Rotating black holes, closed timelike curves, thermodynamics, and the enhançon mechanism

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We reconsider supersymmetric five dimensional rotating charged black holes and their description in terms of D-branes. By wrapping some of the branes on $K3$, we are able to explore the role of the enhançon mechanism in this system. We verify that enhançon loci protect the black hole from violations of the second law of thermodynamics which would have been achieved by the addition of certain D-brane charges. The same charges can potentially result in the formation of closed timelike curves by adding them to holes initially free of them, and so the enhançon mechanism forbids this as well. Although this latter observation is encouraging, it is noted that this mechanism alone does not eliminate closed timelike curves from these systems, but is in accord with earlier suggestions that they may not be manufactured, in this context, by physical processes.

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I. INTRODUCTION AND CONCLUSIONS

As we increase our ability to describe the more exotic types of physics which branes can exhibit (and here we have in mind their ability to change shape, dimension, and other key aspects of their character) we confirm our suspicions that they are part of a fruitful avenue of research into the basic nature of the correct description of spacetime physics and whatever replaces it at the most fundamental level.

A particular example that we have in mind was uncovered last year [1] by reconsidering the case of extremal five dimensional black holes and their microscopic description [2] in terms of D1-branes and D5-branes wrapped on $K3\times S^1$. The intriguing piece of physics observed in that study was the fact that while an analysis of the entropy as a function of the Ramond-Ramond (RR) charges suggests that an approach to the horizon of an additional wrapped D5-brane could decrease the entropy and hence violate the second law of thermodynamics, a further analysis in the light of the results of Ref. [3] shows that the enhançon mechanism—which forces the brane to delocalize, spread out, and cease its approach to the black hole at a specific radius—prevents this from happening. This mechanism extends to the entire family of D5-D1 bound states with particular charge assignments such that they have the potential to allow second law violations, while allowing other types through—there is an enhançon locus for each type of bound state; above the horizon for violators and below it for law abiders.

This result is both satisfying and intriguing, since in ordinary thermodynamics, the preservation of the second law is understood as a coarse-grained statistical outcome, while in this case, since D-branes (in terms of which the black holes find their microscopic description [2]) are the smallest possible objects carrying the RR charges [4], the second law is kept inviolate by a macroscopic supergravity filter which can discriminate at the level of the microscopic constituents. This seems to be a new sharp phenomenon connecting the microscopic to the macroscopic, deserving further investigation in order to be better understood.

The principal reason why we get to address this system quite cleanly in the above terms is because it is a Bogomol'nyi-Prasad-Sommerfield (BPS) system. Therefore, we can separate it into small non-interacting BPS pieces and bring each piece up to it slowly and hence perform adiabatic changes to the thermodynamic quantity of interest, the entropy,[1] without the encumbrance of having to worry about the thermodynamic properties of radiative processes, etc. The purpose of this short note is to extend the result of Ref. [1] to a class of rotating charged black holes, taking advantage of the fact that although naively the more complicated case of a rotating black hole would seem to admit no BPS embedding into string theory, there is indeed such a solution, found in Ref. [7], which owes its BPS nature[2] to an excellent conspiracy of features in five dimensions, discussed in Ref. [9], and further in Refs. [10–12]. We show here that we can carry out a very similar analysis for the rotating black hole to that carried out for the static case, and observe that the potential violations of the second law using the adiabatic addition of constituent parts of the ''incorrect'' charges is again avoided by the enhançon mechanism.

The bottom line, of course, is that the area of the black hole's horizon, to which the entropy is proportional [13] can-

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[1]It is straightforward to see that this sort of reasoning also works very well for the four dimensional Reissner-Nordström black holes which admit a simple description involving D1-branes and D5-branes [and possibly Neveu-Schwarz 5-branes (NS5-branes)] wrapping $K3$ [5], as was confirmed in Ref. [6]. The same sort of BPS processes can be found in those systems too.

[2]All supersymmetric solutions of minimal supergravity in five dimensions were found in Ref. [8].

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not decrease. This follows from an appropriate version of the area theorem [14], adapted to the case in question. Such theorems follow from a weak energy condition, and may also be thought of as a corollary of the cosmic censorship principle [15]. (See Refs. [16,17] for further discussion.) Our goal here is not, therefore, to find violations *per se*, but to study the novel mechanism by which this particular situation involving branes wrapped on $K3$ manages to protect the theorem. So cases which are in the same class of physics as the T^4-wrapped situation—branes with the ''correct'' charges—will be irrelevant to the enhançon mechanism, and will be covered by the area theorem in the usual way. We have nothing to add to the existing discussion for those cases.

As it is a while since many have thought about these models, we review much of the essential material in the short Secs. II, III, and IV, which also allow us to establish our notation and emphasize the crucial differences between the $K3$ and T^4 cases. In Sec. V, we exhibit the basic enhançon locus and in Sec. VI we perform a D-brane probe computation which enjoys a crucial cancellation due to the particular form of its interaction with the background fields representing the rotating solution. This cancellation is crucial for forming the intuition about how to make a rotating BPS black hole, i.e., having no excess energy taking us away from BPS saturation. In that section, we also show that there is a whole family of enhançon loci, one type for each possible arrangement of RR charges that a D-brane probe can carry. Section VII exhibits a supergravity excision computation which demonstrates that the enhançon shells suggested by the probe computation really exist as solutions, and have the required properties. Section VIII then considers the case of a black hole with non-zero entropy, and shows that the class of probes which have RR charges which could potentially reduce the entropy if they merge with the hole are stopped from reaching the horizon by the enhançon mechanism. The remaining sorts of probes are harmless, and the enhançon mechanism has nothing to say about their motion outside the horizon.

