Approach to solve Slavnov-Taylor identities in nonsupersymmetric non-Abelian gauge theories

Igor Kondrashuk,* Gorazd Cvetič,† and Ivan Schmidt‡
Department of Physics, Universidad Técnica Federico Santa María, Valparaíso, Chile
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We present a way to solve Slavnov-Taylor identities in a general nonsupersymmetric theory. The solution can be parametrized by a limited number of functions of spacetime coordinates, so that all the effective fields are dressed by these functions via integral convolution. The solution restricts the ghost part of the effective action and gives predictions for the physical part of the effective action.

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I. INTRODUCTION

The effective action is an important quantity of quantum theory. Defined as the Legendre transformation of the path integral, it provides us with an instrument to find the true vacuum state of the theory under consideration and to study its behavior, taking into account quantum corrections. Slavnov-Taylor (ST) identities are also an important tool to prove the renormalizability of gauge theories in four spacetime dimensions [1,2]. They generalize Ward-Takahashi identities of quantum electrodynamics to the non-Abelian case and can be derived starting from the property of invariance of the tree-level action with respect to Becchi-Rouet-Stora-Tyutin (BRST) symmetry [3,4]. ST identities for the effective action have been derived in Ref. [5].

Slavnov-Taylor identities are equations involving variational derivatives of the effective action. The effective action contains all the information about the quantum behavior of the theory, and in quantum field theory it is the one-particle irreducible diagram generator. Searching for the solution to Slavnov-Taylor identities can be considered as a complementary method to the existing nonperturbative methods of quantum field theory such as the Dyson-Swinger and Bethe-Salpeter equations. A solution to the Slavnov-Taylor identities in four-dimensional supersymmetric theory has been proposed recently [6]. In the procedure to derive that solution, the no-renormalization theorem for the superpotential [7,8] was used extensively. In this paper we will suggest that this point is not crucial and that arguments similar to those given before [6] can be used in the nonsupersymmetric case. In the approach developed below there are no restrictions on the number of dimensions and renormalizability of the theory. We require only that the theory under consideration can be regularized in such a way that the Slavnov-Taylor identities are valid and that BRST symmetry is anomaly-free, as is the case, e.g., in QCD.

We argue that the functional structure of the auxiliary ghost-ghost Lc^2 correlator in nonsupersymmetric gauge theories is fixed by Slavnov-Taylor identities in a unique way. In this correlator L is a nonpropagating background field and it is coupled at the tree level to the BRST transformation of the ghost field c. According to our assumption, the vertex Lc^2 is invariant with respect to ST identities and this then gives the following quantum structure for it:

$$\int dx'dxdydzG_c(x'-x)G_c^{-1}(x'-y)\times G_c^{-1}(x'-z)\frac{i}{2}f^{bca}L^a(x)c^b(y)c^c(z). \qquad (1)$$

As one can see, the main feature of this result is that the effective ghost field c is dressed by the unknown function $G_c^{-1}(x-y)$. This dressing contains all the quantum information about this correlator. We can use the structure of this correlator as a starting point to find the solution for the total effective action.

The solution to the Slavnov-Taylor identities found in the present paper imposes restrictions on the ghost part of the effective action. For example, it means that the gluon-ghost-antighost vertex can be read off from our result for the effective action (67):

$$G_m(q,p)=iq_m\frac{\tilde{G}_A(q^2)}{\tilde{G}_A(k^2)\tilde{G}_c(p^2)}, \qquad (2)$$

where $\tilde{G}_A$ is the Fourier image of a function that dresses the gauge field, while $G_m(q,p)$ is the gluon-ghost-antighost vertex, q is the momentum of the antighost field b and p is the momentum of the ghost field c, and $p+k+q=0$. Another feature of the result obtained here is that the physical part of the effective action (67) is gauge invariant in terms of the effective fields dressed by the dressing functions G. In the result (67) for the effective action information about the quantum behavior of the theory is encoded in a *finite* number of dressing functions and in the running function of the coupling.

The paper is organized in the following way. In Sec. II we review some basic aspects of BRST symmetry and Slavnov-Taylor identities for irreducible vertices. In Sec. III we show how to obtain the functional structure (1) of the Lc^2 correlator. In Sec. IV we obtain the correlator linear in another nonpropagating background field K_m, thus fixing the terms in the effective action that contain ghost and antighost effective fields. In Sec. V we describe higher correlators in K_m and L. In Sec. VI we make a conjecture about the form of the physical (pure gluonic) part of the effective action and then

*Email address: ikondrashuk@fis.utfsm.cl
†Email address: cvetic@fis.utfsm.cl
‡Email address: ischmidt@fis.utfsm.cl

in Sec. VII we consider renormalizing of it to remove infinities. A brief summary is given at the end. The questions of consistency of this effective action within perturbative QCD are investigated in a second paper [9]. For simplicity, in the present paper we focus on pure gauge theories in four space-time dimensions with the $SU(\mathrm{N})$ gauge group. No matter field is included in the consideration, although their addition does not change our results.

II. PRELIMINARIES

We consider the traditional Yang-Mills Lagrangian of the pure gauge theory:

$$S=-\int dx \; \frac{1}{2g^2} \; \mathrm{Tr}[F_{mn}(x)F_{mn}(x)]. \tag{3}$$

The gauge field is in the adjoint representation of the gauge group. A nonlinear local (gauge) transformation of the gauge fields exists which keeps the theory (3) invariant. This symmetry must be fixed, Faddeev-Popov ghost fields [10] must be introduced, and finally the BRST symmetry can be established for a theory that in addition to the classical action (3) contains a Faddeev-Popov ghost action and a gauge-fixing term.

To be specific, we choose the Lorentz gauge-fixing condition

$$\partial_m A_m(x)=f(x). \tag{4}$$

Here f is an arbitrary function in the adjoint representation of the gauge group that is independent of the gauge field. The normalization of the gauge group generators is

$$\mathrm{Tr}(T^aT^b)=\frac{1}{2}\delta^{ab}, \qquad (T^a)^\dagger=T^a, \qquad [T^b,T^c]=if^{bca}T^a,$$

and we use the notation $X=X^aT^a$ for all the fields in the adjoint representation of the gauge group, like the gauge fields themselves, the ghost fields, and their respective sources.

The conventional averaging procedure with respect to f is applied to the path integral with the weight

$$e^{-i\int dx \mathrm{Tr}\frac{1}{\alpha}f^2(x)}$$

and as the result we obtain the path integral

$$Z[J,\eta,\rho,K,L]=\int dA dc db \exp i\Big\{S[A,b,c]+2\,\mathrm{Tr}\Big(\int dx J_m(x)A_m(x)+i\int dx\,\eta(x)c(x)+i\int dx \rho(x)b(x)\Big) + 2\,\mathrm{Tr}\Big(i\int dx K_m(x)\nabla_m c(x)+\int dx L(x)c^2(x)\Big)\Big\}, \tag{5}$$

in which

$$S[A,b,c]=\int dx\Big[-\frac{1}{2g^2}\mathrm{Tr}[F_{mn}(x)F_{mn}(x)] - \mathrm{Tr}\Big(\frac{1}{\alpha}[\partial_m A_m(x)]^2\Big)-2\,\mathrm{Tr}[ib(x)\partial_m\nabla_m c(x)]\Big]. \tag{6}$$

Here the ghost field c and the antighost field b are Hermitian, and $b^\dagger=b$, $c^\dagger=c$ in the adjoint representation of the gauge group. They possess Fermi statistics.

The infinitesimal transformation of the gauge field A_m is defined by the fact that it is a gauge connection,

$$A_m\to A_m-\nabla_m\lambda,$$

where $\lambda(x)$ is an infinitesimal parameter of the gauge transformation. This transformation comes from the transformation of covariant derivatives,

$$\nabla_m\to e^{i\lambda}\nabla_m e^{-i\lambda}, \qquad \nabla_m=\partial_m+iA_m, \qquad \phi\to e^{i\lambda}\phi,$$

where ϕ is some representation of the gauge group. To obtain the BRST symmetry we have to substitute $ic(x)\varepsilon$ for λ. Here ε is the Hermitian Grassmannian parameter, $\varepsilon^\dagger=\varepsilon$, $\varepsilon^2=0$. Thus, the BRST transformation of the gauge field is

$$A_m\to A_m-i\;\nabla_m c\;\;\varepsilon. \tag{7}$$

In order to obtain the BRST transformation of the ghost field c we have to consider two subsequent BRST transformations:

$$\nabla_m\to e^{-c\kappa}e^{-c\varepsilon}\nabla_m e^{c\varepsilon}e^{c\kappa} = e^{-c\varepsilon-c\kappa-(c\varepsilon)(c\kappa)}\nabla_m e^{c\varepsilon+c\kappa+(c\varepsilon)(c\kappa)}, \tag{8}$$

where κ is a Grassmannian parameter too, $\kappa^2=0$. This transformation again is equivalent to an infinitesimal transformation of the gauge field in covariant derivatives,

$$A_m\to A_m-i\nabla_m[c\varepsilon+c\kappa+(c\varepsilon)(c\kappa)].$$

This means that we can consider the inner BRST transformation (with ε) as the substitution (7) in the outer BRST transformation (with κ). The second term after the covariant derivative is a transformation of A_m under the outer BRST transformation while the third term after the covariant derivative is the transformation of $i\nabla_m c\kappa$ and can be cancelled by the transformation of the second term $c\kappa$,

$$c \to c + c^2\varepsilon. \tag{9}$$

Thus, the transformations (7) and (9) together leave the covariant derivative of the ghost field unchanged. Such a symmetry is very general and always exists if the gauge fixing procedure has been performed in the path integral for any theory with nonlinear local symmetry. The noninvariance of the gauge-fixing term is cancelled by the corresponding transformation of the antighost field b.

