$D=4$ supergravity dynamically coupled to a massless superparticle in a superfield Lagrangian approach

Igor A. Bandos,[1,4] José A. de Azcárraga,[1] José M. Izquierdo,[2] and Jerzy Lukierski[3]
[1]*Departamento de Física Teórica and IFIC, 46100-Burjassot (Valencia), Spain*
[2]*Departamento de Física Teórica, Facultad de Ciencias, 47011-Valladolid, Spain*
[3]*Institute for Theoretical Physics, pl. Maxa Borna 9, 50-204 Wroclaw, Poland*
[4]*Institute for Theoretical Physics, NSC KIPT, UA61108, Kharkov, Ukraine*
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We consider the interacting system of $D=4, N=1$ supergravity and the Brink-Schwarz massless superparticle as described by the sum of their superfield actions, and derive the complete set of superfield equations of motion for the coupled dynamical system. These include source terms given by derivatives of a vector superfield current density with support on the worldline. This current density is constructed from the spin 3/2 and spin 2 current density "prepotentials." We analyze the gauge symmetry of the coupled action and show that it is possible to fix the gauge in such a way that the equations of motion reduce to those of the supergravity-bosonic particle coupled system.

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I. INTRODUCTION

There has recently been a search for self-consistent equations for supergravity coupled to a superbrane. They are needed, in particular, for the analysis of anomalies in M theory [1] and in relation to the search [2] for a supersymmetric brane world scenario [3].

In lower dimensions, $D=3,4$ (and for $D=6$ using harmonic superspace [4]), where a superfield action for supergravity exists, one may develop a conventional approach to the supergravity-superbrane systems by using the sum of the superfield action of supergravity and the superbrane action. Such a superfield Lagrangian description of the low-dimensional supergravity-superbrane coupled system provides a possibility to study the structure of the superfield current densities of the supersymmetric extended objects, which might produce some insight in the search for a new superfield approach to higher dimensional supergravity in the line of Ref. [5].

In this paper we give a fully dynamical superfield description of the simplest $D=4,N=1$ supergravity-superparticle interacting system, given by the sum of the superfield action for supergravity [6] and the Brink-Schwarz action for the massless superparticle [7]. We derive the complete set of superfield equations of motion and find that the *superfield* generalizations of the Einstein and Rarita-Schwinger equations acquire source terms. Both sources are determined by the action of the Grassmann spinor covariant derivatives on the vector superfield current density distribution, which, in turn, is constructed from the spin 3/2 and spin 2 current "prepotentials."

The $D=3,4$ superfield supergravity action [6] (see also [8–10]) possesses off-shell supersymmetry and can be written, after integration of Grassmann variables, as a spacetime supergravity action (see, e.g., [11–15]) involving the so-called auxiliary fields (real vector and pseudoscalar for "minimal" supergravity, see, e.g., [8–10]). In higher dimensions, $D=10,11$, neither the superfield action nor the set of auxiliary fields are known (see, however, [16,17] for linearized $D=10,N=1$ supergravity and [5] for recent progress in the superfield description of $D=11$ supergravity). For these cases we proposed in [18] to use the sum of the group manifold action for supergravity [19] and the superbrane action as the basis for a Lagrangian description of *dynamical* supergravity and the superbrane source system. Then it was shown in [20] that the bosonic "limit" of such a dynamical system, provided by the component formulation for supergravity coupled to the bosonic brane, is self-consistent and preserves 1/2 of the local supersymmetry of "free" supergravity (cf. [2], where supergravity interacting with bosonic branes *fixed* at the orbifold fixed "points" is considered).

The approach of [18] is general and could be applied, in principle, to any coupled supergravity-superbrane dynamical system provided that the group manifold approach to the specific supergravity considered exists (this requires its search if it is not known, e.g., for $D=10$ type IIA and type IIB supergravity). On the other hand, the results of [18] (see also [20,21]) were not quite what one would commonly expect. In particular, while the supersymmetric generalization of the Einstein equation acquired the expected source term from the super-p-brane, the superform generalization of the Rarita-Schwinger equation remained sourceless [18]. One might wonder whether these properties would be reproduced by the conventional superfield approach to the dynamically interacting system. Showing that this is indeed the case is an additional motivation for the present study.

In this paper we also analyze the gauge symmetry of the coupled action and find that it is possible to fix a gauge in which the superparticle coordinate function is zero, $\hat{\theta}(\tau)=0$,[1] and that incorporates the Wess-Zumino (WZ) gauge for supergravity. We show that in this gauge the equations of motion for the supergravity-superparticle coupled system reduce to those for the supergravity-bosonic particle coupled

[1]This fact reflects the Goldstone nature of the superparticle coordinate functions [22–24] and is related to the super-Higgs effect [25] (see also [26]).

system derived in [20] (for any dimension D). The superfield action in this gauge should also coincide with the action considered in [20] after integration over the superspace Grassmann coordinates θ [not to be confused with the fermionic *function* $\hat{\theta}\equiv\hat{\theta}(\tau)$] and elimination of the auxiliary fields by using their (purely algebraic) equations of motion. This explains the self-consistency of the supergravity-bosonic particle coupled system, which was studied in [20].

This paper is organized as follows. The first three sections are devoted to the minimal off-shell formulation of simple supergravity in $D=4, N=1$ superspace. In spite of the fact that much of the material in these sections can be found in books [12–15] and original articles [9,11,27,28], we have found it necessary to present it here in a unified notation.

Specifically, we describe in Sec. II the superspace torsion constraints and their consequences derived with the use of Bianchi identities, collect them in compact differential form, and present the expressions for the left-hand sides (lhs's) of the superfield generalizations of the Rarita-Schwinger and Einstein equations in terms of the so-called main superfields and their covariant derivatives. In Sec. III we describe the complete form of the Wess-Zumino gauge (fixed through conditions on the superfield supergravity *potentials*, i.e., on the supervielbein and spin connection) and describe the residual gauge symmetry which preserves this Wess-Zumino gauge.

In Sec. IV we present the Wess-Zumino action for $D=4, N=1$ supergravity and comment on the derivation of "free" superfield equations of motion. Section V describes the $D=4, N=1$ Brink-Schwarz superparticle action in a supergravity *background* (i.e., without assuming any action for superfield supergravity).

In Sec. VI we present the coupled action for the $D=4, N=1$ supergravity-superparticle interacting system and study its gauge symmetry (Sec. VI A) which turns out to be the "direct sum" of supergravity and superparticle gauge symmetries. We derive the superfield equations of motion for the coupled system (Sec. VI B) and study the properties of the superfield current potential and prepotentials (Sec. VI C). We also find the superfield generalizations of the Rarita-Schwinger and Einstein equations, both of which contain source terms.

In Sec. VII we show that the gauge symmetries of the coupled system allow one to fix a gauge in which the superparticle fermionic coordinate functions are set equal to zero. We explain why the coupled action in this gauge reduces to the action of component supergravity interacting with a bosonic particle. We show that the dynamical equations following from the superfield action are reduced to the equations for the supergravity-bosonic particle coupled system [20] in this gauge. We comment briefly on the bosonic counterpart of this gauge in general relativity with sources and on the relation of these results with the (super) Higgs effect in the presence of superbranes, and conclude in Sec. VIII.

Some technical results and additional discussion are given in the Appendices. Appendix A describes the chiral projector in $D=4, N=1$ superspace. In Appendix B we present the complete list of manifest local (gauge) symmetries of the superspace formulation of supergravity. We discuss both the *active* and *passive* form of the *superspace general coordinate transformations*, which we call *general coordinate transformations* and *superdiffeomorphisms*, respectively (see [20,21]). Appendix C contains more details on the Wess-Zumino gauge. We determine there the complete set of residual gauge symmetries which preserve this gauge. Surprisingly, by discussing all the gauge symmetries we find that the Wess-Zumino gauge is invariant under the active form of the superspace general coordinate transformations (in addition to the well known *spacetime local* supersymmetry and Lorentz symmetry as well as *spacetime* diffeomorphisms). We discuss briefly the role of this additional superfield gauge invariance. Finally, Appendix D collects more details about the symmetries of the Brink-Schwarz superparticle action.

II. $D=4, N=1$ SUPERGRAVITY IN SUPERSPACE

In this section we summarize our conventions and some known facts about the off-shell description of $D=4, N=1$ supergravity in superspace. All the formulas in this section coincide with those in [13] up to some signs and numerical coefficients in definitions. However, they are written here in a more compact differential form notation.

A. Superspace constraints for minimal supergravity

Let $\{Z^M\}\equiv\{x^\mu,\theta^{\check{\alpha}}\}$ be the coordinates of curved $D=4, N=1$ superspace $\Sigma^{(4|4)}$. Here $\theta^{\check{\alpha}}$ ($\check{\alpha}=1,2,3,4$) are real Grassmann coordinates (in flat superspace, as well as in the Wess-Zumino gauge, a Majorana spinor $\theta^{\underline{\alpha}}$, $\underline{\alpha}=1,2,3,4$). An unholonomic basis of the cotangent superspace is provided by the supervielbein one-forms

$$E^A\equiv(E^a,E^{\underline{\alpha}})=(E^a,E^\alpha,\bar{E}_{\dot{\alpha}}),$$

$$E^a=dZ^M E_M^a(Z),$$

$$E^{\underline{\alpha}}=dZ^M E_M^{\underline{\alpha}}(Z)\leftrightarrow\begin{cases}E^\alpha=dZ^M E_M^\alpha(Z),\\ \bar{E}^{\dot{\alpha}}=dZ^M \bar{E}_M^{\dot{\alpha}}(Z).\end{cases}\tag{2.1}$$

In this paper we mainly use Weyl spinors notation ($\alpha=1,2$, $\dot{\alpha}=1,2$), except for Secs. III and VII, where Majorana spinors are used [29].

An *off-shell* supergravity multiplet can be extracted from the general superfields $E_M^a(Z)$, $E_M^{\underline{\alpha}}(Z)=(E_M^\alpha(Z),\bar{E}_{M\dot{\alpha}}(Z))$ by imposing the constraints on some components ${T_{CB}}^A$, ${R_{CD}}^{ab}$, of torsion 2-forms,

$$T^a:=\mathcal{D}E^a=dE^a-E^b\wedge {w_b}^a\equiv\frac{1}{2}E^B\wedge E^C {T_{CB}}^a,\tag{2.2}$$

$$T^\alpha:=\mathcal{D}E^\alpha=dE^\alpha-E^\beta\wedge {w_\beta}^\alpha\equiv\frac{1}{2}E^B\wedge E^C T_{CB}^\alpha,\tag{2.3}$$

$$T^{\dot{\alpha}}:=\mathcal{D}\bar{E}^{\dot{\alpha}}=d\bar{E}^{\dot{\alpha}}-\bar{E}^{\dot{\beta}}\wedge {w_{\dot{\beta}}}^{\dot{\alpha}}\equiv\frac{1}{2}E^B\wedge E^C T_{CB}^{\dot{\alpha}},\tag{2.4}$$

and the curvature

$$R^{ab}:=dw^{ab}-w^{ac}\wedge w_c{}^b\equiv\frac{1}{2}E^C\wedge E^D R_{DC}{}^{ab} \tag{2.5}$$

of the spin connection one-form $w^{ab}=dZ^M\, w_M{}^{ab}=-w^{ba}$,

$$w_\beta{}^\alpha=\frac{1}{4}w^{ab}(\sigma_a\tilde{\sigma}_b)_\beta{}^\alpha,\quad w_{\dot\beta}{}^{\dot\alpha}=-\frac{1}{4}w^{ab}(\tilde{\sigma}_a\sigma_b)^{\dot\alpha}{}_{\dot\beta}, \tag{2.6}$$

The constraints of minimal supergravity [30,8,13,9] include $T_{\alpha\dot\beta}{}^a=-2i\sigma^a_{\alpha\dot\beta}$ as well as $T_{\alpha\beta}{}^A=0=T_{\dot\alpha\dot\beta}{}^A$, $T_{\alpha\dot\beta}{}^{\dot\gamma}=0$, $T_{\alpha b}{}^c=0$, and $R_{\alpha\dot\beta}{}^{ab}=0$ (or $T_{ab}{}^c=0$ as, e.g., in [13]).[2] In the presence of the complete set of constraints, the Bianchi identities

$$\mathcal{D}T^A\equiv-E^B\wedge R_B{}^A\Leftrightarrow\begin{cases}\mathcal{D}T^a\equiv-E^b\wedge R_b{}^a,\\ \mathcal{D}T^\alpha\equiv-E^\beta\wedge R_\beta{}^\alpha,\\ \mathcal{D}T^{\dot\alpha}\equiv-E^{\dot\beta}\wedge R_{\dot\beta}{}^{\dot\alpha},\end{cases} \tag{2.7}$$

$$\mathcal{D}R^{ab}\equiv 0\Rightarrow\begin{cases}\mathcal{D}R_\alpha{}^\beta\equiv 0,\\ \mathcal{D}R_{\dot\alpha}{}^{\dot\beta}\equiv 0\end{cases} \tag{2.8}$$

(integrability conditions for Eqs. (2.2)–(2.5)) express the superspace torsion and curvature through the set of "main superfields"

$$G_a:=2i(T_{a\beta}{}^\beta-T_{a\dot\beta}{}^{\dot\beta}), \tag{2.9}$$

$$\bar{R}:=-\frac{1}{3}R_{\alpha\beta}{}^{\alpha\beta}=(R)^*, \tag{2.10}$$

$$W^{\alpha\beta\gamma}:=4i\tilde{\sigma}^{c\dot\gamma\gamma}R_{\dot\gamma c}{}^{\alpha\beta}=W^{(\alpha\beta\gamma)}=(\bar{W}^{\dot\alpha\dot\beta\dot\gamma})^*. \tag{2.11}$$

The constraints of minimal supergravity and their consequences can be collected in the following expressions for the superspace torsion 2-forms (cf. [13])

$$T^a=-2i\sigma^a_{\alpha\dot\alpha}E^\alpha\wedge\bar{E}^{\dot\alpha}+\frac{1}{16}E^b\wedge E^c\varepsilon^a{}_{bcd}G^d, \tag{2.12}$$

$$T^\alpha=\frac{i}{8}E^c\wedge E^\beta(\sigma_c\tilde{\sigma}_d)_\beta{}^\alpha G^d-\frac{i}{8}E^c\wedge\bar{E}^{\dot\beta}\epsilon^{\alpha\beta}\sigma_{c\beta\dot\beta}R+\frac{1}{2}E^c\wedge E^b T_{bc}{}^\alpha, \tag{2.13}$$

$$T^{\dot\alpha}=\frac{i}{8}E^c\wedge E^\beta\epsilon^{\dot\alpha\dot\beta}\sigma_{c\beta\dot\beta}\bar{R}-\frac{i}{8}E^c\wedge\bar{E}^{\dot\beta}(\tilde{\sigma}_d\sigma_c)^{\dot\alpha}{}_{\dot\beta}G^d+\frac{1}{2}E^c\wedge E^b T_{bc}{}^{\dot\alpha}. \tag{2.14}$$

The superspace Riemann curvature 2-form is determined by

$$R^{ab}:=dw^{ab}-w^{ac}\wedge w_c{}^b=\frac{1}{2}R^{\alpha\beta}(\sigma^a\tilde{\sigma}^b)_{\alpha\beta}-\frac{1}{2}R^{\dot\alpha\dot\beta}(\tilde{\sigma}^a\sigma^b)_{\dot\alpha\dot\beta}, \tag{2.15}$$

with

$$R^{\alpha\beta}\equiv dw^{\alpha\beta}-w^{\alpha\gamma}\wedge w_\gamma{}^\beta\equiv\frac{1}{4}R^{ab}(\sigma_a\tilde{\sigma}_b)^{\alpha\beta}=-\frac{1}{2}E^\alpha\wedge E^\beta\bar{R}-\frac{i}{8}E^c\wedge E^{(\alpha}\tilde{\sigma}_c{}^{\dot\gamma\beta)}\bar{\mathcal{D}}_{\dot\gamma}\bar{R}+\frac{i}{8}E^c\wedge E^\gamma(\sigma_c\tilde{\sigma}_d)_\gamma{}^{(\beta}\mathcal{D}^{\alpha)}G^d-\frac{i}{8}E^c\wedge\bar{E}^{\dot\beta}\sigma_{c\gamma\dot\beta}W^{\alpha\beta\gamma}+\frac{1}{2}E^d\wedge E^c R_{cd}{}^{\alpha\beta}, \tag{2.16}$$

and $R^{\dot\alpha\dot\beta}=(R^{\alpha\beta})^*$.

Note that in our conventions the spinor covariant derivatives $\mathcal{D}_\alpha=-(\bar{\mathcal{D}}_{\dot\alpha})^*$ are defined by the following decomposition of the covariant differential $\mathcal{D}$:

$$\mathcal{D}:=E^A\mathcal{D}_A=E^a\mathcal{D}_a+E^{\underline{\alpha}}\mathcal{D}_{\underline{\alpha}}=E^a\mathcal{D}_a+E^\alpha\mathcal{D}_\alpha+\bar{E}^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha} \tag{2.17}$$

[hence $\mathcal{D}_{\underline{\alpha}}=(\mathcal{D}_\alpha,-\bar{\mathcal{D}}^{\dot\alpha})$; note the minus sign]. Then, since, e.g., $\mathcal{D}_\alpha=E_\alpha{}^M\partial_M+w_\alpha$, it is also natural that the spinor components of the spin connection form $w^{ab}=dZ^M w_M^{ab}=E^A w_A^{ab}:=E^c w_c^{ab}+E^\alpha w_\alpha{}^{ab}+\bar{E}^{\dot\alpha}w_{\dot\alpha}{}^{ab}$ be related by $w_\alpha{}^{ab}=-(w_{\dot\alpha}{}^{ab})^*$ [hence $w_{\underline{\alpha}}{}^{ab}=(w_\alpha{}^{ab},-w^{\dot\alpha\, ab})$].

The Bianchi identities (2.7),(2.8) imply as well that the main superfields (2.9),(2.10),(2.11) obey the equations

$$\mathcal{D}_\alpha\bar{R}=0,\quad\bar{\mathcal{D}}_{\dot\alpha}R=0, \tag{2.18}$$

$$\bar{\mathcal{D}}_{\dot\alpha}W^{\alpha\beta\gamma}=0,\quad\mathcal{D}_\alpha\bar{W}^{\dot\alpha\dot\beta\dot\gamma}=0, \tag{2.19}$$

$$\bar{\mathcal{D}}^{\dot\alpha}G_{\alpha\dot\alpha}=\mathcal{D}_\alpha R,\quad\mathcal{D}^\alpha G_{\alpha\dot\alpha}=\bar{\mathcal{D}}_{\dot\alpha}\bar{R}, \tag{2.20}$$

$$\mathcal{D}_\gamma W^{\alpha\beta\gamma}=\bar{\mathcal{D}}_{\dot\gamma}\mathcal{D}^{(\alpha}G^{\beta)\dot\gamma},\qquad \bar{\mathcal{D}}_{\dot\gamma}\bar{W}^{\dot\alpha\dot\beta\dot\gamma}=\mathcal{D}_\gamma\bar{\mathcal{D}}^{(\dot\alpha|}G^{\gamma|\dot\beta)}. \tag{2.21}$$

For the sake of brevity, we will call "constraints" the complete set of relations (2.12)–(2.14),(2.16),(2.18)–(2.21).