We also note the following. While the ten dimensional geometry of the brane configuration giving rise to the black hole in the five dimensional supergravity upon reduction is entirely causal, the reduction process leads to naked closed timelike curves (CTCs) in the geometry if the angular momentum of the solution exceeds a certain bound [7].[3] (See also Refs. [9–11] for more discussion. It was noted in Ref. [8] that CTCs seem to be generic for a wide class of solutions in five dimensions.) In fact, such solutions are not ruled out by supersymmetry, and it is as yet an unresolved question as to whether string theory has any concrete mechanism for rendering CTCs more physically acceptable than they appear to be in field theory.

We do not expect that the enhançon mechanism can be the tool by which we find complete understanding of the role of CTCs since while the former arises in this context from wrapping on $K3$, the latter have nothing to do with the reduction on $K3$, and arise for the case of T^4 as well. However, since the enhançon mechanism seems to be intimately familiar with the physics of the entropy of the black horizon formed upon reduction—enough to be ''mindful'' of the second law—it is not unreasonable to wonder what it has to say about the formation of the CTCs. We find that while it does not rule out the existence of CTCs—the expected result—the same mechanism which prevents the violation of the second law by ''wrong charge'' probes also prevents one from starting with a hole with no CTCs and adding such branes in such as way as to form CTCs. This is in accord with the work presented in Ref. [10], suggesting that these CTCs may not be formed by physical processes.

II. FIVE DIMENSIONS

Consider the following five dimensional action [20]:

$$S_{(5)}=\frac{1}{16\pi G_5}\int d^5x\left(\sqrt{-g}[R-F^2]-\frac{2}{3\sqrt{3}}A\wedge F\wedge F\right), \tag{1}$$

where A is a gauge field and F is its field strength, combining into a Chern-Simons term. This is the bosonic content of an $\mathcal{N}=2$ supergravity theory, which can be embedded into string theory in a number of ways. The metric of the rotating solution is written in the Einstein frame as

$$\begin{aligned} ds^2_{(5)}&=-H^{-2}\left(dt+\frac{J}{2r^2}(\sin^2\theta d\phi_1-\cos^2\theta d\phi_2)\right)^2 \\ &\quad +H(dr^2+r^2d\Omega_3^2), \\ H&=\left(1+\frac{r_0^2}{r^2}\right), \end{aligned} \tag{2}$$

where ϕ_1,ϕ_2 and θ, $(0\leqslant\phi_{1,2}\leqslant 2\pi,\ 0\leqslant\theta\leqslant\pi/2)$ are angles parametrizing a round S^3, with metric:

$$d\Omega_3^2=d\theta^2+\sin^2\theta d\phi_1^2+\cos^2\theta d\phi_2^2. \tag{3}$$

Together with the following gauge field:

$$A=(H^{-1}-1)dt+\frac{J}{2r^2}H^{-1}(\sin^2\theta d\phi_1-\cos^2\theta d\phi_2), \tag{4}$$

the metric (2) represents a very special solution with a regular event horizon located at $r=0$. This solution is special since it is in fact a BPS solution, brought about by the presence of the Chern-Simons term and also a particular (anti) self-duality property of the gauge field [9,10].

One amusing feature of the solution is the fact that although the geometry has non-vanishing asymptotic angular momentum, the angular velocity of the horizon is actually zero. As there is a negative contribution to the angular mo-

[3]This is in contrast with the case of the generalization [18] of these solutions to the case of the analogous gauged five dimensional supergravity. In that case, their lift to ten dimensions still has CTCs [19].

mentum from the spacetime inside the horizon, this vanishing is attributed to the cancellation of opposite "dragging effects" at the horizon [9,12].

The entropy of this solution can be easily computed by use of the Bekenstein-Hawking relation [13] to the horizon area $\mathcal{A}$:

$$S=\frac{\mathcal{A}}{4G_5}=\frac{2\pi^2}{4G_5}\sqrt{r_0^6-\frac{J^2}{4}}. \tag{5}$$

It is very interesting to note that this quantity vanishes for large enough J. In fact, it can be seen that the geometry can develop closed timelike curves if J were to increase further. For example, picking either ϕ_1 or ϕ_2 (and calling it ϕ), an examination of the worst case behavior of the metric for that direction yields

$$g_{\phi\phi}(r)=\frac{1}{(r^2+r_0^2)^2}\left(r^6+3r^4r_0^2+3r^2r_0^4+r_0^6-\frac{J^2}{4}\right), \tag{6}$$

showing that for $J^2>4r_0^6$, the closed loop parametrized by ϕ goes timelike above the horizon (i.e., for $r>0$).

III. TEN DIMENSIONS

The supergravity given in Eq. (1), and the solution (2) can be generalized, as there are more independent gauge fields and a family of scalar fields which can be switched on. These can then be seen to be fields arising from the various geometrical choices to be made in embedding the supergravity into ten dimensional string theory.