To collect everything together, the action (6) is invariant with respect to the BRST symmetry transformation with the Grassmannian parameter ε,

$$\begin{aligned} A_m &\to A_m - i\nabla_m c\varepsilon, \\ c &\to c + c^2\varepsilon, \\ b &\to b - \frac{1}{\alpha}\partial_m A_m \varepsilon. \end{aligned} \tag{10}$$

The external sources K and L of the BRST transformations of the fields are BRST invariant by definition, so the last two lines in Eq. (5) are BRST invariant with respect to the transformations (10).

The effective action Γ is related to $W = i\ln Z$ by the Legendre transformation[1]

$$A_m \equiv -\frac{\delta W}{\delta J_m}, \quad ic \equiv -\frac{\delta W}{\delta \eta}, \quad ib \equiv -\frac{\delta W}{\delta \rho}, \tag{11}$$

$$\begin{aligned} \Gamma = -W - 2\ \mathrm{Tr}\bigg(\int dx J_m(x) A_m(x) + \int dx\, i\eta(x) c(x) \\ + \int dx\, i\rho(x) b(x) \bigg) \\ \equiv -W - 2\ \mathrm{Tr}(X\varphi), \end{aligned} \tag{12}$$

$$(X\varphi) \equiv i^{G(k)}(X^k\varphi^k),$$

$$X \equiv (J_m, \eta, \rho), \quad \varphi \equiv (A_m, c, b),$$

where $G(k)=0$ if φ^k is a Bose field and $G(k)=1$ if φ^k is a Fermi field. We use throughout the paper the notation

$$\frac{\delta}{\delta X} = T^a \frac{\delta}{\delta X^a}$$

for any field X in the adjoint representation of the gauge group. Iteratively, all equations (11) can be reversed,

$$X = X[\varphi, K_m, L],$$

and the effective action is defined in terms of the new variables $\Gamma = \Gamma[\varphi, K_m, L]$. Hence, the following equalities occur:

$$\frac{\delta\Gamma}{\delta A_m} = -J_m, \quad \frac{\delta\Gamma}{\delta K_m} = -\frac{\delta W}{\delta K_m}, \quad \frac{\delta\Gamma}{\delta c} = i\eta, \quad \frac{\delta\Gamma}{\delta b} = i\rho,$$

$$\frac{\delta\Gamma}{\delta L} = -\frac{\delta W}{\delta L}. \tag{13}$$

If the change of fields (10) in the path integral (5) is made, one obtains the Slavnov-Taylor identity as the result of the invariance of the integral (5) under a change of variables,

$$\mathrm{Tr}\bigg[\int dx J_m(x) \frac{\delta}{\delta K_m(x)} - \int dx\, i\eta(x) \bigg(\frac{\delta}{\delta L(x)} \bigg) + \int dx\, i\rho(x) \bigg(\frac{1}{\alpha}\partial_m \frac{\delta}{\delta J_m(x)} \bigg) \bigg] W = 0, \tag{14}$$

or, taking into account the relations (13), we have [2]

$$\mathrm{Tr}\bigg[\int dx \frac{\delta\Gamma}{\delta A_m(x)} \frac{\delta\Gamma}{\delta K_m(x)} + \int dx \frac{\delta\Gamma}{\delta c(x)} \frac{\delta\Gamma}{\delta L(x)} - \int dx \frac{\delta\Gamma}{\delta b(x)} \bigg(\frac{1}{\alpha}\partial_m A_m(x) \bigg) \bigg] = 0. \tag{15}$$

The problem is to find the most general functional Γ of the variables φ, K_m, L that satisfies the ST identity (15). Before doing it, we need in addition to the ST identities also the ghost equation that can be derived by shifting the antighost field b by an arbitrary field $\varepsilon(x)$ in the path integral (5). The consequence of invariance of the path integral with respect to such a change of variable is [in terms of the variables (11)] [2]

$$\frac{\delta\Gamma}{\delta b(x)} + \partial_m \frac{\delta\Gamma}{\delta K_m(x)} = 0. \tag{16}$$

The ghost equation (16) restricts the dependence of Γ on the antighost field b and on the external source K_m to an arbitrary dependence on their combination:

$$\partial_m b(x) + K_m(x). \tag{17}$$

This equation together with the third term in the ST identities (15) is responsible for the absence of quantum corrections to the gauge-fixing term. Stated otherwise, when expressing $\delta\Gamma/\delta b(x)$ in the third term in the ST identity (15) as $-\partial_m[\delta\Gamma/\delta K_m(x)]$ by Eq. (16), the sum of the first and the third terms in Eq. (15) can be rewritten as

$$\mathrm{Tr}\int dx \frac{\delta\Gamma'}{\delta A_m(x)} \frac{\delta\Gamma'}{\delta K_m(x)},$$

[1] We have traditionally used in this paper the same notation for the variable of the effective action and the variable of integration in the path integral coupled to the corresponding source [2].

where $\Gamma' \equiv \Gamma - S^{(\mathrm{gf})}$, and $S^{(\mathrm{gf})} = -(1/\alpha)\mathrm{Tr}\int dx[\partial_m A_m(x)]^2$ is the gauge-fixing part of the classical action (6). In fact, all the other terms in the ST identity (15) can be rewritten with Γ' instead of Γ, yielding

$$\mathrm{Tr}\left[\int dx \frac{\delta\Gamma'}{\delta A_m(x)} \frac{\delta\Gamma'}{\delta K_m(x)} + \int dx \frac{\delta\Gamma'}{\delta c(x)} \frac{\delta\Gamma'}{\delta L(x)}\right] = 0. \tag{18}$$

This shows explicitly that the gauge-fixing part of Γ remains unaffected by quantum corrections ($\Gamma = \Gamma' + \Gamma^{(\mathrm{gf})}; \Gamma^{(\mathrm{gf})} = S^{(\mathrm{gf})}$).

III. FUNCTIONAL STRUCTURE OF Lcc VERTEX

One can consider the part of the effective action that depends only on the fields L and c. We write generally

$$\begin{aligned}\Gamma|_{L,c} = \int dx_1 dy_1 dy_2 \Gamma\Big|_{L,c}^{(a_1;b_1,b_2)}(x_1;y_1,y_2) L^{a_1}(x_1) \\ \times c^{b_1}(y_1) c^{b_2}(y_2) + \cdots \\ + \int dx_1 \cdots dx_n dy_1 \cdots dy_{2n} \Gamma\Big|_{L,c}^{(a_1,\ldots,a_n;b_1,\ldots,b_{2n})} \\ (x_1,\ldots,x_n;y_1,\ldots,y_{2n}) L^{a_1}(x_1)\cdots \\ \times L^{a_n}(x_n) c^{b_1}(y_1)\cdots c^{b_{2n}}(y_{2n}) + \cdots .\end{aligned} \tag{19}$$

We assume that the first term is invariant with respect to the second operator in the identities (15), which is

$$\mathrm{Tr}\int dx \frac{\delta\Gamma}{\delta c(x)} \frac{\delta\Gamma}{\delta L(x)} = 0. \tag{20}$$

This assumption is based on the following. In perturbation theory the first term of Eq. (19) can be understood as the classical term plus a quantum correction to the vertex Lcc (nothing forbids us to consider the auxiliary field L as a nonpropagating background field). The operator (20) can be considered as an infinitesimal substitution in the effective action

$$c(x) \to c(x) + \frac{\delta\Gamma}{\delta L(x)}. \tag{21}$$

In other words, one can consider the result of such a substitution as the difference

$$\Gamma\left[L, c(x) + \frac{\delta\Gamma}{\delta L(x)}\right] - \Gamma[L, c(x)],$$

to linear order in $\delta\Gamma/\delta L(x)$. As one can see, the application of the substitution (21) to the vertex Lcc of the effective action Γ gives a variation of order $Lccc$. Another contribution of the same order $Lccc$ comes into the variation from the monomial $LccA$ of the effective action Γ due to the first term in the ST identity (15). Indeed, one can consider the first term in Eq. (15) as the substitution

$$A_m(x) \to A_m(x) + \frac{\delta\Gamma}{\delta K_m(x)},$$

or, in other words, such a substitution can be considered as the difference

$$\Gamma\left[K_m, A_m(x) + \frac{\delta\Gamma}{\delta K_m(x)}\right] - \Gamma[K_m, A_m(x)]$$

to linear order in $\delta\Gamma/\delta K_m(x)$. Application of such a substitution to the monomial $LccA$ of the effective action Γ gives a contribution of order $Lccc$ in the effective fields and this contribution comes from the full ghost propagator of order in the fields $K_m \partial_m c$,

$$LccA \to Lcc \frac{\delta\Gamma}{\delta K_m} \sim Lcc \frac{\delta(K_m\ \partial_m c)}{\delta K_m} \sim Lccc.$$