[2] A minimal complete set of superspace constraints for the minimal supergravity multiplet [31] can be found, e.g., in [12,13,27]; see [9,12,32,27] and references therein for nonminimal supergravity multiplets, and [33] for a discussion of the algebraic origin of the supergravity constraints.

B. Off-shell nature of the constraints

Using the Bianchi identities (2.7),(2.8), one also finds that the fermionic torsion components $T_{ab}{}^{\alpha}$ $T_{ab}{}^{\dot\alpha}$ entering Eqs. (2.13),(2.14) [which may be regarded as superfield generalizations of the gravitino field strengths, see Eq. (3.6)] are also expressed through the main superfields (2.9), (2.10), (2.11)

$$T_{\alpha\dot\alpha\beta\dot\beta\gamma} \equiv \sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta}\epsilon_{\gamma\delta}T_{ab}{}^{\delta} = -\frac{1}{8}\epsilon_{\alpha\beta}\bar{\mathcal{D}}_{(\dot\alpha|}G_{\gamma|\dot\beta)} - \frac{1}{8}\epsilon_{\dot\alpha\dot\beta}[W_{\alpha\beta\gamma} - 2\epsilon_{\gamma(\alpha}\mathcal{D}_{\beta)}R], \tag{2.22}$$

$$T_{\alpha\dot\alpha\beta\dot\beta\dot\gamma} \equiv \sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta}\epsilon_{\dot\gamma\dot\delta}T_{ab}{}^{\dot\delta} = \frac{1}{8}\epsilon_{\dot\alpha\dot\beta}\mathcal{D}_{(\alpha}G_{\beta)\dot\gamma} - \frac{1}{8}\epsilon_{\alpha\beta}[\bar W_{\dot\alpha\dot\beta\dot\gamma} + 2\epsilon_{\dot\gamma(\dot\alpha}\bar{\mathcal{D}}_{\dot\beta)}\bar R]. \tag{2.23}$$

Equations (2.22), (2.23) imply, in particular,

$$(\sigma^a\tilde\sigma^b)_{\beta}{}^{\gamma}T_{ab\gamma} = \frac{3}{4}\mathcal{D}_{\beta}R,$$

$$(\tilde\sigma^a\sigma^b)^{\dot\gamma}{}_{\dot\beta}T_{ab\dot\gamma} = \frac{3}{4}\bar{\mathcal{D}}_{\dot\beta}\bar R. \tag{2.24}$$

Moreover, the lhs of the Rarita-Schwinger equation can be identified with the leading component (i.e., the $\theta=0$ value) of the superfield expression $\epsilon^{abcd}T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha}$ (see Eq. (4.16) and e.g., [13]). Using the Pauli matrix algebra $(\sigma^a\tilde\sigma^b = \eta^{ab}I + i/2\epsilon^{abcd}\sigma_c\tilde\sigma_d,\ \sigma^{[a}\tilde\sigma^{b]}\sigma_b = 3\sigma^a)$ one finds, from Eq. (2.22),

$$\Psi^a_{\dot\alpha} := \epsilon^{abcd}T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha} = \frac{i}{8}\tilde\sigma^{a\dot\beta\beta}\bar{\mathcal{D}}_{(\dot\beta|}G_{\beta|\dot\alpha)} + \frac{3i}{8}\sigma^a_{\beta\dot\alpha}\mathcal{D}^{\beta}R. \tag{2.25}$$

The *fields* $\mathcal{D}^{\beta}R|_{\theta=0}$ and $\bar{\mathcal{D}}_{(\dot\beta|}G_{\beta|\dot\alpha)}|_{\theta=0}$ are not restricted by the constraints (2.12)–(2.16), (2.18)–(2.21). They are arbitrary fermionic functions, which rather can be identified with the corresponding irreducible parts of the leading component $\Psi^a_{\dot\alpha}|_{\theta=0}$ of the Rarita-Schwinger superfield $\Psi^a_{\dot\alpha}$. Hence $\Psi^a_{\dot\alpha}|_{\theta=0}$ remains arbitrary.

Similarly, the bosonic Riemann curvature tensor superfield is determined by

$$\sigma^c_{\gamma\dot\gamma}\sigma^d_{\delta\dot\delta}R_{cd}{}^{\alpha\beta} = -2\epsilon_{\gamma\delta}r_{\dot\gamma\dot\delta}{}^{\alpha\beta} - 2\epsilon_{\dot\gamma\dot\delta}r_{\gamma\delta}{}^{\alpha\beta}, \tag{2.26}$$

$$r_{\dot\gamma\dot\delta}{}^{\alpha\beta} = \frac{1}{16}\bar{\mathcal{D}}_{(\dot\gamma}\mathcal{D}^{(\alpha}G^{\beta)}{}_{\dot\delta)}, \tag{2.27}$$

$$r_{\gamma\delta}{}^{\alpha\beta} = -\frac{1}{16}\mathcal{D}_{(\gamma}W_{\delta)}{}^{\alpha\beta} - \frac{1}{32}\delta^{\alpha}_{(\gamma}\delta^{\beta}_{\delta)}(\overline{\mathcal{D}\mathcal{D}R} - 2R\bar R). \tag{2.28}$$

In particular, Eq. (2.28) indicates that the superfield generalization of the (spin-tensor components of the) Weyl tensor, $C_{\alpha\beta\gamma\delta} = C_{(\alpha\beta\gamma\delta)}$, is defined through the nonvanishing spinor derivative of $W_{\alpha\beta\gamma}$

$$C_{\alpha\beta\gamma\delta} := r_{(\alpha\beta\gamma\delta)} = -\frac{1}{16}\mathcal{D}_{(\alpha}W_{\beta\gamma\delta)}. \tag{2.29}$$

In this sense one says that $W_{\alpha\beta\gamma}$ and its complex conjugate $\bar W_{\dot\alpha\dot\beta\dot\gamma}$ provide a superfield generalization of the Weyl tensor.

The superfield generalization of the Ricci tensor is given by

$$R_{bc}{}^{ac} = \frac{1}{32}(\mathcal{D}^{\beta}\bar{\mathcal{D}}^{(\dot\alpha|}G^{\alpha|\dot\beta)} - \bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{(\beta}G^{\alpha)\dot\alpha})\sigma^a_{\alpha\dot\alpha}\sigma_{b\beta\dot\beta} - \frac{3}{64}(\overline{\mathcal{D}\mathcal{D}R} + \mathcal{D}\mathcal{D}R - 4R\bar R)\delta^a_b, \tag{2.30}$$

and, henceforth, the scalar curvature superfield is

$$R_{ab}{}^{ab} = -\frac{3}{16}(\overline{\mathcal{D}\mathcal{D}R} + \mathcal{D}\mathcal{D}R - 4R\bar R). \tag{2.31}$$

Hence, once again, one can identify the (arbitrary) leading components of the corresponding second derivatives of main superfields G_a and R [entering the right-hand side (rhs) of Eq. (2.30)] with the irreducible components of the Ricci tensor $R_{bc}{}^{ac}|_{\theta=0}$ [or Einstein tensor $(R_{bc}{}^{ac} - \frac{1}{2}\delta_b{}^aR_{dc}{}^{dc})|_{\theta=0}$] which, thus, remains arbitrary after imposing the constraints (2.12)–(2.16).

This exhibits the well known fact that the constraints (2.12)–(2.16) describe the *off-shell* supergravity multiplet.

III. WESS-ZUMINO (WZ) GAUGE

To move from the superfield formulation of supergravity to the component formulation (i.e., in terms of spacetime fields) [11,13], one fixes the so-called Wess-Zumino (WZ) gauge, where, *in particular*,[3]

$$E_{\breve\alpha}{}^{a}|_{\theta=0} = 0, \quad E_{\breve\alpha}{}^{\underline\beta}|_{\theta=0} = \delta_{\breve\alpha}{}^{\underline\beta}, \quad w_{\breve\alpha}^{ab}|_{\theta=0} = 0, \tag{3.1}$$

while

$$E^a_{\mu}|_{\theta=0} = e^a_{\mu}(x), \quad E^{\underline\alpha}_{\mu}|_{\theta=0} = \psi^{\underline\alpha}_{\mu}, \tag{3.2}$$

$$w^{ab}_{\mu}|_{\theta=0} = \omega^{ab}_{\mu}(x) \tag{3.3}$$

remain unrestricted and are identified with the vielbein, gravitino, and (composed) spin-connection fields of the component formulation of supergravity [11,13].

One can collect the expressions for the supervielbein superfield in Eqs. (3.1),(3.2) in the matrix relation

[3] We mainly use in Sec. III Majorana spinor notation $E^{\underline\alpha} = (E^{\alpha}, E_{\dot\alpha})$ [29]; this also makes all the formulas of this section, except Eq. (3.6), applicable to any dimension D.

$$E_N{}^A|_{\theta=0}=\begin{pmatrix} e_\nu^a(x) & \psi_\nu^{\underline{\alpha}}(x) \\ 0 & \delta_{\check{\beta}}{}^{\underline{\alpha}} \end{pmatrix}. \tag{3.4}$$

Their evident consequences are

$$E_A{}^N|_{\theta=0}=\begin{pmatrix} e_a^\nu(x) & -\psi_a^{\check{\beta}}(x) \\ 0 & \delta_{\underline{\alpha}}{}^{\check{\beta}} \end{pmatrix}, \tag{3.5}$$

where $\psi_a^{\check{\beta}}(x)\equiv e_a{}^\nu\psi_\nu^{\underline{\alpha}}(x)\,\delta_{\underline{\alpha}}{}^{\check{\beta}}$.

Note that already these simple formulas allow one to derive, e.g., the following useful formula:

$$T_{ab}{}^\alpha|_{\theta=0}=2e_a^\mu e_b^\nu\mathcal{D}_{[\mu}\psi_{\nu]}^\alpha(x)-\frac{i}{4}(\psi_{[a}\sigma_{b]})_{\dot\beta}G^{\alpha\dot\beta}\Big|_{\theta=0} - \frac{i}{4}(\tilde\sigma_{[a}\bar\psi_{b]})^\alpha R\Big|_{\theta=0}, \tag{3.6}$$

where $\mathcal{D}_{[\mu}\psi_{\nu]}^\alpha(x)=\partial_{[\mu}\psi_{\nu]}^\alpha(x)-\psi_{[\nu}^\beta(x)w_{\mu]\beta}{}^\alpha|_{\theta=0}$ is the gravitino fields strength [though with the nonstandard spin connection which, in general, due to Eq. (2.12), involves the term proportional to $G_a|_{\theta=0}$ into the spacetime torsion]. Thus one can call $T_{ab}{}^\alpha$ the superfield generalization of the gravitino field strength.

One more simple but useful equation which is valid due to Eqs. (3.1),(3.2) is

$$E|_{\theta=\bar\theta=0}\equiv \mathrm{sdet}[E_M^A(x,0,0)]=\det(e_\mu^a)\equiv e(x). \tag{3.7}$$

A. Complete description of the Wess-Zumino gauge

As it was early recognized [28,34,35], the WZ gauge is the fermionic counterpart of the normal coordinate system in general relativity (see Refs. [10,36,28] for the so-called normal gauge in supergravity, which is the complete superspace generalization of the normal coordinate frame). This observation suggested to collect [28] the complete set of the conditions of the WZ gauge in[4]

$$\theta^{\check\alpha}E_{\check\alpha}^a=0,\quad \theta^{\check\alpha}(E_{\check\alpha}^{\underline{\beta}}-\delta_{\check\alpha}^{\underline{\beta}})=0, \tag{3.8}$$
$$\theta^{\check\alpha}w_{\check\alpha}^{ab}=0.$$

Using the inner product notation [see Eqs. (B10),(B11)], the WZ gauge may be equivalently defined by

$$i_\theta E^a=0, \tag{3.9}$$

$$i_\theta E^{\underline{\alpha}}=\theta^{\check\beta}\delta_{\check\beta}^{\underline{\alpha}}\equiv\theta^{\underline{\alpha}}, \tag{3.10}$$

$$i_\theta w^{ab}=0, \tag{3.11}$$

[4]Note that there exists another ("prepotential") form of the Wess-Zumino gauge which is fixed through a condition for the Ogievetsky-Sokatchev auxiliary vector *prepotential*, giving $\mathcal{H}^\mu=\theta\sigma^a\bar\theta e_a^\mu(x)+\bar\theta\bar\theta\theta^\alpha\psi_\alpha^\mu(x)+\text{c.c.}+\theta\theta\bar\theta\bar\theta A^\mu(x)$ [8], and for the chiral compensator, $\Phi=e^{1/3}(1-\frac{2}{3}\theta\sigma^a\bar\psi_a+\cdots)$ (see, e.g., [9,28]).

where $\theta^{\underline{\alpha}}$ is a Grassmann coordinate with a tangent space spinor index,

$$\theta^{\underline{\beta}}\equiv\theta^{\check\alpha}\delta_{\check\alpha}{}^{\underline{\beta}}. \tag{3.12}$$

One of the characteristic properties of the WZ gauge (3.8) is that the Grassmann coordinate (3.12) coincides with the contraction of the fermionic supervielbein form, (3.10). The next observation is that in the gauge (3.8)

$$\theta^{\check\alpha}\mathcal{D}_{\check\alpha}=\theta^{\underline{\beta}}\mathcal{D}_{\underline{\beta}}=\theta^{\check\alpha}\partial_{\check\alpha}\equiv\theta\partial. \tag{3.13}$$

With this in mind one can find that the decomposition of the supervielbein and spin connection superfields can be expressed in terms of the physical graviton and gravitino fields [Eq. (3.2)], the leading components of the torsion and curvature superfields and their covariant derivatives. A convenient way of reproducing these decompositions is by using the following recurrent relations (cf. [28]):

$$(1+\theta\partial)E^a=i_\theta T^a+dx^\mu E_\mu^a, \tag{3.14}$$

$$(1+\theta\partial)E^{\underline{\alpha}}=\mathcal{D}\theta^{\underline{\alpha}}+i_\theta T^{\underline{\alpha}}+dx^\mu E_\mu^{\underline{\alpha}}, \tag{3.15}$$

$$(1+\theta\partial)w^{ab}=i_\theta R^{ab}+dx^\mu w_\mu^{ab}, \tag{3.16}$$

together with Eq. (3.13). Equations (3.14),(3.15),(3.16) are obtained by taking the external derivative of the defining relations of the WZ gauge, Eqs. (3.8). There

$$\mathcal{D}\theta^{\underline{\beta}}=d\theta^{\underline{\beta}}-\theta^{\underline{\gamma}}w_{\underline{\gamma}}{}^{\underline{\beta}}, \tag{3.17}$$

$$i_\theta T^A\equiv E^C\theta^{\underline{\beta}}T_{\underline{\beta}C}{}^A,\quad i_\theta R^{ab}\equiv E^D\theta^{\underline{\gamma}}R_{\underline{\gamma}D}{}^{ab}. \tag{3.18}$$

Equations (3.14),(3.15),(3.16) do not restrict the physical fields (3.2), as the terms containing dx^μ in lhs's are canceled by the last terms in the rhs's. Thus the leading ($\theta=0$) components of Eqs. (3.14),(3.15),(3.16) reproduce Eq. (3.1).

A discussion of the decomposition of the superfields, i.e., of the solutions to Eqs. (3.14), (3.15), and (3.16), can be found in Appendix C 1.

B. Symmetries preserving the Wess-Zumino gauge

In the consideration of the coupled system it is important to know the subset of superspace local symmetries preserving the gauge (3.8). The subset of superspace diffeomorphisms [parameter $b^M(Z)$] and local Lorentz [$L^{ab}(Z)$] transformations preserving this gauge is singled out by the equations (see Appendix C 2 for details and further discussion)

$$\theta\partial(b^A)=(b^B)\theta^{\underline{\gamma}}T_{\underline{\gamma}B}{}^A+\theta^{\underline{\gamma}}(L_{\underline{\gamma}}{}^{\underline{\beta}}-b^Mw_{M\underline{\gamma}}{}^{\underline{\beta}})\delta_{\underline{\beta}}{}^A, \tag{3.19}$$

$$\theta\partial(L^{ab}(Z)-b^Mw_M{}^{ab})=-b^D\theta^{\underline{\gamma}}R_{\underline{\gamma}D}{}^{ab}, \tag{3.20}$$

where

$$L_B{}^A(Z)=\begin{pmatrix} L_b{}^a & 0 \\ 0 & L_{\underline{\beta}}{}^{\underline{\alpha}} \end{pmatrix}, \quad L^{ab}=-L^{ba},$$

$$L_{\underline{\beta}}{}^{\underline{\alpha}}=\frac{1}{4}L^{ab}\gamma_{ab\,\underline{\beta}}{}^{\underline{\alpha}}, \tag{3.21}$$

and the parameter b^M can be conventionally decomposed into a fermionic spinor and a bosonic vector part

$$b^A := b^M(Z)E_M^A(Z)\equiv(b^a(Z),\varepsilon^{\underline{\alpha}}(Z)). \tag{3.22}$$

It is instructive to write Eqs. (3.19),(3.20) in the weak field approximation. At zero-order one finds the set of equations

$$\theta\partial(b^a)=-2i\varepsilon^{\underline{\beta}}\gamma^a_{\underline{\beta\gamma}}\theta^{\underline{\gamma}}, \tag{3.23}$$

$$\theta\partial(\varepsilon^{\underline{\alpha}})=\theta^{\underline{\beta}}L_{\underline{\beta}}{}^{\underline{\alpha}}, \tag{3.24}$$

$$\theta\partial L^{ab}(Z)=0, \tag{3.25}$$

which can be easily solved,

$$b^a(Z)=b_0^a(x)+2i\theta\gamma^a\varepsilon_0(x)+\frac{i}{4}\theta(\gamma_{bc}\gamma^a)\theta l^{bc}(x), \tag{3.26}$$

$$\varepsilon^{\underline{\alpha}}(Z)=\varepsilon_0^{\underline{\alpha}}(x)-\theta^{\underline{\beta}}l_{\underline{\beta}}{}^{\underline{\alpha}}(x), \tag{3.27}$$

$$L^{ab}(Z)=l^{ab}(x), \tag{3.28}$$

where $b_0^a(x)$, $\varepsilon_0^{\underline{\alpha}}(x)$ are arbitrary vector and spinor functions and $l^{ab}(x)$ are local Lorentz parameters.

In the general case the WZ gauge is also preserved, in particular (see Appendix C 2) by *spacetime* diffeomorphisms [with parameters $b^a(Z)|_{\theta=0}$], as well as by Lorentz $[l^{ab}(x)]$ and local supersymmetry $[\varepsilon^{\underline{\alpha}}(x)=\varepsilon^{\underline{\alpha}}(Z)|_{\theta=0}]$ transformations.

IV. SUPERFIELD ACTION FOR "FREE" $D=4, N=1$ SUPERGRAVITY

A. Superfield action and variational problem with constraints

The $D=4,N=1$ supergravity action can be written [6] as an integral over superspace $\Sigma^{(4|4)}$ of the Berezenian (superdeterminant) $E:=\mathrm{sdet}(E_M^A)$ of the supervielbein $E_M^A(Z)$,

$$S_{SG}=\int d^4x\,\tilde{d}^4\theta\,\mathrm{sdet}(E_M^A)\equiv\int d^8Z\,E, \tag{4.1}$$

where $E_M^A(Z)$ are assumed to be subject to the constraints (2.12),(2.13),(2.14),(2.16). This action is evidently invariant under the superdiffeomorphisms and local Lorentz symmetries (further discussion of its gauge symmetries can be found in Appendix B 1).