The five dimensional Einstein frame metric for the more general solution is [7]:

$$\begin{aligned}ds^2_{(5)}=&-(H_1H_5H_P)^{-2/3}\left(dt+\frac{J}{2r^2}(\sin^2\theta d\phi_1\right.\\&\left.-\cos^2\theta d\phi_2)\right)^2+(H_1H_5H_P)^{1/3}(dr^2+r^2d\Omega_3^2),\\H_5=1+\frac{r_5^2}{r^2},&\quad H_1=1+\frac{r_1^2}{r^2},\quad H_P=1+\frac{r_P^2}{r^2},\end{aligned} \tag{7}$$

and the previous case corresponded to $r_1^2=r_5^2=r_P^2=Q$. [We will shortly identify the origin of the different scales in equations (12).] Now we can write a more general formula for the entropy-area relation:

$$S=\frac{\mathcal{A}}{4G_5}=\frac{2\pi^2}{4G_5}\sqrt{r_1^2r_5^2r_P^2-\frac{J^2}{4}}, \tag{8}$$

with an obvious bound on the angular momentum: $r_1^2r_5^2r_P^2\geqslant J^2/4$. A quick computation shows that this is the same bound which, when violated, gives CTCs above the horizon.

Once we begin to work with this more general case, we must note that are three scalar fields (we will not give their forms here) which can be chosen as corresponding to the dilaton, the radius of a circle and the volume of a four-surface $\mathcal{M}$; these are the five extra dimensions taking us back to ten dimensions. Now $\mathcal{M}$ can be T^4 or $K3$, but very soon we will be focusing on the case of $K3$. We will wrap D5-branes on $\mathcal{M}$ and combine them with D1-branes transverse to $\mathcal{M}$, and subsequently the resulting stringlike object will be wrapped upon the circle. There will be non-trivial momentum in that circle. The gauge field separates into two independent one-forms and a two-form potential (the latter arrived at by $D=5$ Hodge dualization) and these naturally have an interpretation as a Kaluza-Klein gauge field from reduction on the circle, the unwrapped part of the RR 2-form gauge field coming from wrapping the D1-branes, and the $(D=10)$ Hodge dual of the RR 6-form coupling to the D5-branes.

The full ten dimensional geometry is given, in string frame, by [11]

$$\begin{aligned}ds^2=&H_5^{-1/2}H_1^{-1/2}\left(-dt^2+\frac{r_P^2}{r^2}(dt-dz)^2+dz^2\right.\\&\left.+\frac{J}{r^2}(\sin^2\theta d\phi_1-\cos^2\theta d\phi_2)(dz-dt)\right)\\&+H_5^{-1/2}H_1^{1/2}V^{1/2}ds^2_{\mathcal{M}}+H_5^{1/2}H_1^{1/2}(dr^2+r^2d\Omega_3^2).\end{aligned} \tag{9}$$

Here, z parametrizes our circle, and $ds^2_{\mathcal{M}}$ is the metric on the manifold $\mathcal{M}$, of unit volume. We denote the volume element on it as $\varepsilon_{\mathcal{M}}$. $\mathcal{M}$'s volume varies with the radial coordinate of the transverse space as

$$V(r)=V\frac{H_1}{H_5}, \tag{10}$$

reaching the asymptotic value V at spatial infinity. The dilaton and RR potentials are [11]

$$\begin{aligned}e^{2\Phi}=&g_s^2\frac{H_1}{H_5},\\C^{(6)}=&g_s^{-1}H_5^{-1}dt\wedge dz\wedge\varepsilon_{\mathcal{M}}+\frac{J}{2r^2}H_5^{-1}\\&\times(\sin^2\theta d\phi_1-\cos^2\theta d\phi_2)\wedge dz\wedge\varepsilon_{\mathcal{M}},\\C^{(2)}=&g_s^{-1}H_1^{-1}dt\wedge dz+\frac{J}{2r^2}H_1^{-1}\\&\times(\sin^2\theta d\phi_1-\cos^2\theta d\phi_2)\wedge dz,\end{aligned} \tag{11}$$

and we remind the reader that the harmonic functions pertaining to D5-, D1-branes and the pp wave (representing the momentum in the z circle) are given in Eqs. (7). The scales given in those equations are set by the string coupling g_s, string length ℓ_s, $\mathcal{M}$'s asymptotic volume V, and the radius, R_z, of the circle parametrized by z:

$$r_5^2 = g_s \ell_s^2 Q_5, \quad r_1^2 = g_s \ell_s^2 \frac{V_\star}{V} Q_1, \quad r_P^2 = g_s^2 \ell_s^2 \frac{V_\star}{V} \frac{\ell_s^2}{R_z^2} Q_P, \tag{12}$$

where $V_\star = (2\pi\ell_s)^4$. Q_1 and Q_5 are integer amounts of the basic RR two-form and RR six-form charges present in the system. Note also that Q_P is an integer, parametrizing the discrete amounts of momentum that we can have in the compact direction, z.

Later we will define the ten dimensional Einstein frame metric G_{MN}, in terms of the string frame metric g_{MN} in Eq. (9) by $G_{MN} = e^{-\Phi/2} g_{MN}$. Note also that the ten dimensional Newton constant G_{10} is given by the relation $16\pi G_{10} = (2\pi)^7 \ell_s^8 g_s^2$. The five dimensional Newton's constant is related to it by $G_5 = G_{10}/2\pi R_z V$.

