Double-trace operators and one-loop vacuum energy in AdS/CFT

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We perform a one-loop calculation of the vacuum energy of a tachyon field in anti–de Sitter space with boundary conditions corresponding to the presence of a double-trace operator in the dual field theory. Such an operator can lead to a renormalization group flow between two different conformal field theories related to each other by a Legendre transformation in the large N limit. The calculation of the one-loop vacuum energy enables us to verify the holographic c theorem one step beyond the classical supergravity approximation.

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I. INTRODUCTION

The AdS conformal field theory (CFT) correspondence [1,2,3] (for reviews see [4,5]) relates a d-dimensional quantum field theory to a $(d+1)$-dimensional gravitational theory, the most notable example being $\mathcal{N}=4$, $d=4$ super-Yang-Mills theory and type IIB string theory on $AdS_5\times S^5$. Most of the checks and predictions of this duality have been at the level of classical supergravity. It is particularly difficult to carry out meaningful loop computations in AdS, corresponding to $1/N$ corrections in the gauge theory, simply because the supergravity theory is highly nonrenormalizable, and the Ramond-Ramond fields make computations in the string genus expansion unwieldy at best. The aim of this note is to obtain a simple one-loop result in AdS that is finite in any dimension. The result is an expression for the difference of the vacuum energies that arises from changing boundary conditions on a tachyonic scalar field with mass in a particular range.

The inspiration for this computation came from Witten's treatment [6] of multitrace deformations of the gauge theory Lagrangian and their dual descriptions in asymptotically anti–de Sitter space. Such a dual description was also discussed in [7]; however, our treatment will follow [6] more closely. Earlier work describing the same gauge theory deformations in terms of nonlocal terms in the string worldsheet action appeared in [8,9]. To be definite, suppose one were to add to the gauge theory Lagrangian a term $(f/2)\mathcal{O}^2$ where $\mathcal{O}$ is a single trace operator with dimension 3/2, dual to a scalar field ϕ whose mass satisfies $m^2L^2=-15/4$.[1] The coefficient f has dimensions of mass, so $(f/2)\mathcal{O}^2$ is a relevant deformation, and there is a renormalization group (RG) flow starting from a UV fixed point where $f=0$. The end point of this flow is, plausibly, an IR fixed point whose correlators are related to those of the original $f=0$ theory, in the large N limit, by a Legendre transformation in a manner explained in [11].[2] In particular, the scalar that was for $f=0$ related to the operator $\mathcal{O}$ of dimension 3/2 is at the IR fixed point related to an operator $\tilde{\mathcal{O}}$ of dimension 5/2.

How is all this reflected in AdS? According to [6], the addition of $(f/2)\mathcal{O}^2$ amounts to specifying particular linear boundary conditions on the scalar ϕ at the boundary of AdS. At the classical level, these boundary conditions are consistent with the original AdS_5 solution with $\phi=0$. Superficially, this looks like a puzzle, since we were expecting a RG flow. In fact, conformal invariance is violated by the $\mathcal{O}^2$ deformation, but at leading order in N its effects are restricted to certain correlators that we will describe in Sec. II. The crux of the matter is that it is impossible to satisfy the boundary conditions on ϕ with a SO(4,2)-invariant bulk-to-bulk propagator, except when $f=0$ or ∞. This gives rise to one-loop effects that cause deviations from AdS_5.

Although we will not obtain the full one-loop corrected solution corresponding to RG flow due to the $(f/2)\mathcal{O}^2$ deformation, we will consider its end points and perform a one-loop supergravity check of the c theorem. This "theorem," conjectured in four dimensions by Cardy [12] as a generalization of Zamolodchikov's celebrated two-dimensional c theorem [13], has been shown to follow from AdS/CFT correspondence at the level of classical supergravity provided the null energy condition holds [14,15] (see also [16] for earlier work in this direction). The magnitude of the vacuum energy of AdS_5, measured in five-dimensional Planck units, is proportional to an appropriate central charge raised to the $-2/3$ power. So the vacuum energy should be more negative in the infrared than in the ultraviolet, and at the classical level, that is what is shown in [14,15] (actually, the arguments on the AdS are dimension independent, though it is not entirely clear how to translate the "holographic" central charge into field theory language in the case of odd-dimensional CFT's). At the quantum level, the arguments of [14,15] have no force because it is not clear that the null energy condition is valid or even relevant. So an explicit loop calculation is appropriate. All that is needed is the one-loop contribution of the scalar ϕ to the vacuum energy. This quantity is divergent, but the difference between imposing the two simple boundary conditions (described above as $f\to 0$ and $f\to\infty$) gives a finite result. The contributions of all other fields can be ignored because they do not change at the one-loop level as one changes the boundary conditions on ϕ. Also, because we only desire a one-loop vacuum amplitude, we may entirely ignore interactions of the scalar with other

[1]Such a situation could arise in the theory dual to D3-branes at the tip of a conifold [10], where there are indeed dimension 3/2 color singlet operators.

[2]We will discuss further in Sec. II the reasoning behind the claim that the flow ends at an IR fixed point, as well as some caveats.

fields, and work simply with the free action

$$S=\int d^5z\sqrt{g}\left(-\frac{1}{2}(\partial\phi)^2-\frac{1}{2}m^2\phi^2\right), \tag{1}$$

where we work in mostly plus signature, so that the metric of AdS_5 on the Poincaré patch is

$$ds^2=\frac{L^2}{z^2}(-dt^2+d\vec{x}^2+dz^2). \tag{2}$$

For definiteness, our discussion has focused on AdS_5 and a scalar with a particular mass; however, the results we will obtain can be presented with considerable generality for AdS_{d+1}, as we will describe. For odd d, the formulas for the vacuum energy are much more complicated, and for the sake of efficiency we check the sign via numerics.

The organization of the paper is as follows. In Sec. II we briefly review the prescription of [6] for treating multitrace operators, and we demonstrate that general boundary conditions are incompatible with SO(4, 2) invariance of the scalar propagator. In Sec. III we compute the finite change in the one-loop vacuum energy discussed above, and make some remarks on the interpolating geometry connecting the two anti–de Sitter end points. We conclude in Sec. IV by extracting the prediction for the central charge, and observing that the c theorem is obeyed.

