warping). We will comment more on these corrections later.

A general flux configuration will lift the complex-structure moduli z^α and fix the dilaton τ. In order to find this potential, we assume a general metric that is *constant* in x,

$$ds^2=e^{2A(y)}g_{\mu\nu}dx^\mu dx^\nu+e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n; \tag{65}$$

the moduli potential is exhibited from dependence of the action on the Calabi-Yau metric $\tilde{g}_{mn}$, as well as the dilaton. Specifically, the effective potential for these is computed from the $\mathcal{R}$, $|G_{(3)}|^2$, and $\tilde{F}^2_{(5)}$ terms in the action (28); these are terms with explicit dependence on the metric in the compact directions.[8]

For the metric (65), the Einstein-Hilbert term can be shown to give

$$\int d^{10}x\sqrt{-g}\,\mathcal{R}=\int d^4x\sqrt{-g_4}\int d^6x\sqrt{g_6}[-8(\nabla A)^2e^{4A}]. \tag{66}$$

The action for $\tilde{F}_5$ is more subtle: for a self-dual field, it vanishes. This is part of the usual problem for formulating an action for self-dual p-form field strengths. One way to obtain a consistent dimensionally reduced action is to double the coefficient on the 5-form term, but restrict to components of $\tilde{F}_{(5)}$ with indices along R^4 (or equivalently, restricting it only to components with no indices along R^4). It is readily checked that this prescription yields the correct dimensionally reduced equations of motion for the metric. Using the expression (33), we find

$$\int d^{10}x\sqrt{-g}\,\frac{\tilde{F}_5^2}{4\cdot 5!}\to\int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\,\frac{e^{-4A}}{2}(\partial_m\alpha)^2. \tag{67}$$

Then from the relation (36), we find a contribution equivalent to Eq. (66). This can be rewritten in terms of the fluxes using the Bianchi identity (39), which takes the form

$$\Box A=\frac{iG_{mnp}*\bar{G}^{mnp}}{48\,\mathrm{Im}\,\tau}+\text{local} \tag{68}$$

[8]As usual, in the F-theory case the dilaton term must be added to the Ricci term to get the desired result.

[the localized source terms cancel for sources saturating the pseudo-BPS condition (35)]. The first term is a total derivative when integrated. We therefore find

$$\int d^{10}x\sqrt{-g_{10}}\left[\mathcal{R}-\frac{\tilde{F}_5}{4\cdot 5!}\right]=\int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\,\frac{ie^{4A}G_{mnp}*_6\bar{G}^{mnp}}{12\,\mathrm{Im}\,\tau}. \tag{69}$$

Combining this with the G_3 term then gives

$$S_{\mathcal{V}}=\frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g_4}\int\frac{e^{4A}}{2\,\mathrm{Im}\,\tau}G_{(3)}\wedge(*_6\bar{G}_{(3)}+i\bar{G}_{(3)}). \tag{70}$$

Defining imaginary self- and anti-self-dual parts of the flux $G_{(3)}$,

$$G_{(3)}^{\pm}=\tfrac{1}{2}(G_{(3)}\pm i*_6G_{(3)}),\quad *_6G_{(3)}^{\pm}=\mp iG_{(3)}^{\pm}, \tag{71}$$

we can write the potential (70) as

$$S_{\mathcal{V}}=\frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g_4}\int\frac{ie^{4A}}{\mathrm{Im}\,\tau}G_{(3)}\wedge\bar{G}_{(3)}^{+}=\frac{1}{2\kappa_{10}^2}\int d^4x\sqrt{-g_4}\int\frac{e^{4A}}{\mathrm{Im}\,\tau}G_{(3)}^{+}\wedge*_6\bar{G}_{(3)}^{+}, \tag{72}$$

where in the second line we used the self-duality properties (71) to relate $\bar{G}_{(3)}^{+}$ to $*_6\bar{G}_{(3)}^{+}$ and to show $G_{(3)}^{-}\wedge\bar{G}_{(3)}^{+}=0$. The potential (72) has the form anticipated in [4], but with warping included.