In the case when $\mathcal{M}$ is $K3$, we must be careful [1]. Wrapping a D5-brane on $K3$ induces precisely minus one units of D1-brane charge [22]. So defining N_5 and N_1 to be the numbers of D5- and D1-branes, the charges in Eq. (12) are

$$Q_5 = N_5, \quad Q_1 = N_1 - N_5. \tag{13}$$

The configuration preserves 1/8 of the original type IIB supersymmetry: 1/2 is broken by having D5-branes, another 1/2 by wrapping them on $K3$ (*or* combining them with D1-branes), and finally a pp wave in the z direction with purely right-moving momentum excited breaks 1/2 of the remaining supersymmetry. Rotation does not break an extra amount of supersymmetry, but for the solution to be regular the linear combination of angular momenta in ϕ_1 and ϕ_2 directions should vanish [7,21], and $J_{\phi_1} = -J_{\phi_2}$.

IV. TWO DIMENSIONS

There is a $(1+1)$-dimensional superconformal field theory living on the world volume of the stringlike intersection of the D5-branes and D1-branes with a number of interesting properties relevant to the spacetime physics. We present it here, following closely the original Ref. [7]. The theory lives on the cylinder $S_z^1 \times \mathbb{R}$, and has four supercharges. It has states coming from the massless strings stretching between the various D1- and D5-branes. The usual 1–5 and 5–1 strings give the net contribution to the degrees of freedom ($N_B = N_F = 4Q_1Q_5$), giving a central charge $c = N_B + N_F/2 = 6Q_1Q_5$. The R symmetry of this theory is the $SO(4)$ isometry of the $\mathbb{R}^4$ transverse to the intersection. The relation between the coordinates (x_1, x_2, x_3, x_4) of this $\mathbb{R}^4$ and the coordinates $(r, \theta, \phi_1, \phi_2)$ we have been using so far is

$$x_1 = r \sin\theta \cos\phi_1, \quad x_3 = r \cos\theta \cos\phi_2,$$
$$x_2 = r \sin\theta \sin\phi_1, \quad x_4 = r \cos\theta \sin\phi_2. \tag{14}$$

The maximal Abelian subgroup of this $SO(4)$ is $U(1)_{12} \times U(1)_{34}$ corresponding, say, to rotations in the two orthogonal planes indicated by the labeling. These also give the two independent angular momenta (J_{12}, J_{34}) that a pointlike object in five dimensions are allowed to have. The BPS condition on the spacetime solution allows only one linear combination of these two angular momenta to be non-zero, and there is an analogous situation in the two dimensional CFT corresponding to a condition on the allowed masses and charges of states (J_{12}, J_{34}) which are excited there, restricting them to be BPS states.

The convention is that the linear combination $J_{12} - J_{34}$ is called J_L and the other is $J_R = J_{12} + J_{34}$. We have $J_L = 0$ and the non-zero J_R is related to the spacetime J by

$$J_R = \frac{\pi}{4G_5} J. \tag{15}$$

In the conformal field theory on the cylinder, the L_0 energy eigenvalue of a state is Q_P. Unitarity, and an examination of the superconformal algebra requires that the energy (conformal weight) of a state of R charge J_R is bounded: $Q_P \geqslant 3J_R^2/(2c)$, which translates into our bound from before arising from the spacetime entropy (8), or absence of spacetime CTCs:

$$Q_1 Q_5 Q_P \geqslant \frac{J_R^2}{4} \;\Leftrightarrow\; r_1^2 r_5^2 r_P^2 \geqslant \frac{J^2}{4}, \tag{16}$$

where we have converted the charges using the relation in Eqs. (12) and (15).

The entropy of the black hole for large charges Q_1, Q_5 is just given by the logarithm of the standard formula (see, e.g., Ref. [23]) for the asymptotic level density of states, $d(n,c)$, of charge J_R as the level $n = Q_P - 3J_R^2/(2c)$ becomes large:

$$d(n,c) \simeq \exp\left(2\pi\sqrt{\frac{nc}{6}}\right) \Rightarrow S = 2\pi\sqrt{Q_1 Q_5 Q_P - \frac{J_R^2}{4}}, \tag{17}$$

which easily converts [using Eqs. (12) and (15)] to the entropy (8) computed from the supergravity.

V. THE BASIC ENHANÇON LOCUS

The ten dimensional solution exhibits a naked repulson singularity [24], at the place where the $K3$ volume shrinks to zero [3]. This unphysical behavior is repaired by the enhançon mechanism [3]: this repulson part of the geometry must be a supergravity artifact, since new degrees of freedom must have come to play at the radius where the $K3$ volume $V(r)$ reached the special value $V_\star = (2\pi\ell_s)^4$. A quick computation shows that this "enhançon" radius is precisely the same as in the non-rotating black hole case of Ref. [1]:

$$r_e^2 = g_s \ell_s^2 \frac{V_\star}{(V - V_\star)} (2N_5 - N_1), \tag{18}$$

where negative r_e^2 means that the enhançon lies inside the event horizon (located at $r^2 = 0$). This is the case of $2N_5 < N_1$.

Note that the enhançon locus is spherically symmetric. This is a feature of the geometry which is very useful in much of our analysis, allow for much simplification, as we

will comment further later. The harmonic functions $H_{1,5}$ in Eq. (10) have no angular dependence at all, despite the rotation.