II. MULTITRACE OPERATORS AND SCALAR PROPAGATORS

The proposal of [6] is a natural generalization of the original prescription for computing correlators [2,3], and it should in principle be derivable from it: see [17] for a more precise discussion. Suppose one starts with the complete set $\mathcal{O}_a$ of independent, local, color-singlet, normalized, single-trace operators: for $\mathcal{N}=4$ super-Yang-Mills theory these would include, for example, $(1/N)\mathrm{tr}X_1X_2$ and $(1/N)\mathrm{tr}F_{\mu\nu}\nabla_\rho\lambda_1$. The action can be written as $I=N^2W(\mathcal{O}_a)$ for some functional W, which for $\mathcal{N}=4$ super-Yang-Mills theory would be the integral of a linear function of those $\mathcal{O}_a$ which are Lorentz scalars. The general belief is that the $\mathcal{O}_a$ can be put into one-to-one correspondence with the quantum states of type IIB string theory in AdS_5.[3] Restricting ourselves to scalars in AdS_5, we have the standard relation $\Delta_a(\Delta_a-d)=m_a^2L^2$ relating the dimension of $\mathcal{O}_a$ to the mass of the field ϕ_a. Writing the metric for the Poincaré patch of AdS_5 as

$$ds^2=\frac{L^2}{r^2}\left(-dt^2+\sum_{i=0}^{d-2}dx_i^2+dr^2\right), \tag{3}$$

we have boundary asymptotics for ϕ_a as follows:

$$\phi_a\sim\alpha_a(x)r^{d-\Delta_a}+\beta_a(x)r^{\Delta_a}\quad\text{for}\ \ r\to 0. \tag{4}$$

The prescription of [6] is to replace $W(\mathcal{O}_a)$ by $W(\beta_a)$ and impose the following boundary conditions:

$$\alpha_a(x)=\frac{\delta W}{\delta\beta_a(x)}. \tag{5}$$

The partition function of the gravitational theory in AdS, subject to the boundary conditions (5), is then supposed to equal the partition function of the gauge theory.

The simplest nontrivial example is double-trace operators: most simply, $\mathcal{O}^2$ where the scalar operator $\mathcal{O}$ has dimension Δ between $d/2-1$ and $d/2$. Precisely in this range, unitarity bounds are satisfied, and both power law behaviors in Eq. (4) are normalizable. Then W includes a term $(f/2)\int d^dx\,\mathcal{O}^2$. This brings us back to the discussion initiated in the Introduction: Nonzero f plausibly drives the field theory from a UV fixed point where the boundary conditions are $\alpha=0$ to an IR fixed point where the boundary conditions are $\beta=0$. Since these two fixed points will be the focus of Sec. III, let us introduce an additional convenient notation: Δ_+ and Δ_- are the two solutions to $\Delta(\Delta-d)=m^2L^2$, with Δ_- being the lesser of the two (and thus in the aforementioned range, from $d/2-1$ to $d/2$). Clearly $\Delta_+=d-\Delta_-$.

When $\Delta_-<d/2$, the addition of a trace-squared operator $\mathcal{O}^2$, where $\mathcal{O}$ has dimension $\Delta=\Delta_-$, is a relevant deformation, so conformal invariance must be broken in the gauge theory. The results of [6] for $d=4$ and $\Delta_-=2$ suggest that even when $\Delta_-=d/2$ there is a logarithmic RG flow. The simplest indication of the breaking of conformal invariance in supergravity is that the bulk-to-bulk propagator for the scalar ϕ dual to $\mathcal{O}$ cannot be SO(4, 2) invariant. We will now demonstrate this claim.

The propagator in question can be defined as

$$iG(z,z')=\langle 0|T\{\phi(z)\phi(z')\}|0\rangle, \tag{6}$$

and it satisfies the equation of motion

$$(\Box-m^2)G(z,z')=\delta^{d+1}(z-z'), \tag{7}$$

where $\Box=g^{\mu\nu}\nabla_\mu\nabla_\nu$, and the delta function includes a $1/\sqrt{g}$ in its definition, so that

$$\int d^{d+1}z\sqrt{g}f(z)\,\delta(z-z')=f(z') \tag{8}$$

for any continuous function $f(z)$. If the propagator is to respect SO(4, 2) invariance, it must be a function only of the geodesic distance $\sigma(z,z')$, which is known to be

[3]There is considerable subtlety in this claim. It has been demonstrated that the Kaluza-Klein tower of supergravity modes in $AdS_5\times S^5$ is in correspondence with the chiral primaries of $\mathcal{N}=4$ super-Yang-Mills theory and their descendants; and the duals of certain nonperturbative states have been found, such as dibaryons (see, for example, [18]) and giant gravitons [19]. Evidence is growing that the operator-state map extends faithfully to excited string states (see, for example, [20,21]). Since the states in question can sometimes be extended across most of AdS_5 (as in [21]), it is not entirely clear that a second quantized treatment in terms of local fields is appropriate; but this is scarcely relevant to the situation at hand, since extended states are very massive, and we are interested only in tachyons.

$$\sigma(z,z') = L \log\left(\frac{1+\sqrt{1-\zeta^2}}{\zeta}\right) \quad \text{where}$$

$$\zeta = \frac{2rr'}{r^2+r'^2-(t-t')^2+(\vec{x}-\vec{x}')^2}, \tag{9}$$

where L is the radius of AdS. The only solutions to Eq. (8) which are functions only of ζ are $G(z,z') = pG_{\Delta_-} + (1-p)G_{\Delta_+}$ where for any Δ (cf. [22,23])[4]

$$iG_\Delta = \frac{\Gamma(\Delta)}{2^\Delta \pi^{d/2} L^{d-1}(2\Delta-d)\Gamma(\Delta-d/2)} \times \zeta^\Delta F\left(\frac{\Delta}{2}, \frac{\Delta+1}{2}; \Delta - \frac{d}{2}+1; \zeta^2\right). \tag{10}$$