We also anticipate that we should be able to write this potential in terms of the Kähler potential (63) and a superpotential via the usual $\mathcal{N}=1$ formula

$$\mathcal{V}=\kappa_4^2\int d^4x\sqrt{-g_4}e^{\mathcal{K}}\{(\mathcal{G}^{-1})^{A\bar{B}}D_AW\overline{D_{\bar{B}}W}-3|W|^2\}, \tag{73}$$

and it is interesting to check whether the Gukov-Vafa-Witten form (59) persists in the presence of warping. For simplicity we specialize to the case $\tau=\mathrm{const}$. The equation of motion for $G_{(3)}$ is [4]

$$d\Lambda+\frac{i}{\mathrm{Im}\,\tau}d\tau\wedge\mathrm{Re}\Lambda=0,\quad \Lambda\equiv e^{4A}*_6G_{(3)}-i\alpha G_{(3)}, \tag{74}$$

which then becomes

$$de^{4A}G_{(3)}^{+}=0=d*_6e^{4A}G_{(3)}^{+}. \tag{75}$$

Consequently $e^{4A}G_{(3)}^{+}$ is harmonic on the Calabi-Yau metric, and we can expand it in a basis of harmonic three-forms. The analysis now proceeds analogously to that in [4]. Only the (3,0) and (1,2) forms have the correct self-duality properties to appear in the expansion, so we find

$$e^{4A}G_{(3)}^{+}=\frac{1}{\omega_w}\left(\Omega\int G_{(3)}\wedge\bar{\Omega}+\mathcal{G}^{\alpha\bar{\beta}}\bar{\chi}_{\bar{\beta}}\int G_{(3)}\wedge\chi_\alpha\right), \tag{76}$$

where we used $\int G_{(3)}^{+}\wedge\bar{\Omega}=\int G_{(3)}\wedge\bar{\Omega}$ and an analogous expression for the basis of (2,1) forms χ_α, and where $\mathcal{G}^{\alpha\bar{\beta}}$ is the inverse to the metric

$$\mathcal{G}_{\alpha\bar{\beta}}=-\frac{1}{\omega_w}\int e^{-4A}\chi_\alpha\wedge\bar{\chi}_{\bar{\beta}}, \tag{77}$$

which follows from the Kähler potential (63).

Hence the potential is (restoring factors of ρ)

$$\begin{aligned}S_{\mathcal{V}}&=\frac{1}{2\kappa_{10}^2}\frac{1}{\omega_w^2}\int d^4x\sqrt{-g_4}\frac{1}{(\mathrm{Im}\,\rho)^3}\int\frac{e^{-4A}}{\mathrm{Im}\,\tau}\Big[\Omega\wedge\bar{\Omega}\int G_{(3)}\wedge\bar{\Omega}\int\bar{G}_{(3)}\wedge\Omega+(\mathcal{G}^{-1})^{\alpha\bar{\gamma}}(\mathcal{G}^{-1})^{\delta\bar{\beta}}\bar{\chi}_{\bar{\beta}}\wedge\chi_\alpha\int G_{(3)}\wedge\chi_\delta\int\bar{G}_{(3)}\wedge\bar{\chi}_{\bar{\gamma}}\Big]\\&=\frac{1}{2\kappa_4^2}\frac{1}{V_w\omega_w}\int d^4x\sqrt{-g_4}\frac{1}{(\mathrm{Im}\,\rho)^3}\frac{1}{\mathrm{Im}\,\tau}\Big[\int G_{(3)}\wedge\bar{\Omega}\int\bar{G}_{(3)}\wedge\Omega+(\mathcal{G}^{-1})^{\alpha\bar{\beta}}\int G\wedge\chi_\alpha\int\bar{G}\wedge\bar{\chi}_{\bar{\beta}}\Big].\end{aligned} \tag{78}$$

It is not hard to show that this form can be derived from the Kähler potential (63), together with an unwarped GVW superpotential of the GVW form (59). Using the identity $\partial_\alpha\Omega=k_\alpha\Omega+\chi_\alpha$, where k_α is a moduli-dependent constant, one may show that

$$D_\tau W=-\frac{1}{(\tau-\bar{\tau})}\frac{a}{\kappa_4^8}\int\Omega\wedge\bar{G},\quad D_\alpha W=\frac{a}{\kappa_4^8}\int\chi_\alpha\wedge G,\quad D_\rho W=-\frac{3W}{\rho-\bar{\rho}}. \tag{79}$$

The potential (73) may then be computed. As in the large-volume case, the $|D_\rho W|^2$ term cancels $-3|W|^2$, producing a no-scale potential. The other terms then reproduce Eq. (78), with the overall factors arising from the Kähler potential.