Thus, there are only these two possible contributions to the variation $Lccc$. Schematically, the total $Lccc$ variation can be presented as

$$\langle Lcc\rangle \times \langle Lcc\rangle + \langle LccA\rangle \times \langle K_m \partial_m c\rangle = 0 \tag{22}$$

where the angular brackets mean the vacuum expectation values of the vertices. This is a schematic form of the ST identity relating the Lcc and $LccA$ field monomials. The precise form of this relation can be obtain by differentiating the identity (15) with respect to L and three times with respect to c and then by setting all the variables of the effective action to zero. The angular brackets in Eq. (22) mean that we have taken the functional derivatives with respect to the fields in the corresponding brackets and then have put all the effective fields to zero. Of course, this sum (22) should be zero since on the right hand side of the ST identity (15) we have zero. One can consider the identity (22) order by order in g^2. At the tree level, the second contribution is absent since the $LccA$ term is absent in the classical action. For the first one we obtain the Jacobi identity. At one-loop level, we have one-loop Lcc times tree level Lcc plus one-loop $LccA$ times tree level $K\partial c$. However, one-loop $LccA$ is superficially convergent and does not depend on the normalization point μ. In the asymptotic region one-loop Lcc depends on the first degree of $\ln(p^2/\mu^2)$ where we have taken the symmetric point in momentum space, that is, all the external momenta of the vertex Lcc are $\sim p^2$. This means that the first degree of $\ln(p^2/\mu^2)$ in one-loop Lcc is invariant with respect to the operator (20). In other words, the dependence on $\ln(p^2/\mu^2)$ is cancelled within the first term of the identity (22). We can consider the two-loop approximation for the identity (22) in the same manner. Indeed, at the two-loop level of the identity (22) one has two-loop Lcc times tree level Lcc plus one-loop Lcc times one-loop Lcc plus two-loop $LccA$ times tree level $K\partial c$ plus one-loop $LccA$ times one-loop $K\partial c$ and all this should be zero. However, one can see that the second degree of $\ln(p^2/\mu^2)$ is determined again by only the first term in the schematic identity (22) since two-loop $LccA$ does not have superficial divergences and is divergent only in subgraphs. Thus, the second degree of

$\ln(p^2/\mu^2)$ is also determined by the invariance with respect to the first term in the identity (22). We can go further in this logical chain and we will always conclude that the highest degree of $\ln(p^2/\mu^2)$ in Lcc is invariant itself with respect to the ST identity. This is the main source of the intuitive motivation for considering the Lcc correlator separately from the other field monomial $LccA$.

In such a case, it will be shown below that the only solution for this Lcc term of the effective action is

$$\int dx dx_1 dy_1 dy_2 G_c(x-x_1) G_c^{-1}(x-y_1) \times G_c^{-1}(x-y_2) 2\,\mathrm{Tr}[L(x_1)c(y_1)c(y_2)]. \quad (23)$$

To prove Eq. (23), we consider the proper correlator

$$\Gamma = \int dx dy dz \Gamma(x,y,z) T^{abc} L^a(x) c^b(y) c^c(z). \quad (24)$$

As we have already noted, in perturbation theory it can be understood as a correction to the vertex Lc^2 and we consider the auxiliary field L as a nonpropagating background field. T^{abc} is some group structure. Equation (24) is just a general parametrization of the proper correlator Lc^2 and nothing more. Equation (24) says that $\Gamma^{(a;b,c)}(x,y,z) = \Gamma(x,y,z)T^{abc}$, where T^{abc} is a three-tensor in the adjoint representation of the gauge group. This reflects the fact that the global symmetry of the gauge group must be conserved in the effective action. With respect to that symmetry the auxiliary fields K^a and L^a are vectors in the adjoint representation of the gauge group. Also,

$$\Gamma(x,y,z)T^{abc} = -\Gamma(x,z,y)T^{acb}. \quad (25)$$

This is a direct consequence of the Grassmannian nature of the ghost fields. It follows from the parametrization (24). Further, from Eq. (24) it follows that

$$\frac{\delta\Gamma}{\delta L^a(x)} = \int dy dz \Gamma(x,y,z) T^{abc} c^b(y) c^c(z).$$

By substituting this expression in the Slavnov-Taylor identity (15) we have

$$\begin{aligned}
\int dx \frac{\delta\Gamma}{\delta c^a(x)} \frac{\delta\Gamma}{\delta L^a(x)} &= \int dx dy' dz' \Gamma(y',x,z') T^{dab} L^d(y') \frac{\delta\Gamma}{\delta L^a(x)} c^b(z') \\
&\quad - \int dx dy' dz' \Gamma(y',z',x) T^{dba} L^d(y') c^b(z') \frac{\delta\Gamma}{\delta L^a(x)} \\
&= \int dx dy dz dy' dz' \Gamma(y',x,z') T^{dab} L^d(y') \Gamma(x,y,z) T^{amn} c^m(y) c^n(z) c^b(z') \\
&\quad - \int dx dy dz dy' dz' \Gamma(y',z',x) T^{dba} L^d(y') c^b(z') \Gamma(x,y,z) T^{amn} c^m(y) c^n(z) \\
&= \int dx dy dz dy' dz' \Gamma(y',x,z') \Gamma(x,y,z) T^{dab} T^{amn} L^d(y') c^m(y) c^n(z) c^b(z') \\
&\quad - \int dx dy dz dy' dz' \Gamma(y',y,x) \Gamma(x,z,z') T^{dma} T^{anb} L^d(y') c^m(y) c^n(z) c^b(z') \\
&= \int dx dy dz dy' dz' [\Gamma(y',x,z') \Gamma(x,y,z) T^{dab} T^{amn} \\
&\quad - \Gamma(y',y,x) \Gamma(x,z,z') T^{dma} T^{anb}] L^d(y') c^m(y) c^n(z) c^b(z') = 0.
\end{aligned}$$

Taking into account Eq. (25) the last two lines can be rewritten as

$$\begin{aligned}
&\int dx dy dz dy' dz' [\Gamma(y',x,z') \Gamma(x,y,z) T^{dab} T^{amn} - \Gamma(y',y,x) \Gamma(x,z,z') T^{dma} T^{anb}] L^d(y') c^m(y) c^n(z) c^b(z') \\
&= \int dx dy dz dy' dz' [\Gamma(y',x,z') \Gamma(x,y,z) T^{dab} T^{amn} - \Gamma(y',x,y) \Gamma(x,z',z) T^{dam} T^{abn}] L^d(y') c^m(y) c^n(z) c^b(z') \\
&= 2 \int dx dy dz dy' dz' \Gamma(y',x,z') \Gamma(x,y,z) T^{dab} T^{amn} L^d(y') c^m(y) c^n(z) c^b(z') = 0.
\end{aligned}$$

Now one can make a total symmetrization with respect to the pairs (m,y), (n,z), and (b,z'). It results in

$$\int dxdydzdy'dz'[\Gamma(y',x,z')\Gamma(x,y,z)T^{dab}T^{amn} + \Gamma(y',x,y)\Gamma(x,z,z')T^{dam}T^{anb} + \Gamma(y',x,z)\Gamma(x,z',y)T^{dan}T^{abm}] \times L^d(y')c^m(y)c^n(z)c^b(z')=0.$$

Thus, one comes to the equation

$$\int dx\Gamma(y',x,z')\Gamma(x,y,z)T^{dab}T^{amn} + \int dx\Gamma(y',x,y)\Gamma(x,z,z')T^{dam}T^{anb} + \int dx\Gamma(y',x,z)\Gamma(x,z',y)T^{dan}T^{abm}=0. \quad (26)$$

As one can see, at the tree level $T^{dab}\sim f^{abd}$ and

$$\Gamma_{tree}(x,y,z)=\int dx'\,\delta(x'-x)\,\delta(x'-y)\,\delta(x'-z) \quad (27)$$

and, hence, the identity (26) is a Jacobi identity. We consider in this paper gauge theories with the $SU(N)$ gauge group and we noted this in the Introduction. The structure constants f^{abc} are completely antisymmetric in such a case. With the help of the identities

$$f^{ABC}f^{CDE}f^{EBF}=-\frac{1}{2}Nf^{ADF}$$

which are consequences of the Jacobi identity, one can reduce the group structure of the one-loop diagram Lcc to f^{ABC} and that is true for all loops. Thus, it is natural to assume that $T^{abc}\sim f^{bca}$ and the identity (26) is

$$\int dx\Gamma(y',x,z')\Gamma(x,y,z)f^{abd}f^{mna} + \int dx\Gamma(y',x,y)\Gamma(x,z,z')f^{amd}f^{nba} + \int dx\Gamma(y',x,z)\Gamma(x,z',y)f^{and}f^{bma}=0.$$

Because of the Jacobi identity only two group structures are independent here:

$$\left[\int dx\Gamma(y',x,z')\Gamma(x,y,z) - \int dx\Gamma(y',x,y)\Gamma(x,z,z')\right]f^{abd}f^{mna} + \left[\int dx\Gamma(y',x,z)\Gamma(x,z',y) - \int dx\Gamma(y',x,y)\Gamma(x,z,z')\right]f^{and}f^{bma}=0.$$

Since these two group structures are independent, we come to the equations

$$\int dx\Gamma(y',x,z')\Gamma(x,y,z)=\int dx\Gamma(y',y,x)\Gamma(x,z,z') =\int dx\Gamma(y',x,z)\Gamma(x,z',y). \quad (28)$$

We can start by solving the first one:

$$\int dx\Gamma(y',x,z')\Gamma(x,y,z)=\int dx\Gamma(y',y,x)\Gamma(x,z,z'), \quad (29)$$

and then check that the second equality is also satisfied. In writing this equation we have used the symmetry properties (25). We introduce the Fourier transformations[2]

$$\Gamma(x,y,z)=\int dp_1dq_1dk_1\delta(p_1+q_1+k_1) \times\tilde{\Gamma}(p_1,q_1,k_1)\exp(ip_1x+iq_1y+ik_1z),$$

$$\Gamma(y',x,z')=\int dp_2dq_2dk_2\delta(p_2+q_2+k_2) \times\tilde{\Gamma}(p_2,q_2,k_2)\exp(ip_2y'+iq_2x+ik_2z'),$$

$$\Gamma(y',y,x)=\int dp_3dq_3dk_3\delta(p_3+q_3+k_3) \times\tilde{\Gamma}(p_3,q_3,k_3)\exp(ip_3y'+iq_3y+ik_3x),$$

$$\Gamma(x,z,z')=\int dp_4dq_4dk_4\delta(p_4+q_4+k_4) \times\tilde{\Gamma}(p_4,q_4,k_4)\exp(ip_4x+iq_4z+ik_4z').$$