By keeping z' fixed while z approaches the boundary of AdS, it is straightforward to verify that for no choice of $p \in (0,1)$ and $f \in (0,\infty)$ does the propagator $G(z,z') = pG_{\Delta_-} + (1-p)G_{\Delta_+}$ satisfy the boundary conditions (5), which in our case amount to $\alpha = f\beta$. For $p=0$ and $f=0$ the boundary conditions are satisfied with SO(4, 2) invariance preserved, corresponding to a fixed point of the RG where ϕ is dual to an operator $\mathcal{O}$ with dimension Δ_-. Let us call this the Δ_- theory. And for $p=1$ and $f=\infty$ (formally speaking) again the boundary conditions are satisfied with SO(4, 2) invariance, and now ϕ corresponds to an operator $\tilde{\mathcal{O}}$ with dimension Δ_+: this we will call the Δ_+ theory.

It was already remarked in [6] that a renormalization group flow should interpolate between the Δ_- theory in the UV and the Δ_+ theory in the IR. This is in fact a somewhat subtle claim: Why should we think that the RG flow initiated by adding $(f/2)\mathcal{O}^2$ ends up at a nontrivial IR fixed point? We can argue as follows:[5] The Legendre transformation prescription of [11] guarantees that the IR fixed point exists, at least in the large N limit. The existence of a fixed point of the RG is a generic phenomenon, so $1/N$ corrections should not spoil the claim, nor should they greatly alter the location of the fixed point in the space of possible couplings. Since a naive scaling argument (just looking at the dimension of f) tells us that the RG flow should end up at the desired IR fixed point if we ignore all $1/N$ corrections, it should be that *some* RG flow exists close to the approximate one we naively identified, ending at the nontrivial IR fixed point. A significant caveat to this reasoning is that AdS/CFT examples often (in fact, nearly always in the literature so far) have exactly marginal deformations. A *line* of fixed points of the RG is *not* a generic phenomenon, and $1/N$ effects in the absence of supersymmetry generically could destroy such a line. Only one point could be left after $1/N$ effects are included; or, worse yet, only a point infinitely far out in coupling space could be left. Translated into supergravity terms, these remarks mean that the one-loop contribution to the potential could source the dilaton or other moduli, possibly leaving no extrema at finite values of the fields. If there are no such moduli in the first place (as perhaps one would expect for a truly *generic* nonsupersymmetric quantum field theory with an AdS dual), then this caveat is not a problem. In practice, however, it is likely to interfere with constructing explicit string theory examples of the RG flow discussed in this paper. For the remainder of our discussion, we will ignore the caveat.

Since the renormalization group flow is nontrivial, it is natural to expect that the supergravity geometry deviates from AdS. The surprise is that this does *not* happen classically. Roughly, this can be understood in field theory terms as a reflection of the fact that n-point functions involving only the stress-energy tensor do not receive corrections at leading order in N.[6] At subleading order in N, or at one loop in supergravity, deviations from AdS must occur, simply because a one-loop diagram where the SO(4, 2)-noninvariant scalar propagator closes upon itself must give rise to an effective potential that varies over spacetime. Entertainingly, there is no classical scalar field which is varying; rather, the variation in the potential arises on account of proximity to the boundary. This is in contrast to previously studied examples of RG flow in AdS_5 (for instance, [14,15]), where the flow is described in terms of scalars in the five-dimensional supermultiplet of the graviton with nontrivial dependence on the radius.

There should be a solution to the one-loop-corrected supergravity Lagrangian interpolating between one asymptotically AdS region near the boundary, corresponding to the Δ_- UV fixed point, and a different one in the interior, corresponding to the Δ_+ IR fixed point. For instance, one could require that the symmetries of $\mathbf{R}^{3,1}$ be preserved in the solution, which must then have the form

$$ds^2 = e^{2A(r)}(-dt^2 + d\vec{x}^2) + dr^2, \tag{11}$$

where $A(r) \to r/L_\mp$ as $r \to \pm\infty$. (Another choice would be to require the symmetries of $\mathbf{S}^3 \times \mathbf{R}$, which should lead to a solution with the conformal structure of global AdS.) We will not find the full interpolating solution, but we will explore some properties of its AdS end points. We will be particularly interested in the central charge of the CFT's dual to the two end points. To the leading nontrivial order, these may be computed as a one-loop saddle-point approximation to the supergravity "path integral" (supposing that such an object exists), but without deforming the AdS background itself.

III. ONE-LOOP VACUUM ENERGY FOR THE TACHYON FIELD

The full classical action that we wish to consider is

[4] The expression for $G(z,z')$ above differs by a sign from that in [23,22] because the latter define the Green's function as $-iG(z,z') = \langle 0|T\phi(z)\phi(z')|0\rangle$.

[5] S.S.G. thanks E. Silverstein for a discussion in which the following line of reasoning arose.

[6] Correlation functions which *do* receive corrections at leading order in N when $(f/2)\mathcal{O}^2$ is added to the Lagrangian are precisely those which pick up contributions from factorized forms $\langle\mathcal{O}\cdots\rangle\langle\mathcal{O}\cdots\rangle$, where the dots indicate any arrangement of the operators involved in the original correlator.

$$S=\frac{1}{2\kappa^2}\int d^{d+1}z\sqrt{g}(R-\Lambda_0)$$
$$+\int d^{d+1}z\sqrt{g}\left(-\frac{1}{2}(\partial\phi)^2-\frac{1}{2}m^2\phi^2\right). \tag{12}$$

Here Λ_0 is a negative constant. The scalar is subject to the boundary conditions

$$\phi\sim\alpha r^{d-\Delta}+\beta r^{\Delta}\quad\text{where}\quad\alpha=f\beta. \tag{13}$$