Notice that the subtlety of distinguishing ω_w (which depends on the complex structure moduli) from V_w (which depends on the Kähler moduli) was essential in making this identification. In the large-volume case in [4] this subtlety was not clearly treated.

A check of our derivation of the Kähler potential (63) and superpotential (59) can be obtained by reproducing the gravitino mass (57) from the formula (58). We indeed reproduce the correct form. This gives us confidence in our results, as well as reinforcing the ubiquity of the Gukov-Vafa-Witten superpotential. Although it is generally believed not to receive corrections from the warp factor, this is to our knowledge the first demonstration that this is the case.

D. Estimating gravitino and moduli masses

Having obtained an analytic expression for the gravitino and moduli masses, we would like to estimate their values. One might intuit that a bulk field like the gravitino gaining mass from SUSY breaking in the bulk will have a mass of the same order as the effective scale of gravity, namely M_4. Indeed this proves generally to be the case. In calculating bulk quantities it is more convenient to use the conventions where the warp factor is 1 at the top of the throat rather than at the bottom, which we shall do below.

Having succeeded in expressing the gravitino mass (57) in terms of a topological integral independent of the warping, we can evaluate it in terms of the moduli of the Calabi-Yau space in straightforward fashion. For the Klebanov-Strassler fluxes (42) this was already done in [4], with the result (using, in our conventions, $\int_A \Omega = V^{1/2} z$ where $V=\int d^6 y \sqrt{\tilde{g}_6}$ is the unwarped volume)

$$\int \Omega \wedge G = (2\pi)^2 (\alpha') V^{1/2} [K\tau z - M\mathcal{G}(z)], \tag{80}$$

where $\mathcal{G}(z) = z \log z/(2\pi i)$ + holomorphic. Although the complex structure modulus z is the source of the hierarchy and is fixed to be exponentially small, the holomorphic part of $\mathcal{G}(0)$ is generically $\mathcal{O}(1)$. Consequently Eq. (80) is just $(\alpha')V^{1/2}$ times factors of order unity. Cases with more general flux configurations will behave similarly: exponentially small terms in $W(z,\tau)$ will be washed out by $\mathcal{O}(1)$ terms, and the overall dimensionful constants will not change.

The expression (57) for $m_{3/2}$ also involves background values of the moduli Im τ and Im ρ. The axion-dilaton Im τ is fixed by the superpotential to be of order unity.[9] The volume modulus Im ρ has a flat potential at tree level. We have chosen units, however, where the background value is $\langle \mathrm{Im}\,\rho \rangle = 1$; the overall size of the compact manifold is then given by values for the integrals such as V and V_w. Thus we see that

$$m_{3/2} \sim \frac{(\alpha')V^{1/2}}{V_w}. \tag{81}$$

When the volume and the warped volume are of the fundamental scale $M_D \sim M_4$, we find that $m_{3/2} \sim M_4$.

One can estimate the moduli masses in similar fashion. From Eq. (73) we read off the form for the moduli potential

$$\mathcal{V} \sim \frac{1}{\kappa_4^2} m_{3/2}^2 \mathcal{G}^{i\bar{j}} \frac{D_i W \overline{D_j W}}{|W|^2}. \tag{82}$$

[9]One needs a slightly more involved set of fluxes than Eq. (42) to fix the dilaton; see [4].

Hence the potential for the moduli is also generically of the scale M_4.