The condition (29) in momentum space is

[2]We do not write factors 2π in these Fourier transformations since at the end of the calculations we will go back to coordinate space, in which all the factors 2π will disappear.

$$\int dx dp_1 dq_1 dk_1 dp_2 dq_2 dk_2 \delta(p_1+q_1+k_1)\,\delta(p_2+q_2+k_2)\tilde{\Gamma}(p_1,q_1,k_1)\tilde{\Gamma}(p_2,q_2,k_2)$$
$$\times \exp(ip_1x+iq_1y+ik_1z+ip_2y'+iq_2x+ik_2z')$$
$$=\int dx dp_3 dq_3 dk_3 dp_4 dq_4 dk_4 \delta(p_3+q_3+k_3)\,\delta(p_4+q_4+k_4)\tilde{\Gamma}(p_3,q_3,k_3)\tilde{\Gamma}(p_4,q_4,k_4)$$
$$\times \exp(ip_3y'+iq_3y+ik_3x+ip_4x+iq_4z+ik_4z').$$

It can be transformed to

$$\int dp_1 dq_1 dk_1 dp_2 dk_2 \delta(p_1+q_1+k_1)\,\delta(p_2-p_1+k_2)\tilde{\Gamma}(p_1,q_1,k_1)\tilde{\Gamma}(p_2,-p_1,k_2)\exp(iq_1y+ik_1z+ip_2y'+ik_2z')$$
$$=\int dp_3 dq_3 dk_3 dq_4 dk_4 \delta(p_3+q_3+k_3)\,\delta(-k_3+q_4+k_4)\tilde{\Gamma}(p_3,q_3,k_3)\tilde{\Gamma}(-k_3,q_4,k_4)\exp(ip_3y'+iq_3y+iq_4z+ik_4z'),$$

and then by momentum redefinitions in the second integral one obtains

$$\int dp_1 dq_1 dk_1 dp_2 dk_2 \delta(p_1+q_1+k_1)\,\delta(p_2-p_1+k_2)\tilde{\Gamma}(p_1,q_1,k_1)\tilde{\Gamma}(p_2,-p_1,k_2)\exp(iq_1y+ik_1z+ip_2y'+ik_2z')$$
$$=\int dp_2 dq_1 dk_3 dk_1 dk_2 \delta(p_2+q_1+k_3)\,\delta(-k_3+k_1+k_2)\tilde{\Gamma}(p_2,q_1,k_3)\tilde{\Gamma}(-k_3,k_1,k_2)\exp(iq_1y+ik_1z+ip_2y'+ik_2z').$$

By removing one of the delta functions in each part one obtains

$$\int dq_1 dk_1 dp_2 dk_2 \delta(p_2+k_2+q_1+k_1)\tilde{\Gamma}(p_2+k_2,q_1,k_1)\tilde{\Gamma}(p_2,-p_2-k_2,k_2)\exp(iq_1y+ik_1z+ip_2y'+ik_2z')$$
$$=\int dp_2 dq_1 dk_1 dk_2 \delta(p_2+q_1+k_1+k_2)\tilde{\Gamma}(p_2,q_1,k_1+k_2)\tilde{\Gamma}(-k_1-k_2,k_1,k_2)\exp(iq_1y+ik_1z+ip_2y'+ik_2z').$$

By making the last simplification one obtains

$$\int dk_1 dp_2 dk_2 \tilde{\Gamma}(p_2+k_2,-p_2-k_2-k_1,k_1)\tilde{\Gamma}(p_2,-p_2-k_2,k_2)\exp[i(-p_2-k_2-k_1)y+ik_1z+ip_2y'+ik_2z']$$
$$=\int dp_2 dk_1 dk_2 \tilde{\Gamma}(p_2,-p_2-k_2-k_1,k_1+k_2)\tilde{\Gamma}(-k_1-k_2,k_1,k_2)\exp[i(-p_2-k_2-k_1)y+ik_1z+ip_2y'+ik_2z'].$$

Thus, finally, the condition (29) takes the form

$$\tilde{\Gamma}(p_2+k_2,-p_2-k_2-k_1,k_1)\tilde{\Gamma}(p_2,-p_2-k_2,k_2)$$
$$=\tilde{\Gamma}(p_2,-p_2-k_2-k_1,k_1+k_2)\tilde{\Gamma}(-k_1-k_2,k_1,k_2). \tag{30}$$

This is an equation for a function of three variables, which will be solved below. First we show that there is a simple ansatz that satisfies Eq. (30). Indeed, by choosing the ansatz

$$\tilde{\Gamma}(p,q,k)=\frac{\tilde{G}(q^2)\tilde{G}(k^2)}{\tilde{G}(p^2)}, \tag{31}$$

where $\tilde{G}$ is the Fourier image of some function G_c^{-1}, we can substitute this expression in Eq. (30):

$$\frac{\tilde{G}((p_2+k_2)^2)\tilde{G}(k_2^2)}{\tilde{G}(p_2^2)}\times\frac{\tilde{G}((p_2+k_2+k_1)^2)\tilde{G}(k_1^2)}{\tilde{G}((p_2+k_2)^2)}$$
$$=\frac{\tilde{G}((p_2+k_2+k_1)^2)\tilde{G}((k_2+k_1)^2)}{\tilde{G}(p_2^2)}\times\frac{\tilde{G}(k_1^2)\;\;\tilde{G}(k_2^2)}{\tilde{G}((k_1+k_2)^2)}. \tag{32}$$

This is an identity. That is, for the ansatz (31), Eq. (30) is valid. Now we will demonstrate that this anzatz is a unique solution.

In general, the function $\tilde{\Gamma}(p,q,k)$ is a function of three independent Lorentz invariants, since the moments p, q, and k are not independent but related by conservation of the moments, $p+q+k=0$. We can choose p^2, q^2, and k^2 as those independent invariants,

$$\tilde{\Gamma}(p,q,k) \equiv f(p^2,q^2,k^2).$$

Therefore, we can rewrite the basic equation (30) as

$$f((p_2+k_2)^2,(p_2+k_2+k_1)^2,k_1^2)\times f(p_2^2,(p_2+k_2)^2,k_2^2) \\ =f(p_2^2,(p_2+k_2+k_1)^2,(k_1+k_2)^2)\times f((k_1+k_2)^2,k_1^2,k_2^2). \tag{33}$$

Let us introduce into Eq. (33) new independent variables,

$$(p_2+k_2)^2=x, \quad (p_2+k_2+k_1)^2=y,$$

$$k_1^2=z, \quad p_2^2=u, \tag{34}$$

$$k_2^2=v, \quad (k_1+k_2)^2=w.$$

The number of independent variables is six, since in Eq. (33) we have only three independent Lorentz vectors p_2,k_2,k_1. Using these vectors we can construct the six Lorentz-invariant values above. In terms of these new independent variables the basic equation (33) looks like

$$f(x,y,z)\times f(u,x,v)=f(u,y,w)\times f(w,z,v). \tag{35}$$

We consider Eq. (35) as an equation for an analytical function of three variables in R^3 space. We observe that the RHS of Eq. (35) does not depend on x for *any* values of y,z,u,v. There is a unique solution to this—the dependence on x must be factorized in the following way:

$$f(x,y,z)=\frac{1}{\varphi(x)}F_1(y,z), \quad f(u,x,v)=\varphi(x)F_2(u,v), \tag{36}$$

where $\varphi(x)$ is some function, and $F_1(y,z)$ and $F_2(u,v)$ are other functions. The rigorous proof of this statement is given below. The two equations in (36) imply

$$f(x,y,z)=\frac{\varphi(y)}{\varphi(x)}\times F(z),$$

where $F(z)$ is some function. By substituting this in Eq. (35) we immediately infer that $F(z)=\text{const}\times\varphi(z)$. Rescaling $\varphi(z)$ by an appropriate constant, we obtain

$$f(x,y,z)=\frac{\varphi(y)\varphi(z)}{\varphi(x)}. \tag{37}$$

Let us give a rigorous proof that the factorization (36) of the x dependence is the unique solution to Eq. (35). Set $h \equiv \ln f$. Applying the logarithm to Eq. (35), we have

$$h(x,y,z)=-h(u,x,v)+\text{ terms independent of } x. \tag{38}$$

Applying d^m/dx^m, $m=1,2,\ldots,$ to Eq. (38), we obtain

$$\left.\frac{\partial^m h(x_1,y,z)}{\partial x_1^m}\right|_{x_1=x}=-\left.\frac{\partial^m h(u,x_2,v)}{\partial x_2^m}\right|_{x_2=x}.$$

This means that the Taylor expansions in x around the point $x=0$ for the functions $h(x,y,z)$ and $h(u,x,v)$ are

$$h(x,y,z)=h(0,y,z)-\tilde{\varphi}(x,y,z), \tag{39}$$

$$h(u,x,v)=h(u,0,v)+\tilde{\varphi}(x,y,z), \tag{40}$$

where

$$\tilde{\varphi}(x,y,z)=-\sum_{n=1}^{\infty}\frac{1}{n!}x^n\left.\frac{\partial^n h(x_1,y,z)}{\partial x_1^n}\right|_{x_1=0}.$$