As remarked previously, AdS_{d+1} with $\phi=0$ and $1/L^2=-\Lambda_0/d(d-1)$ is a classical solution to the equations of motion from Eq. (13), but we expect that once one-loop effects are accounted for, this solution is corrected to an interpolation between AdS_{d+1} spaces in the UV and IR with slightly different radii. The one-loop scalar bubble diagram corrects the gravitational Lagrangian by an amount $\delta\mathcal{L}$, where

$$-\sqrt{g}^{-1}\delta\mathcal{L}=V=-\frac{i}{2}\operatorname{tr}\log(-\Box+m^2). \tag{14}$$

Our main computation will be to evaluate this correction in the unperturbed background. In principle, one could go on to find the interpolating geometry perturbatively in the small parameter $\kappa\Lambda_0^{(d-1)/2}$. This would require separating $\delta\mathcal{L}$ into contributions to the cosmological term and two- and four-derivative expressions in the metric—a much more involved computation than simply evaluating Eq. (14) in the unperturbed background. For brevity, we will use the notation V in preference to $\delta\mathcal{L}$ for the scalar self-energy (14), despite the fact that the full background-independent form involves derivative terms as well as finite nonlocal terms. V is divergent, but we assume that the action (12) is part of a well-defined theory of quantum gravity (presumably, a compactification of string theory or M theory), so that all loop divergences are canceled in some physical way, leaving only finite renormalization effects. It may be that in the full theory Λ_0 is just the extremal value of a classical potential function of several scalars; if so, then we are operating on the understanding that the second derivative of this potential function with respect to ϕ vanishes at $\phi=0$ [that is, we have soaked up any such second derivative into what we call m^2 in Eq. (12)].

In general, it is difficult to compute one-loop corrections in an effective theory without knowing precisely how the full theory cancels divergences. Results obtained for a chiral anomaly in supergravity [24] for $AdS_5\times S^5$ can be used to show that the central charge is corrected at one loop in supergravity, leading to $c\propto N^2-1$, as appropriate for SU(N) super Yang-Mills theory, rather than $c\propto N^2$ (the leading order result). Thus in this case, the difficulties were overcome. Our situation is more generic, in that we do not depend on supersymmetry or a special spectrum of operators. What we are nevertheless able to do is to determine the finite difference between V in the case where $f=0$ in Eq. (13) and the case where $f=\infty$. This we will then translate into a change in the central charge as one flows from the UV (the Δ_- theory) to the IR (the Δ_+ theory). What makes the computation clean is that at one loop, we do not have to worry about interactions of the scalar with other fields, and the only relevant diagram is the one where a single scalar propagator closes on itself, with no vertices.

A. Vacuum energy in limiting regions of AdS

The computation of the one-loop contribution to the vacuum energy by a scalar in curved space, as in flat space, amounts to summing the logarithm of the eigenvalues of the Klein-Gordon operator. A more easily computable expression is obtained by expressing the result in terms of an integral of the Green's function with respect to some parameter such as proper time or mass.[7] All of this is quite standard, so we just write down the result, referring the reader to [[26], pp. 156–158] for a derivation: If the propagator $G(z,z';m^2,f)$ is defined by

$$(\Box_z-m^2)G(z,z';m^2,f)=\delta^{d+1}(z-z') \tag{15}$$

[with the delta function including a $\sqrt{g}$ factor as in Eq. (8)] together with boundary conditions (13), as discussed in Sec. II, then formally

$$V(z;m^2,f)=-\frac{i}{2}\lim_{z\to z'}\int_{m^2}^{\infty}d\tilde{m}^2G(z,z';\tilde{m}^2,f), \tag{16}$$

and for the cases $f=0,\infty$, the fact that we can make the scalar propagator SO(4, 2) invariant means that V will be independent of the position z.[8] The formula (16) is problematic because, for large masses, $G(z,z',\tilde{m}^2,0)$ diverges at the boundary of AdS. This is unusual: The typical situation for quantum field theory in curved spacetime is that quantities become well defined in the limit where masses are much larger that the inverse radius of curvature. Thus, instead of using Eq. (16), a well-defined procedure is to integrate down to the Breitenlohner-Freedman bound which is the smallest mass possible with normalizable modes in AdS. Thus we obtain

$$V(z;m^2,f)=V(z;m_{\rm BF}^2,f)+\frac{i}{2}\lim_{z\to z'}\int_{m_{\rm BF}^2}^{m^2}d\tilde{m}^2G(z,z';\tilde{m}^2,f), \tag{17}$$

where $m_{\rm BF}^2L^2=-d^2/4$ is the Breitenlohner-Freedman (BF) bound. (For a derivation, see the Appendix.) It is possible to argue that $V(z;m_{\rm BF}^2,f)$ is the same for $f=0$ and $f=\infty$. Indeed, the eigenmodes for a tachyon of mass m^2 with boundary conditions specified by $f=0$ are given by $\omega=\Delta_-+\ell+2n$ and that specified by $f=\infty$ is given by $\omega=\Delta_++\ell$

[7] For a different method of computing the effective potential based on the technique of Zeta-function regularization, see [25].

[8] Actually, we have tucked an additional complication into our notation: V is, more properly, minus the one-loop correction to the full gravitational Lagrangian, and as such includes not just a scalar piece, but also terms depending on curvatures. For the central charge computation, as we shall explain, the relevant quantity is the sum of all these terms evaluated on AdS.

$+2n$ [27], where ℓ is the orbital angular momentum quantum number and n is the radial quantum number. But for a scalar with mass saturating the BF bound, $\Delta_+=\Delta_-=d/2$. So from the viewpoint of canonical quantization it seems inevitable that $V(z;m_{\rm BF}^2,0)-V(z;m_{\rm BF}^2,\infty)=0$. We can argue further that for general f the eigenfunctions would be a linear combination of those with $f=0$ and $f=\infty$. That would again imply that for $\Delta=d/2$, the eigenvalues are unchanged. So we conclude that the $V(z;m_{\rm BF}^2,f)-V(z;m_{\rm BF}^2,0)=0$ for all values of f.