The above radius (18) is the enhançon radius uncovered in Ref. [3], where the tension of a wrapped D5-brane would fall to zero. Such a brane cannot proceed further inside the geometry as its tension would go negative. Its motion ends at r_e^2, and such probes can form a shell of tensionless branes at this radius. We will write a new supergravity solution for this possibility later. As observed in Ref. [1], the presence of other species of brane, allowing for the formation of D5-D1 bound states, gives a much richer behavior. There are other enhançon loci corresponding to the place where these bound states can become massless and can proceed no further into the interior. Let us study these probes in the background of this geometry next.

VI. PROBING THE GEOMETRY

Let us probe the geometry with a bound state of n_5 D5-branes and n_1 D1-branes. The effective world-sheet action for such a composite brane probe is [1]

$$S=-\int_\Sigma d^2\xi e^{-\Phi}[n_5\tau_5 V(r)+(n_1-n_5)\tau_1](-\det g_{ab})^{1/2} + n_5\mu_5\int_{\Sigma\times K3}C^{(6)}+(n_1-n_5)\mu_1\int_\Sigma C^{(2)}, \tag{19}$$

where n_5 and n_1 denote the number of D5- and D1-branes we have assembled to make up the probe. We have called the world sheet Σ. It is assumed that $n_5 \ll N_5$ and $n_1 \ll N_1$.

In the static situations the terms from the Wess-Zumino (WZ) part of the effective action (second line) are cancelled by contributions from the Dirac-Born-Infeld (DBI) part (first line), so that only the kinetic term remains. This happens due to the BPS condition—the ''electric'' Coulomb repulsion is balanced by the gravitational attraction. In the rotating case one would expect that the analogue of frame dragging effects give some additional terms to the DBI part of the effective action. Happily, the RR potential in Eq. (11) is endowed with additional ''magnetic'' components, and so the WZ part of the action also gets an extra contribution. They cancel, as we see immediately below, and this is a consequence of the fact that we have a BPS system.

We adopt a static gauge, defining the coordinates on the probe brane world volume Σ to be $\xi^0=t, \xi^1=z$. The probe brane can move in the transverse directions $x^i=x^i(t)$, $x^i=(r,\theta,\phi_1,\phi_2)$, but is frozen on $K3$. Pulling back the string frame metric to the probe world volume and expanding the square root gives

$$\mathcal{L}_{\rm DBI}=-(n_5\tau_5 H_5^{-1}V+(n_1-n_5)\tau_1 H_1^{-1})\left(1+\frac{J}{2r^2}(\sin^2\theta\dot\phi_1 - \cos^2\theta\dot\phi_2)\right)+\frac{1}{2}(n_5\tau_5 H_1 V+(n_1-n_5)\tau_1 H_5)H_P v^2, \tag{20}$$

where we have assumed slow motion of the probe, i.e.,

$$v^2=\dot r^2+r^2(\dot\theta^2+\sin^2\theta\dot\phi_1^2+\cos^2\theta\dot\phi_2^2) \tag{21}$$

is taken be small. Similarly the WZ part of the effective action is given by

$$\mathcal{L}_{WZ}=(n_5\tau_5 H_5^{-1}V+(n_1-n_5)\tau_1 H_1^{-1})\left(1+\frac{J}{2r^2}(\sin^2\theta\dot\phi_1 - \cos^2\theta\dot\phi_2)\right). \tag{22}$$

As we can see, not only do the potential terms of the DBI and WZ parts of the effective action cancel, but the J terms linear in angular velocity cancel as well—the effects of gravitational frame dragging are neatly balanced by the ''magnetic'' force induced by rotation.[4]

Putting Eqs. (20) and (22) together we find that only the kinetic term survives in the effective Lagrangian

$$\mathcal{L}=\frac{1}{2}(n_5\tau_5 H_1 V+(n_1-n_5)\tau_1 H_5)H_P v^2. \tag{23}$$

The prefactor of the kinetic energy gives the effective tension of the probe. This tension is positive as long as

$$r^2>r_e^2[n_1,n_5]=g_s\ell_s^2 V_\star\frac{(2N_5-N_1)n_5-N_5 n_1}{(V-V_\star)n_5+V_\star n_1}, \tag{24}$$

while the locus of the vanishing tension indicates the position of the enhançon for the (n_1,n_5) bound state, generalizing the expression given in Eq. (18), which is the case $n_1=0$. Note that at these radii, the volume of $K3$ is below $V_\star$.

We will find out in the next Sec. VII that the lower bound where the tension vanishes agrees perfectly with the results of supergravity computations, where we build new geometries containing the shell formed by bringing up probes to the locus of points where their tension vanishes.

For a probe made up of D1-branes only (i.e., $n_5=0$), the tension remains positive everywhere and hence D1-brane probes can make their way freely down to $r=0$ without being forced to stop by an enhançon locus. For the case $n_5=n_1$, the tension of such a probe is also positive everywhere. This is how we can imagine constructing a black hole of arbitrary charges. The result of the previous subsection might have suggested that we cannot successfully bring individual D5-branes up to the horizon at $r^2=0$, depending upon the charges already present. This result for the $n_1=n_5$ case gives us an avenue around this [1], as we can move the D1-branes already present to the D5-brane enhançon locus, bind them

[4]Otherwise it has been noticed in e.g., Ref. [30], that in the non-extremal rotating D3-brane background the effective action of a probe D3-brane contained a term proportional to the probe angular velocity.

with the D5-branes already there, and then move them in as bound states, thereby constructing a hole with arbitrary charges of one's choice.