One of the ways to obtain the superfield equations of motion from this action is to solve the constraints in terms of unconstrained superfields (prepotentials): axial vector superfield $\mathcal{H}^\mu(x,\theta)$ [8] and chiral compensator Φ [9] (in this way the local symmetries of the complete superfield formulation are partially gauge fixed).

Alternatively, following [6], one can keep $E_M^A(Z)$ as the basic variable, but take the constraints into account when searching for the independent variations. Namely, one denotes the general variation of the supervielbein and spin connections by [6]

$$\delta E_M^A(Z)=E_M^B\mathcal{K}_B^A(\delta), \quad \delta w_M^{ab}(Z)=E_M^C u_C^{ab}(\delta), \tag{4.2}$$

and obtains the equations to be satisfied by $\mathcal{K}_B{}^A(\delta),u_C^{ab}(\delta)$ from the requirement that the constraints (2.12),(2.13) are preserved under (4.2). Then one solves these equations in terms of some set of independent variations. Straightforward but quite involved calculations (the results of which were partially given in [6]) show that the constraints of minimal supergravity (2.12)–(2.16) are preserved by a set of *superfield variations* (superspace coordinates are not affected) which include:

(i) the local Lorentz transformations $\delta_L(L^{ab})$, Eq. (B1);

(ii) the variational version of the superspace general coordinate transformations [6] $[\tilde{\delta}_{gc}(t^A)$, Eqs. (B15),(B16),(B17) in Appendix B]; and

(iii) the set of transformations with parameters $\delta H^a=\frac{1}{2}\sigma^a_{\alpha\dot{\alpha}}\delta H^{\alpha\dot{\alpha}}$, $\delta\mathcal{U},\delta\bar{\mathcal{U}}$, under which the supervielbein transforms as[5]

$$\delta E^a=E^a[\Lambda(\delta)+\bar{\Lambda}(\delta)]-\frac{1}{4}E^b\tilde{\sigma}_b^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a +iE^\alpha\mathcal{D}_\alpha\delta H^a-i\bar{E}^{\dot{\alpha}}\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a, \tag{4.3}$$

$$\delta E^\alpha=E^a\Xi_a^\alpha(\delta)+E^\alpha\Lambda(\delta)+\frac{1}{8}\bar{E}^{\dot{\alpha}}R\sigma_{a\dot{\alpha}}{}^\alpha\delta H^a. \tag{4.4}$$

In Eqs. (4.3),(4.4), $\Lambda(\delta),\bar{\Lambda}(\delta)$ are given by

$$\Lambda(\delta)=\frac{1}{24}\tilde{\sigma}_a^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a+\frac{i}{4}\mathcal{D}_a\delta H^a+\frac{1}{24}G_a\delta H^a +2(\mathcal{DD}-\bar{R})\delta\mathcal{U}-(\overline{\mathcal{DD}}-R)\delta\bar{\mathcal{U}} \tag{4.5}$$

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[5]This procedure can be regarded as a linearized counterpart of solving the superspace constraints in terms of the prepotentials [9] (the price to achieve linearity, however, is that we have to deal with the covariant derivatives $\mathcal{D}_A$ rather than with the holonomic ones, ∂_M). So, the counterpart δH^a of the variation of the Ogievetsky-Sokatchev auxiliary vector superfield [8] $\mathcal{H}^\mu$, as well as the counterparts of the variation of the complex chiral compensators Φ [9], $(\mathcal{DD}-\bar{R})\delta\mathcal{U}$, are involved in the solution of these equations. [The (anti)chiral superfield $\bar{\Phi}$ satisfies $\mathcal{D}_\alpha\bar{\Phi}=0$ and can be expressed through the independent superfield $\mathcal{U}$ by $\bar{\Phi}=(\mathcal{DD}-\bar{R})\mathcal{U}$. Then the variation of $\bar{\Phi}$ is $\delta\bar{\Phi}=(\mathcal{DD}-R)\delta\mathcal{U}$.]

$$\Lambda(\delta)+\bar{\Lambda}(\delta)=\frac{1}{12}\tilde{\sigma}_a^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a+\frac{1}{12}G_a\delta H^a+(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}+(\bar{\mathcal{D}}\bar{\mathcal{D}}-R)\delta\bar{\mathcal{U}}; \quad (4.6)$$

the explicit expression for $\Xi_a^\alpha(\delta)$ in Eq. (4.4) will not be needed below. It reads

$$\Xi_a^\alpha(\delta)=-\frac{i}{4}\sigma_{a\beta\dot{\gamma}}u^{\dot{\gamma}\alpha\beta}(\delta)-\frac{i}{4}\tilde{\sigma}_a^{\dot{\alpha}\alpha}\bar{\mathcal{D}}_{\dot{\alpha}}\Lambda(\delta)-\frac{i}{32}\sigma_{a\beta\dot{\beta}}\mathcal{D}^\beta R\,\delta H^{\alpha\dot{\beta}}-\frac{i}{16}\sigma_{a\beta\dot{\beta}}R\mathcal{D}^\beta\delta H^{\alpha\dot{\beta}}-\frac{i}{32}\tilde{\sigma}_a^{\dot{\beta}\beta}G^{\alpha\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\beta}}\delta H_{\beta\dot{\gamma}}, \quad (4.7)$$

where

$$u_{\dot{\gamma}}^{\alpha\beta}(\delta)=-\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}^{(\alpha}\delta H^{\beta)}{}_{\dot{\gamma}}+\frac{3}{8}R\mathcal{D}^{(\alpha}\delta H^{\beta)}{}_{\dot{\gamma}}-\frac{1}{8}G^{(\alpha}{}_{\dot{\beta}}\bar{\mathcal{D}}_{\dot{\gamma}}\delta H^{\beta)\dot{\beta}}+\frac{1}{16}D^{(\alpha}R\,\delta H^{\beta)}{}_{\dot{\gamma}}-\frac{1}{8}\mathcal{D}_{(\dot{\gamma}}G^{(\alpha}{}_{\dot{\beta})}\delta H^{\beta)\dot{\beta}}+\frac{1}{8}W^{\alpha\beta\gamma}\delta H_{\gamma\dot{\gamma}}. \quad (4.8)$$

Equation (4.8), together with

$$u_\gamma^{\alpha\beta}(\delta)=\frac{1}{8}G^{(\alpha}{}_{\dot{\beta}}\mathcal{D}_\gamma\delta H^{\beta)\dot{\beta}}-\frac{1}{8}G_{\delta\dot{\delta}}\delta_\gamma^{(\alpha}\mathcal{D}^{\beta)}\delta H^{\delta\dot{\delta}}+2\delta_\gamma^{(\alpha}\mathcal{D}^{\beta)}\Lambda(\delta), \quad (4.9)$$

$$\sigma^a_{\gamma\dot{\gamma}}u_a^{\alpha\beta}(\delta)=-\frac{i}{2}[\mathcal{D}_\gamma u_{\dot{\gamma}}^{\alpha\beta}(\delta)+\bar{\mathcal{D}}_{\dot{\gamma}}u_\gamma^{\alpha\beta}(\delta)]-\frac{i}{16}R\bar{R}\delta_\gamma^{(\alpha}\delta H^{\beta)}{}_{\dot{\gamma}}-\frac{i}{16}\bar{\mathcal{D}}_{\dot{\beta}}\bar{R}\,\delta_\gamma^{(\alpha}\bar{\mathcal{D}}_{\dot{\gamma}}\delta H^{\beta)\dot{\beta}}+\frac{i}{16}\mathcal{D}^{(\alpha}G^{\beta)\dot{\beta}}\bar{\mathcal{D}}_{\dot{\gamma}}\delta H_{\gamma\dot{\beta}}+\frac{i}{16}W^{\alpha\beta\delta}\mathcal{D}_\gamma\delta H_{\delta\dot{\gamma}}, \quad (4.10)$$

define the variation of the spin connection through the second equation in Eq. (4.2).

B. Superfield action and "free" equations of motion

The nontrivial dynamical equations of motion should follow from the variations (4.3),(4.4) with (4.5),(4.6) only. The variation of the superdeterminant $E=\mathrm{sdet}(E_M^A)$ under Eqs. (4.3), (4.4), has the form (see [6])

$$\delta E=E\left[-\frac{1}{12}\tilde{\sigma}_a^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a+\frac{1}{6}G_a\delta H^a+2(\bar{\mathcal{D}}\bar{\mathcal{D}}-R)\delta\bar{\mathcal{U}}+2(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}\right]. \quad (4.11)$$

In the light of the identity (B35), all the terms with derivatives can be omitted in Eq. (4.11) when one considers the variation of the action (4.1). Hence

$$\delta S_{SG}=\int d^8Z\,\delta E=\int d^8Z\,E\left[\frac{1}{6}G_a\delta H^a-2R\,\delta\bar{\mathcal{U}}-2\bar{R}\,\delta\mathcal{U}\right] \quad (4.12)$$

and one arrives at the following *superfield equations of motion for "free," simple $D=4,N=1$ supergravity*:

$$\frac{\delta S_{SG}}{\delta H^a}=0 \quad\Rightarrow\quad G_a=0, \quad (4.13)$$

$$\frac{\delta S_{SG}}{\delta\bar{\mathcal{U}}}=0 \quad\Rightarrow\quad R=0, \quad (4.14)$$

$$\frac{\delta S_{SG}}{\delta\mathcal{U}}=0 \quad\Rightarrow\quad \bar{R}=0. \quad (4.15)$$

Then the "free" *superfield Rarita-Schwinger equations*,

$$\epsilon^{abcd}T_{bc}{}^\gamma\sigma_{d\gamma\dot{\gamma}}=0, \quad \epsilon^{abcd}T_{bc}{}^{\dot{\gamma}}\sigma_{d\gamma\dot{\gamma}}=0, \quad (4.16)$$

follow from the constraints (2.22),(2.23) with $G_a=0=R$,

$$T_{\alpha\dot{\alpha}\beta\dot{\beta}\gamma}=-\frac{1}{8}\epsilon_{\dot{\alpha}\dot{\beta}}W_{\alpha\beta\gamma} \;\Leftrightarrow\; T_{ab}{}^\gamma=\frac{1}{32}(\sigma_a\tilde{\sigma}_b)_{\alpha\beta}W^{\alpha\beta\gamma}. \quad (4.17)$$

The superfield generalization of the free Einstein equation $R_{ac}{}^{bc}=\frac{1}{2}\delta_a^bR_{cd}{}^{cd}=0$ follows from setting $G_a=0$ [Eq. (4.13)] and $R=0$ [Eq. (4.16)] in Eq. (2.30).

V. BRINK-SCHWARZ SUPERPARTICLE IN A SUPERGRAVITY BACKGROUND

The superparticle dynamical variables are the supercoordinate functions $\hat{Z}^M(\tau)$ defined by the map

$$\hat{\phi}: W^1\rightarrow\Sigma^{(4|4)}, \quad \tau\mapsto\hat{Z}^M(\tau)=[\hat{x}^\mu(\tau),\hat{\theta}^{\check{\alpha}}(\tau)], \quad (5.1)$$

defining a worldline $\mathcal{W}^1$ in $\Sigma^{(4|4)}$ parametrized by the proper time τ,

$$\mathcal{W}^1\subset\Sigma^{(4|4)}, \quad Z^M=\hat{Z}^M(\tau). \quad (5.2)$$

The actual superparticle worldline is determined by the equations of motion. For the massless superparticle these equations follow from the Brink-Schwarz action

$$S_{sp}=\int_{W^1}\hat{\mathcal{L}}_1=\frac{1}{2}\int_{W^1}l(\tau)\hat{E}^a\hat{E}^b_\tau\eta_{ab}, \quad (5.3)$$

which involves the pull-back $\hat{E}^a\equiv\hat{E}^a(\tau)=d\tau\,\hat{E}^a_\tau(\tau)$ to W^1 of the bosonic supervielbein form E^a [Eq. (2.1)] on $\Sigma^{(4|4)}$,

$$\hat{E}^a=d\hat{Z}^M(\tau)E_M^a(\hat{Z})\equiv d\tau\,\hat{E}^a_\tau, \quad (5.4)$$

$$\hat{E}^a_\tau = \partial_\tau \hat{Z}^M E^a_M(\hat{Z}),$$

and the Lagrange multiplier (worldline einbein) $l(\tau)$. Note that the pull-backs of the fermionic supervielbein forms

$$\hat{E}^\alpha = d\hat{Z}^M(\tau) E^\alpha_M(\hat{Z}) \equiv d\tau \hat{E}^\alpha_\tau,$$

$$\hat{\bar{E}}^{\dot\alpha} = d\hat{Z}^M(\tau) \overline{E}^{\dot\alpha}_M(\hat{Z}) \equiv d\tau \hat{\bar{E}}^{\dot\alpha}_\tau,$$

$$\hat{E}^\alpha_\tau = \partial_\tau \hat{Z}^M E^\alpha_M(\hat{Z}), \quad \hat{\bar{E}}^{\dot\alpha}_\tau = \partial_\tau \hat{Z}^M \overline{E}^{\dot\alpha}_M(\hat{Z}) \tag{5.5}$$

are not involved in the superparticle action (5.3) explicitly. This is a general property of the D-dimensional super-p-brane actions that reflects an especial role for the bosonic ''directions'' in superspace.

A. Equations of motion

The equations of motion for a superparticle moving in a supergravity *background* follow from the variation of the action (5.3) with respect to the Lagrange multiplier $\delta l(\tau)$ and the supercoordinate functions $\delta\hat{Z}$. The corresponding variation of the pull-back (5.4) of the bosonic supervielbein form (2.1) is

$$\delta_{\hat{Z}}\hat{E}^a \equiv \delta_{\hat{Z}} E^a(\hat{Z}) := E^a(\hat{Z}+\delta\hat{Z}) - E^a(\hat{Z})$$
$$= i_{\delta\hat{Z}}\hat{T}^a + \mathcal{D}(i_{\delta\hat{Z}}\hat{E}^a) + \hat{E}^b i_{\delta\hat{Z}} w_b{}^a, \tag{5.6}$$

$$i_{\delta\hat{Z}} E^a(\hat{Z}) := \delta\hat{Z}^M E^a_M(\hat{Z}), \tag{5.7}$$

$$i_{\delta\hat{Z}} w^{ab} := \delta\hat{Z}^M w^{ab}_M(\hat{Z}) \tag{5.8}$$

(note in passing that these transformations coincide with the pull-back of superspace general coordinate transformations δ_{gc}, Eqs. (B7),(B8), to W^1, $\delta_{\hat{Z}}\hat{E}^a = \hat{\phi}^*[\delta_{gc}E^a(Z)]$).

The last term in Eq. (5.6) does not contribute to the action variation[6]

$$\delta S_{sp} = \int_{W^1}\left[\frac{1}{2}\delta l(\tau)\hat{E}_{\tau a}\hat{E}^a + l(\tau)\hat{E}_{\tau a}\delta_{\hat{Z}}\hat{E}^a\right], \tag{5.9}$$

because $\hat{E}_{\tau a}\hat{E}_{\tau b} i_{\delta\hat{Z}} w^{ba} \equiv 0$ due to $i_{\delta\hat{Z}} w^{ba} = -i_{\delta\hat{Z}} w^{ab}$.

When the background obeys the constraints (2.12) the superparticle equations of motion become

$$\hat{E}^\alpha \sigma^a_{\alpha\dot\alpha}\hat{E}_{a\tau} = 0, \quad \hat{E}_{a\tau}\sigma^a_{\alpha\dot\alpha}\hat{\bar{E}}^{\dot\alpha} = 0, \tag{5.10}$$

$$\mathcal{D}(l\hat{E}_{a\tau}) = 0, \tag{5.11}$$

$$\hat{E}^a_\tau \hat{E}_{a\tau} = 0. \tag{5.12}$$

Indeed, Eq. (2.12) implies

$$i_{\delta\hat{Z}}\hat{T}^a = -2i\sigma^a_{\alpha\dot\alpha}\hat{E}^\alpha i_{\delta\hat{Z}}\hat{\bar{E}}^{\dot\alpha} - 2i\sigma^a_{\alpha\dot\alpha}\hat{\bar{E}}^{\dot\alpha} i_{\delta\hat{Z}}\hat{E}^\alpha$$
$$-\frac{1}{8}\hat{E}^b \varepsilon^a{}_{bcd} G^c(\hat{Z}) i_{\delta\hat{Z}}\hat{E}^d. \tag{5.13}$$

The last term does not contribute to the contraction $\hat{E}_{\tau a} i_{\delta\hat{Z}}\hat{T}^a$. Hence, after integration by parts, the expression (5.9) with Eq. (5.6) becomes

$$\delta S_{sp} = \int_{W^1}\left[\frac{1}{2}\delta l \hat{E}_{\tau a}\hat{E}^a - \mathcal{D}(l\hat{E}_{\tau a}) i_{\delta\hat{Z}}\hat{E}^a - 2il\hat{E}_{\tau a}(\sigma^a_{\alpha\dot\alpha}\hat{E}^\alpha i_{\delta\hat{Z}}\hat{\bar{E}}^{\dot\alpha} + \sigma^a_{\alpha\dot\alpha}\hat{\bar{E}}^{\dot\alpha} i_{\delta\hat{Z}}\hat{E}^\alpha)\right], \tag{5.14}$$

which implies the equations of motion (5.12) ($\delta S_{sp}/\delta l = 0$), (5.11) [$\delta S_{sp}/\delta\hat{Z}^M E^M_a(\hat{Z}) = 0$], and (5.10) [$\delta S_{sp}/\delta\hat{Z}^M E^M_{\dot\alpha}(\hat{Z}) = 0$ and its complex conjugate].

Let us stress that we derived the superparticle equations of motion (5.11),(5.10) from an arbitrary variation of the supercoordinate functions $\delta\hat{Z}$, which is tantamount to saying that they were obtained *from the general coordinate transformations* δ_{gc}, (B7),(B8), *pulled-back* to W^1. This reflects a *spontaneous* (partial) breaking of the superspace general coordinate symmetry δ_{gc} of the background by the superparticle worldline. The part of the general coordinate symmetry δ_{gc} of the supergravity background which is preserved by the worldline can be identified with the gauge fermionic κ-symmetry [37] and reparametrization symmetry (more rigorously, the variational version of the worldline general coordinate symmetry) [21].