Applying exponents to both sides of Eqs. (39) and (40), we obtain

$$f(x,y,z)=\frac{f(0,y,z)}{\varphi(x,y,z)}, \tag{41}$$

$$f(u,x,v)=f(u,0,v)\times\varphi(x,y,z), \tag{42}$$

where $\varphi(x,y,z)=\exp\tilde{\varphi}(x,y,z)$. In Eq. (42) the LHS is y and z independent. Hence, $\varphi(x,y,z)$ is also y and z independent: $\varphi(x,y,z)\equiv\varphi(x)$. Thus, we can rewrite Eqs. (41) and (42) as Eq. (36), where

$$F_1(y,z)\equiv f(0,y,z), \quad F_2(u,v)\equiv f(u,0,v).$$

This proves Eq. (36) and thus Eq. (37). Thus, we can conclude from Eq. (37) that Eq. (31) is the unique solution for $\tilde{\Gamma}(p,q,k)$. To go back to the coordinate representation, we have to perform a Fourier transformation of Eq. (31),

$$\Gamma(x,y,z)=\int dpdqdk\,\delta(p+q+k)\tilde{\Gamma}(p,q,k)\exp(ipx+iqy+ikz)$$
$$=\int dpdqdk\,\delta(p+q+k)\frac{\tilde{G}(q^2)\tilde{G}(k^2)}{\tilde{G}(p^2)}\exp(ipx+iqy+ikz)$$
$$=\int dx'dpdqdk\exp[-i(p+q+k)x']\frac{\tilde{G}(q^2)\tilde{G}(k^2)}{\tilde{G}(p^2)}\exp(ipx+iqy+ikz)$$
$$=\int dx' G_c(x'-x)G_c^{-1}(x'-y)G_c^{-1}(x'-z). \tag{43}$$

By substituting this result in the second of the equalities (28), we can see that it is also satisfied by this solution. One can take the correct tree level normalization of T^{abc},

$$T^{abc}=\frac{i}{2}f^{bca}, \tag{44}$$

and present the final result for the functional structure of the Lc^2 proper correlator in the following form:

$$\int dxdydz\Gamma(x,y,z)T^{abc}L^a(x)c^b(y)c^c(z)$$
$$=\int dx'dxdydzG_c(x'-x)G_c^{-1}(x'-y)$$
$$\times G_c^{-1}(x'-z)\frac{i}{2}f^{bca}L^a(x)c^b(y)c^c(z).$$

As we have mentioned above, the natural assumption (44) about the group structure of the proper correlator Lc^2 has been made. However, we could avoid this assumption. Indeed, if all the group structures in Eq. (26) are independent, we obtain from there, instead of Eq. (28), three equations,

$$\int dx\Gamma(y',x,z')\Gamma(x,y,z)=\int dx\Gamma(y',y,x)\Gamma(x,z,z')$$
$$=\int dx\Gamma(y',x,z)\Gamma(x,z',y)$$
$$=0$$

which are not true even at the tree level as can be seen from Eq. (27). This means that at most two of the group structures must be independent to have a consistent solution. In such a case we come again to the necessity of solving Eq. (29), which has the unique solution (43) as we have demonstrated above. Substituting this solution in Eq. (26) we obtain Jacobi identities for T^{abc} which means that they are structure constants. In detail, this procedure can be done as follows. We can substitute the result (43) in Eq. (24):

$$\int dxdydz\Gamma(x,y,z)T^{abc}L^a(x)c^b(y)c^c(z)$$
$$=\int dx'dxdydzG_c(x'-x)G_c^{-1}(x'-y)$$
$$\times G_c^{-1}(x'-z)T^{abc}L^a(x)c^b(y)c^c(z) \tag{45}$$

and then redefine the fields L and c,

$$c^a(x)=\int dx' G_c(x-x')\tilde{c}^a(x')$$
$$L^a(x)=\int dx' G_c^{-1}(x-x')\tilde{L}^a(x'),$$
$$\int dx' G_c^{-1}(x-x')G_c(x'-x'')=\delta(x-x'').$$

The second term in the Slavnov-Taylor identity (15) is covariant with respect to this change of variables,

$$\int dx\frac{\delta\Gamma[L,c]}{\delta c^a(x)}\frac{\delta\Gamma[L,c]}{\delta L^a(x)}$$
$$=\int dx\frac{\delta\Gamma[L(\tilde{L}),c(\tilde{c})]}{\delta\tilde{c}^a(x)}\frac{\delta\Gamma[L(\tilde{L}),c(\tilde{c})]}{\delta\tilde{L}^a(x)}, \tag{46}$$

as can be explicitly checked, but the expression (45) takes the local form,

$$\Gamma=\int dxT^{abc}\tilde{L}^a(x)\tilde{c}^b(x)\tilde{c}^c(x).$$

By substituting this in the ST operator (46) one concludes that

$$T^{abc}=\frac{i}{2}f^{bca} \tag{47}$$

solves it. The reason for this is that this f^{abc} structure appears also at the level of the classical action

$$2\mathrm{Tr}\int dx L(x)c^2(x)=\frac{i}{2}f^{bca}L^a(x)c^b(x)c^c(x),$$

and we already know that this structure satisfies the ST operator (46). Furthermore, there can be no other solution for T^{abc}, because Eq. (47) is the only three-tensor in the adjoint representation of the gauge group that is antisymmetric in the last two indices and satisfies the Jacobi identities. Thus, the final result for the functional structure of the Lc^2 proper correlator is

$$\int dxdydz\Gamma(x,y,z)T^{abc}L^a(x)c^b(y)c^c(z)$$
$$=\int dx'dxdydzG_c(x'-x)G_c^{-1}(x'-y)$$
$$\times G_c^{-1}(x'-z)\frac{i}{2}f^{bca}L^a(x)c^b(y)c^c(z). \quad (48)$$

In concluding of this section we present arguments that the form (48) of the Lcc correlator remains unchanged even if corrections from the $LccA$ correlator are allowed to contribute to the $\sim Lc^3$ term in the ST equation, i.e., the first term in the ST identity (15) contributes as well. This results in corrections to Eq. (26). In this case we can demonstrate that the basic equation (35) will be modified to the following form:

$$f(x,y,z)\times f(u,x,v)-f(u,y,w)\times f(w,z,v)$$
$$=f_2(u,z,v,y,x,w)-f_2(u,v,z,y,u+z+y+v-x-w,w). \quad (49)$$

The new function f_2 of the variables (34) parametrizes the contribution from the $LccA$ correlator. As one can see, there is a four-dimensional subspace of the six-dimensional space (34) with coordinates x,y,z,u,v,w which is the intersection of two hyperplanes $x=u+z+y+v-x-w$ and $v=z$ where the contribution of $LccA$ in Eq. (49) disappears. In this four-dimensional subspace Eq. (49) takes the same form that the basic equation (35) takes in the six-dimensional space,

$$f\left(\frac{u+2z+y-w}{2},y,z\right)\times f\left(u,\frac{u+2z+y-w}{2},z\right)-f(u,y,w)$$
$$\times f(w,z,z)=0. \quad (50)$$

Unfortunately, at present we do not have a clear proof that the factorization (37) is the only solution to this equation. However, there are several strong indications in favor of the uniqueness of the factorization. Indeed, one of them is that if we reduce the subspace under consideration further to $u=y=z$ and $w=4\alpha z$, where α is an arbitrary real parameter, we obtain

$$f(2(1-\alpha)z,z,z)\times f(z,2(1-\alpha)z,z)-f(z,z,4\alpha z)$$
$$\times f(4\alpha z,z,z)=0.$$

This suggests

$$\frac{d}{d\alpha}f(z,z,\alpha z)\times f(\alpha z,z,z)=0.$$

As we have shown above, the factorization (37) is the only solution for this type of equation.

Another indication in favor of the factorization (37) is that for the region of the four-dimensional subspace under consideration where z is much larger than each of u, y, and w we have in the leading order of u/z and y/z the equation

$$f(z,y,z)\times f(u,z,z)-f(u,y,w)\times f(w,z,z)=0,$$

which also requires the factorization (37) as the only solution, since the information about w disappears on the LHS.

As the third indication, we can decompose the logarithm of Eq. (50) in the Taylor expansion in the vicinity of any point in the four-dimensional subspace with coordinates u,y,z,w. We then obtain, for the function $h=\ln f$ at the quadratic order of the Taylor expansion, separability of the variables as the only solution. But separability for h means factorization for f. Further, we have indications that the separability must occur at any order of the Taylor expansion.

Thus, we have shown that there are at least three arguments in favor of the factorization (37) being the only solution also for Eq. (50), where this latter equation takes into account possible corrections from the $LccA$ correlator to the basic equation (35).

IV. SOLUTION TO THE CORRELATOR OF K_mA_mc TYPE

Starting from this point we can repeat the method that was used in Ref. [6] for deriving the solution to the ST identities for supersymmetric theories. As was noted at the end of the Introduction, the antighost equation (16) restricts the dependence of Γ on the antighost field b and on the external source K_m to an arbitrary dependence on their combination,

$$\partial_m b(x)+K_m(x).$$

We can present this dependence of the effective action on the external source K_m in terms of a series,

$$\Gamma=\mathcal{F}_0+\sum_{n=1}\int dx_1dx_2\cdots dx_n\mathcal{F}_n^{m_1m_2\cdots m_n}(x_1,x_2,\ldots,x_n)$$
$$\times[\partial_{m_1}b(x_1)+K_{m_1}(x_1)][\partial_{m_2}b(x_2)+K_{m_2}(x_2)]\cdots$$
$$\times[\partial_{m_n}b(x_n)+K_{m_n}(x_n)], \quad (51)$$

where we assume contractions in the spacetime indices m_j. The coefficient functions of this expansion are in their turn functionals of the other effective fields (11),

$$\mathcal{F}_n^{m_1m_2\cdots m_n}=\mathcal{F}_n^{m_1m_2\cdots m_n}[A_m,c,L],$$

whose coefficient functions, for example, in the case $L=0$, are ghost-antighost-vector correlators. $\mathcal{F}_0$ is a K_m-independent part of the effective action. The spacetime indices m_j of $\mathcal{F}_n$ will be omitted everywhere below since they are not important in the present analysis.