VII. EXCISION

Although by the processes described immediately above we can take D5-branes inside the enhançon radius to construct the geometry described in solution (9), we learn that there can be geometries where branes with a certain set of charges (n_1,n_5) can remain "hung up" at a specific radius. They form a shell at that radius, made of zero tension branes. In fact, using purely supergravity techniques, we can check that this is a consistent picture by building the suggested geometry. We glue two geometries together at some radius $r=r_{\rm i}$, with excess charge in the outer geometry and check what the conditions at the junction between the two suggest for the physics there. We confirm that it is consistent with the probe picture, with particular attention given to the case $r_{\rm i}^2=r_{\rm e}^2[n_1,n_5]$. The procedure goes through pretty much along the same lines as in the static case studied in Refs. [25,1]. The shells we have here are all spherically symmetric, despite the rotation. As already stated above, this is because the loci of equal $K3$ volumes [given by Eq. (10)] are spherical, a very special feature of the rotating geometry. This is in contrast to other recent cases studied in the literature, where the BPS enhançon shells are highly non-spherical [27,28], and even disconnected [29].

We keep the total number of D-branes as measured at infinity as N_1 and N_5, as before. Now, however, certain numbers, δN_5 and δN_1, of D5-branes and of D1-branes respectively are located on the shell at $r_{\rm i}$, while $N_5'=N_5-\delta N_5$ of D5-branes and $N_1'=N_1-\delta N_1$ of D1-branes are located in the interior. The solution describing the interior region has the same form as Eq. (9) above, only the harmonic functions H_1, H_5 are now substituted by

$$h_1=1+\frac{r_1^2-\tilde{r}_1^2}{r_{\rm i}^2}+\frac{\tilde{r}_1^2}{r^2},\quad h_5=1+\frac{r_5^2-\tilde{r}_5^2}{r_{\rm i}^2}+\frac{\tilde{r}_5^2}{r^2},\qquad (25)$$

with the scales

$$\tilde{r}_1^2=g_s\ell_s^2\frac{V_\star}{V}Q_1',\quad \tilde{r}_5^2=g_s\ell_s^2Q_5',\qquad (26)$$

being proportional to the number of branes inside, i.e., $Q_5'=N_5'$, $Q_1'=N_1'-N_5'$. We keep $Q_1'\geqslant 0$ to avoid a repulson singularity. The functions h_1 and h_5 in definition (25) are chosen so that the metric of the corrected solution is continuous at $r_{\rm i}$. The discontinuity in the extrinsic curvature on the junction surface at $r_{\rm i}$ has an interpretation as the surface stress-energy tensor of this thin shell [16,26]. After some algebra, we find that the stress-energy tensor of the gluing surface is of the same form as in the static case [1], but now with additional (t,ϕ_i) and (z,ϕ_i) terms:

$$S_{\mu\nu}=\frac{1}{2\kappa^2\sqrt{G_{rr}}}\left(\frac{H_1'}{H_1}+\frac{H_5'}{H_5}-\frac{h_1'}{h_1}-\frac{h_5'}{h_5}\right)G_{\mu\nu},$$

$$S_{\mu\phi_i}=\frac{1}{2\kappa^2\sqrt{G_{rr}}}\left(\frac{H_1'}{H_1}+\frac{H_5'}{H_5}-\frac{h_1'}{h_1}-\frac{h_5'}{h_5}\right)G_{\mu\phi_i},$$

$$S_{ij}=0,$$

$$S_{ab}=\frac{1}{2\kappa^2\sqrt{G_{rr}}}\left(\frac{H_5'}{H_5}-\frac{h_5'}{h_5}\right)G_{ab},\qquad (27)$$

where indices μ, ν denote the t and z directions, a, b denote the $K3$ directions, i, j denote the angular directions along the junction S^3. The Einstein frame metric G_{MN}, natural in this computation, is related to the string frame metric g_{MN} in Eq. (9) by $G_{MN}=e^{-\Phi/2}g_{MN}$, and $2\kappa^2=16\pi G_{10}$.

The tension along the angular directions vanishes, since despite rotation we still have a BPS system that does not need some force between the branes to support the shell at arbitrary radius. In the $K3$ directions the tension depends only on the harmonic functions of the D5-branes as only they wrap these directions. Finally in the t and z directions as well as $t-\phi_i$ and $z-\phi_i$ directions the surface stress-energy is proportional to a tension

$$T_{\rm eff}=\frac{1}{2\kappa^2\sqrt{G_{rr}}}\left(\frac{H_1'}{H_1}+\frac{H_5'}{H_5}-\frac{h_1'}{h_1}-\frac{h_5'}{h_5}\right).\qquad (28)$$

These results are consistent with what one would expect from the fact that the shell is built of D5- and D1-brane sources [25].

If there are no D1-branes on the shell ($\delta N_1=0$), the tension (28) vanishes precisely at the basic enhançon radius given in Eq. (18). Alternatively if some D1-branes stay on the shell, the tension is positive down to

$$\tilde{r}_{\rm e}^2=g_s\ell_s^2V_\star\frac{(2N_5-N_1)\delta N_5-N_5\delta N_1}{(V-V_\star)\delta N_5+V_\star\delta N_1},\qquad (29)$$

and we note that $\tilde{r}_{\rm e}^2<r_{\rm e}^2$. Satisfyingly, this lower bound where the tension vanishes agrees perfectly with the (n_1,n_5) probe computation result for the enhançon radii $r_{\rm e}^2[n_1,n_5]$ given in Eq. (24) if one substitutes $\delta N_1\to n_1$, $\delta N_5\to n_5$. We see that the consistency conditions derived here and the probe results of the previous section are in perfect agreement with each other.