The supersymmetry breaking may be heuristically thought of as coming from the region around the top of the throat. The G flux vanishes when the warp factor stops varying, so the source of SUSY breaking is concentrated in the throat; however it is not localized at the bottom of the throat, but instead receives its dominant contribution where the warp factor is largest, which is near the top. From the point of view of the earlier discussion on supersymmetry breaking, one may interpret our result $m_{3/2} \sim M_4$ as Eqs. (24),(25) with $\Lambda_{SUSY} = M_4$ since the breaking is fundamental scale, $M_p = M_4$ since the SUSY breaking is well separated from the visible sector, and $e^{A_{SUSY}} \sim 1$ since it is near the top of the throat.

One may be puzzled that the gravitino mass rises so far above the scale of bulk KK excitations (44). However, since the massless graviton stays massless even with the addition of G flux, the higher excitations of the graviton are protected by 4D general covariance from receiving mass corrections from the fluxes, and consequently get mass only from their shape in the compact geometry. The gravitini have no such protection.

Note that the broken gravitino will also generically receive mixing terms with the other 7 massive gravitini; for the case we have outlined all will have masses like M_4, the (0,3) flux will be just as large as the (2,1) flux, and there will not be a region of energies where $\mathcal{N}=1$ supersymmetry is a good description. One can speculate as to whether one of these other IIB gravitini could come down in mass as the contribution from the Calabi-Yau compactification is canceled by the contribution from fluxes (or more generally, whether an eigenvalue of the gravitino mass matrix might be particularly small). Although such a cancellation could conceivably be engineered at tree level, there is no reason why the mass should remain small once quantum corrections are included.

All our results hold at leading order in the α' expansion. It is likely that α' corrections will destroy the no-scale structure, giving a potential to the overall volume ρ. A computation of the first subleading order was performed by Becker *et al.* [43], where a correction to the Kähler potential was found (neglecting warping). The leading order correction was not enough to isolate an extremum of the ρ potential, but the corrections involve additional factors of the superpotential and the volume, which presumably becomes warped. The corrections to the potential are of order

$$\delta\mathcal{V} \sim \frac{e^{\mathcal{K}}|W|^2}{M_4^2} \sim m_{3/2}^2 M_4^2. \tag{83}$$

This suggests that the induced potential for ρ is also of order M_4; whether there is any regime where the no-scale structure is approximately preserved is not known and would be an important question to answer.

E. Brane matter and sequestering

We have estimated the value of the gravitino mass $m_{3/2}$ to be of the order of the 4D Planck scale or slightly less. At first, this seems to be a phenomenological disaster, since symmetry breaking effects in visible sector fields might be expected to be as large. Indeed, it is easy to see that generic scalar fields ϕ with canonical Kähler potential $\mathcal{K}_\phi \sim \overline{\phi}\phi$ and no quadratic contribution to the superpotential receive masses from supersymmetry breaking on the order of the gravitino mass:

$$D_\phi W \sim W\overline{\phi} + \mathcal{O}(\phi^2) \rightarrow \mathcal{V} \supset e^K |W|^2 \overline{\phi}\phi. \tag{84}$$

One might naively believe that brane matter will couple in this fashion, in which case bulk supersymmetry breaking by ISD fluxes in the pseudo-BPS spacetimes, despite other nice features, would not be a viable candidate for phenomenology.

However, one may explicitly calculate the mass induced by the fluxes for brane fields. The action for a D-brane is given by the sum of Born-Infeld and Wess-Zumino actions, given here in the string frame,

$$S_{D3} = -T_3 \int d^4x e^{-\phi} \sqrt{\det[P(G_{ab}+B_{ab}) + 2\pi\alpha' F_{ab}]} + \mu_3 \int \sum_i P[C_{(i)}] \wedge e^{2\pi\alpha' F - B}, \tag{85}$$

where P denotes the pullback of a spacetime quantity to the brane, F is the worldvolume gauge field and μ_3 and T_3 are the D3-brane charge and tension. In the absence of $G_{(3)}$, the D3-brane preserves the same supersymmetries as the warped geometry, and thus there is no potential generated; any potential must appear with the SUSY breaking. However, $G_{(3)}$ appears in the D3-brane action solely through the pullback of the potentials $B_{(2)}$ and $C_{(2)}$. Since neither potential is polarized along the D3-brane, it is not hard to convince oneself that all nonvanishing terms in their pullbacks involve at least one derivative of the brane fields, and hence cannot generate a potential. Indeed, one may explicitly check by examining the three-brane action (85) that the relation (36) between the warp factor and five-form guarantees a no-force condition on D3-branes, with gravitational and RR 5-form potentials canceling.