Our purpose is to restrict the expansion (51) by using the ST identities (15). Let us consider for the moment the terms of Eq. (51) without the field L. The noninvariance of these terms with respect to the ST identities (15) must be compensated by the first term (23) of the series (19) or possible interactions of this term with physical effective fields because $\delta\Gamma/\delta L(x)$ for such terms only has no L. The total degree of the ghost fields c in $\mathcal{F}_n$ must be equal to n since each proper graph contains an equal number of ghost and antighost fields among its external legs.

Let us consider terms in the effective action whose variations are cancelled by variations of the ghost field caused by the first term (23) of the series (19). To start we consider the $\mathcal{F}_1(x_1)$ coefficient function in the expansion (51). The corresponding term of lowest order in the fields in Eq. (51) is

$$\int dxdx' 2i \,\mathrm{Tr}\{[\partial_m b(x)+K_m(x)]\partial_m G(x-x')c(x')\}, \tag{52}$$

where $-i\partial^2 G(x-x')$ is a two-point ghost-antighost proper correlator. It is a Hermitian kernel of the above integral,

$$G^\dagger = G.$$

We can make any change of variables in the effective action Γ. Let us make the following change of variables:

$$A_m(x)=\int dx' G_A(x-x')\tilde{A}_m(x'),$$
$$K_m(x)=\int dx' G_A^{-1}(x-x')\tilde{K}_m(x'),$$
$$c(x)=\int dx' G_c(x-x')\tilde{c}(x'),$$
$$L(x)=\int dx' G_c^{-1}(x-x')\tilde{L}(x'), \tag{53}$$
$$b(x)=\int dx' G_A^{-1}(x-x')\tilde{b}(x').$$

Here $G_X(x-x')$ are some dressing functions,[3]

$$\int dx' G_X^{-1}(x-x')G_X(x'-x'')=\delta(x-x''). \tag{54}$$

In terms of the new variables the effective action

$$\tilde{\Gamma}[\tilde{\varphi},\tilde{K}_m,\tilde{L}]=\Gamma[\varphi(\tilde{\varphi}),K_m(\tilde{K}_m),L(\tilde{L})]$$

must satisfy the identity

$$\mathrm{Tr}\Bigg[\int dx\frac{\delta\tilde{\Gamma}}{\delta\tilde{A}_m(x)}\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}_m(x)}+\int dx\frac{\delta\tilde{\Gamma}}{\delta\tilde{c}(x)}\frac{\delta\tilde{\Gamma}}{\delta\tilde{L}(x)} - \int dxdx'dx''\frac{\delta\tilde{\Gamma}}{\delta\tilde{b}(x')}G_A(x-x') \times\left(\frac{1}{\alpha}\partial_m\tilde{A}_m(x'')G_A(x-x'')\right)\Bigg]=0, \tag{55}$$

which is the identity (15) reexpressed in terms of the new variables according to Eq. (53). As one can see the ST operator is covariant with respect to this change of variables except for the gauge-fixing term, which remains unaffected by quantum corrections anyway as mentioned earlier.

One can make the change of variables (53) in the integral (52):

$$\int dxdx'dx''dx''' 2i\,\mathrm{Tr}\{[\partial_m\tilde{b}(x'')+\tilde{K}_m(x'')] \times G_A^{-1}(x''-x)G(x-x')G_c(x'-x''')\partial_m\tilde{c}(x''')\}. \tag{56}$$

While the dressing function $G_c(x-x')$ has been defined through the solution (23) to the operator (20), the dressing function $G_A(x-x')$ has not been defined yet. We define it from the requirement

$$\int dxdx' G_A^{-1}(x''-x)G(x-x')G_c(x'-x''')=\delta(x''-x''').$$

In this case the term (56) after the change of variables (53) simplifies to

$$\int dx 2i\,\mathrm{Tr}\{[\partial_m\tilde{b}(x)+\tilde{K}_m(x)]\partial_m\tilde{c}(x)\}. \tag{57}$$

The first term in the ST identities (55) can also be expanded in terms of $\partial_m\tilde{b}(x)+\tilde{K}_m(x)$,

$$\int dx\frac{\delta\tilde{\Gamma}}{\delta\tilde{A}_m(x)}\frac{\delta\tilde{\Gamma}}{\delta\tilde{K}_m(x)} =\mathcal{M}_0+\sum_{n=1}\int dx_1dx_2\ldots dx_n \times\mathcal{M}_n^{m_1m_2\ldots m_n}(x_1,x_2,\ldots,x_n) \times[\partial_{m_1}\tilde{b}(x_1)+\tilde{K}_{m_1}(x_1)][\partial_{m_2}\tilde{b}(x_2)+\tilde{K}_{m_2}(x_2)]\cdots \times[\partial_{m_n}\tilde{b}(x_n)+\tilde{K}_{m_n}(x_n)], \tag{58}$$

where we assume contractions in the spacetime indices m_j. Again, the spacetime indices m_j of $\mathcal{M}_n$ will be omitted everywhere below since they are not important in the present analysis. $\mathcal{M}_0$ is the $\tilde{K}_m$-independent part of Eq. (58). We can

[3]The formula (54) does not mean that both the functions $G_X^{-1}(x-x')$ and $G_X(x'-x'')$ are δ functions. It means only that the product of their Fourier transforms is equal to 1.

consider that the LHS of Eq. (58) is the result of an infinitesimal transformation in $\tilde{\Gamma}$, in which instead of $\tilde{A}_m(x)$ we have substituted

$$\tilde{A}_m(x) \to \tilde{A}_m(x) + \frac{\delta \tilde{\Gamma}}{\delta \tilde{K}_m(x)}. \tag{59}$$

In other words, one can consider the result of such a substitution as the difference

$$\Gamma\left[\tilde{K}_m, \tilde{A}_m(x) + \frac{\delta \Gamma}{\delta \tilde{K}_m(x)}\right] - \Gamma[\tilde{K}_m, \tilde{A}_m(x)]$$

to linear order in $\delta\Gamma/\delta\tilde{K}_m(x)$. Equation (57) implies that the "gauge" transformation (59) can be rewritten as

$$\delta \tilde{A}_m(x) \sim i\partial_m \tilde{c}(x) + \text{higher terms.}$$

The sum of the part quadratic in $\tilde{A}$ of $\mathcal{F}_0$ and the $\mathcal{F}_1$-type term (57) contributes to $\mathcal{M}_0$ by yielding a term $\sim \tilde{A}\tilde{c}$. However, $\mathcal{M}_0$ must be equal to zero.[4] Hence, the part quadratic in $\tilde{A}$ of $\mathcal{F}_0$ must be invariant, at quadratic order, under the aforementioned "gauge" transformation, implying the form

$$-\int dx Z_{g^2} \frac{1}{2g^2} \mathrm{Tr}(\partial_m \tilde{A}_n(x) - \partial_n \tilde{A}_m(x)) \times \mathcal{O}(\partial_m \tilde{A}_n(x) - \partial_n \tilde{A}_m(x)), \tag{60}$$

where Z_{g^2} is a number that depends on the couplings and regularization parameter of the theory, and $\mathcal{O}$ is some differential operator. Later we will see how the ST identities put restrictions on such an operator.

Having fixed the form of the quadratic term (57) in $\mathcal{F}_1$, we consider the vertex of next order in the fields in $\mathcal{F}_1$, which looks like $\sim(\partial_m \tilde{b} + \tilde{K}_m)\tilde{A}_m \tilde{c}$. We will show now that the structure of the vertex $\sim(\partial_m \tilde{b} + \tilde{K}_m)\tilde{A}_m \tilde{c}$ is fixed completely by the quadratic term (57) and by the term (23). According to the Slavnov-Taylor identity (55), the contribution of $\sim(\partial_m \tilde{b} + \tilde{K}_m)\tilde{A}_m \tilde{c}$ of the $\mathcal{F}_1$ part of the effective action into $\mathcal{M}_1$ caused by the quadratic term (57) due to the substitution (59) must be cancelled by the variation of the ghost field caused in Eq. (57) by the first term (23) of the series (19) due to the substitution (21). According to our conjecture, the term (23) has the form

$$2\,\mathrm{Tr}\int dx \tilde{L}(x)\tilde{c}^2(x).$$

[4] In principle, another term $\sim \tilde{A}\tilde{c}$ can appear in the third term there, coming from the $\sim \tilde{b}\partial_m \tilde{\nabla}_m \tilde{c}$ part of $\tilde{\Gamma}$. However, the third term in Eq. (55) [and in Eq. (15)] is only responsible for the absence of corrections to the gauge-fixing term in Γ, as we already noted at the end of Sec. II.