We note that as we replace the geometry inside of the enhançon radius with a repaired geometry given by the harmonic functions (25) the running $K3$ volume in the interior is now

$$V(r)=\frac{h_1}{h_5}V_\star,\qquad (30)$$

and in particular at the horizon

$$V(r=0)=\frac{\tilde{r}_1^2}{\tilde{r}_5^2}V_\star=\frac{N_1'-N_5'}{N_5'}V_\star.\qquad (31)$$

The volume at the enhançon radius is still $V_\star$, but now we have a possibility that inside, i.e., for $r<\tilde{r}_e^2$, it can actually grow larger. Therefore means that some of the D5-branes can actually pass the enhançon radius and move in as their tension does not become negative in the process. This is subject to the condition $N_1'>2N_5'$, apparent also from Eq. (31), which shows $V(r=0)>V_\star$ if the condition holds. In fact, in the limit $N_1'=2N_5'$ the $K3$ volume is $V(r)=V_\star$ uniformly everywhere in interior up to the enhançon radius.

Notice that at arbitrary excision radius, the $S_{\mu\phi_i}$ components of the surface stress-energy tensor depends on $G_{\mu\phi_i}$ and hence on J, while at the enhançon radius these components of stress-energy vanish. This indicates that the zero-tension enhançon shells also have vanishing angular momentum. (We shall see that this is consistent with a probe's motion in the geometry in the next section: If they have non-zero angular momentum the probe computations show that they cannot stay in the shell.) This result is reminiscent of one of the already mentioned special features that these black hole solutions have [9] which is a vanishing angular velocity of the horizon. This analogy should not be stretched too far, however, since in that case, the vanishing is taken to be a result of a cancellation of opposite dragging effects: there is a opposite sense of rotation between the two sides of the horizon. Here, the senses of rotation on either side of a generic enhançon shell are the same.

VIII. THE SECOND LAW AND CTCs

So, as we have already computed, the entropy of our five dimensional black hole given in Eq. (7) is given by

$$S=\frac{\mathcal{A}}{4G_N^{(5)}}=\frac{2\pi^2}{4G_N^{(5)}}\sqrt{r_1^2r_5^2r_P^2-\frac{J^2}{4}}$$
$$=2\pi\sqrt{(N_1-N_5)N_5Q_P-\frac{J_R^2}{4}} \tag{32}$$

where in the last term we have written it in terms of the actual *number* of each type of brane, as opposed to the net charges. The essential novelty here is the presence of minus signs in the part involving the charges, which is due to wrapping our branes on $K3$ instead of T^4, where we would have had simply $N_1N_5Q_P$.

The key point, observed first in Ref. [1], is the fact that for a probe with the correct choice of D1- and D5-brane charges, bringing it to the horizon would *reduce* the entropy. This process can be done as slowly as we like (given our probe computations in Sec. VI), and the resulting adiabaticity gives us a very clear violation of the second law of thermodynamics. The area theorem assures us that this cannot happen, and the novelty we wish to observe is that the enhançon mechanism operates precisely to ensure that the area theorem—and hence the second law—is inviolate.

The first order change in the entropy is

$$\delta S=2\pi^2S^{-1}\left\{Q_P(N_5\delta N_1+(N_1-2N_5)\delta N_5)-\frac{J_R\delta J_R}{2}\right\}$$
$$=2\pi^2S^{-1}\left\{Q_P(n_1N_5+n_5(N_1-2N_5))-\frac{J_R\delta J_R}{2}\right\}, \tag{33}$$

and we have inserted $\delta N_1=n_1$ and $\delta N_5=n_5$ for the charges on the probe. We have neglected the change in Q_P since it could only decrease S by itself decreasing. This obviously cannot be achieved with a probe while retaining the saturation of the BPS condition which takes us out of the class of processes which we wish to consider.

Let us consider first probes with no angular momentum, and so we set $\delta J_R=0$. In such cases then, the key observation is that the entropy change is negative if $n_1N_5+n_5(N_1-2N_5)$ is negative, which is the same condition for an enhançon locus to appear above the horizon [see Eq. (29) or (24)], stopping that particular probe from reaching the horizon. This is how the enhançon mechanism protects the second law from dangerous probes.

We can of course have probes with non-zero angular momentum,[5] j_R. In fact, we must, or we cannot actually construct the BPS black hole with non-zero angular momentum at all. There are again two cases: Dangerous probes, for which the RR charges are such that they can reduce $(N_1-N_5)N_5Q_P$, and probes for which the RR charges cannot decrease $(N_1-N_5)N_5Q_P$. For the latter sort, there is no novel enhançon physics to be found, since the physics there is exactly the same as in the case of having made a hole by wrapping on T^4. The area theorem is protected in the usual way and we consider them no further.