B. Local fermionic κ-symmetry and reparametrization invariance of superparticle action

It is not hard to see that the superparticle action (5.3) is invariant under the gauge fermionic κ-symmetry [37] that acts on the coordinate functions and the Lagrange multiplier l by

$$\delta_\kappa\hat{Z}^M = \hat{E}^a_\tau \tilde{\sigma}^{\dot\alpha\alpha}_a[\overline{\kappa}_{\dot\alpha}(\tau)E^M_\alpha(\hat{Z}) + \kappa_\alpha(\tau)\overline{E}^M_{\dot\alpha}(\hat{Z})], \tag{5.15}$$

$$\delta_\kappa l(\tau) = 4il[\hat{E}^\alpha_\tau\kappa_\alpha(\tau) + \hat{\bar{E}}^{\dot\alpha}_\tau\overline{\kappa}_{\dot\alpha}(\tau)], \tag{5.16}$$

To this end, it is convenient to write Eq. (5.15) in the form

$$i_\kappa\hat{E}^a \equiv \delta_\kappa\hat{Z}^M E^a_M(\hat{Z}) = 0, \tag{5.17}$$

$$i_\kappa\hat{E}^\alpha \equiv \delta_\kappa\hat{Z}^M E^\alpha_M(\hat{Z}) = \overline{\kappa}_{\dot\alpha}(\tau)\tilde{\sigma}^{\dot\alpha\alpha}_a\hat{E}^a_\tau,$$
$$i_\kappa\hat{\bar{E}}^{\dot\alpha} \equiv \delta_\kappa\hat{Z}^M E^{\dot\alpha}_M(\hat{Z}) = \hat{E}^a_\tau\tilde{\sigma}^{\dot\alpha\alpha}_a\kappa_\alpha(\tau), \tag{5.18}$$

[6]This reflects the invariance of the action under a Lorentz rotation of the supervielbein, which can be considered as a pull-back of the local Lorentz transformation of the supergravity background. Such transformations cannot be treated as *gauge* symmetries of the superparticle in a supergravity *background*. However, they *are* gauge symmetries of the interacting system of *dynamical* supergravity and the superparticle.

substitute these $i_\kappa \hat{E}^A$ and $\delta_\kappa l(\tau)$ [Eq. (5.16)] for $i_{\delta\hat{Z}}\hat{E}^A$ and $\delta l(\tau)$ in Eq. (5.14), and observe that, due to the identity

$$\hat{E}_{a\tau}\hat{E}_{b\tau}(\sigma^a\tilde{\sigma}^b)_\alpha{}^\beta=\hat{E}^a_\tau\hat{E}_{a\tau}\delta_\alpha{}^\beta, \tag{5.19}$$

the contribution $l\hat{E}_{a\tau}\delta_\kappa\hat{E}^a=-2il\hat{E}_{a\tau}\hat{E}^a_\tau\hat{E}^{\underline{\alpha}}\kappa_{\underline{\alpha}}+$c.c. can be compensated by the variation of the Lagrange multiplier $\delta_\kappa l$ (5.16).

In the same manner, one finds that the following transformations of the supercoordinate functions

$$\delta_r\hat{Z}^M=r(\tau)\hat{E}^a_\tau E_a^M(\hat{Z}), \tag{5.20}$$

or, equivalently,

$$i_r\hat{E}^a\equiv\delta_r\hat{Z}^ME_M^a(\hat{Z})=r(\tau)\hat{E}^a_\tau,$$

$$i_r\hat{E}^\alpha=0,\quad i_r\hat{\bar{E}}^{\dot\alpha}=0, \tag{5.21}$$

can be compensated by[7]

$$\delta_r l(\tau)=l\partial_\tau r-r\partial_\tau l. \tag{5.22}$$

This proves the so-called reparametrization symmetry of the superparticle action (see also Appendix D).

VI. COMPLETE LAGRANGIAN DESCRIPTION OF THE SUPERGRAVITY-SUPERPARTICLE INTERACTING SYSTEM

A fully dynamical description of $D=4,N=1$ supergravity and the massless superparticle source interacting system can be achieved by means of the action

$$S=S_{SG}+S_{sp}=\int d^8z\,E+\frac{1}{2}\int_{W^1}l(\tau)\hat{E}^a\hat{E}^b_\tau\eta_{ab}, \tag{6.1}$$

where $E=\mathrm{sdet}(E_M^A)$ and the supervielbein in superspace is assumed to be restricted by the constraints (2.12),(2.13),(2.14).

A. Gauge symmetries of the coupled system

As the superparticle coordinate functions $\hat{Z}^M\equiv\hat{Z}^M(\tau)$ do not enter in the supergravity part of the action, Eqs. (5.10),(5.11),(5.12) remain the same as in the interacting system (6.1),

$$\hat{E}^\alpha\sigma^a_{\alpha\dot\alpha}\hat{E}_{a\tau}=0, \tag{6.2}$$

$$\hat{E}_{a\tau}\sigma^a_{\alpha\dot\alpha}\hat{\bar{E}}^{\dot\alpha}=0, \tag{6.3}$$

$$\mathcal{D}(l\hat{E}_{a\tau})=0, \tag{6.4}$$

$$\hat{E}^a_\tau\hat{E}_{a\tau}=0. \tag{6.5}$$

Moreover, κ-symmetry [Eqs. (5.17),(5.18),(5.16)] and re parametrization symmetry [Eqs. (5.21),(5.22)] are preserved by the interaction.

The coupled action is evidently invariant under superdiffeomorphisms δ_{diff},

$$Z'^M=Z^M+b^M(Z):\quad\begin{cases}x'^\mu=x^\mu+b^\mu(x,\theta),\\ \theta'^{\check\alpha}=\theta^{\check\alpha}+\varepsilon^{\check\alpha}(x,\theta),\end{cases} \tag{6.6}$$

$$E'^A(Z')=E^A(Z),\quad w'^{ab}(Z')=w^{ab}(Z), \tag{6.7}$$

now supplemented by the corresponding transformations for the superparticle variables $\hat{Z}'^M=\hat{Z}'^M(\tau)$

$$\hat{Z}'^M=\hat{Z}^M+b^M(\hat{Z}):\quad\begin{cases}\hat{x}'^\mu(\tau)=\hat{x}^\mu+b^\mu(\hat{x},\hat{\theta}),\\ \hat{\theta}'^{\check\alpha}(\tau)=\hat{\theta}^{\check\alpha}+\varepsilon^{\check\alpha}(\hat{x},\hat{\theta}),\end{cases} \tag{6.8}$$

so that

$$\delta_{diff}Z^M=Z'^M-Z^M=b^M(Z), \tag{6.9}$$

$$\delta_{diff}\hat{Z}^M=b^M(\hat{Z}), \tag{6.10}$$

and $\delta_{diff}S=0$ (see Appendix B 2, where further discussion on the gauge symmetries of the coupled system can be found).

B. Equations of motion of the coupled system

As mentioned above, the superparticle equations $\delta S/\delta\hat{Z}^M=0$, $\delta S/\delta l=0$ for the coupled dynamical system remain the same as those for the system in a supergravity *background*, $\delta S_{sp}/\delta\hat{Z}^M=0$, $\delta S_{sp}/\delta l=0$ [Eqs. (6.2), (6.3), (6.4), and (6.5)]. Let us now see how the supergravity equations of motion are modified by the inclusion of the superparticle source.

Denoting the variation of the action induced by the constraints preserving variations (4.3)–(4.6) by δ' , one concludes that

$$\delta' S=\int d^8Z\,E\left[\frac{1}{6}G_a\delta H^a-2\,R\,\delta\bar{\mathcal{U}}-2\bar{R}\,\delta\mathcal{U}\right]+\delta' S_{sp}, \tag{6.11}$$

where

$$\delta' S_{sp}=\int_{W^1}l(\tau)\hat{E}_{a\tau}\delta'\hat{E}^a=\int_{W^1}l(\tau)\hat{E}_{a\tau}d\hat{Z}^M\,\delta' E_M^a(\hat{Z}) \tag{6.12}$$

and $\delta'\hat{E}^a$ is the pull-back of Eq. (4.3) to W^1. To have a well posed variational problem, we extend the integration in Eq. (6.12) to superspace by introducing the superspace delta function

$$\delta^8(Z-\hat{Z}):=\delta^4(x-\hat{x})(\theta-\hat{\theta})^4 \tag{6.13}$$

where

[7]On the worldvolume, acting on the pull-back of the superforms, $\mathcal{D}=d\hat{Z}^M\mathcal{D}_M=d\tau\mathcal{D}_\tau$, where $\mathcal{D}_\tau=\partial_\tau+connection\ term(s)$. Integrating by parts one arrives at the terms involving $\partial_\tau l$ in the worldvolume action variations. Note that $\mathcal{D}_\tau l=\partial_\tau l$, because the einbein $l(\tau)$ does not have Lorentz group indices.

$$(\theta-\hat{\theta})^4:=\frac{1}{4!}\epsilon_{\check{\alpha}_1\cdots\check{\alpha}_4}(\theta-\hat{\theta})^{\check{\alpha}_1}\cdots(\theta-\hat{\theta})^{\check{\alpha}_4}. \quad (6.14)$$

Namely, we insert $1=\int d^8Z\,\delta^8(Z-\hat{Z})$ into Eq. (6.12) and use the identity $\delta' E_M^a(\hat{Z})\delta^8(Z-\hat{Z})\equiv\delta' E_M^a(Z)\delta^8(Z-\hat{Z})$ to arrive at

$$\delta' S_{sp}=\int d^8Z\left[\int_{W^1} l(\tau)\hat{E}_{a\tau}d\hat{Z}^M\,\delta^8(Z-\hat{Z})\right]\delta' E_M^a(Z). \quad (6.15)$$

Now Eq. (4.3) can be straightforwardly inserted into Eq. (6.15) and, using

$$d\hat{Z}^M\delta^8(Z-\hat{Z})E_M^A(Z)\equiv\hat{E}^A\delta^8(Z-\hat{Z})\equiv E(Z)\frac{1}{\hat{E}}\hat{E}^A\delta^8(Z-\hat{Z}),$$

$$E(Z)\equiv\mathrm{sdet}[E_M^A(Z)],\quad \hat{E}=E(\hat{Z}), \quad (6.16)$$

one finds

$$\begin{aligned}\delta' S_{sp}=&\int d^8Z\,E\left[\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^\alpha\delta^8(Z-\hat{Z})\right]i\mathcal{D}_\alpha\delta H^a\\&+\int d^8Z\,E\left[\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^{\dot{\alpha}}\delta^8(Z-\hat{Z})\right](-i)\bar{\mathcal{D}}_{\dot{\alpha}}\delta H^a\\&-\int d^8Z\,E\left[\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^b\delta^8(Z-\hat{Z})\right]\\&\times\frac{1}{4}\tilde{\sigma}_b^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\delta H^a\\&+\int d^8Z\,E\left[\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^a\delta^8(Z-\hat{Z})\right]\\&\times[\Lambda(\delta)+\bar{\Lambda}(\delta)]. \quad (6.17)\end{aligned}$$

The extraction of the superdeterminant in Eq. (6.17) permits integrating by parts using the identity (B35) in Appendix B. Thus Eqs. (6.11),(6.17) allows us a direct derivation of the coupled equations of motion.

Note that the scalar variations $\delta\mathcal{U},\delta\bar{\mathcal{U}}$ are involved only in the last term of Eq. (6.11), through $[\Lambda(\delta)+\bar{\Lambda}(\delta)]$ defined by Eq. (4.6).

Let us now compute the $\delta\mathcal{U}$ variation of the coupled action, $\delta_{\mathcal{U}}S=\delta_{\mathcal{U}}S_{SG}+\delta_{\mathcal{U}}S_{sp}$. The variation of the supergravity part reads $\delta_{\mathcal{U}}S_{SG}=-\frac{1}{2}\int d^8Z\,E\bar{R}\delta\mathcal{U}$ [see Eq. (4.12)], while, due to $[\Lambda(\delta_{\mathcal{U}})+\bar{\Lambda}(\delta_{\mathcal{U}})]=(\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}$ [see Eq. (4.6)],

$$\begin{aligned}\delta_{\mathcal{U}}S_{sp}=&\int d^8Z\,E\left[\int_{W^1}\frac{l}{\hat{E}}\hat{E}_{a\tau}\hat{E}^a\delta^8(Z-\hat{Z})\right](\mathcal{D}\mathcal{D}-\bar{R})\delta\mathcal{U}\\=&\int d^8Z\,E\left[\int_{W^1}\frac{l}{\hat{E}}\hat{E}_{a\tau}\hat{E}^a(\mathcal{D}\mathcal{D}-\bar{R})\delta^8(Z-\hat{Z})\right]\delta\mathcal{U}. \quad (6.18)\end{aligned}$$

Thus, at a first look, Eq. (4.15) acquires a source term

$$\frac{\delta S}{\delta\mathcal{U}}=0\quad\Rightarrow\quad\bar{R}=\mathcal{J}_0, \quad (6.19)$$

$$\mathcal{J}_0=\int_{W^1}\frac{2l}{\hat{E}}\hat{E}_{a\tau}\hat{E}^a(\mathcal{D}\mathcal{D}-R)\delta^8(Z-\hat{Z}).$$

However, one immediately observes that this source vanishes due the superparticle equation of motion (6.5)

$$\frac{\delta S}{\delta l(\tau)}=0\quad\Rightarrow\quad\hat{E}_\tau^a\hat{E}_{a\tau}=0\quad\Rightarrow\quad\mathcal{J}_0=0. \quad (6.20)$$

Hence *the scalar superfield equations for the coupled dynamical system are the same as in the "free" supergravity case,*

$$\frac{\delta S}{\delta\mathcal{U}}=0\quad\Rightarrow\quad\bar{R}=0, \quad (6.21)$$

$$\frac{\delta S}{\delta\bar{\mathcal{U}}}=0\quad\Rightarrow\quad R=0. \quad (6.22)$$

Moreover, the above observation implies that the last term in the superparticle action variation does not contribute to the equations of motion,

$$\int d^8Z\,E\left[\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^a\delta^8(Z-\hat{Z})\right][\Lambda(\delta)+\bar{\Lambda}(\delta)]=0, \quad (6.23)$$

due to Eq. (6.5), $\hat{E}^a\hat{E}_{a\tau}=0$. Hence after an integration by parts using the identity (B35), and taking into account Eq. (6.5), the variation (6.17) of the superparticle action reads

$$\begin{aligned}\delta' S_{sp}=&\int d^8Z\,E\Big\{i\mathcal{D}_\alpha\mathcal{K}_a^{\ \alpha}-i\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{K}}_a^{\dot{\alpha}}\\&+\frac{1}{4}\tilde{\sigma}_b^{\dot{\alpha}\alpha}[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\mathcal{K}_a^{\ b}\Big\}\delta H^a, \quad (6.24)\end{aligned}$$

where the "spin 3/2" and "spin 2" "current prepotentials," $\mathcal{K}_a^{\ \alpha}$, $\bar{\mathcal{K}}_a^{\dot{\alpha}}=(\mathcal{K}_a^{\ \alpha})^*$, and $\mathcal{K}_a^{\ b}$, are defined by

$$\mathcal{K}_a^\alpha(Z):=\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^\alpha\delta^8(Z-\hat{Z}), \quad (6.25)$$

$$\bar{\mathcal{K}}_a^{\dot{\alpha}}(Z):=\int_{W^1}\frac{l(\tau)}{\hat{E}}\hat{E}_{a\tau}\hat{E}^{\dot{\alpha}}\delta^8(Z-\hat{Z}), \quad (6.26)$$

$$\mathcal{K}_a{}^b(Z) := \int_{W^1} \frac{l(\tau)}{\hat{E}} \hat{E}_{a\tau} \hat{E}^b \delta^8(Z-\hat{Z}). \tag{6.27}$$

Equations (6.24) and (6.11) imply the appearance of a current potential superfield $\mathcal{J}_a$ (which is a vector density distribution with support on the worldline), in the vector superfield equation of the coupled system [cf. (4.13)],

$$\frac{\delta S}{\delta H^a} = 0 \quad \Rightarrow \quad G_a = \mathcal{J}_a . \tag{6.28}$$

This vector current potential is constructed from the vector-spinor and tensor densities Eqs. (6.25),(6.26),(6.27) (hence the ''current prepotential'' name for $\mathcal{K}_a^B$) as follows:

$$\frac{1}{6}\mathcal{J}_a = -i\mathcal{D}_\alpha \mathcal{K}_a^{\ \alpha} + i\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{K}}_a^{\dot\alpha} + \frac{1}{4}\tilde{\sigma}_b^{\dot\alpha\alpha}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}]\mathcal{K}_a{}^b. \tag{6.29}$$

The preservation of the scalar superfield equation $R=0$ in the interacting dynamical system (6.1), Eq. (6.21), immediately implies the vanishing of the spin 1/2 part of the superfield generalization of the gravitino field strength,

$$(\sigma^a\tilde{\sigma}^b)_\beta{}^\gamma T_{ab\gamma} = 0, \quad (\tilde{\sigma}^a\sigma^b)^{\dot\gamma}{}_{\dot\beta} T_{ab\dot\gamma} = 0 \tag{6.30}$$

[see Eq. (2.24)]. However, the above equation is only a part of the content of the superfield generalization of the free Rarita-Schwinger equation.[8] The complete *superfield generalization of the Rarita-Schwinger equation* for the coupling system can be obtained from Eq. (2.22) with $R=0, G_a=\mathcal{J}_a$ and possesses the source term

$$\epsilon^{abcd} T_{bc}{}^\alpha \sigma_{d\alpha\dot\alpha} = \frac{i}{8}\tilde{\sigma}^{a\dot\beta\beta}\bar{\mathcal{D}}_{(\dot\beta|}\mathcal{J}_{\beta|\dot\alpha)} . \tag{6.31}$$

Using Eq. (2.31) and Eq. (6.21), $R=0$, one finds that the superfield generalization of the scalar curvature vanishes in the supergravity-superparticle interacting system,

$$R_{ab}{}^{ab} = 0. \tag{6.32}$$

However, in accordance with Eq. (2.30), the Ricci tensor is expressed not only through $R, \bar{R}$, but also through the G_a superfield. Hence in the interacting system the *superfield Einstein equation* acquires a source term which is expressed through a second derivative of the current potential superfield $\mathcal{J}^{\alpha\dot\beta} = \mathcal{J}^a \tilde{\sigma}_a^{\alpha\dot\beta}$:

$$R_{bc}{}^{ac} = \frac{1}{32}(\mathcal{D}^\beta \bar{\mathcal{D}}^{(\dot\alpha|}\mathcal{J}^{\alpha|\dot\beta)} - \bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{(\beta}\mathcal{J}^{\alpha)\dot\alpha})\sigma^a_{\alpha\dot\alpha}\sigma_{b\beta\dot\beta}. \tag{6.33}$$

[8]Indeed, the linear approximation equation $\epsilon^{abcd}\partial_b\psi_c^\alpha\sigma_{d\alpha\dot\alpha}=0$ is equivalent to the equations $(\sigma^a\tilde{\sigma}^b)_{(\beta\gamma)}\partial_b\psi_a^\gamma = 0$ [which is a counterpart of Eq. (6.30)] *and* $\partial^c\psi_c^\alpha = 0$.