Indeed, the only contribution to $\mathcal{M}_1$ of the order of $\sim(\partial_m \tilde{b} + \tilde{K}_m)\partial_m \tilde{c}^2$ in $\mathcal{M}_1$ comes from this $\sim(\partial_m \tilde{b} + \tilde{K}_m)\tilde{A}_m \tilde{c}$ term in $\mathcal{F}_1$:

$$\int dx \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_1}}{\delta \tilde{A}_m(x)} \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_1}}{\delta \tilde{K}_m(x)} \sim [(\partial_m \tilde{b} + \tilde{K}_m)\tilde{c}]\partial_m \tilde{c} \sim (\partial_m \tilde{b} + \tilde{K}_m)\partial_m \tilde{c}^2,$$

where $\tilde{\Gamma}|_{\mathcal{F}_1}$ is the $\mathcal{F}_1$ part of the effective action. One could think at first that the $\mathcal{F}_0$- and $\mathcal{F}_2$- type terms $\tilde{\Gamma}|_{\mathcal{F}_0}$, $\tilde{\Gamma}|_{\mathcal{F}_2}$ of Eq. (19) might also contribute to the term of order $\sim(\partial_m \tilde{b} + \tilde{K}_m)\partial_m \tilde{c}^2$ in $\mathcal{M}_1$ via

$$\int dx \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_0}}{\delta \tilde{A}_m(x)} \frac{\delta \tilde{\Gamma}|_{\mathcal{F}_2}}{\delta \tilde{K}_m(x)}$$

because

$$\frac{\delta \tilde{\Gamma}|_{\mathcal{F}_2}}{\delta \tilde{K}_m(x)} \sim (\partial_m \tilde{b} + \tilde{K}_m)\mathcal{F}_2[A_m, c].$$

However, $\delta\tilde{\Gamma}|_{\mathcal{F}_0}/\delta\tilde{A}_m(x)$ starts with terms linear in $\tilde{A}_m(x)$. Thus, the $\mathcal{F}_2$ part of the effective action does not contribute to the term of the order of $\sim(\partial_m \tilde{b} + \tilde{K}_m)\partial_m \tilde{c}^2$ in $\mathcal{M}_1$; only the $\mathcal{F}_1$ part of the effective action does. Hence, the term of order $\sim(\partial_m \tilde{b} + \tilde{K}_m)\tilde{A}_m \tilde{c}$ in $\mathcal{F}_1$ is the term of the same order that is contained in $\tilde{K}_m(x)\tilde{\nabla}_m \tilde{c}(x)$ because only in this case will the terms $\sim(\partial_m \tilde{b} + \tilde{K}_m)\partial_m \tilde{c}^2$ in $\mathcal{M}_1$ be cancelled by the second term in the ST identities (55), which will result in

$$\int dx 2i \mathrm{Tr}[(\partial_m \tilde{b}(x) + \tilde{K}_m(x))\partial_m \tilde{c}^2(x)]$$

due to the substitution (21). Thus, the term of lowest order in the fields in $\mathcal{F}_1$ is

$$2i\ \mathrm{Tr}[(\partial_m \tilde{b}(x) + \tilde{K}_m(x))\tilde{\nabla}_m \tilde{c}(x)], \quad \tilde{\nabla}_m = \partial_m + i\tilde{A}_m. \tag{61}$$

All the terms in $\mathcal{F}_0$ of higher orders in $\tilde{A}_m(x)$ are fixed by themselves in an iterative way due to the requirement that $\mathcal{F}_0$ must be invariant with respect to the substitution (59). Taking into account Eq. (61), we see that the first invariant term is

$$-\int dx Z_{g^2} \frac{1}{2g^2} \mathrm{Tr}\,\tilde{F}_{mn}(x)\tilde{F}_{mn}(x),$$

where $\tilde{F}_{mn}(x)$ is the Yang-Mills tensor of $\tilde{A}_m(x)$. That is, the physical part of the effective action can be restored from the requirement of its invariance with respect to the gauge invariance in terms of the gauge field dressed by the dressing function. Here we see that the differential operators $\mathcal{O}$ in Eq.

(60) between two Yang-Mills tensors must be covariant derivatives. For example, the following term is allowed,

$$\int dx f_2 \frac{1}{\Lambda^2} \mathrm{Tr}\, \tilde{F}_{mn}(x) \tilde{\nabla}^2 \tilde{F}_{mn}(x), \tag{62}$$

where f_2 is another number that depends on couplings, and Λ is a regularization parameter of the theory. Starting from the fourth degree of $\tilde{A}_m(x)$, higher order gauge invariant contributions like

$$\int dx f_3 \frac{1}{\Lambda^4} \mathrm{Tr}\, \tilde{F}_{mn}(x) \tilde{F}_{mn}(x) \tilde{F}_{kl}(x) \tilde{F}_{kl}(x) \tag{63}$$

to $\mathcal{F}_0$ are allowed. Here f_3 is another number that depends on couplings.

V. FURTHER STEPS FOR HIGHER CORRELATORS IN K_m AND L

We consider now the coefficient functions $\mathcal{F}_n$ with $n>1$ in Eq. (51) for $L=0$. There are two possibilities here. The first possibility is that these terms of higher degree in $\tilde{K}$ do not respect the gauge invariance of the physical part of Eq. (51) created by the $\mathcal{F}_1$ term. In the case $\mathcal{F}_2$ contributes to $\mathcal{M}_1$ but we do not have anything that can compensate this contribution by ghost transformations induced by the second term in the ST identities (55). Hence, $\mathcal{F}_2=0$. If we consider $\mathcal{F}_3$, it contributes to $\mathcal{M}_2$ and, in general, could be compensated by ghost transformations in $\mathcal{F}_2$. But $\mathcal{F}_2$ is zero; hence, $\mathcal{F}_3$ is also zero. We can repeat the former argument for all higher numbers n of $\mathcal{F}_n$. All coefficient functions $\mathcal{F}_n$ with $n>1$ are equal to zero in the first possibility. The second possibility is that the terms of higher degree in $\tilde{K}$ respect the gauge invariance of the physical part of Eq. (51). In this case $\mathcal{F}_n$ with $n>1$ does not contribute to $\mathcal{M}_n$ for any n. In supersymmetric theories this possibility does not exist [6] because of the chiral nature of the ghost superfields. However, in the nonsupersymmetric case one can invent, for example, $\mathcal{F}_2$ constructions such as the following one:

$$\int dx\; \mathrm{Tr}\{[\tilde{c}(x)\tilde{\nabla}_m(\partial_m \tilde{b}(x)+\tilde{K}_m(x))] \times[\tilde{c}(x)\tilde{\nabla}_m(\partial_m \tilde{b}(x)+\tilde{K}_m(x))]\}. \tag{64}$$

Such a term gives zero contribution to $\mathcal{M}_1$, since its variation with respect to $\tilde{K}$ is proportional to $\tilde{\nabla}_m$(scalar function) and its contribution to $\mathcal{M}_2$ can be cancelled by the transformation of the ghost field in $\mathcal{F}_2$ if the coefficient before Eq. (64) has been fixed in an appropriate way. This can be proved in the same way [Eq. (8)] that was used to derive the BRST transformation in Sec. II.

We have considered the terms in the effective action whose variations are cancelled by variations of the ghost field caused by the first term (23) of the series (19). In general, some sophisticated interactions of the term (23) with physical fields can be introduced. However, again we can state that the higher order terms must respect the already established invariance with respect to the Slavnov-Taylor operator for the terms of lowest degree in the fields. In our case, for example, we can write the result for interactions of the term (23) with physical fields by using the following substitution:

$$\tilde{L}\tilde{c}^2 \to \tilde{L}\tilde{c}^2\left(1+f_4\, \frac{1}{\Lambda^4} \mathrm{Tr}\, \tilde{F}_{mn}(x)\tilde{F}_{mn}(x)\right),$$

and then making a substitution in Eq. (61):

$$\tilde{c} \to \tilde{c}\left(1+f_4 \frac{1}{\Lambda^4} \mathrm{Tr}\, \tilde{F}_{mn}(x)\tilde{F}_{mn}(x)\right). \tag{65}$$

However, these terms cannot change the structure of the physical part of the effective action since it is already determined by the terms of first order in the auxiliary field $\tilde{K}_m$.

One can consider possible terms with higher degrees of L. For example, the sum of Eq. (23) and

$$\int dx \sum_{a;b_1,b_2,\ldots,b_{4k}} (\tilde{L}^a(x)\tilde{L}^a(x))^k \tilde{c}^{b_1}(x) \ldots \times \tilde{c}^{b_{4k}}(x)\, \epsilon_{b_1 b_2 \ldots b_{4k}} \tag{66}$$

satisfies the identity (20) if $4k$ is the rank of the gauge group. If these terms exist it is also necessary to consider the dependence of $\mathcal{F}_n$ on the auxiliary field L, since the substitution due to the second term in the ST identities would produce these terms. However, at the end we put all the auxiliary fields equal to zero, and therefore all the terms with higher degrees of $\tilde{L}$ do not have any importance. In comparison, the situation with the $\tilde{K}_m$ field is different. Indeed, terms with zero $\tilde{K}_m$ are still important since they are responsible for higher degrees of ghost-antighost correlators which may have applications in some models.

VI. CONJECTURE FOR THE PHYSICAL PART OF THE ACTION

Taking into account the structure (61) of the term linear in $\tilde{K}_m$, one can come to a natural conjecture about the form of the part of the effective action that depends only on the gauge effective field A_m; namely, due to the ST identity (55) in terms of the dressed fields, the structure of the effective action is

$$\Gamma[A_m,b,c] = \int dx \left\{ -\frac{1}{2g^2} Z_{g^2} \mathrm{Tr}\left[\tilde{F}_{mn}(x)\; \mathcal{G}\left(\frac{\tilde{\nabla}^2}{\Lambda^2}\right) \tilde{F}_{mn}(x)\right] - \mathrm{Tr}\left(\frac{1}{\alpha}[\partial_m A_m(x)]^2\right) - 2i\,\mathrm{Tr}\, \tilde{b}(x)\partial_m \tilde{\nabla}_m \tilde{c}(x)\right\} + \text{irrelevant part}, \tag{67}$$

where all auxiliary fields K and L are set equal to zero. It is necessary to make three comments here. The function $\mathcal{G}$ is a series in terms of covariant derivatives with a dressed gauge connection. The part of this series without gauge connection $\mathcal{G}(\partial^2/\Lambda^2)$ has a logarithmic asymptotic in the momentum space at high momentum, $\mathcal{G}(-p^2/\Lambda^2)\sim\ln(-p^2/\Lambda^2)$, while at low momentum it may be represented e.g., by powers of p^2/Λ^2_{QCD} with $\Lambda_{QCD}\sim 0.1$ GeV [9].