Thus we are led to the formula that we will really use for computation:

$$V_+-V_-=\frac{i}{2}\int_{m_{\rm BF}^2}^{m^2}d\tilde{m}^2[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]+V(z;m_{\rm BF}^2,\infty)-V(z;m_{\rm BF}^2,0), \tag{18}$$

where $V_+=V(z,m^2,\infty)$ and $V_-=V(z,m^2,0)$. We have used the fact that $G_{\tilde{\Delta}_+}(z,z')$, as defined in Eq. (10), is precisely $G(z,z';\tilde{m}^2,\infty)$, while $G_{\tilde{\Delta}_-}(z,z')=G(z,z';\tilde{m}^2,0)$. In light of the argument of the previous paragraph, the terms outside the integral cancel. The advantage of Eq. (18) is that $G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)$ is finite, so that the final answer is also manifestly finite. We have confidence that no other finite renormalization effects can slip in to the calculation, because the only thing that changes between the Δ_- and Δ_+ vacua is the boundary condition on ϕ.

As a warm-up let us first carry out the computation for AdS_5. To get the value of $G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)$ for coincident points one has to first express the Green's functions in terms of the geodesic distance σ. From Eq. (9) we see that in terms of the variable ζ the geodesic separation is given by $\cosh(\sigma/L)=1/\zeta$, so we rewrite the propagator (10) in terms of σ and then expand $i[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]$ in a power series in powers of σ/L. The answer is finite and in the limit $\sigma/L\to 0$, for AdS_5 we obtain the simple expression

$$i[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]=-i[G_{\tilde{\Delta}}(z,z)-G_{4-\tilde{\Delta}}(z,z)]=-\frac{(\tilde{\Delta}-1)(\tilde{\Delta}-2)(\tilde{\Delta}-3)}{12\pi^2L^3}. \tag{19}$$

The difference in the vacuum energies using Eq. (18) is therefore

$$\begin{aligned}V_+-V_-&=\frac{i}{2}\int_{m_{\rm BF}^2}^{m_0^2}d\tilde{m}^2[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]\\&=-\frac{1}{2}\int_2^{\Delta_-}\frac{d\tilde{\Delta}}{L^2}\left[2(\tilde{\Delta}-2)\frac{(\tilde{\Delta}-1)(\tilde{\Delta}-2)(\tilde{\Delta}-3)}{12\pi^2L^3}\right]\\&=-\frac{1}{12\pi^2L^5}\int_0^{\Delta_--2}d\tilde{\nu}[\tilde{\nu}^2(\tilde{\nu}^2-1)]\\&=\frac{1}{12\pi^2L^5}\left[\frac{(\Delta_--2)^3}{3}-\frac{(\Delta_--2)^5}{5}\right],\end{aligned} \tag{20}$$

where in the second equality we have used $\tilde{m}^2L^2=\tilde{\Delta}(\tilde{\Delta}-4)$ and the fact that $\Delta_{\rm BF}=d/2=2$. Since $\Delta_-<2$ we find that $V_+-V_-<0$, and therefore $c_->c_+$ in agreement with the field theory prediction.

It is straightforward to generalize this for any odd-dimensional anti–de Sitter spacetime because for d even the difference $i[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]$ is quite simple in form. Before writing this down, for convenience, let us define $d\equiv 2k$ so that the spacetime is AdS_{2k+1}. In terms of k, $i[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]$ is

$$i[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]=-i[G_{\tilde{\Delta}}(z,z)-G_{d-\tilde{\Delta}}(z,z)]=-\frac{(-1)^k}{n_k\pi^kL^{d-1}}\prod_{i=1}^{2k-1}(\tilde{\Delta}-i), \tag{21}$$

where $n_k=2^k(2k-1)!!$

The difference in the vacuum energies is therefore

$$\begin{aligned}V_+-V_-&=\frac{i}{2}\int_{m_{\rm BF}^2}^{m_0^2}d\tilde{m}^2[G_{\tilde{\Delta}_+}(z,z)-G_{\tilde{\Delta}_-}(z,z)]\\&=\frac{1}{2}\int_{\Delta_-}^{k}\frac{d\tilde{\Delta}}{L^2}\left[2(\tilde{\Delta}-k)\frac{(-1)^k}{n_k\pi^kL^{d-1}}\prod_{i=1}^{2k-1}(\tilde{\Delta}-i)\right],\end{aligned} \tag{22}$$

where in the second equality we have used $\tilde{m}^2L^2=\tilde{\Delta}(\tilde{\Delta}-d)$ and the fact that $\Delta_{\rm BF}=d/2=k$. Shifting the variable of integration by introducing a new variable $\tilde{\nu}\equiv\tilde{\Delta}_--k$, the integrand can be written down in a terms of the Pochhammer symbol $(a)_n=\Gamma(a+n)/\Gamma(n)$:

$$\begin{aligned}2(\tilde{\Delta}-k)\frac{(-1)^k}{n_k\pi^kL^{d+1}}\prod_{i=1}^{2k-1}(\tilde{\Delta}-i)&=\frac{(-1)^k}{n_k\pi^kL^{d+1}}\prod_{i=0}^{k-1}(\tilde{\nu}^2-i^2)\\&=\frac{1}{n_k\pi^kL^{d+1}}(\tilde{\nu})_k(-\tilde{\nu})_k.\end{aligned} \tag{23}$$

The factor $(-1)^k$ was nullified by an extra factor of $(-1)^k$ from the product. Assembling all of this, we finally have

$$V_+-V_-=\frac{1}{2n_k\pi^kL^{d+1}}\int_{\nu}^{0}d\tilde{\nu}[(\tilde{\nu})_k(-\tilde{\nu})_k], \tag{24}$$

where we recall that $n_k=2^k(2k-1)!!$ The lower limit of integration ν depends on the value of Δ_-. Since $k\leqslant\Delta_-\leqslant k-1$, the range of ν is $-1\leqslant\nu\leqslant 0$. The function $(\nu)_k(-\nu)_k<0$ for all k and $-1\leqslant\nu\leqslant 0$. So for any odd-dimensional anti–de Sitter spacetimes we have shown that $V_+-V_-<0$.