For the dangerous sort of angular momentum carrying probes, we must consider the ways in which they can bring angular momentum into the hole: (a) They can have intrinsic angular momentum j_R, which means that their world-sheet CFT of Sec. IV has states with non-trivial R-charge, (b) they can have non-zero impact parameter contained in the geometry of their approach, giving them ''orbital'' angular momentum, and (c) they can have some mixture of the two previous cases. The effective Lagrangian for a probe's slow motion in the geometry is given by Eq. (23), where v^2 is given by Eq. (21). The orbital angular momenta are the conjugate momenta to ϕ_1 and ϕ_2:

$$j_{\phi_1}\equiv\frac{\partial\mathcal{L}}{\partial\dot{\phi}_1}=r^2F(r)\sin^2\theta\dot{\phi}_1,\quad j_{\phi_2}\equiv\frac{\partial\mathcal{L}}{\partial\dot{\phi}_2}=r^2F(r)\cos^2\theta\dot{\phi}_2, \tag{34}$$

where

[5]We are of course assuming that the charges are such that $4(n_1-n_5)n_5q_P\geq j_R^2$ on an individual probe, where j_R is its angular momentum and q_P is its z momentum, otherwise the theory on the probe is nonunitary at the outset. See Sec. IV.

$$F(r)=(n_5\tau_5 H_1 V+(n_1-n_5)\tau_1 H_5)H_P\,. \tag{35}$$

As these momenta are conserved, we can initially reduce our problem to a two dimensional one in r and θ, with an effective potential set by the angular momenta:

$$\mathcal{L}=\frac{1}{2}F(r)(\dot{r}^2+r^2\dot{\theta}^2)+\frac{1}{2r^2F(r)}\left(\frac{j_{\phi_1}^2}{\sin^2\theta}+j_{\phi_2}^2\cos^2\theta\right). \tag{36}$$

The case (a) above corresponds to vanishing j_{ϕ_1} and j_{ϕ_2}, for which it is clear again that we have the same discussion as above: Either $F(r)=0$ above the horizon and the enhançon mechanism forbids further approach, or the mechanism is not relevant and the area is understood to be non-decreasing in the conventional fashion.

Cases (b) or (c) are no longer BPS. We need to have equal angular momenta in the 1–2 and 3–4 planes and to satisfy the BPS condition, with the angular momentum tensor $M^{\mu\nu}$ in the form

$$M\sim\begin{pmatrix}0 & j & 0 & 0\\ -j & 0 & 0 & 0\\ 0 & 0 & 0 & j\\ 0 & 0 & -j & 0\end{pmatrix}. \tag{37}$$

We can place such an equality condition on the orbital angular momenta of the probe in the 1–2 and 3–4 planes, but we cannot avoid a centripetal potential being generated. There is no assignment of orbital angular momenta to a single probe which will give a vanishing potential which is a consequence of the fact that for a single probe we can easily find a rotation (e.g., combining one in the 1–3 with one in the 2–4 plane) to find a frame in which the associated angular momentum tensor is just[6]

$$M\sim\begin{pmatrix}-0 & j & 0 & 0\\ -j & 0 & 0 & 0\\ -0 & 0 & 0 & 0\\ -0 & 0 & 0 & 0\end{pmatrix}. \tag{38}$$

In retrospect, this is clearly a result of the fact that in any dimension, a particle with conserved orbital angular momentum will remain constrained to move in a single plane. In order to get a BPS configuration with the correct combination of orbital momentum, one would have to consider two probes, moving such that they have angular momentum in the two independent planes. Counting parameters and available rotations shows that it should be possible to achieve an angular momentum tensor of the form given in Eq. (37). On the other hand, from this point of view and also from that of the required probe action, this would seem to be requiring them to be coupled in an interesting (and apparently non-local) manner. This is a probe problem which we will not pursue here, since it will take us beyond the matters we wish to discuss in the present work. In any event, such a BPS case would then become rather similar to that of case (a), where we have only intrinsic angular momentum, in the sense that if the probes have the ''wrong'' charges, they will be subject to the appearance of an enhançon locus above the horizon to stop their approach.

Further to the discussion in the previous paragraph, note that from Eqs. (34) we see that for fixed angular momenta, we must ensure that $r^2F(r)\sin^2\theta$ and $r^2F(r)\cos^2\theta$ stay away from zero, otherwise the velocities $\dot{\phi}_{1,2}$ diverge, taking us out of the slow-probe limit of Sec. VI. We must avoid the neighborhood of the planes ($\theta=0,\pi/2$) lest we violate this, or the probe will encounter additional forces from the background which we previously neglected.

Looking further at the non-trivial effective potential given in Eq. (36) (and even staying sufficiently far away from the special planes $\theta=0,\pi/2$), we see that there is an infinitely repulsive wall at the enhançon locus for such cases of non-zero impact parameter, naturally induced by the vanishing kinetic term there. Near there, the velocities cannot be small, and so there will be further terms which we neglected which introduce more forces on the probe due to the background. Unless there is a remarkable conspiracy, it is unlikely that these terms can soften this infinite repulsion at the enhançon locus, and so our result is consistent with the conclusions reached for a BPS approach.

Finally, we note that the impossibility of adding certain charges on D-brane probes to the hole in order to reduce the entropy also means something for the occurrence of CTCs: If we start with a hole with no CTCs above the horizon, we simply cannot introduce a probe to the black hole which will create a CTC, since this would require S in Eq. (32) to reduce, and the enhançon mechanism forbids that. This is in accord with existing discussion in the literature [10] about not being able to manufacture CTCs (at least for this class of geometries) by a physical process. As we pointed out in Sec. I, this does not rule out considering a CTC-endowed object with this geometry, since they do not owe their existence to the presence of $K3$, while the enhançon (in this example) does.

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