Furthermore, it was found by Graña [9] that the D3-brane fermionic terms do not couple to the imaginary self-dual part of $G_{(3)}$. Although other kinds of $G_{(3)}$ flux can lead to various masses and couplings for brane fermions, the brane is entirely insensitive to ISD flux. Consequently, we arrive at the result that supersymmetry breaking by (0,3) fluxes induces no tree level masses at all for D3-brane fields.

Vanishing of scalar masses arises from the no-scale structure of the theory; the additional feature of vanishing fermion masses is analogous to the sequestered structure proposed in [10], and we shall refer to it as sequestering in what follows. The no-scale structure is characterized by the Kähler potential

$$\mathcal{K} = -3 \log[f_{visible}(X,\overline{X}) + g_{hidden}(\rho,\overline{\rho})], \tag{86}$$

where X are visible sector fields and ρ are hidden-sector fields; supersymmetry breaking in the hidden sector will not be communicated to the visible sector scalars at tree level.

Reference [10] suggested the naturalness of sequestering when the visible sector lives on a brane and the SUSY-breaking sector is physically separated from it in a higher-dimensional space. However, Anisimov, Dine, Graesser, and Thomas (ADGT) [44,45] have pointed out several examples from string and M theory, including type I, Hořava-Witten, and Dp-Dp' systems, where sequestering is not generic despite the physical separation of sectors on two different branes. The reason can be traced to the exchange of bulk (closed-string) modes at tree level, which can generate contact terms between the sectors of the order of the gravitino mass.

Our scenario is the first example we know of sequestering in a string theory background, at least to leading order in the α' expansion. ADGT [44,45] were aware of the no-scale Kähler potential of the pseudo-BPS solutions of [4], but speculated that even were sequestering to arise in such models with brane back reaction neglected, such back reaction would destroy the sequestered form. This is not the case for our scenario. As remarked previously, the pseudo-BPS solutions can include the presence of certain localized sources—including D3-branes—in the background. Hence, although the back reaction of the D3-branes in the throat will locally change the specific form of the solution, it will not bring it outside the pseudo-BPS class, and our conclusions about the lack of tree level masses will persist.

This raises the question as to whether another type of brane known to sit in the almost-BPS class of objects, such as the 7-brane wrapped on a 4-cycle, also has worldvolume excitations sequestered from bulk supersymmetry breaking. If so, it would provide a richer set of possibilities for engineering visible sector matter, with the wealth of possible cycles in the compact space to wrap. We leave this question for the future.

In previous sections, we established the warped Kähler potential (63) for the bulk moduli. The D3-brane matter must enter into the Kähler potential as well, and owing to the sequestered form it must enter in a nontrivial fashion. A natural guess is something of the form

$$\mathcal{K} = -3 \log(-i(\rho - \overline{\rho}) + K(\overline{X},X)) + \mathcal{K}(\tau,\overline{\tau}) + \mathcal{K}(z,\overline{z}), \tag{87}$$

where $K(\overline{X},X)$ is related to the spacetime Kähler potential for the Calabi-Yau space. This modified Kähler potential preserves the no-scale structure: one may verify that for arbitrary K, the contributions to $|DW|^2$ from ρ and X (including off-diagonal terms) always combine to give precisely $3|W|^2$.

The expression (87) also leads to a coupling to the radial modulus, at leading order, of the form

$$T_{D3} \int d^4x \sqrt{-g_4} \frac{1}{\mathrm{Im}\,\rho} e^{2A} \tilde{g}_{i\overline{j}} \partial_\mu X^i \partial^\mu \overline{X}^{\overline{j}}, \tag{88}$$

which is the correct power of Im ρ arising in the Born-Infeld (BI) action. The lack of coupling of the dilaton that appears is also correct for the Einstein-frame action. We leave further exploration of this Kähler potential, including the coupling of the complex structure moduli, for future work.