For even-dimensional spacetimes, an analytic proof seems cumbersome, so we resorted to numerics. As an explicit ex-

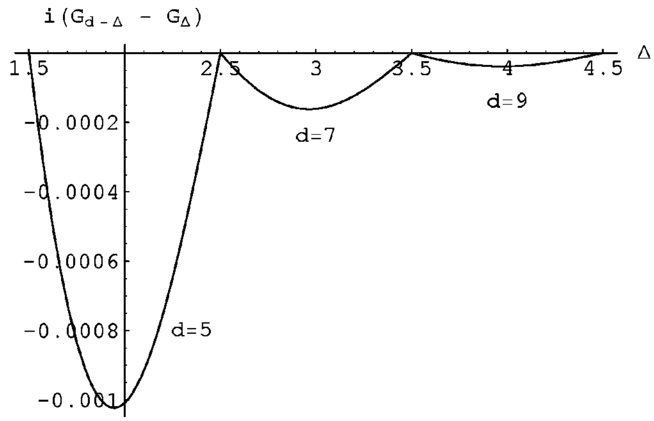

FIG. 1. $i(G_{d-\Delta}-G_{\Delta})$ as a function of Δ, for AdS_6, AdS_8, and AdS_{10} (corresponding to $d=5$, 7, and 9), in units where $L=1$.

ample, Fig. 1 shows a plot of $i(G_{d-\Delta}-G_{\Delta})$ as a function of Δ for several even-dimensional anti–de Sitter spacetimes. In each dimension, we have plotted the integrand of Eq. (18) for $d/2-1<\Delta<d/2$. Since the integrand is always negative on this range, we conclude that $V_+<V_-$ in accordance with the c-theorem intuition. This is also true for $d=3$, and we believe it is true generally.

B. Vacuum energy throughout AdS

The results of the previous section were stated in terms of $V_+-V_-=V(z;m^2,\infty)-V(z;m^2,0)$ (both terms were in fact independent of the position z in AdS). Here we would like to investigate $V(z;m^2,f)$ for finite f. This quantity diverges, but $V(z;m^2,f)-V_-$ is finite. We will be able to verify the formulas

$$\lim_{z_0\to 0}[V(z;m^2,f)-V_-]=0,$$

$$\lim_{z_0\to\infty}[V(z;m^2,f)-V_-]=V_+-V_-, \tag{25}$$

which we consider intuitively obvious since $(f/2)\mathcal{O}^2$ is a relevant operator in the CFT, and therefore unimportant in the UV but important in the IR.

As a first step, one needs the Green's function for the scalar obeying mixed boundary conditions for all values of f (not just the ones for $f=0$ and $f=\infty$ that we wrote down earlier). This would be needed to compute the vacuum energy contribution due to the bubble diagram. The one-loop corrected action would then induce corrections in the geometry which can be computed from the Einstein equations. Let us work in Euclidean AdS to get the Green's function $G_E(x,y;f)$ which we shall Wick rotate to obtain $G(x,y;f)$ in the Minkowski signature. We shall follow the canonical method of obtaining Green's functions. In Poincaré coordinates the scalar wave equation is

$$[x_0^2(\vec{\partial}^2+\partial_0^2)-x_0(d-1)\partial_0-m^2]\phi(x_0,\vec{x})=0, \tag{26}$$

where from now on we shall denote the radial direction by x_0 or y_0 and $\vec{x}$ is a vector with components along the d remaining directions. The two linearly independent solutions to this equation are: $\phi_1=x_0^2e^{-i\vec{k}\cdot\vec{x}}I_\nu(kx_0)$ and $\phi_2=x_0^2e^{-i\vec{k}\cdot\vec{x}}I_{-\nu}(kx_0)$ where $\nu=\sqrt{m^2L^2+d^2/4}$. In the notation of our previous sections, the Green's function obeys the equation

$$(\Box-m^2)G_E(x,y;f)=\delta^{d+1}(x-y), \tag{27}$$

where we remind ourselves that the delta function includes a $1/\sqrt{g}$ in its definition. The right hand side is zero for $x_0\neq y_0$, so we have

$$G_E(x,y)=\begin{cases}A_1\phi_1(x)+A_2\phi_2(x) & \text{for } x_0<y_0,\\ B_1\phi_1(x)+B_2\phi_2(x) & \text{for } x_0>y_0.\end{cases} \tag{28}$$

The boundary behavior of the scalar we are interested in is $\phi(x_0,\vec{x})=f\beta(\vec{x})x_0^{d/2+\nu}+\beta(\vec{x})x_0^{d/2-\nu}$. We choose our ϕ_1 and ϕ_2 so that they have the right boundary behavior and also require that the Green's function not diverge in the bulk (large values of the radial coordinate x_0) for two noncoincident points. One convenient choice of ϕ_1 and ϕ_2 is

$$\phi_1=x_0^{d/2}e^{-i\vec{k}\cdot\vec{x}}\left[I_\nu(kx_0)+f\left(\frac{2}{k}\right)^{2n}\frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}I_\nu(kx_0)\right]$$

and

$$\phi_2=x_0^{d/2}e^{-i\vec{k}\cdot\vec{x}}K_\nu(kx_0), \tag{29}$$

so that ϕ_1 satisfies the boundary condition for small x_0 and ϕ_2 is finite in the bulk. From the asymptotics of Bessel functions, we see that ϕ_1 diverges as $x_0\to\infty$ and ϕ_2 diverges as $x_0\to 0$. This forces us to set $A_2=B_1=0$ in Eq. (28). The remaining two constants are determined by integrating Eq. (27) twice which gives us two conditions: (i) the Green's function is continuous at $x_0=y_0$ and (ii) its radial derivative has a jump discontinuity of $1/x_0^{d-1}$ at $x_0=y_0$. This yields

$$A_1=\frac{\phi_2(y_0)}{\mathcal{W}[\phi_1(y_0),\phi_2(y_0)]},\quad B_2=\frac{\phi_1(y_0)}{\mathcal{W}[\phi_1(y_0),\phi_2(y_0)]}, \tag{30}$$

where $\mathcal{W}[\phi_1(y_0),\phi_2(y_0)]$ is the Wronskian. For our choice of ϕ_1 and ϕ_2 the Wronskian is