C. Properties of current potential $\mathcal{J}_a$ and $\mathcal{K}_a{}^B$ prepotentials

Thus the vector superfield supergravity equation acquires the source (6.29) (''current potential'') from the Brink-Schwarz superparticle action S_{sp}, Eq. (6.28), while the scalar superfield equations (6.21),(6.22) remain sourceless as in free supergravity. Then the identities (2.20) immediately result in

$$\mathcal{D}^\alpha \mathcal{J}_{\alpha\dot\alpha} = 0, \quad \bar{\mathcal{D}}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = 0, \tag{6.34}$$

which imply the supercurrent conservation

$$\mathcal{D}^a \mathcal{J}_a = 0. \tag{6.35}$$

In accordance with Eq. (6.29), the superparticle current is constructed from the current prepotentials (6.25),(6.26),(6.27). Moreover, Eq. (6.29) can be presented in the form

$$\frac{1}{6}\mathcal{J}_a = -i\mathcal{D}_\alpha\left(\mathcal{K}_a^{\ \alpha} + \frac{i}{4}\tilde{\sigma}_b^{\dot\alpha\alpha}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{K}_a{}^b\right) + i\bar{\mathcal{D}}_{\dot\alpha}\left(\bar{\mathcal{K}}_a^{\dot\alpha} + \frac{i}{4}\tilde{\sigma}_b^{\dot\alpha\alpha}\mathcal{D}_\alpha\mathcal{K}_a{}^b\right). \tag{6.36}$$

It is interesting that the spinor-vector and tensor current prepotential carry only the irreducible spin 3/2 and spin 2 representation of the Lorentz group, respectively. Indeed, extracting the worldline measure $d\tau$ in Eq. (6.27), $\mathcal{K}^{ab}(Z) = \int_{W^1} d\tau (l(\tau)/\hat{E}) E_\tau^a \hat{E}_\tau^b \delta^8(Z-\hat{Z})$, one easily sees that the tensor $\mathcal{K}^{ab}(Z)$ is symmetric, and traceless due to Eq. (6.5). In this sense one can say that the current potential contains only a spin 2 irreducible part,

$$\mathcal{K}^{ab}(Z) = \mathcal{K}^{ba}(Z), \quad \mathcal{K}_b{}^b(Z) = 0. \tag{6.37}$$

The spinor-vector current prepotentials carry spin 3/2, because, due to Eq. (5.10), their spin 1/2 irreducible parts vanish,

$$\mathcal{K}_a^{\ \alpha}(Z)\sigma^a_{\alpha\dot\alpha} \equiv \mathcal{K}_{\alpha\dot\alpha}{}^\alpha = 0, \tag{6.38}$$

$$\sigma^a_{\alpha\dot\alpha}\bar{\mathcal{K}}_a^{\dot\alpha}(Z) \equiv \bar{\mathcal{K}}_{\alpha\dot\alpha}{}^{\dot\alpha}(Z) = 0. \tag{6.39}$$

Finally, using Eq. (5.11), together with the identities

$$E_A{}^M(Z)\partial_M\delta(Z-\hat{Z}) = E_A{}^M(\hat{Z})\partial_M\delta(Z-\hat{Z}) - (-1)^{M+AM}\partial_M E_A{}^M(Z)\delta(Z-\hat{Z}),$$

$$\partial_M\delta(Z-\hat{Z}) = -\partial/\partial\hat{Z}^M\delta(Z-\hat{Z}),$$

and $(-1)^{M+AM}\mathcal{D}_M(EE_A{}^M) \equiv E(-1)^B T_{AB}{}^B = 0$ (the last part of the last identity is valid due to the supergravity constraints), one finds the relation

$$(-)^B\mathcal{D}_B\mathcal{K}_a^{\ B} \equiv \mathcal{D}_b\mathcal{K}_a{}^b - \mathcal{D}_\beta\mathcal{K}_a^{\ \beta} - \bar{\mathcal{D}}_{\dot\beta}\bar{\mathcal{K}}_a^{\dot\beta} = 0, \tag{6.40}$$

which completes the list of the properties of the superparticle current prepotentials (6.25),(6.26),(6.27).

Contracting the vector indices of the current prepotentials with the σ matrices, one can write the irreducibility conditions (6.37),(6.38),(6.39) in the form

$$\mathcal{K}^{\alpha\beta\dot\alpha\dot\beta}\equiv\mathcal{K}^{ab}\tilde\sigma_a^{\dot\alpha\alpha}\tilde\sigma_b^{\dot\beta\beta}=\mathcal{K}^{(\alpha\beta)(\dot\alpha\dot\beta)}, \tag{6.41}$$

$$\mathcal{K}^{\alpha\beta\dot\beta}\equiv\mathcal{K}_a^{\ \alpha}\tilde\sigma_a^{\dot\beta\beta}=\mathcal{K}^{(\alpha\beta)\dot\alpha}, \tag{6.42}$$

$$\overline{\mathcal{K}}^{\dot\alpha\dot\beta\beta}\equiv\overline{\mathcal{K}}_a^{\dot\alpha}\tilde\sigma_a^{\dot\beta\beta}=\overline{\mathcal{K}}^{(\dot\alpha\dot\beta)\beta}. \tag{6.43}$$

Then, relation (6.40) reads

$$\frac{1}{2}\mathcal{D}_{\beta\dot\beta}\mathcal{K}^{\alpha\beta\dot\alpha\dot\beta}-\mathcal{D}_\beta\mathcal{K}^{\alpha\beta\dot\alpha}-\overline{\mathcal{D}}_{\dot\beta}\overline{\mathcal{K}}^{\dot\alpha\dot\beta\alpha}=0, \tag{6.44}$$

or, equivalently,

$$\mathcal{D}_\beta\left(\mathcal{K}^{\alpha\beta\dot\alpha}+\frac{i}{4}\overline{\mathcal{D}}_{\dot\beta}\mathcal{K}^{\alpha\beta\dot\alpha\dot\beta}\right)=-\overline{\mathcal{D}}_{\dot\beta}\left(\overline{\mathcal{K}}^{\dot\alpha\dot\beta\alpha}+\frac{i}{4}\mathcal{D}_\beta\mathcal{K}^{\alpha\beta\dot\alpha\dot\beta}\right). \tag{6.45}$$

Equations (6.44), (6.45) allow us to write the expression (6.36) for supercurrent in two other equivalent forms

$$\frac{1}{6}\mathcal{J}^{\alpha\dot\alpha}\equiv\frac{1}{6}\mathcal{J}_a\tilde\sigma^{a\dot\alpha\alpha}=-2i\mathcal{D}_\beta\left(\mathcal{K}^{(\beta\alpha)\dot\alpha}+\frac{i}{4}\overline{\mathcal{D}}_{\dot\beta}\mathcal{K}^{(\alpha\beta)(\dot\alpha\dot\beta)}\right) \tag{6.46}$$

$$=2i\overline{\mathcal{D}}_{\dot\alpha}\left(\overline{\mathcal{K}}^{(\dot\alpha\dot\beta)\beta}+\frac{i}{4}\mathcal{D}_\beta\mathcal{K}^{(\alpha\beta)(\dot\alpha\dot\beta)}\right). \tag{6.47}$$

Now one can easily derive Eq. (6.34) using Eqs. (6.46) and (6.47). To this aim one uses the algebra of spinor derivatives of the same chirality, Eq. (A4),

$$R=\overline{R}=0 \quad\Rightarrow\quad \{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=0. \tag{6.48}$$

Then the current potential conservation, Eq. (6.35), follows from Eqs. (6.34) and Eq. (A5).

The properties (6.34) imply $\overline{\mathcal{D}}_{\dot\alpha}\mathcal{J}_{\beta\dot\beta}=\overline{\mathcal{D}}_{(\dot\beta|}\mathcal{J}_{\beta|\dot\alpha)}$ and, hence, allow us to write the rhs of the *superfield Rarita-Schwinger equation* as the fermionic covariant derivative of the current potential

$$\Psi_{\dot\alpha}^{a}\equiv\epsilon^{abcd}T_{bc}{}^{\alpha}\sigma_{d\alpha\dot\alpha}=\frac{i}{4}\overline{\mathcal{D}}_{\dot\alpha}\mathcal{J}^a. \tag{6.49}$$

The *superfield Einstein equation* (6.33) can be written as

$$R_{bc}{}^{ac}=\frac{1}{16}\tilde\sigma_b^{\dot\beta\beta}[\mathcal{D}_\beta,\overline{\mathcal{D}}_{\dot\beta}]\mathcal{J}^a. \tag{6.50}$$

Equations (6.49),(6.50) exhibit an interdependence of the Einstein and Rarita-Schwinger superfield equations,

$$R_{bc}{}^{ac}=-\frac{i}{4}\tilde\sigma_b^{\dot\beta\beta}(\mathcal{D}_\beta\Psi_\beta^a+\overline{\mathcal{D}}_{\dot\beta}\overline{\Psi}_\beta^a). \tag{6.51}$$

VII. GAUGE FIXING AND EQUATIONS OF MOTION FOR THE INTERACTING SYSTEM

A. Gauge fixing

As all the gauge symmetries of the "free" superfield supergravity are still present in the interacting system, one can fix first the WZ gauge (3.8). This would be the first step towards the component description of the interacting system in terms of the usual graviton and gravitino spacetime fields.

As was shown in Sec. III B and in Appendix C 2, the WZ gauge is preserved by some specific superdiffeomorphisms and superspace local Lorentz transformations with free parameters $b_0^A(x)=(b_0^a(x),\varepsilon_0^{\alpha}(x))=b^A(Z)|_{\theta=0}$ and $l^{ab}(x)=L^{ab}(Z)|_{\theta=0}$. In accordance with Eqs. (6.10) and (5.15), the transformation of the fermionic coordinate function $\hat\theta^{\check\alpha}(\tau)$ under superdiffeomorphisms and worldline κ transformations acquires the form

$$\delta\hat\theta^{\check\alpha}(\tau)=b^{\check\alpha}(\hat Z)+\delta_\kappa\hat\theta^{\check\alpha}(\tau), \tag{7.1}$$

where $\delta_\kappa\hat\theta^{\check\alpha}(\tau)$ is defined by the Eq. (5.15) with $M=\check\alpha$. This transformation rule reflects the Goldstone nature of the superparticle (or superbrane) coordinate function [22] (see also [23,24] and references therein).

In the WZ gauge (3.8), Eq. (7.1) can be written in the form [see Eqs. (3.12),(3.22)]

$$\delta\hat\theta^{\underline\alpha}(\tau)\equiv\delta\hat\theta^{\check\alpha}(\tau)\,\delta_{\check\alpha}{}^{\underline\alpha}=\varepsilon^{\underline\alpha}(\hat Z)+\delta_\kappa\hat\theta^{\underline\alpha}(\tau). \tag{7.2}$$

Decomposing the rhs of Eq. (7.2) in power series in $\hat\theta(\tau)$ one writes

$$\delta\hat\theta^{\underline\alpha}(\tau)=\varepsilon^{\underline\alpha}(\hat Z)|_{\hat\theta=0}+\delta_\kappa\hat\theta^{\underline\alpha}(\tau)|_{\hat\theta=0}+\mathcal{O}(\hat\theta)$$
$$=\varepsilon_0^{\underline\alpha}(\hat x)+\delta_\kappa\hat\theta^{\underline\alpha}(\tau)|_{\hat\theta=0}+\mathcal{O}(\hat\theta), \tag{7.3}$$

where the arbitrary fermionic field parameter $\varepsilon_0^{\underline\alpha}(\hat x)$ is defined as in Eqs. (3.22), (3.27) and *corresponds to one of the symmetries that preserve the WZ gauge*.

Thus we can fix the gauge (simultaneously with the WZ gauge)

$$\hat\theta^{\underline\alpha}(\tau)=0 \tag{7.4}$$

(cf. the description of super-Higgs effect in [25]) *by using the freedom in the fermionic parameters* $\varepsilon_0^{\underline\alpha}(\hat x)$ [but *not* the pullback $\varepsilon_0^{\underline\alpha}(\hat x,\hat\theta)$ of the complete superfield $\varepsilon_0^{\underline\alpha}(Z)$]. The gauge (7.4) is preserved by transformations such that

$$\varepsilon_0^{\underline\alpha}(\hat x)=-\delta_\kappa\hat\theta^{\underline\alpha}(\tau)|_{\hat\theta=0}=-\kappa^{\underline\beta}\gamma_{a\,\underline\beta}{}^{\underline\alpha}\hat E_\tau^a|_{\hat\theta=0}, \tag{7.5}$$

where we have written the form of the κ-symmetry transformations (5.15) explicitly, in Majorana spinor notation, by using the WZ gauge relations.

B. On the Goldstone nature of the (super)brane coordinate functions

Since the possibility of fixing the gauge (7.4) might look unexpected, we now discuss its physical meaning.

First, let us note that similar considerations show that the bosonic counterpart of the gauge (7.4) can also be fixed on the bosonic coordinate functions. It reads[9]

$$\hat{x}^{\mu}(\tau)=(\tau,0,0,0) \tag{7.6}$$

[or $x^{\mu}(\tau)=(\tau,0,0,\pm\tau)$ if one identifies x^0 with the time-like dimension in the flat (super) space limit]. In general, for a D-dimensional p-brane interacting with dynamical gravity one can fix *locally* the following counterpart of the gauge (7.6) (static gauge)

$$\hat{x}^{\mu}(\tau,\vec{\sigma})=(\tau,\sigma^1,\ldots,\sigma^p,0,\ldots,0), \tag{7.7}$$

where the first $(p+1)$ of the D coordinate functions are identified with the local worldvolume coordinates $\xi^m=(\tau,\sigma^1,\ldots,\sigma^p)$, and the remaining coordinate functions are set to zero (see [20]).

Clearly, the gauge (7.6) or (7.7) can be fixed also in a dynamical system of pure bosonic gravity interacting with a bosonic particle or brane. As such, this phenomenon should be known in general relativity, and this is indeed the case. The pure gauge nature of the coordinate functions describing the motion of a dynamical source (particle) was already known in general relativity, see, e.g., [38,39]. The presence of the gauge symmetry allowing one to fix *locally* the gauge (7.7) for branes or (7.6) for a particle is reflected in the language of the second Noether theorem (see [20,21]) by stating that the brane or particle equations of motion can be derived as a consequence of the field equations for gravity. This type of statement can be found in books (see, e.g., p. 240 in [39], pp. 19, 44–48, and Eq. (1.6.13) in [38]) and comes back to the original paper by Einstein and Grommer [40]. Namely, one can derive the equations of motion of the particle source from the covariant conservation of the particle energy-momentum tensor in the rhs of the Einstein field equation. So, the statement of [38] is that we do not need to vary the action with respect to the matter (particle) variables because we can obtain the equations of motion for the matter part as a consequence of the Einstein equations. These, by their geometric structure, imply the covariant energy-momentum conservation which in turn is equivalent to the matter equations of motion.

Clearly, for the case of the brane source the same arguments result in the derivation of the equations of motion for the brane variables from the conservation of an energy-momentum tensor with support on worldvolume (see [20] for an explicit proof). Then the choice of *local* coordinate system allows one to fix the gauge (7.7) *locally*. Certainly, for topologically nontrivial and/or closed worldvolume this gauge cannot be fixed globally. In contrast, one immediately notices that there are no restrictions on a global fixing of the fermionic gauge (7.4) as no way of introducing topology on a Grassmann algebra is known.

Thus one can state that both the fermionic and bosonic coordinate functions of superbrane are pure gauge (can be gauged away) when the interacting system of *dynamical* (*not* background) *supergravity* and a dynamical superbrane is considered.

Actually, the above statement is tantamount to saying that the coordinate functions of superbranes are *Goldstone fields*.[10] In flat superspace these Goldstone fields correspond to the translational symmetry and global supersymmetry that are broken by the superbrane worldvolume [22,23] i.e., by the position of the superbrane in superspace. Then, when a brane or particle interacting with (super)gravity is considered and, *moreover*, (super)gravity is described by an action on the same footing as the (super)brane, the global translations and global supersymmetry are replaced by *superdiffeomorphism* symmetry, which is the *gauge* symmetry of the coupled action [e.g., of the action (6.1); see also Appendix B 2). Thus the coordinate functions in such a dynamical system should be considered as *Goldstone fields for gauge symmetries*.

The Goldstone fields for the *gauge* symmetries are *always* pure gauge fields (compensators in the supergravity language). The ''unitary'' gauge where the Goldstone degrees of freedom are set to zero is always assumed in the consideration of Higgs phenomenon. For the case of spontaneously broken *internal* gauge symmetry, the only trace of the interaction with the Goldstone fields in this gauge turns out to be *the mass terms* in the gauge field equations. This is just the content of the standard Higgs phenomenon.

Now, when the Goldstone fields for *spacetime* (or superspace) gauge symmetry live on a subspace of spacetime (superspace), i.e., on the (super)brane worldvolume or (super)particle worldline, we may also expect a modification of the equations for the spacetime (or superspace) gauge fields. However, such a modification will only be produced by terms with support on the worldvolume or worldline. Hence these new terms modifying the gauge field equations should be just the *source terms*, like the rhs of Eq. (7.16) below [in particular, for $\hat{x}$ given by Eq. (7.6)]. Summarizing, *when the Goldstone fields are worldvolume fields, the counterpart of the mass terms appearing in the gauge field equa-*

[9]Note that the gauge with all components of $\hat{x}^{\mu}(\tau)$ equal to zero cannot be fixed due to the restrictions on the pure bosonic sector of the transformations since the diffeomorphism transformations have to be invertible and it is clear that a (world)line could not be represented by one point in any nondegenerate coordinate system. In contrast, the nondegeneracy of superdiffeomorphisms implies $\det(\delta_{\check{\alpha}}{}^{\check{\beta}}+\partial b^{\check{\beta}}(x,\theta)/\partial\theta^{\check{\alpha}})\neq 0$, which does not restrict the field parameter $b^{\check{\beta}}(x,0)$ and, hence, $\varepsilon_0^{\alpha}(x)$ [see Eq. (3.22)] in Eq. (7.3). This allows us to use the pull-back $\hat{\varepsilon}_0^{\alpha}:=\varepsilon_0^{\alpha}(\hat{x})$ of $\varepsilon_0^{\alpha}(x)$ to fix the gauge (7.4), where all the components of $\hat{\theta}^{\check{\alpha}}(\tau)$ are set to zero.

[10]More precisely, the bosonic and fermionic Goldstone fields are identified, respectively, with the bosonic coordinate functions corresponding to the directions orthogonal to the worldvolume and with a half of fermionic coordinate functions.

tions as a result of the usual Higgs mechanism are precisely the source terms in the Einstein equation and in some other gauge (super)field equations.

In complete correspondence with the usual Higgs phenomenon, the bosonic "unitary" gauge (7.7) clearly cannot remove the source from the Einstein equation. However, the super-Higgs effect [25] may be subtler when we have *fermionic* Goldstone fields defined on a surface in superspace (i.e., on the superbrane worldvolume). The gauge field equations that acquire a source term as a result of the super-Higgs effect would be the *superfield generalizations* of the Einstein equations and other gauge field equations, including that for the gravitino, $\Psi = J_\Psi$ [see Eq. (6.49)]. Let us discuss the fermionic *superfield* source term J_Ψ. In the "unitary" gauge $\hat{\theta}^{\check{\alpha}}(\xi)=0$ one can expect that $J_\Psi \propto \theta$ (we show below that this is indeed the case for a $D=4,N=1$ supergravity-superparticle interacting system). Now let us recall that the spacetime fermionic gauge field equation (the gravitino equation) is given by the *leading component* of the superfield equation $\Psi = J_\Psi$, i.e., by $\Psi|_{\theta=0} = J_\Psi|_{\theta=0}$. Thus, if $J_\Psi \propto \theta$, this gives $J_\Psi|_{\theta=0}=0$. This means that the *spacetime* equation for fermionic gauge field becomes sourceless, $\Psi|_{\theta=0}=0$, in the "unitary" gauge $\hat{\theta}^{\check{\alpha}}(\xi)=0$ [Eq. (7.4) for the superparticle].