The physical part of the action is gauge invariant in terms of the dressed field $\widetilde{A}_m(x)$.

We do not write in the physical part terms like (63) since finally we are going to take the regularization mass Λ to infinity. Terms like (63), (64), (65) are called irrelevant in (67).

VII. REGULARIZATION AND RENORMALIZATION

In a general nonsupersymmetric four-dimensional gauge theory which is regularized in a way that preserves gauge (and BRST) symmetry, the dressing functions are of the following form:

$$G_X^{-1}(x-x')=z_X\delta(x-x')+\frac{C_1(\Lambda^2,\mu^2)}{\mu^2}(\partial^2-\mu^2)\delta(x-x')+\frac{C_2(\Lambda^2,\mu^2)}{(\mu^2)^2}(\partial^2-\mu^2)^2\delta(x-x')+\cdots. \tag{68}$$

This representation means that we have expanded the Fourier transformed dressing function $\widetilde{G}_X^{-1}(p^2)=1/\widetilde{G}_X(p^2)$, $X=A,c$, in the vicinity of the point $p^2=-\mu^2$. Here z_X is a constant that goes to infinity if the regularization is removed, and C_1, C_2 are finite constants.[5] For instance, z_A is a renormalization constant of the gauge field. To renormalize the theory we have to introduce counterterms into the classical action (6) [11]. This is equivalent to a change of the field in the classical action (6). For example, in the case of the pure gauge theory, to remove divergences from $G_A^{-1}(x-x')$ we have to make the following redefinition of the gauge field in the classical action:

$$A_m^{\rm bare}\rightarrow\frac{A_m^{\rm phys}}{z_A}. \tag{69}$$

The motivation for the terminology ''bare'' and ''physical'' for the fields in the path integral is that introducing counterterms into the classical action (6) by the rescaling (69) of fields and couplings will result in an effective action without divergences (a renormalized effective action). We can show that by such a redefinition we can make the dressing function G_A^{-1} finite. Indeed, if we represent the term with the source of the gauge field in the path integral (5) as

$$J_mA_m=(J_mz_A)\frac{A_m}{z_A},$$

then the path integral for the theory with counterterms (69) can be transformed to the form (5) by substitution of variables of the integral $A_m=A'_mz_A$. This means that all the previous construction can be reproduced without any change but taking into account the redefinition $J_m\rightarrow J_mz_A$. In turn, such a redefinition, according to the definitions (11), means nothing else but that the effective fields are also redefined as in Eq. (69), which is equivalent to the redefinition of the dressing function

$$G_A^{-1}(x-x')\rightarrow\frac{1}{z_A}G_A^{-1}(x-x'). \tag{70}$$

One can consider Eq. (70) in momentum space,

$$\begin{aligned}\frac{1}{z_A}\frac{1}{\widetilde{G}_A(p^2)}&=\frac{\widetilde{G}_A(-\mu^2,\Lambda^2)}{\widetilde{G}_A(p^2,\Lambda^2)}\\&=(1+\alpha g^2+\beta(g^2)^2+\gamma(g^2)^3+\cdots)\\&\quad\times(1+\widetilde{G}_1(p^2)g^2+\widetilde{G}_2(p^2)(g^2)^2+\cdots)\\&=1+(\alpha+\widetilde{G}_1(p^2))g^2+(\beta+\alpha\widetilde{G}_1(p^2)+\widetilde{G}_2(p^2))\\&\quad\times(g^2)^2+\cdots,\end{aligned} \tag{71}$$

where we have presented both factors on the LHS as a series in terms of the coupling constant. In this expansion g^2 is the physical coupling that stays in the classical action according to the counterterm approach [11]. All these dressing functions parametrize our result (67) for the effective action, that is, they parametrize the irreducible vertices that contain divergences. Divergences from the dressing functions must be removed. We can remove the divergences at each order in the coupling constant by choosing the divergent coefficients α, β, γ in $1/z_A$ in an appropriate way, because each coefficient $\widetilde{G}_n(p^2)$ of the decomposition $\widetilde{G}_A^{-1}(p^2)$ in terms of the coupling constant is in its turn a series in terms of p^2 with only the zero order in p^2, terms being divergent. This is due to the fact that

$$\lim_{\Lambda\rightarrow\infty}\frac{\widetilde{G}_A(p^2,\Lambda^2)}{\widetilde{G}_A(-\mu^2,\Lambda^2)}$$

is finite. As to divergent coefficients before the relevant operators, they will be compensated by counterterms from the bare couplings.[6]

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[5] $G_X^{-1}(x-x')=(2\pi)^{-4}\int dp\,\exp[-ip(x-x')][1/\widetilde{G}_X(p^2)]$, i.e., $G_X^{-1}(x-x')\neq 0$ for $x-x'\neq 0$ in general, although the expansion (68) might suggest otherwise.

[6] Even if the renormalization (71) has been done and the dressing functions are finite, the theory still has divergences in the coefficients of the relevant operators. These divergences are absorbed by the bare couplings.

Until this moment we did not specify which regularization is used. Regularization by higher derivatives is the most convenient from the point of view of the theoretical analysis [2]. It provides strong suppression of ultraviolet divergences by introducing additional terms with higher degrees of covariant derivatives acting on the Yang-Mills tensor in the classical action (6), which are suppressed by appropriate degrees of the regularization scale Λ. In addition to this it is necessary to introduce a modification of the Pauli-Villars regularization to guarantee the convergence of the one-loop diagrams [2]. To regularize the fermion cycles, the usual Pauli-Villars regularization can be used.[7]

Thus in the case of four-dimensional QCD without quarks the classical action (6) is

$$S_{QCD}[A,b,c]=\int dx\bigg[-\frac{1}{2g^2}\mathrm{Tr}[F_{mn}(A(x))F_{mn}(A(x))] - \mathrm{Tr}\bigg(\frac{1}{\alpha}[\partial_m A_m(x)]^2\bigg) - 2\,\mathrm{Tr}[ib(x)\partial_m\nabla_m(A)c(x)]\bigg].$$

In the counterterm technique [11] the coupling constant here is the physical coupling constant. The classical action with the counterterms is

$$S_{QCD}[A,b,c]=\int dx\bigg\{-\frac{1}{Z_{g^2}}\frac{1}{2g^2} \times\mathrm{Tr}\bigg[F_{mn}\bigg(\frac{A}{z_A}(x)\bigg)F_{mn}\bigg(\frac{A}{z_A}(x)\bigg)\bigg] - \mathrm{Tr}\bigg(\frac{(z_A)^2}{\alpha}\bigg[\partial_m\frac{A_m}{z_A}(x)\bigg]^2\bigg) - 2\,\mathrm{Tr}\bigg[iz_A b(x)\partial_m\nabla_m\bigg(\frac{A}{z_A}\bigg)\frac{c}{z_c}(x)\bigg]\bigg\},$$

where the fields are "physical" in the sense that this classical action together with counterterms results in an effective action in which divergences are removed. Thus, we come to the conclusion that the *renormalized* effective action takes the form

$$\Gamma_{QCD}[A_m,b,c]=\int dx\bigg\{-\frac{1}{2g^2}\mathrm{Tr}\bigg[\tilde{F}_{mn}(x)\mathcal{G}_2\bigg(\frac{\tilde{\nabla}^2}{\mu^2}\bigg)\tilde{F}_{mn}(x)\bigg] - \mathrm{Tr}\bigg(\frac{1}{\alpha}[\partial_m A_m(x)]^2\bigg) - 2i\,\mathrm{Tr}[\tilde{b}(x)\partial_m\ \tilde{\nabla}_m\ \tilde{c}(x)]\bigg\}, \tag{72}$$

where all the auxiliary fields K and L are set equal to zero. Here the function $\mathcal{G}_2$ is defined as

$$\mathcal{G}_2\bigg(\frac{\tilde{\nabla}^2}{\mu^2}\bigg)\equiv\lim_{\Lambda\to\infty}\mathcal{G}\bigg(\frac{\tilde{\nabla}^2}{\Lambda^2}\bigg)\bigg/\mathcal{G}\bigg(\frac{\mu^2}{\Lambda^2}\bigg). \tag{73}$$

VIII. SUMMARY

In this work we proposed a solution to the Slavnov-Taylor identities for the effective action of nonsupersymmetric non-Abelian gauge theory without matter. The solution is expressed in terms of gauge A_m and (anti)ghost effective fields (c,b) convoluted with unspecified dressing functions:

$$\tilde{A}_m(x)=\int dx' G_A^{-1}(x-x')A_m(x')$$

$$\tilde{c}(x)=\int dx' G_c^{-1}(x-x')c(x'),$$

$$\tilde{b}(x)=\int dx' G_A(x-x')b(x').$$

Further, the solution is invariant under the gauge (BRST) transformation of the convoluted fields. We gave arguments which show that, under a specific plausible assumption, the terms of the effective action containing (anti)ghost fields must have the same form as those in the classical action, but under the substitution $X\to\tilde{X}(X=c,b,A_m)$. Further, we conjectured a rather general form of the terms of the effective action which contain only the effective gauge fields and involve an additional function $\mathcal{G}$. We briefly described how regularization and renormalization are reflected in the dressing functions. The effective action obtained is assumed to contain the quantum contributions of the gauge theory, perturbative and nonperturbative, but not including the soliton-like vacuum effects. Stated otherwise, all these effects are assumed to be contained in a limited number of dressing functions $(G_A,G_c,\mathcal{G})$. The application and consistency checks of this effective action for the case of high-momentum QCD are presented elsewhere [9].

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[7]A somewhat different regularization approach is applied in Ref. [9] where explicit QCD one-loop dressing functions are obtained.

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