$$\mathcal{W}[\phi_1(y_0),\phi_2(y_0)]=-\frac{\Gamma(1-\nu)+f(2/k)^{2\nu}\Gamma(1+\nu)}{\Gamma(1-\nu)}\left(\frac{L}{y_0}\right)^{d-1}, \tag{31}$$

so combining Eqs. (28), (30), and (31) we obtain the Green's function

$$G_E(x,y;f)=-\int \frac{d\kappa_E d^{d-1}k}{(2\pi)^d}\frac{e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}(x_0y_0)^{d/2}K_\nu(ky_0)}{[1+(2/k)^{2\nu}f\,\Gamma(1+\nu)/\Gamma(1-\nu)]L^{d-1}}\left[I_{-\nu}(kx_0)+f\frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}\left(\frac{2}{k}\right)^{2\nu}I_\nu(kx_0)\right] \quad (32)$$

for $x_0<y_0$ and a similar expression for $x_0>y_0$. In the above equation, κ_E is the temporal component of momentum. Finally, we Wick rotate this component $\kappa_E=ik$ to get the Green's function in the Minkowski signature:

$$iG(x,y;f)=\int \frac{d^dk}{(2\pi)^d}\frac{e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}(x_0y_0)^{d/2}K_\nu(ky_0)}{[1+(2/k)^{2\nu}f\,\Gamma(1+\nu)/\Gamma(1-\nu)]L^{d-1}}\left[I_{-\nu}(kx_0)+f\frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}\left(\frac{2}{k}\right)^{2\nu}I_\nu(kx_0)\right]. \quad (33)$$

The integral for general values of f, d, and ν is hard. For $f=0$ and $f=\infty$ it can be evaluated and the result is an expression which is related to Eq. (10) by a quadratic hyper-geometric transformation [28]. A little bit more can be said about the radial dependence of the one-loop vacuum energy. This latter quantity depends on the Green's function for coincident points $G(x,x;f)$. We saw before that this divergent quantity was best handled by subtracting out $G(x,x;0)$. The result is then finite:

$$i[G(x,x;f)-G(x,x;0)]=-\frac{1}{2^{d-2}\pi^{d/2}L^{d-1}\Gamma(\nu)\Gamma(1-\nu)\Gamma(d/2)}\int_0^\infty d\tilde{k}\,\tilde{k}^{d-1}\frac{\tilde{f}}{\tilde{k}^{2\nu}+\tilde{f}}[K_\nu(\tilde{k})]^2, \quad (34)$$

where $\tilde{f}=2^{2\nu}[\Gamma(1+\nu)/\Gamma(1-\nu)]fx_0^{2\nu}$ and $\tilde{k}=kx_0$. Note that the excess vacuum energy depends on the radial coordinate x_0 in the particular combination $fx_0^{2\nu}$.

In order to make any further progress, one would need to first compute the momentum integral and then integrate over ν to obtain the vacuum energy. We argued earlier that $V(x;m^2_{\rm BF},f)-V(x;m^2_{\rm BF},0)=0$ for all values of f, so using Eqs. (18) and (34) we have

$$V(x;m^2,f)-V(x;m^2,0)=\frac{i}{2}\int_{m^2_{\rm BF}}^{m_0^2}d\tilde{m}^2[G(x,x;f)$$
$$-G(x,x;0)]=\frac{i}{2}\int_0^\nu\frac{d\tilde{\nu}^2}{L^2}[G(x,x;f)-G(x,x;0)]$$
$$=-\frac{1}{2^{d-2}\pi^{d/2}\Gamma(d/2)L^{d+1}}\int_0^\nu d\tilde{\nu}\frac{\tilde{\nu}}{\Gamma(\tilde{\nu})\Gamma(1-\tilde{\nu})}$$
$$\times\int_0^\infty d\tilde{k}\frac{\tilde{k}^{d-1}\tilde{f}}{\tilde{k}^{2\nu}+\tilde{f}}[K_\nu(\tilde{k})]^2, \quad (35)$$

where we remind ourselves that $\tilde{f}=2^{2\nu}[\Gamma(1+\nu)/\Gamma(1-\nu)]fx_0^{2\nu}$. The double integral is difficult to perform explicitly. However, it is not hard to show from Eq. (35) that $V(x;m^2,f)$ decreases monotonically as f increases from 0 to ∞. To see this we note that the integrand depends on x_0 only through $\tilde{f}$ and since the integrand is a monotonic function of $\tilde{f}$, clearly $V(x;m^2,f)$ decreases monotonically with increasing f.

IV. CONCLUSIONS

The upshot of Sec. III A was an evaluation of the change in the one-loop self-energy, V_+-V_-, between the IR and UV end points of a holographic RG flow. We would now like to convert this into a change in the central charge of the dual field theory.

In [29], the central charge was obtained by holographically computing the Weyl anomaly: on the field theory side,

$$\delta W[g_{\mu\nu}]=\frac{1}{2}\int d^4x\sqrt{g}\,\omega\langle T^\mu_\mu\rangle \quad (36)$$

upon a conformal variation $g_{\mu\nu}\to e^{2\omega}g_{\mu\nu}$, where W is the generating functional for connected Green's functions. At the one-loop level, the prescription of [2,3] asserts that W is the classical supergravity action. The exact statement is that the partition functions of string theory and gauge theory coincide (subjected to boundary conditions and source terms in the usual way). In the calculation of [29], the supergravity action integral is evaluated with a radial cutoff, where the choice of radius amounts to a choice of metric within a conformal class. The supergravity Lagrangian evaluates to a constant in AdS, and the central charge is proportional to this constant.[9] All that we need to do in order to correct the central charge computation at one loop is to ask by how much the one-loop-corrected Lagrangian differs from the tree-level Lagrangian, when evaluated in AdS. The tree level and one-loop Lagrangians will stand in the same ratio as the leading large N central charge and its $1/N$-corrected counterpart.