We hope to return to the discussion of the fate of the superbrane degrees of freedom and other issues of the (super)Higgs phenomenon in the presence of superbranes in a future publication. Here our goal is more immediate: to find the explicit form of the equations of motion of the supergravity-superparticle interacting system in the fermionic "unitary" gauge (7.4).

C. Gauge fixed form of the equations of motion of the coupled system

In the WZ gauge supplemented by the condition (7.4), *the coupled system action is reduced to the action for supergravity interacting with a bosonic particle*. After integration on Grassmann variable θ in the supergravity part of the coupled action (6.1) this coupled action should become basically the same as the $D=4$ case of the action for the *supergravity-bosonic particle* interacting system considered in Ref. [20]. The only expected difference is the presence of the auxiliary fields, $G_a|_{\theta=0}, R|_{\theta=0}, \bar{R}|_{\theta=0}$, which are not essential as they appear in the component action only through quadratic combinations, without derivatives [9] and can be removed using their algebraic equations of motion. Furthermore, passing to the component approach to supergravity, which deals with fields on spacetime, one excludes the superspace diffeomorphisms $\delta_{diff}(b^M)$ with $\theta^{\check{\alpha}} \to \theta^{\check{\alpha}} + b^{\check{\alpha}}(x,\theta)$ from consideration. Then Eq. (7.5) is treated as the partial breaking of the local *spacetime* supersymmetry [20] [originating in $\tilde{\delta}_{gc}$ and given by Eqs. (B20),(B21),(B22) with $\theta=0$ and $G_a|_{\theta=0}=0=R|_{\theta=0}$].

Having in mind the results of [20], one would expect that, in the light of above correspondence, the auxiliary fields should have vanishing values in the gauge (7.4) and that the spacetime Rarita-Schwinger equations following from the superfield action for the interacting system, Eq. (6.1), would be sourceless in this gauge.

The analysis indicates that this is indeed the case. First, in the coupled system the scalar main superfields (2.10) are equal to zero on the mass shell, $R=0=\bar{R}$, Eqs. (6.21), (6.22). Thus $R|_{\theta=0}=0, \bar{R}|_{\theta=0}=0$. In contrast, the vector main superfield (2.9) becomes equal to the current potential (6.29), Eq. (6.28). Hence $G_a|_{\theta=0}=\mathcal{J}_a|_{\theta=0}$. However, it is seen that $\mathcal{J}_a|_{\theta=0}=0$ in the gauge (7.4). Indeed, $\mathcal{J}_a$ is constructed from the current prepotentials (6.25),(6.26),(6.27), which involve $\delta^8(Z-\hat{Z}) \equiv [\theta - \hat{\theta}(\tau)]^4 \delta^4(x-\hat{x})$, Eqs. (6.13),(6.14). In the gauge (7.4)

$$\mathcal{K}_a^{\ \alpha}(Z) = (\theta)^4 \int_{W^1} l(\tau) \left[\frac{1}{\hat{E}} \hat{E}_{a\tau} \hat{E}^\alpha \right] \Bigg|_{\hat{\theta}=0} \delta^4(x-\hat{x}), \quad (7.8)$$

$$\bar{\mathcal{K}}_a^{\dot{\alpha}}(Z) = (\theta)^4 \int_{W^1} l(\tau) \left[\frac{1}{\hat{E}} \hat{E}_{a\tau} \hat{E}^{\dot{\alpha}} \right] \Bigg|_{\hat{\theta}=0} \delta^4(x-\hat{x}), \quad (7.9)$$

$$\mathcal{K}_a^{\ b}(Z) = (\theta)^4 \int_{W^1} l(\tau) \left[\frac{1}{\hat{E}} \hat{E}_{a\tau} \hat{E}^b \right] \Bigg|_{\hat{\theta}=0} \delta^4(x-\hat{x}), \quad (7.10)$$

i.e., all current prepotentials become proportional to the highest possible power in the superspace Grassmann coordinates,

$$\hat{\theta}=0 \quad \Rightarrow \quad \begin{cases} \mathcal{K}_a^{\ \beta}(Z) \propto (\theta)^4, \\ \bar{\mathcal{K}}_a^{\ \dot{\beta}}(Z) \propto (\theta)^4, \\ \mathcal{K}_a^{\ b}(Z) \propto (\theta)^4. \end{cases} \quad (7.11)$$

Thus only the action of *four* Grassmann covariant derivatives on $\mathcal{K}_a^A(Z) = (\mathcal{K}_a^{\ b}, \mathcal{K}_a^{\ \alpha}, \bar{\mathcal{K}}_a^{\dot{\alpha}})$ can produce an expression which has a nonvanishing value for $\theta=0$. In particular,

$$\hat{\theta}=0 \;\Rightarrow\; \mathcal{J}_a \propto (\theta)^2, \quad (7.12)$$

and, hence, the auxiliary vector field of the minimal $D=4,N=1$ supergravity vanishes on the mass shell in the gauge (7.4),

$$\hat{\theta}=0, \quad \Rightarrow \quad G_a|_{\theta=0} = \mathcal{J}_a|_{\theta=0} = 0. \quad (7.13)$$

The Rarita-Schwinger equation can be derived setting $\theta=0$ in the superfield equation (6.49). However, in accordance with Eq. (7.11), $\mathcal{D}_A \mathcal{J}_a|_{\theta=0}=0$. Hence *the spacetime Rarita-Schwinger equation derived from the superfield action for the interacting supergravity-superbrane system becomes sourceless in the gauge (7.4),*

$$\hat{\theta}=0 \;\Rightarrow\; \Psi^a_{\dot{\alpha}}|_{\theta=0} \equiv \epsilon^{abcd} T_{bc}{}^{\alpha} \sigma_{d\alpha\dot{\alpha}}|_{\theta=0} = \frac{i}{4} \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{J}^a{}_{\theta=0} = 0. \quad (7.14)$$

One can verify using Eq. (3.6) that, due to Eqs. (7.13), (6.21), $T_{ab}{}^{\alpha}|_{\theta=0} = 2 e_a^\mu e_b^\nu \mathcal{D}_{[\mu} \psi^\alpha_{\nu]}(x)$. Hence the above statement is related to the true component gravitino equation.

The component Einstein equation for the coupled system can be obtained by setting $\theta=0$ in Eq. (6.50). Clearly, it possesses a source term, but only from the spin 2 current prepotential, Eq. (6.27),

$$
\begin{aligned}
R_{bc}{}^{ac}\Big|_{\theta=0} &= \frac{1}{16}\tilde{\sigma}_b^{\dot{\beta}\beta}[[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}]\mathcal{J}^a]_{\theta=0}\\
&= \frac{1}{64}\tilde{\sigma}_b^{\dot{\beta}\beta}\tilde{\sigma}_c^{\dot{\alpha}\alpha}[[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}][\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}]\mathcal{K}^{ac}]_{\theta=0}\\
&= \frac{1}{64}\tilde{\sigma}_b^{\dot{\beta}\beta}\tilde{\sigma}_c^{\dot{\alpha}\alpha}[[\mathcal{D}_\beta,\bar{\mathcal{D}}_{\dot{\beta}}][\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot{\alpha}}](\theta)^2(\bar{\theta})^2]_{\theta=0}\\
&\quad\times\int_{W^1} l(\tau)\left[\frac{1}{\hat{E}}\hat{E}^c_\tau\hat{E}^a\right]_{\hat{\theta}=0}\delta^4(x-\hat{x}).
\end{aligned}
\tag{7.15}
$$

In the WZ gauge (3.8) [recall that it can be fixed simultaneously with the gauge (7.4)], where Eqs. (3.1),(3.2) as well as (3.7) and $E_{\underline{\alpha}}{}^{\check{\beta}}|_{\hat{\theta}=0}=\delta_{\underline{\alpha}}{}^{\check{\beta}}$ are valid, Eq. (7.15) reads

$$
e(x)R_{bc}{}^{ac}|_{\theta=0}=c\int_{W^1} l(\tau)[\hat{e}_{b\tau}\hat{e}^a]\delta^4(x-\hat{x}), \tag{7.16}
$$

where c is a constant and $\hat{e}^a\equiv d\tau\, e^a_\tau=d\hat{x}^\mu(\tau)e^a_\mu(\hat{x})$ is the pull-back to the worldline of the bosonic form $e^a=dx^\mu e^a_\mu(x)=E^a|_{\theta=0}$. Equation (7.16) coincides with the one obtained from the supergravity-bosonic particle coupled action provided by the sum of the component action for supergravity and the bosonic particle action [20], for the case $D=4$.

VIII. CONCLUSIONS

We have provided in this paper a fully dynamical description of the $D=4, N=1$ supergravity and the massless superparticle coupled system. It is given by the sum of the superfield supergravity action [6] and the Brink-Schwarz action [7] for the massless superparticle. We have derived the complete set of superfield equations of motion for such a dynamical system.

The *superfield* generalizations of the Rarita-Schwinger (gravitino) equation and of the Einstein equation both acquire source terms. These sources are determined by the Grassmann spinor covariant derivatives of one vector superfield $\mathcal{J}_a$, the current "potential," which is a current density distribution with support on the worldline that appears at the right-hand side of the vector superfield equation (2.9) for the supergravity-superparticle coupled system.

The current potential $\mathcal{J}_a$ is covariantly conserved, Eqs. (6.34),(6.35), and turns out to be constructed from the spin 3/2 and spin 2 distributions (6.27),(6.25), which we call "current prepotentials." These current prepotentials obey Eqs. (6.37),(6.38),(6.40), as a result of the superparticle equations of motion.

In the interacting system with dynamical supergravity, the Goldstone nature of the superparticle coordinate functions $\hat{Z}^M(\tau)$ [22–24] allows one to fix the gauge (7.4) that sets the Grassmann coordinate function equal to zero, $\hat{\theta}(\tau)=0$ (cf. [25]). The analysis of the local (gauge) symmetries of the coupled system shows that it is possible to fix simultaneously $\hat{\theta}(\tau)=0$ and the Wess-Zumino gauge for the supergravity variables. Clearly, with these gauge fixing conditions, after integration over the superspace Grassmann coordinates θ and the elimination of the auxiliary fields $G_a|_{\theta=0}, R|_{\theta=0}, \bar{R}|_{\theta=0}$ by means of their (algebraic) equations of motion, the supergravity-superparticle interacting action (6.1) should reduce to the action for the supergravity-bosonic particle system investigated in [20]. To verify this conclusion we have studied the component equations of motion derived from the superfield equations for the supergravity-superparticle system and shown that they do coincide with the supergravity bosonic particle equations from [20] when both the WZ gauge and the gauge (7.4) are used. In particular, in the resulting gauge the component Rarita-Schwinger equations remain sourceless while the Einstein equations acquire a source term from the (super)particle.

The net outcome of our analysis is that the complete superfield action for the supergravity-superparticle interacting system has the supergravity-bosonic particle system as its gauge fixed version, as it is also the case for the group-manifold based action for the coupled system [18].

The applications of the present approach to the case of $D=4$ supergravity-superstring and supergravity-supermembrane systems requires previous knowledge of $D=4$ superspace supergravity with additional two-form and three-form in superspace (cf. [41]). This, as well as an analysis of the (super-)Higgs effect in the presence of superbranes and the study of the interaction of supergravity with more than one superbrane, will be the subject of future work.

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APPENDIX A: CHIRAL PROJECTORS

The algebra of covariant derivatives $\mathcal{D}_A$, Eq. (2.17), is encoded in the Ricci identities

$$
\mathcal{D}\mathcal{D}V_A=R_A^B V_B \leftrightarrow \begin{cases} \mathcal{D}\mathcal{D}V_a=R_a^b V_b, \\ \mathcal{D}\mathcal{D}V_\alpha=R_\alpha^\beta V_\beta, \\ \mathcal{D}\mathcal{D}V_{\dot{\alpha}}=R_{\dot{\alpha}}^{\dot{\beta}} V_{\dot{\beta}}, \end{cases} \tag{A1}
$$

where $V_A=(V_a, V_\alpha, V^{\dot{\alpha}})$ is an arbitrary supervector with tangent superspace Lorentz indices. Decomposing Eq. (A1) on the basic two-forms $E^A\wedge E^B$, one finds (see [12,13])

$$[\mathcal{D}_A,\mathcal{D}_B\}V_C=-T_{AB}{}^D\mathcal{D}_DV_C+R_{ABC}{}^DV_D. \tag{A2}$$

When the constraints (2.12),(2.13),(2.14),(2.16),(2.18),(2.19),(2.20) are taken into account, Eqs. (A1) [or (A2)] implies

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}V_\gamma=-\bar{R}\epsilon_{\gamma(\alpha}V_{\beta)}, \tag{A3}$$

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}V^\gamma=-\bar{R}V_{(\alpha}\delta_{\beta)}{}^\gamma, \tag{A4}$$

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\}=2i\sigma^a_{\alpha\dot\beta}\mathcal{D}_a\equiv 2i\mathcal{D}_{\alpha\dot\beta}, \quad \text{etc.} \tag{A5}$$

In their turn, Eqs. (A3), (A4) and their complex conjugates determine the form of the chiral projectors, i.e., they can be used to prove the identities

$$(\mathcal{D}\mathcal{D}-\bar{R})\mathcal{D}_\alpha\xi^\alpha:=(\mathcal{D}^\beta\mathcal{D}_\beta-\bar{R})\mathcal{D}_\alpha\xi^\alpha=0, \tag{A6}$$

$$(\overline{\mathcal{D}\mathcal{D}}-R)\bar{\mathcal{D}}_{\dot\alpha}\bar{\xi}^{\dot\alpha}:=(\bar{\mathcal{D}}_{\dot\beta}\bar{\mathcal{D}}^{\dot\beta}-R)\bar{\mathcal{D}}_{\dot\alpha}\bar{\xi}^{\dot\alpha}=0,$$

$$\mathcal{D}_\alpha(\mathcal{D}\mathcal{D}-\bar{R})U=0, \tag{A7}$$

$$\bar{\mathcal{D}}_{\dot\alpha}(\overline{\mathcal{D}\mathcal{D}}-R)U=0,$$

where $\xi^\alpha,\bar{\xi}^{\dot\alpha}$ are arbitrary spinor superfields and U is an arbitrary scalar superfield. Note that the chiral projectors are different when acting on superfields with Lorentz group indices, e.g.,

$$\left(\mathcal{D}\mathcal{D}+\frac{1}{2}\bar{R}\right)\mathcal{D}_\alpha U\equiv 0. \tag{A8}$$

APPENDIX B: ON SUPERDIFFEOMORPHISM INVARIANCE AND SUPERSPACE GENERAL COORDINATE INVARIANCE

In this appendix we present a complete account of all the manifest superfield gauge symmetries of superfield supergravity. We discuss separately the *active* and *passive* forms of *general coordinate invariance* which we call *general coordinate symmetry*, δ_{gc}, and *superdiffeomorphism symmetry*, δ_{diff}, respectively. Although both symmetries are known, usually only one of these two symmetries are considered in the literature. The reason is that the invariance of the Lagrangian form in a field theory (or of the Lagrangian integral form in a superfield theory) under δ_{diff} implies immediately the invariance under δ_{gc} (see Appendix A 1 for further discussion). However, when dealing with a new type of system where some of the (super)fields live on a submanifold of (super)space (e.g., on the superparticle worldline) while others are defined on whole superspace, it is important to take into account that δ_{diff} and δ_{gc} act differently. In fact, this difference is already seen even for "free" supergravity where we show (Appendix B 2) that the Wess-Zumino gauge is invariant under δ_{gc}, whereas the δ_{diff} transformations are broken by the Wess-Zumino gauge fixing conditions down to *spacetime* local supersymmetry and *spacetime* diffeomorphisms.

First, let us note that the set of superspace local Lorentz transformations

$$\delta_LE^A=E^BL_B{}^A(Z)$$

$$\Leftrightarrow \begin{cases} \delta_LE^a=E^bL_b{}^a(Z), \;\; L^{ab}=-L^{ba}:=L^{ab}(Z), \\ \delta_LE^\alpha=E^\beta L_\beta{}^\alpha, \;\; L_\beta{}^\alpha=\frac{1}{4}L^{ab}(\sigma_a\tilde{\sigma}_b)_\beta{}^\alpha, \\ \delta_LE^{\dot\alpha}=E^{\dot\beta}L_{\dot\beta}{}^{\dot\alpha}, \;\; L_{\dot\beta}{}^{\dot\alpha}=-\frac{1}{4}L^{ab}(\sigma_a\tilde{\sigma}_b)^{\dot\alpha}{}_{\dot\beta}, \end{cases}$$

$$\delta_Lw^{ab}=\mathcal{D}L^{ab}, \tag{B1}$$

is a manifest symmetry of the constraints. Clearly, they do not act on the superspace coordinates $\delta_LZ^M=0$.

Second, the constraints (2.12),(2.13),(2.14),(2.16),(2.18),(2.19),(2.20), as relations among differential forms, are independent on the choice of a superspace local coordinate system. This evident statement can be formulated as an invariance under *superdiffeomorphism* (i.e., superspace diffeomorphism) transformations δ_{diff} (see [20]),

$$Z'^M=Z^M+b^M(Z): \quad \begin{cases} x'^\mu=x^\mu+b^\mu(x,\theta), \\ \theta'^{\check\alpha}=\theta^{\check\alpha}+\varepsilon^{\check\alpha}(x,\theta), \end{cases} \tag{B2}$$

$$E'^A(Z')=E^A(Z), \quad w'^{ab}(Z')=w^{ab}(Z). \tag{B3}$$

The statement of the invariance of differential forms, Eqs. (6.7),

$$\delta_{diff}Z^M=Z'^M-Z^M=b^M(Z), \tag{B4}$$

$$\delta_{diff}E^A=E'^A(Z')-E^A(Z)=0, \tag{B5}$$

$$\delta_{diff}w^{ab}=w'^{ab}(Z')-w^{ab}(Z)=0, \quad \text{etc.}, \tag{B6}$$

just implies that Eq. (B2) [or Eq. (6.9)] describes a change of local coordinates, but does not act on the superspace "points."[11] Thus δ_{diff} invariance can be treated as the *passive* form of the general coordinate symmetry in superspace.

Third, the set of constraints is invariant under *general coordinate transformations of superspace* δ_{gc} [6,13,20] (*active* form of general coordinate symmetry). δ_{gc} is the symmetry under an arbitrary change of superspace "points" (in contrast to a change of local coordinates as in the case of δ_{diff})

$$\delta_{gc}Z^M=t^M(Z^M). \tag{B7}$$

[11]The prime under differential form means, e.g., $E'^A(Z')\equiv E^A[Z(Z')]$. Thus Eq. (B5) is the trivial identity $E^A(Z)\equiv E^A[Z(Z')]$ reflecting the freedom of choosing an arbitrary local coordinate system. Nevertheless, the *form invariance* of an action or of an equation under δ_{diff} requires the model to be formulated using the supervielbein superfield (in the bosonic case, when spinor fields are absent, it is enough to introduce a metric field). Thus δ_{diff} can be used as a gauge principle for gravity and supergravity models.