Spartner masses vanish at tree level, but as discussed in Sec. IV, should receive corrections at loop level. From the point of view of an observer on the brane, supersymmetry is broken — the gravitino is eliminated from the low energy spectrum. Thus generic loop corrections are expected to raise mass scales to the cutoff scale. However, as we have emphasized, for an observer on an IR brane in a TeV-scale gravity scenario, the fundamental scale is $\mathcal{O}(\mathrm{TeV})$, and this is where the cutoff on loop momenta should be placed: above this scale, one encounters strongly coupled gravitational physics. Thus spartner masses are generically expected to be around a TeV in such a scenario, which is a reasonable answer for phenomenology.

Note that part of the original motivation for the sequestered scenarios of [10] was to have a situation where the dominant contribution to spartner masses was through anomaly mediated supersymmetry breaking (AMSB) [10,46], with mass scale

$$m_{AMSB} \sim b_0\left(\frac{g^2}{16\pi^2}\right)m_{3/2}, \tag{89}$$

where b_0 is the one-loop beta function coefficient (for the scalars there is an an additional constant from the anomalous dimensions). Given that in the present case the gravitino mass is far above the effective cutoff scale of $\mathcal{O}(\mathrm{TeV})$, it seems that this formula cannot give the correct masses here; rather it appears that the masses arise from generic loop corrections. A better understanding of the role of AMSB in this model could be of interest.

Another possible way to exploit the no-scale structure is to find a background in which the (0,3) component of $G_{(3)}$ can be switched off, and to break SUSY on another set of branes situated in the middle of the throat. The Kähler potential becomes

$$\mathcal{K} = -3\log(-i(\rho-\overline{\rho}) + f(\overline{X},X) + g(\overline{Y},Y)) + \mathcal{K}(\tau,\overline{\tau}) + \mathcal{K}(z,\overline{z}), \tag{90}$$

where Y are the hidden sector fields. Again, as far as SUSY breaking is concerned this has the no-scale form. The location of the hidden sector brane could be tuned to provide the right amount of SUSY breaking; this is a fine-tuning, but it preserves the other advantage of AMSB, that it addresses the supersymmetric flavor problem. This sort of ‘‘brane’’ SUSY breaking is much more prevalent in the literature than the ‘‘bulk’’ SUSY breaking we have examined for much of this paper.

F. Summary of phenomenology

Since this section has been rather long and technical, we give an overview of its essential results here.

String theory solutions found in [4] provide a non-trivial example of many of the warped compactification ideas discussed in the first four sections, and in particular can be arranged to generate a hierarchy through warping and thus produce a TeV-scale gravity scenario. This means that geometrical scales will be realized as was discussed in Secs. II and III. In particular, for an observer on the IR brane where we imagine standard model physics residing, the fundamental Planck scale will be reached at scattering energies $\mathcal{O}(\mathrm{TeV})$, and we can envision string and black hole production taking place at such energies. Kaluza-Klein masses are even lighter, and are given in terms of the flux quanta by Eq. (44).

These solutions arise by considering close analogues of Calabi-Yau manifolds with three-form fluxes frozen into their geometry. These fluxes break supersymmetry. They also generate a potential for many of the moduli fields that would otherwise be massless in a standard Calabi-Yau compactification. The gravitino mass is given in Eq. (57), and can be estimated to be of the order of the four-dimensional Planck scale, 10^{19} GeV. The moduli kinetic terms are given in Eq. (61), and the potential for moduli in Eq. (72). This lifts the complex structure moduli and the dilaton to have masses also generically of the order of 10^{19} GeV. The action for the moduli, and for the gravitino, can be conveniently summarized in supergravity language in terms of a Kähler potential, Eq. (63), and a superpotential, given by Eq. (59). These explicitly include the effects of the warping.