The tree level Lagrangian is

$$\sqrt{g}^{-1}\mathcal{L}_{\rm tree}=\frac{1}{\kappa^2_{d+1}}(R-\Lambda_0)=-\frac{2d}{\kappa^2_{d+1}L^2}. \quad (37)$$

[9] *A priori*, one might worry that boundary terms in the supergravity action also contribute to the central charge. That this does not happen depends on the circumstance, noted in [29], that the only log-divergent terms in the supergravity calculation arise from the integral of the bulk action.

The calculation indicated by the discussion in the previous paragraph is

$$\frac{c_{\text{corrected}}}{c_{\text{tree}}}=\frac{\mathcal{L}_{\text{tree}}+\delta\mathcal{L}}{\mathcal{L}_{\text{tree}}}, \tag{38}$$

where $\delta\mathcal{L}=-\sqrt{g}V$ is the one-loop correction to the Lagrangian that we computed in Sec. III. Because we are only able to compute V up to an additive constant that is independent of boundary conditions, the only meaningful ratio that we can form is

$$\frac{c_+}{c_-}=\frac{\mathcal{L}_{\text{tree}}-\sqrt{g}V_+}{\mathcal{L}_{\text{tree}}-\sqrt{g}V_-}=1+\frac{V_--V_+}{\sqrt{g}^{-1}\mathcal{L}_{\text{tree}}}, \tag{39}$$

so that

$$\frac{c_+-c_-}{c_-}=(V_+-V_-)\left(\frac{\kappa_{d+1}^2L^2}{2d}\right). \tag{40}$$

To check if c_- is indeed greater than c_+, all that we have to show is that $V_+<V_-$. But that is exactly what we saw above.

As an example, in AdS_5, we obtain from Eqs. (22) and (40) the result

$$\frac{c_+-c_-}{c_-}=\frac{\kappa_5^2}{192\pi^2L^3}\left[\frac{(\Delta_--2)^3}{3}-\frac{(\Delta_--2)^5}{5}\right]. \tag{41}$$

One can go further and translate the function $V(z;m^2,f)-V_-$ into a correction to the central charge whose scale dependence is monotonic. It is not clear how well defined such a function can be on the supergravity side: because the bulk theory includes gravity, it has no local observables. Poetically, we would like to relate this to the fact that renormalization group effects in field theory are scheme dependent—but it is difficult to make this precise.

It would be interesting to see how the construction discussed in this paper might be realized as part of a compactification of string theory to four dimensions, along the lines of [30,31]. One of the most interesting questions in that context is one that we glossed over here: before considering the loop effects in supergravity, one generally expects a moduli space of vacua, and this statement probably translates into field theory terms as the existence of a line of fixed points. Mapping the lifting of moduli into field theory terms might at least gain us a restatement of the moduli problem in terms of the existence of isolated fixed points of the renormalization group.

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APPENDIX

In this appendix we shall sketch the derivation of Eq. (17). Our starting point is the familiar field theory result that the one-loop effective potential is

$$V(z;m^2,f)=-\frac{i}{2}\operatorname{tr}\log(-\Box+m^2). \tag{A1}$$

We shall denote the Klein-Gordon operator $(-\Box+m^2)$ by $\hat{K}(m^2,f)$ and as an operator, it is related to our definition of the Green's function (15) by $\hat{G}(m^2,f)=-[\hat{K}(m^2,f)]^{-1}$. The representations of operators such as $\hat{G}(m^2,f)$ in an orthonormal basis will be denoted by the obvious notation $\langle z|\hat{G}(m^2,f)|z\rangle=G(z,z';m^2,f)$. In terms of the Green's function, the effective potential is then

$$V(z;m^2,f)=\frac{i}{2}\lim_{z'\to z}\log[-G(z,z';m^2,f)]. \tag{A2}$$

We shall use the Schwinger proper-time formalism to evaluate this. One needs two simple operator relations both of which follow from the relation between $\hat{G}(m^2,f)$ and $\hat{K}(m^2,f)$:

$$\hat{G}(m^2,f)=-i\int_0^\infty e^{-is\tilde{K}(m^2,f)}ds,$$

$$\log[-\hat{G}(m^2,f)]=\int_0^\infty\frac{e^{-is\hat{K}(m^2,f)}}{is}i\,ds+\gamma, \tag{A3}$$

where γ is Euler's constant. For the effective potential, we see from Eq. (A2) that we need $\log[-\hat{G}(m^2,f)]$ which differs by a factor of is from the integral representation of $\hat{G}(m^2,f)$ above.

To proceed any further we need the DeWitt-Schwinger representation of the Green's function (the reader is referred to [26], p. 75 for a derivation)

$$G(z,z';m^2,f)=-i\frac{\sqrt{M(z,z')}}{(4\pi is)^{(d+1)/2}}\times\int_0^\infty i\,ds\,e^{-im^2s+\eta(z,z')/2is}F(z,z';is), \tag{A4}$$

where $\eta(z,z')$ is one-half the proper distance between the points z and z', and $M(z,z')=-\det[\partial_\mu\partial_\nu\eta(z,z')]$. For our purposes we shall just need to use the fact that the only place where the mass appears is in the exponent and integrating with respect to m^2 will bring down an extra factor of is that we need. So integrating both sides of Eq. (A4) between two arbitrary masses m_1^2 and m_2^2 and using Eq. (A3) we obtain

$$\int_{m_1^2}^{m_2^2} d\tilde{m}^2[-G(z,z';\tilde{m}^2,f)] = -\log[-G(z,z';m_2^2,f)] + \log[-G(z,z';m_1^2,f)]. \tag{A5}$$

In the usual treatment one chooses one of the masses to be infinite, but as we explained in the main text, this cannot be done here. Instead of integrating toward heavier masses, we integrate in the opposite direction down to the Breitenlohner-Freedman bound. Therefore, we set $m_1^2 = m_{\mathrm{BF}}^2$ and $m_2^2 = m^2$ in Eq. (A5) and use Eq. (A2) to get

$$V(z;m^2,f) = \frac{i}{2} \lim_{z\to z'} \int_{m_{\mathrm{BF}}^2}^{m^2} d\tilde{m}^2 G(z,z';m^2,f) + V(z;m_{\mathrm{BF}}^2,f). \tag{A6}$$

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