The transformation of differential forms under the change of arguments (B7) is given by the Lie derivative $\mathcal{L}_t \equiv i_t d + d i_t$, i.e.,

$$\delta_{gc} E^A(Z) := E^A(Z+t) - E^A(Z) = \mathcal{L}_t E^A(Z) = i_t T^A + \mathcal{D}(i_t E^A) + E^B i_t w_B{}^A, \tag{B8}$$

$$\delta_{gc} w^{ab}(Z) := w^{ab}(Z+t) - w^{ab}(Z) = \mathcal{L}_t w^{ab}(Z) = i_t R^{ab} + \mathcal{D}[i_t w^{ab}(Z)], \quad \text{etc.}, \tag{B9}$$

where

$$i_t E^A = t^M E_M^A =: t^A, \tag{B10}$$

$$i_t w^{ab}(Z) = t^M(Z) w_M^{ab}(Z) = t^A(Z) w_A^{ab}(Z), \tag{B11}$$

$$i_t T^A = E^B t^C T_{CB}{}^A, \quad i_t R^{ab} = E^B t^C R_{CB}^{ab}. \tag{B12}$$

The last terms in Eqs. (B8),(B9) can be regarded as a Lorentz transformation (B1) induced by δ_{gc}, $\delta_L(L^{ab} = i_t w^{ab})$ and, thus, they can be conventionally ignored in a manifestly Lorentz invariant theory. In other words, one may consider, equivalently, the superposition of transformations $\delta_{gc}(t) + \delta_L(L^{ab} = -i_t w^{ab})$ instead of the original $\delta_{gc}(t)$. These transformations were called *supergauge transformations* in [13].

The simplest way to see that the constraints are invariant under the superspace general coordinate transformations δ_{gc} is to recall that δ_{gc} implies moving from one superspace "point" to another one and that, since the constraints are satisfied at any superspace "point," they are invariant.[12]

Note also that the transformations of superforms, (B8), (B9), $\delta_{gc} T^A = \mathcal{D} i_t T^A + i_t \mathcal{D} T^A$, etc., imply the usual transformation rules for the (super)tensors (zero forms). For instance, for $T_{CB}{}^A$ defined by Eqs. (2.2)–(2.4), $T^A := \frac{1}{2} E^C \wedge E^B T_{BC}{}^A$, one obtains $\delta_{gc} T_{CB}{}^A = t^D \mathcal{D}_D T_{CB}{}^A$.

The fermionic general coordinate transformations (B7), (B8), with parameter $t^M(Z) = \epsilon^{\underline{\alpha}}(Z) E_{\underline{\alpha}}{}^M(Z)$, i.e., [see Eq. (B10)],

$$i_\epsilon E^a = 0, \quad i_\epsilon E^{\underline{\alpha}} = \epsilon^{\underline{\alpha}}(Z), \tag{B13}$$

can be treated as a local supersymmetry [13], while the bosonic transformations (B7) with parameter $t^M(Z) = t^a(Z) E_a{}^M(Z)$ provide the superfield generalization of the spacetime general coordinate transformations. However, with such treatment, the origin of the local supersymmetry of the *component* formulation of supergravity, i.e., of supergravity formulated as a theory of fields on spacetime, becomes slightly obscure. The following observation helps to make the above-mentioned relation clearer.

[12] Denote the set of constraints by $C_2^{\mathcal{A}}(Z) \equiv \frac{1}{2} E^B \wedge E^C C_{CB}{}^{\mathcal{A}}(Z) = 0$. They are satisfied at any superspace point Z^M. Thus $C_2^{\mathcal{A}}[Z + t(Z)] = 0$ too and $\delta_{gc} C_2^{\mathcal{A}}(Z) = C_2^{\mathcal{A}}[Z + t(Z)] - C_2^{\mathcal{A}}(Z) = 0$.

Since diffeomorphism invariance $\delta_{diff}(b^M)$ [Eqs. (B2), (B3)] is guaranteed, one can consider, instead of Eq. (B7), the *variational version of the general coordinate transformations* [6] $\tilde{\delta}_{gc}$ with parameter $t^A(Z) = t^M E_M^A := i_t E^A$, defined by (see [21])

$$\tilde{\delta}_{gc}(t^A) = \delta_{gc}(t^M) + \delta_{diff}(b^M = -t^M) + \delta_L(L^{ab} = -i_t w^{ab}). \tag{B14}$$

$\tilde{\delta}_{gc}(t)$ does not act on the superspace coordinates and acts on superforms through the covariant Lie derivative

$$\tilde{\delta}_{gc} Z^M = 0, \tag{B15}$$

$$\tilde{\delta}_{gc} E^A(Z) = i_t T^A + \mathcal{D} t^A, \tag{B16}$$

$$\tilde{\delta}_{gc} w^{ab}(Z) = i_t R^{ab}, \quad \text{etc.} \tag{B17}$$

The *superfield* local supersymmetry $\delta_{ls}(\epsilon^{\underline{\alpha}})$ can be identified with the variational version $\tilde{\delta}_{gc}(\epsilon^{\underline{\alpha}})$ of the fermionic general coordinate transformations (B15),(B16):

$$\delta_{ls}(\epsilon^{\underline{\alpha}}) = \tilde{\delta}_{gc}[t^a = 0, t^\alpha = \epsilon^\alpha(Z), t^{\dot\alpha} = \bar{\epsilon}^{\dot\alpha}(Z)]. \tag{B18}$$

Then the relation with the local supersymmetry of the component formulation of supergravity becomes especially transparent.

Indeed, Eqs. (B16),(B17) with the torsion and curvature two-forms from Eqs. (2.12), (2.13), (2.14), (2.16), and $t^A = [0, \epsilon^{\underline{\alpha}}(Z), \bar{\epsilon}^{\dot\alpha}(Z)]$ provide us with the following local superspace supersymmetry transformations

$$\delta_{ls} Z^M = 0 \quad \Leftrightarrow \quad \begin{cases} \delta_{ls} x^\mu = 0, \\ \delta_{ls} \theta^{\check\alpha} = 0, \end{cases} \tag{B19}$$

$$\delta_{ls} E^a = -2i E^\alpha \sigma^a_{\alpha\dot\beta} \bar{\epsilon}^{\dot\beta}(Z) - 2i \bar{E}^{\dot\alpha} \sigma^a_{\beta\dot\alpha} \epsilon^\beta, \tag{B20}$$

$$\delta_{ls} E^\alpha = \mathcal{D}\epsilon^\alpha + \frac{i}{8} E^a [(\epsilon \sigma_a \tilde{\sigma}_b)^\alpha G^b + (\bar{\epsilon} \tilde{\sigma}_a)^\alpha R], \tag{B21}$$

$$\delta_{ls} \bar{E}^{\dot\alpha} = \mathcal{D}\bar{\epsilon}^{\dot\alpha} - \frac{i}{8} E^a [(\tilde{\sigma}_b \sigma_a \bar{\epsilon})^{\dot\alpha} G^b + (\tilde{\sigma}_a \epsilon)^{\dot\alpha} \bar{R}], \tag{B22}$$

$$\delta_{ls} w^{\alpha\beta} = -E^{(\alpha} \epsilon^{\beta)} \bar{R} - \frac{i}{8} E^a [(\tilde{\sigma}_a)^{\dot\gamma(\alpha} \epsilon^{\beta)} \bar{\mathcal{D}}_{\dot\gamma} \bar{R} + (\epsilon \sigma_a \tilde{\sigma}_b)^{(\alpha} \mathcal{D}^{\beta)} G^b]. \tag{B23}$$

The superspace local supersymmetry transformations δ_{ls} of the main superfields (2.9)–(2.11) are determined by

$$\delta_{ls} R = \epsilon^\alpha \mathcal{D}_\alpha R, \quad \delta_{ls} \bar{R} = \bar{\epsilon}^{\dot\alpha} \bar{\mathcal{D}}_{\dot\alpha} \bar{R}, \tag{B24}$$

$$\delta_{ls} G^a = \epsilon^\alpha \mathcal{D}_\alpha G^a + \bar{\epsilon}^{\dot\alpha} \bar{\mathcal{D}}_{\dot\alpha} G^a, \tag{B25}$$

$$\delta_{ls} W^{\alpha\beta\gamma} = \epsilon^\delta \mathcal{D}_\delta W^{\alpha\beta\gamma}, \quad \delta_{ls} \bar{W}^{\dot\alpha\dot\beta\dot\gamma} = \bar{\epsilon}^{\dot\delta} \bar{\mathcal{D}}_{\dot\delta} \bar{W}^{\dot\alpha\dot\beta\dot\gamma}. \tag{B26}$$

Setting $\theta=0$ in the δ_{ls} transformations (B20)–(B26) we arrive at the transformation rules of the *off-shell* supersymmetry characteristic of the minimal formulation of the $D=4, N=1$ supergravity. To this end one needs the expression of the spinor derivatives of the main superfields in terms of the Riemann curvature $[R_{cd}{}^{ab}$, Eqs. (2.15),(2.26)] and the gravitino field strengths $[T^{\alpha}_{bc}, T^{\dot\alpha}_{bc}$, Eqs. (2.22),(2.23)] with the use of the consequences of the constraints (2.12),(2.13), (2.14),(2.16),(2.18),(2.19),(2.20). For instance, $\mathcal{D}_\alpha R$ and $\overline{\mathcal{D}}_{\dot\alpha}\bar{R}$ are expressed through the gravitino field strength $T_{ab\beta}$ with the use of Eq. (2.24).

The variational version of the superspace general coordinate transformations $\tilde{\delta}_{gc}$ with bosonic parameters t^a can be called "local translations," $\delta_{lt}=\tilde{\delta}_{gc}(t^a, t^\alpha=0)$

$$\delta_{lt}Z^M=0 \quad \Leftrightarrow \quad \begin{cases}\delta_{lt}x^\mu=0,\\ \delta_{lt}\theta^{\check\alpha}=0,\end{cases} \tag{B27}$$

$$\delta_{lt}E^a=\mathcal{D}t^a+\frac{1}{8}E_b\varepsilon^{abcd}t_c(Z)G_d, \tag{B28}$$

$$\delta_{lt}E^\alpha=-\frac{i}{8}E^\beta t^a(\sigma_a\tilde{\sigma}_b)_\beta{}^\alpha G^b+\frac{i}{8}\bar{E}^{\dot\beta}\epsilon^{\alpha\beta}t^a\sigma_{a\beta\dot\beta}R$$
$$+E^b t^a T_{ab}{}^\alpha, \tag{B29}$$

etc.

In the pure bosonic case it is precisely this symmetry (this form of the spacetime general coordinate invariance) that provides the possibility of treating general relativity as a gauge theory of the Poincaré group [42] (see [21] for further discussion).

1. Gauge symmetries of the "free" supergravity superfield action

The action (4.1) is evidently invariant under the superdiffeomorphisms (B2),(B3),

$$\delta_{diff}S_{SG}=0. \tag{B30}$$

This invariance is a simple consequence of the possibility of changing variables in *any* integral (see footnote 11); but moreover, in our case the action is *form invariant* as the theory is formulated in terms of the supervielbein.

In the pure bosonic case, where the counterpart of the above statement means that the action is an integral of a differential form (Lagrangian form), $S=\int_{M^D}L_D$, the general coordinate invariance follows then from the simple observation that the variation of the Lagrangian form under δ_{gc}, as well as under $\tilde{\delta}_{gc}$, is given (see [21]) by a Lie derivative: $\delta_{gc}L_D\equiv\tilde{\delta}_{gc}L_D=i_t dL_D+d(i_t L_D)$. Then the first term vanishes as it contains the exterior derivative of D-form on a D-dimensional manifold, while the second term is a total derivative which does not contribute for a spacetime M^D without boundary. This statement is usually treated as a manifestation of the equivalence between the active and passive forms of general coordinate transformations. However, although these symmetries imply each other in field theories, their role is different as we show in Appendix C [Eqs. (C2), (C3)].

In the case of superspace the action is written in terms of an integral (Berezin) form. Nevertheless, the general coordinate invariance of the superdiffeomorphism invariant action can be also established easily. For instance, to prove the invariance of the action (4.1) under the variational version $\tilde{\delta}_{gc}(t^A)$ (B15),(B16),(B17) of the superspace general coordinate transformations, including local supersymmetry [Eqs. (B19),(B20)–(B26)], one has to use the identity

$$\int d^4x\,\tilde{d}^4\theta\,E(\mathcal{D}_A\xi^A+\xi^B T_{BA}{}^A)(-1)^A\equiv 0, \tag{B31}$$

which is valid for any complex superfield $\xi^A=[C^a(Z),\nu^\alpha(Z),\overline{\mu}^{\dot\alpha}(Z)]$.

A variation of superdeterminant has the form

$$\delta E\equiv E E_A{}^M\delta E_M{}^A(-1)^A. \tag{B32}$$

To compute $\tilde{\delta}_{gc}E$ one substitutes $i_M(\tilde{\delta}_{gc}E^A)$ from Eq. (B16) for $\delta E_M{}^A$, and finds

$$\tilde{\delta}_{gc}E=E(-1)^A\mathcal{D}_A t^A+E(-1)^A t^B T_{BA}{}^A. \tag{B33}$$

Then the identity (B31) implies

$$\tilde{\delta}_{gc}S_{SG}=\int d^8Z\,E(\mathcal{D}_A t^A+t^B T_{BA}{}^A)(-1)^A=0. \tag{B34}$$

This completes the proof of general coordinate symmetry.

Note that, as the constraints of minimal supergravity (2.12),(2.13) imply $(-1)^A T_{BA}{}^A=0$, the identity (B31) simplifies to

$$\int d^8Z E\mathcal{D}_A\xi^A(-1)^A=0. \tag{B35}$$

Since $\delta_{ls}=\tilde{\delta}_{gc}[t^A=(0,\epsilon^{\underline{\alpha}})]$, Eq. (B18), this proves, in particular, the invariance under the local supersymmetry transformations (B19)–(B26) [which imply, e.g., $\delta_{ls}E^a_M=-2iE^\alpha_M\sigma^a_{\alpha\dot\beta}\bar{\epsilon}^{\dot\beta}(Z)+2i\bar{E}^{\dot\alpha}_M\sigma^a_{\beta\dot\alpha}\epsilon^\beta$]. Specifically, one finds

$$\delta_{ls}S_{SG}=-\int d^8Z\,E\mathcal{D}_{\underline{\alpha}}\epsilon^{\underline{\alpha}}=\int d^8Z\,\mathcal{D}_M(EE^M_{\underline{\alpha}})\epsilon^{\underline{\alpha}}$$
$$\equiv-\int d^8Z\,T_{\underline{\alpha}A}{}^A(-1)^A\epsilon^{\underline{\alpha}}=0. \tag{B36}$$

2. On the gauge symmetries of the supergravity-superparticle coupled system

The invariance of the coupled action under superspace diffeomorphisms δ_{diff} follows from the fact that Eqs. (6.10), (6.9),(B5) imply

$$\delta_{diff}\hat{E}^a=\hat{E}'^a(\hat{Z}+\delta_{diff}\hat{Z})-\hat{E}(\hat{Z})=0. \tag{B37}$$

Thus

$$\delta_{diff} S_{sp} = 0 \tag{B38}$$

and, since $\delta_{diff} S_{SG} = 0$ [Eq. (B30)] we find

$$\delta_{diff} S = 0. \tag{B39}$$

On the other hand, as the superspace coordinates Z^M [not to be confused with $\hat{Z}^M(\tau)$] do not enter in the superparticle action, the general coordinate transformations δ_{gc} [Eqs. (B7),(B8)] supplemented by the definition

$$\delta_{gc} \hat{Z}^M(\tau) = 0, \tag{B40}$$

trivially give $\delta_{gc} S_{sp} = 0$, and the invariance of the supergravity action $\delta_{gc} S_{SG} = 0$ gives

$$\delta_{gc} S = 0. \tag{B41}$$

Then the invariance under the variational copy of the superspace general coordinate transformations, $\tilde{\delta}_{gc}$, Eqs. (B15), (B16) supplemented by the definition

$$\tilde{\delta}_{gc} \hat{Z}^M(\tau) = -t^M(\hat{Z}) \equiv -t^a(\hat{Z}) E_a^M(\hat{Z}) - \epsilon^\alpha(\hat{Z}) E_\alpha^M(\hat{Z}) - \bar{\epsilon}^{\dot{\alpha}}(\hat{Z}) E_{\dot{\alpha}}^M(\hat{Z}), \tag{B42}$$

follows from the δ_{gc} and δ_{diff} invariances,[13]

$$\tilde{\delta}_{gc} S \equiv \tilde{\delta}_{gc} S_{sp} = 0. \tag{B43}$$

Note that in the "superparticle sector" of the configuration space of the interacting system the action of $\tilde{\delta}_{gc}$, Eq. (B42), coincides with the action of diffeomorphism transformations.

In particular, the transformation of the fermionic coordinate function $\hat{\theta}^{\check{\alpha}}(\tau)$ under the full set of local symmetries of the interacting system [including $\delta_{diff}(b)$ Eq. (6.10) and $\tilde{\delta}_{gc}(t)$, Eq. (B42) and the worldline κ symmetry] acquires the form

$$\delta \hat{\theta}^{\check{\alpha}}(\tau) = b^{\check{\alpha}}(\hat{Z}) - t^{\check{\alpha}}(\hat{Z}) + \delta_\kappa \hat{\theta}^{\check{\alpha}}(\tau), \tag{B44}$$

where $\delta_\kappa \hat{\theta}^{\check{\alpha}}(\tau)$ is defined by the Eq. (5.15) with $M = \check{\alpha}$.

APPENDIX C: MORE ON THE WESS-ZUMINO GAUGE

1. Decomposition of superfields in the Wess-Zumino gauge

The decomposition of the superfields $E_{\check{\alpha}}{}^a, E_{\check{\alpha}}{}^{\underline{\alpha}}, w_{\check{\alpha}}{}^{ab}$ in power series on θ is completely determined by Eqs. (3.14), (3.15),(3.16). To make such an expansion explicit one can use the formal operator [28]

$$\frac{1}{(1+\theta\partial)} = \frac{1}{(1+\theta^{\underline{\alpha}}\mathcal{D}_{\underline{\alpha}})}. \tag{C1}$$

The action of such an operator is well defined on superfields (as they are polynomials in θ) and produces expressions involving *covariant* Grassmann derivatives $\mathcal{D}_{\underline{\alpha}}$ when Eq. (C1) acts on the torsion and curvature superfields. For instance, from Eq. (3.14) one finds

$$E_{\check{\alpha}}^a(Z) = \frac{1}{(1+\theta\partial)} \theta^{\underline{\beta}} T^a_{\underline{\beta}\check{\alpha}} = \theta^{\underline{\beta}} \frac{1}{(2+\theta^{\underline{\alpha}}\mathcal{D}_{\underline{\alpha}})} (E^C_{\check{\alpha}} T^a_{C\underline{\beta}}), \tag{C2}$$

where we use Eq. (3.13) and the identity

$$\frac{1}{(k+\theta\partial)} \theta^{\underline{\beta}} \equiv \theta^{\underline{\beta}} \frac{1}{(k+1+\theta\partial)} \tag{C3}$$

[which follows from $(k+\theta\partial)\theta^{\underline{\beta}} = \theta^{\underline{\beta}}(k+1+\theta\partial)$].