Although supersymmetry is broken at a high scale, at the tree level the cosmological constant vanishes and matter fields on an IR brane have vanishing masses. For scalars, this statement corresponds to the fact that we are dealing with a *no-scale* model. This structure also extends to fermion matter, resulting in a *sequestered* structure. This structure survives brane back reaction. Spartners are however expected to get masses from loop corrections, but since the fundamental scale for brane matter, and hence the relevant cutoff, lies at the TeV scale, these masses are expected to be TeV size.

VI. CONCLUSIONS

We have discussed a number of generic features of the scales and thresholds in warped compactifications, and illustrated them in the special case of the solutions of type IIB string theory given in [4]. The latter solutions in particular offer possible solutions to some of the difficult problems of string phenomenology. Supersymmetry is broken by three-form fluxes frozen into the geometry, and a potential for a large number of otherwise problematic moduli is generated at the same time. Spartner masses are not generated at tree level, but in such a TeV-scale gravity scenario are expected to receive loop corrections of TeV magnitude.

While these certainly seem like interesting successes, it should be borne in mind that there are a number of other problems that must be resolved in order to find solutions of string theory that realize TeV-scale gravity and reproduce a realistic phenomenology including the standard model. (Several of these are also problems also for more traditional Planck-scale compactifications of string theory, so do not discriminate against TeV-scale scenarios.) One obvious question is how to realize the structure of the standard model within the general framework of this kind of solution. Many ideas have occurred in the literature, involving intersecting branes and branes at singularities, and it may be possible to combine these scenarios with a framework like that presented here, but clearly there is some non-trivial work to be done; some interesting recent progress in this direction includes [47,48]. Particularly challenging issues include reproducing the gauge groups and matter representations, with reasonable couplings, of the standard model; addressing baryon and lepton number violation, and reproducing the relation between the gauge coupling constants that can otherwise be taken to indicate matching via renormalization group running to a grand unified scale. A second problem is that of the remaining moduli; in particular, Kähler moduli are not stabilized by the fluxes we consider, and thus must be fixed by another mechanism. This is a generic problem, since the overall scale of the compact manifold is generically a Kähler modulus. (For another approach, see [49].) Corrections at higher order in string loops or α' (see e.g., [43]) may play a role, but it is difficult to see they do so *and* maintain reasonable mass scales. In particular, we must ultimately face the thorny problem of the cosmological constant, which here as in other scenarios with broken supersymmetry would appear to take a value that is far too large.

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APPENDIX A: CONVENTIONS

We work in mostly plus signature in both ten and four dimensions. We use M,N for 10D indices, μ,ν for 4D indices, and m,n for generic 6D indices; the last can in turn be divided into holomorphic i,j and antiholomorphic $\overline{\imath,\jmath}$ indices with respect to the complex structure of the Calabi-Yau threefold.

Ten dimensional gamma matrices Γ^M are 32×32 matrices. They decompose into a product of 4×4 4D matrices γ^μ and 8×8 6D matrices $\tilde{\gamma}^i$ as follows:

$$\Gamma^\mu = e^{-A}\gamma^\mu \otimes I, \quad \Gamma^m = e^A \gamma_5 \otimes \tilde{\gamma}^m, \tag{A1}$$

where

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}, \quad \{\tilde{\gamma}^m,\tilde{\gamma}^n\} = 2\tilde{g}^{mn}. \tag{A2}$$

The chirality matrices are related as $\Gamma_{11} = \gamma_5 \tilde{\gamma}_{\mathcal{M}}$, and obey $\gamma_5^2 = \tilde{\gamma}_{\mathcal{M}}{}^2 = \Gamma_{11}{}^2 = 1$.

The ten-dimensional gravitino Ψ_M is Weyl (but not Majorana):

$$\Gamma_{11}\Psi_M = -\Psi_M. \tag{A3}$$

It decomposes into the 4D gravitino as $\Psi_\mu = \psi_\mu \otimes e^{A/2}\chi$, for which we have for our class of solution

$$\gamma_5\psi_\mu = \psi_\mu \quad \tilde{\gamma}_{\mathcal{M}}\chi = -\chi. \tag{A4}$$

APPENDIX B: FERMIONIC EQUATIONS OF MOTION FOR TYPE IIB SUPERGRAVITY

The fermionic equations of motion for type IIB supergravity (to linear order in fermions) are presented in Eqs. (4.6),(4.12) of [41] with coefficients given by Eqs. (4.8),(4.14). However, the derivative in these equations contains supercovariantizations involving three- and five-form fluxes not explicitly recorded there. The complete definition is however implicit in other expressions given in [41], notably the supersymmetric variations of the fermionic equations of motion (4.7), (4.10), (4.13), and (4.15), and can be deduced from these. We collect the complete equations here for future convenience.