As one more example, let us present the explicit form of the $d\theta$ component of the expression (3.17), which enters Eq. (3.15):

$$\begin{aligned} \mathcal{D}_{\check{\alpha}} \theta^{\underline{\beta}} &= \delta_{\check{\alpha}}^{\underline{\beta}} + \theta^{\underline{\gamma}} w_{\check{\alpha}\underline{\gamma}}{}^{\underline{\beta}} \\ &= \delta_{\check{\alpha}}^{\underline{\alpha}} \left(\delta_{\underline{\alpha}}^{\underline{\beta}} + \frac{1}{4} \theta^{\underline{\gamma}} \Gamma_{ab\underline{\gamma}}{}^{\underline{\beta}} \frac{1}{(1+\theta\partial)} \theta^{\underline{\beta}} R_{\underline{\beta}\underline{\alpha}}{}^{ab} \right) \\ &= \delta_{\check{\alpha}}^{\underline{\alpha}} \left(\delta_{\underline{\alpha}}^{\underline{\beta}} + \frac{1}{4} \theta^{\underline{\gamma}} \Gamma_{ab\underline{\gamma}}{}^{\underline{\beta}} \theta^{\underline{\beta}} \frac{1}{(2+\theta^{\underline{\epsilon}}\mathcal{D}_{\underline{\epsilon}})} R_{\underline{\beta}\underline{\alpha}}{}^{ab} \right). \end{aligned} \tag{C4}$$

The complete decomposition of the dx^μ components of the forms E^a, $E^{\underline{\alpha}}$, w^{ab} is governed by the dx^μ components of Eqs. (3.14), (3.15), (3.16), e.g.,

$$\theta\partial E_\mu^a = E_\mu^B \theta^{\underline{\beta}} T_{\underline{\beta}B}{}^a = -\theta^{\underline{\beta}} E_\mu^B T_{B\underline{\beta}}{}^a. \tag{C5}$$

Clearly, Eq. (C5) involves the nilpotent operator $\theta\partial \equiv \theta^{\check{\alpha}}\partial_{\check{\alpha}} = \theta^{\underline{\alpha}}\mathcal{D}_{\underline{\alpha}}$. This nilpotent operator has an evident kernel: the leading component of the superfield, e.g., $E_m^a|_{\theta=0} = e_m^a(x)$. However, as it was observed in [28], this operator can be considered as *invertible in the space of superfields with the vanishing leading components*. Thus one can write as well the formal expansion for E_μ^a by subtracting the kernel, $E_\mu^a(Z) \rightarrow [E_\mu^a(Z) - E_\mu^a|_{\theta=0}]$ (thus arriving at a superfield with a vanishing leading component) and using the formal relation (C3) with $k=0$ (which is meaningful in the space of superfields with vanishing leading components) to arrive at

$$E_\mu^a(Z) = E_\mu^a|_{\theta=0} - \theta^{\underline{\beta}} \frac{1}{(1+\theta^{\underline{\alpha}}\mathcal{D}_{\underline{\alpha}})} [E_\mu^B(Z) T_{B\underline{\beta}}{}^a]. \tag{C6}$$

---

[13]The breaking of $\tilde{\delta}_{gc}$ invariance, discussed in [20,21], is a spontaneous symmetry breaking.

2. Gauge symmetries preserving the Wess-Zumino gauge

To find the full set of local symmetries that preserve the WZ gauge (3.8)[14] one may write the infinitesimal variations $\delta_{diff}(b^M), \tilde{\delta}_{gc}(t^A), \delta_L(L^{ab})$ of the WZ conditions (3.8) and require their preservation,

$$[\theta^{\breve{\alpha}}+b^{\breve{\alpha}}(Z)][E'^{A}_{\breve{\alpha}}(Z')+\delta_L E^A_{\breve{\alpha}}(Z)+\tilde{\delta}_{gc}E^A_{\breve{\alpha}}(Z)] =[\theta^{\breve{\alpha}}+b^{\breve{\alpha}}(Z)]\delta^A_{\breve{\alpha}}, \tag{C7}$$

$$[\theta^{\breve{\alpha}}+b^{\breve{\alpha}}(Z)][w'^{ab}_{\breve{\alpha}}(Z')+\tilde{\delta}_{gc}w^{ab}_{\breve{\alpha}}(Z)+\delta_L w^{ab}_{\breve{\alpha}}(Z)]=0. \tag{C8}$$

Here $\tilde{\delta}_{gc}$ is defined by Eqs. (B15),(B16),(B17) and the primes reflect the superdiffeomorphism transformations, Eqs. (B15),(6.7), or (B5),(6.9). Hence

$$E'^{A}_{\breve{\alpha}}(Z')=E^A_{\breve{\alpha}}(Z)-\partial_{\breve{\alpha}}b^M E^A_M(Z), \tag{C9}$$

$$w'^{ab}_{\breve{\alpha}}(Z')=w^{ab}_{\breve{\alpha}}(Z)-\partial_{\breve{\alpha}}b^M w^{ab}_M(Z). \tag{C10}$$

The terms $\tilde{\delta}_{gc}E^A_{\breve{\alpha}}(Z)$ and $\tilde{\delta}_{gc}w^{ab}_{\breve{\alpha}}(Z)$ in Eqs. (C7),(C8) are defined by the contraction of Eqs. (B16),(B17),

$$\tilde{\delta}_{gc}E^A_{\breve{\alpha}}(Z)=t^B T^A_{B\breve{\alpha}}+\mathcal{D}_{\breve{\alpha}}t^A, \tag{C11}$$

$$\tilde{\delta}_{gc}w^{ab}_{\breve{\alpha}}(Z)=t^D R^{ab}_{D\breve{\alpha}}. \tag{C12}$$

Finally, the Lorentz transformations have the standard form (B1),(3.21),

$$\delta_L E^A_{\breve{\alpha}}(Z)=E^B_{\breve{\alpha}}(Z)L_B{}^A(Z), \tag{C13}$$

$$\delta_L w^{ab}_{\breve{\alpha}}(Z)=\mathcal{D}_{\breve{\alpha}}L^{ab}(Z), \tag{C14}$$

$$L_B{}^A(Z)=\begin{pmatrix} L_b{}^a & 0 \\ 0 & L_{\underline{\beta}}{}^{\underline{\alpha}}\end{pmatrix}, \quad L^{ab}=-L^{ba},$$

$$L_{\underline{\beta}}{}^{\underline{\alpha}}=\tfrac{1}{4}L^{ab}\gamma_{ab\,\underline{\beta}}{}^{\underline{\alpha}}. \tag{C15}$$

By algebraic manipulation with the use of the recurrent relations (3.14),(3.15),(3.16), one can present Eqs. (C7), (C8) in the form

$$\theta\partial(b^A-t^A)=(b^B-t^B)\theta^{\underline{\gamma}}T_{\underline{\gamma}B}{}^A+\theta^{\underline{\gamma}}(L_{\underline{\gamma}}{}^{\underline{\beta}}-b^M w_{M\underline{\gamma}}{}^{\underline{\beta}}), \tag{C16}$$

$$\theta\partial[L^{ab}(Z)-b^M w_M{}^{ab}]=-(b^D-t^D)\theta^{\underline{\gamma}}R_{\underline{\gamma}D}{}^{ab}. \tag{C17}$$

Setting $t^A=0$ in Eqs. (C16),(C17), one arrives at Eqs. (3.19), (3.20).

At zero order of the weak field approximation one finds the set of equations [cf. (3.23)–(3.25)]

$$\theta\partial(b^a-t^a)=-2i(\varepsilon^{\underline{\beta}}-\epsilon^{\underline{\beta}})\gamma^a_{\underline{\beta\gamma}}\theta^{\underline{\gamma}}, \tag{C18}$$

$$\theta\partial(\varepsilon^{\underline{\alpha}}-\epsilon^{\underline{\alpha}})=\theta^{\underline{\beta}}L_{\underline{\beta}}{}^{\underline{\alpha}}, \tag{C19}$$

$$\theta\partial L^{ab}(Z)=0, \tag{C20}$$

which are solved by

$$t^a(Z)-b^a(Z)=t^a_-(x)-2i\theta\gamma^a\epsilon_-(x)-\frac{i}{4}\theta(\gamma_{bc}\gamma^a)\theta l^{bc}(x), \tag{C21}$$

$$\epsilon^{\underline{\alpha}}(Z)-\varepsilon^{\underline{\alpha}}(Z)=\epsilon^{\underline{\alpha}}_-(x)-\theta^{\underline{\beta}}l_{\underline{\beta}}{}^{\underline{\alpha}}(x), \tag{C22}$$

$$L^{ab}(Z)=l^{ab}(x), \tag{C23}$$

where $t^a_-(x), \epsilon^{\underline{\alpha}}_-(x)$ are arbitrary vector and spinor functions and $l^{ab}(x)$ are local Lorentz parameters.

In the general case the WZ gauge is preserved by the part of the original superspace local symmetry corresponding to the parameters that are not restricted by Eqs. (3.19),(3.20). These are the *superfield* parameter

$$t^A_+(Z)=b^A(Z)+t^A(Z), \tag{C24}$$

the vector and spinor *field* parameters

$$t^A_-(x)=[t^a_-(x),\epsilon^{\underline{\alpha}}_-(x)]=[t^A(Z)-b^A(Z)]|_{\theta=0}, \tag{C25}$$

and the antisymmetric tensor *field* parameter

$$l^{ab}(x)=L^{ab}(Z)|_{\theta=0}. \tag{C26}$$

In particular, both the spacetime diffeomorphisms and general coordinate transformations [with parameters $b^a(Z)|_{\theta=0}, t^a(Z)|_{\theta=0}$], as well as Lorentz [$l^{ab}(x)$] and local supersymmetry [$\epsilon^{\underline{\alpha}}(x)=\epsilon^{\underline{\alpha}}(Z)|_{\theta=0}$] transformations preserve the WZ gauge.

3. On the general coordinate invariance of the Wess-Zumino gauge

The fact that the conditions (3.19), (3.20) on the parameters of the symmetry that preserve the WZ gauge do not restrict also the superfield parameter (C24) requires some comments. Equation (B14) can be rewritten as

$$\delta_{gc}(t^M)=\tilde{\delta}_{gc}(t^A)+\delta_{diff}(b^M=t^M)+\delta_L(L^{ab}=i_t w^{ab}). \tag{C27}$$

Then, Eqs. (3.19),(3.20) become the identity $0\equiv 0$ when $b^M=t^M$, $L^{ab}=i_t w^{ab}$. This observation shows that the superfield symmetry which preserves the WZ gauge is just the

[14]Note that we do not use here the "prepotential" form of the WZ gauge described in footnote 4, and shall not address the issues of residual symmetries in such gauge. This requires a separate study as a number of gauge symmetries have to be fixed before one arrives at the expression in terms of auxiliary vector superfield and chiral compensator, and, on the other hand, the solutions of the constraints are defined modulo additional gauge symmetry transformations. Thus all our statements below are for the Wess-Zumino gauge (3.8) fixed through the conditions on the *potentials* of the superfield supergravity.

general coordinate symmetry in its original form δ_{gc}, Eqs. (B7),(B8),(B9). This can be also verified straightforwardly.

Actually, the general coordinate invariance of the WZ gauge (3.8) is natural and should be expected if one has in mind that the general coordinate transformations imply passing from one ''point'' of superspace to another, while the WZ gauge (3.8) is valid at any superspace ''point.''[15]

It is instructive to understand how this symmetry is realized in the spacetime supergravity action. Let us consider first a superfield action with a full superspace (Berezin) measure [e.g., the functional (4.1)] which possesses superspace general coordinate invariance. Then one can integrate over the Grassmann variables and arrive at a component action written as the integral over spacetime of a Lagrangian form expressed in terms of spacetime fields. However, as this is still *the same* action, it should still possess the *superspace* general coordinate invariance. But, on the other hand, it is independent of the Grassmann variables after the Berezin integration. The resolution of this apparent paradox is that on the component fields the *superspace* general coordinate transformations (B7),(B8) are realized nonlinearly, with only the subgroup of *spacetime* general coordinate transformations acting linearly. For instance, on the spacetime vielbein form $e^a(x)=E^a(Z)|_{\theta=0,d\theta=0}$ the superspace general coordinate symmetry with parameters $t^M(Z)=(t^\mu(x,\theta),\epsilon^{\check{\alpha}}(x,\theta))$ acts as $e^a(x)\to e^a[x+t(x,\theta)]|_{\theta+\epsilon(x,\theta)=0}$ (cf., the nonlinear realization of the superspace supergravity supergroups in [26]; it is instructive to note that the above expression simplifies if the superfield $\epsilon^{\check{\alpha}}$ is assumed to be independent of θ, $\epsilon^{\check{\alpha}}(x,\theta)=\epsilon_0^{\check{\alpha}}(x)$; in this case one finds $e^a(x)\to e^a\{x+t[x,-\epsilon_0(x)]\}$).

The role of superdiffeomorphism symmetry is different. It allows us to choose a coordinate system in superspace (the WZ gauge) where all the higher terms in the decomposition of supervielbein superfields on powers of Grassmann coordinates are expressed in terms of leading components of supertensors (torsion, curvature, and their covariant derivatives).

The additional hidden superspace general coordinate invariance of the component supergravity action may shed some light on the transition from the superfield action to its component form that uses ''Ectoplasm'' ideas [43,15], as well as on the existence of the rheonomic or group manifold approach to supergravity [19][16] and of a related treatment of the $D=10$ superfield superstring action [45].

[15]A bosonic counterpart of the above statement is that the defining conditions of the normal coordinate system in general relativity, $x^\mu[e^a_\mu(x)-\delta^a_\mu]=0$, $x^\mu\omega^{ab}_\mu=0$, are invariant under the active form of spacetime general coordinate transformations $x^\mu\to x^\mu+t^\mu(x)$, $e^a(x):=dx^\mu e^a_\mu(x)\to e^a(x+t)=e^a(x)+i_t de^a+di_t e^a$. This again can be easily explained by observing that the above conditions are valid at any spacetime point and that the active form of the general coordinate transformation implies just replacement of one spacetime point by another.

APPENDIX D: ON WORLDLINE SYMMETRIES OF THE BRINK-SCHWARZ SUPERPARTICLE ACTION

The reparametrization symmetry δ_r, Eqs. (5.21),(5.22), is the gauge symmetry of the superparticle action which can be identified with the *variational version of the worldline general coordinate transformations*, $\tilde{\delta}_{wgc}$, because the transformations (5.20), (5.21), (5.22) do not act on the proper time τ. Note that, actually, as the natural definition of $\tilde{\delta}_{wgc}(s(\tau))$ [cf. (B15),(B16)] is provided by

$$\tilde{\delta}_{wgc}\tau:=0, \tag{D1}$$

$$\tilde{\delta}_{wgc}\hat{Z}^M(\tau)=s(\tau)\partial_\tau\hat{Z}^M(\tau), \tag{D2}$$

$$\tilde{\delta}_{wgc}l(\tau)=l\partial_\tau s-s\partial_\tau l, \tag{D3}$$

the transformation $\tilde{\delta}_{wgc}$ differs from δ_r by one more local symmetry, $\delta_h(h(\tau))$,

$$\delta_h\hat{Z}^M(\tau)=h(\tau)[\hat{E}^\alpha_\tau E^M_\alpha(\hat{Z})+\hat{\bar{E}}_\tau{}^{\dot{\alpha}}\overline{E}^M_{\dot{\alpha}}(\hat{Z})] \tag{D4}$$

$$\Leftrightarrow \begin{cases} i_h\hat{E}^a=0, \\ i_h\hat{E}^\alpha=h(\tau)\hat{E}^\alpha_\tau, \\ i_h\hat{\bar{E}}{}^{\dot{\alpha}}=h(\tau)\hat{\bar{E}}{}^{\dot{\alpha}}_\tau, \end{cases} \tag{D5}$$

namely,

$$\delta_r(r)=\tilde{\delta}_{wgc}(s=r)+\delta_h(h=-r). \tag{D6}$$

Note that $\delta_h(h(\tau))$ is present in the Brink-Schwarz superparticle in any spacetime dimension D where the gamma-matrices can be chosen to be symmetric.

[16]The rheonomic approach to supergravity [19] is based on a generalized action principle constructed in accordance with the following prescriptions (see [44,18]): (i) one takes the usual component action, (ii) writes it in the first order form, without using the Hodge duality operator, and (iii) replaces all the fields by superfields, but taken on the surface $\mathcal{M}^D$ in D-dimensional superspace defined parametrically by $\theta=\tilde{\theta}(x)$, where $\tilde{\theta}(x)$ is an arbitrary fermionic function of spacetime coordinates. Such an action is clearly invariant under superspace general coordinate transformations pulled back onto the surface $\mathcal{M}^D$. On the other hand, setting $\tilde{\theta}(x)=0$ one arrives at the first order form of the component action, where the superspace general coordinate invariance is not manifest, but it is a hidden symmetry allowing one to go back to an arbitrary surface $\mathcal{M}^D$ in superspace (cf. the *rheonomic principle* of [19]).

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[29] The relation between $D=4$ Majorana ($\Psi_{\underline{\alpha}}$) and Weyl $[\Psi_\alpha, \bar{\Psi}_{\dot\alpha}=(\Psi_\alpha)^*]$ spinor notations are

$$\Psi_{\underline{\alpha}} \equiv \begin{pmatrix} \Psi_\alpha \\ \bar{\Psi}^{\dot\alpha} \end{pmatrix}, \quad \Psi_\alpha \equiv (\bar{\Psi}_{\dot\alpha})^* . \tag{R1}$$

For $D=4$ fermionic supervielbein form $E^{\underline{\alpha}}$,

$$E^{\underline{\alpha}} = (E^\alpha, \bar{E}_{\dot\alpha}), \tag{R2}$$

and we can define

$$E_{\underline{\alpha}} = C_{\underline{\alpha}\,\underline{\beta}} E^{\underline{\beta}} = \begin{pmatrix} E_\alpha \\ \bar{E}^{\dot\alpha} \end{pmatrix}, \tag{R3}$$

where

$$C_{\underline{\alpha\beta}} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta} \end{pmatrix}, \tag{R4}$$

$$E_\alpha = \epsilon_{\alpha\beta} E^\beta, \quad \bar{E}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta} \bar{E}_{\dot\beta}, \tag{R5}$$

$$\epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta_\beta^\alpha, \quad \epsilon^{12} = 1 = -\epsilon_{12}.$$

The Majorana spinor (R2) satisfies

$$(E^{\underline{\beta}})^\dagger \mathcal{B}_{\underline{\beta}*}{}^{\underline{\alpha}} = E^{\underline{\alpha}}, \quad \mathcal{B}_{\underline{\beta}*}{}^{\underline{\alpha}} = \begin{pmatrix} 0 & \delta^{\dot\alpha}{}_{\dot\beta} \\ \delta_\alpha^\beta & 0 \end{pmatrix}. \tag{R6}$$

Note that, as a matrix, $\mathcal{B} = \gamma^0$, where

$$\gamma_{\underline{\alpha}}^{a\underline{\beta}}=\begin{pmatrix} 0 & \sigma^{a}_{\alpha\dot{\beta}} \\ \tilde{\sigma}^{a\dot{\alpha}\beta} & 0 \end{pmatrix}. \tag{R7}$$

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