In this appendix we use the conventions of [41] for the fields, although we use our index conventions. We indicate how to pass to our field conventions at the end.

The dilatino equation of motion (4.6) of [41] is

$$\Gamma^M \hat{D}_M \lambda = \frac{i\kappa}{240}\Gamma^{M_1\cdots M_5}\lambda F_{M_1\ldots M_5} + \mathcal{O}(\Psi^3), \tag{B1}$$

where the supercovariant derivative of the dilatino is

$$\hat{D}_M\lambda = D_M\lambda - \kappa T\Psi_M - \kappa U\Psi_M^*, \tag{B2}$$

$$T = -\frac{i}{24}\Gamma^{MNP}G_{MNP}, \quad U = \frac{i}{\kappa}\Gamma^M P_M. \tag{B3}$$

Here $D_M = \nabla_M - (i/2)Q_M$ contains the ordinary covariant derivative including the spin connection ∇_M, and a composite connection Q_M composed of the complex scalar, while P_M is the field strength for the complex scalar. The gravitino equation of motion is

$$\Gamma^{MNP}\hat{D}_N\Psi_P = -\frac{i}{2}\Gamma^P\Gamma^M\lambda^* P_P - \frac{i\kappa}{48}\Gamma^{NPQ}\Gamma^M\lambda G^*_{NPQ} + \mathcal{O}(\Psi^3), \tag{B4}$$

where the supercovariant derivative acting on the gravitino is

$$\hat{D}_N\Psi_P = D_N\Psi_P - \kappa R_P\Psi_N - \kappa S_P\Psi_N^*, \tag{B5}$$

$$R_M = \frac{i}{480}(\Gamma^{M_1\cdots M_5}F_{M_1\cdots M_5})\Gamma_M,$$

$$S_M=\frac{1}{96}(\Gamma_M{}^{NPQ}G_{NPQ}-9\Gamma^{NP}G_{MNP}). \quad \text{(B6)}$$

A supercovariant derivative in a general supergravity theory consists of the ordinary covariant derivative supplemented with terms involving the gravitino such that the supersymmetry variation of the combined terms does not contain any derivatives of the supersymmetry parameter ε. These expressions arise naturally in supergravity equations of motion, as the variation of a one-derivative fermionic equation must be a bosonic equation with two derivatives on the fields, and hence a derivative may not be spared to act on ε. Equations (B2),(B5) constitute the general form for supercovariantization of the derivative in an arbitrary supergravity theory with fermionic supersymmetry variations

$$\delta\Psi_M=\frac{1}{\kappa}D_M\varepsilon+R_M\varepsilon+S_M\varepsilon^*, \quad \text{(B7)}$$

$$\delta\lambda=T\varepsilon+U\varepsilon^*. \quad \text{(B8)}$$

In our conventions, Schwarz's constant $\kappa=1$, and should not be confused with our κ_{10} which is an overall coefficient in the action and does not appear in the equations of motion. The relations between Schwarz's F and G and the $\tilde{F}_5$ and G_3 of this paper are

$$F_{Sch}=-\frac{1}{4}\tilde{F}_5, \quad G_{Sch}=\frac{ie^{i\theta}}{\sqrt{\mathrm{Im}\,\tau}}G_3, \quad e^{i\theta}\equiv\left(\frac{1+i\bar{\tau}}{1-i\tau}\right)^{1/2}. \quad \text{(B9)}$$

For relations involving the complex scalar, see [42]; note that the authors of [42] use an $F=4F_{sch}$, and consequently for them $\alpha=-e^{4A}$.

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