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Can the Chaplygin gas be a plausible model for dark energy?

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In this paper two cosmological models representing the flat Friedmann universe filled with a Chaplygin fluid, with or without dust, are analyzed in terms of the recently proposed "statefinder" parameters. Trajectories of both models in the parameter plane are shown to be significantly different with respect to the "quiessence" and "tracker" models. The generalized Chaplygin gas model with an equation of state of the form $p=-A/\rho^{\alpha}$ is also analyzed in terms of the statefinder parameters.

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In the search for cosmological models describing the observed cosmic acceleration [1–3], the inspiration coming from inflation has suggested mainly models making use of scalar fields [4–8]. There are of course alternatives; in particular, in [9–11] an elementary model has been presented describing a Friedmann universe filled with a perfect fluid obeying the Chaplygin equation of state

$$p=-\frac{A}{\rho}, \tag{1}$$

where A is a positive constant (for a thorough review, see Ref. [12]). The interesting feature of this model is that it naturally provides a universe that undergoes a transition from a decelerating phase, driven by dust-like matter, to a cosmic acceleration at later stages of its evolution (see [9] for details). An interesting attempt to justify this model [13] makes use of an effective field theory for a three-brane universe [14].

In the flat case, the model can be equivalently described in terms of a homogeneous minimally coupled scalar field ϕ, with the potential [9]

$$V(\phi)=\frac{1}{2}\sqrt{A}\left(\cosh 3\phi+\frac{1}{\cosh 3\phi}\right). \tag{2}$$

However, since models trying to provide a description (if not an explanation) of the cosmic acceleration are proliferating, there exists the problem of discriminating between the various contenders. To this aim a new proposal introduced in [15] makes use of a pair of parameters $\{r,s\}$, called the "statefinder." The relevant definition is as follows:

$$r\equiv\frac{\dddot{a}}{aH^3},\quad s\equiv\frac{r-1}{3(q-1/2)}, \tag{3}$$

where $H\equiv\dot{a}/a$ is the Hubble constant and $q\equiv-\ddot{a}/aH^2$ is the deceleration parameter. The new feature of the statefinder is that it involves the third derivative of the cosmological radius.

Trajectories in the $\{s,r\}$ plane corresponding to different cosmological models exhibit qualitatively different behaviors. The cold dark matter model with a cosmological constant (ΛCDM) diagrams correspond to the fixed point $s=0$, $r=1$. The so-called "quiessence" models [15] are described by vertical segments with r decreasing from $r=1$ down to some definite value. Tracker models [16] have typical trajectories similar to arcs of a parabola lying in the positive quadrant with a positive second derivative.

The current location of the parameters s and r in these diagrams can be calculated in models (given the deceleration parameter); it may also be extracted from data coming from SNAP (SuperNovae Acceleration Probe) type experiments [15]. Therefore, the statefinder diagnostic combined with future SNAP observations may possibly be used to discriminate between different dark energy models.

In this note we apply the statefinder diagnostic to Chaplygin cosmological models. A direct comparison of the present available type Ia supernovae data with Chaplygin gas models was undertaken recently in [17–19]. Constraints on the parameters of the models arising from estimates of the age of the universe and from statistics of gravitationally lensed optical quasars were studied in [20], while constraints coming from the study of the cosmic microwave background acoustic peak location were analyzed in [21].

We consider the one-fluid pure Chaplygin gas model and a two-fluid model where dust is also present, as well a generalized Chaplygin gas model [9,22] without and with dust. We show that these models are different from those considered in [15] and they are worthy of further study.

To begin with, let us rewrite the formulas for the statefinder parameters [15] in a form convenient for our pur-

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poses. We shall need the Friedmann equation for the flat universe,

$$H^2=\frac{\dot{a}^2}{a^2}=\rho, \tag{4}$$

and the energy conservation equation

$$\dot{\rho}=-3H(\rho+p). \tag{5}$$

Using these two equations it is easy to find that

$$q=\frac{1}{2}+\frac{3}{2}\frac{p}{\rho}, \tag{6}$$

and then

$$r=1-\frac{3\dot{p}}{2\rho\sqrt{\rho}}, \qquad s=-\frac{\dot{p}}{3p\sqrt{\rho}}. \tag{7}$$

For a one-component fluid[1] these formulas become especially simple. Since

$$\dot{p}=\frac{\partial p}{\partial\rho}\dot{\rho}=-3\sqrt{\rho}(\rho+p)\frac{\partial p}{\partial\rho}, \tag{8}$$

we easily get

$$r=1+\frac{9}{2}\left(1+\frac{p}{\rho}\right)\frac{\partial p}{\partial\rho}, \quad s=\left(1+\frac{\rho}{p}\right)\frac{\partial p}{\partial\rho}. \tag{9}$$

For the Chaplygin gas one has simply that

$$v_s^2=\frac{\partial p}{\partial\rho}=\frac{A}{\rho^2}=-\frac{p}{\rho}=1+s \tag{10}$$

(v_s^2 is the square of the velocity of sound) and therefore

$$r=1-\frac{9}{2}s(1+s). \tag{11}$$

Thus, the curve of $r(s)$ is an arc of a parabola. To find the admissible values of s, we note that Eqs. (1), (4), and (5) easily give the following dependence of the energy density on the cosmological scale factor [9]:

$$\rho=\sqrt{A+\frac{B}{a^6}}, \tag{12}$$

where B is an integration constant; therefore

$$v_s^2=1+s=\frac{A}{A+B/a^6}. \tag{13}$$

[1]We confine ourselves to the case, which is satisfied in our models, of a fluid for which the equation of state has the form $p=p(\rho)$.

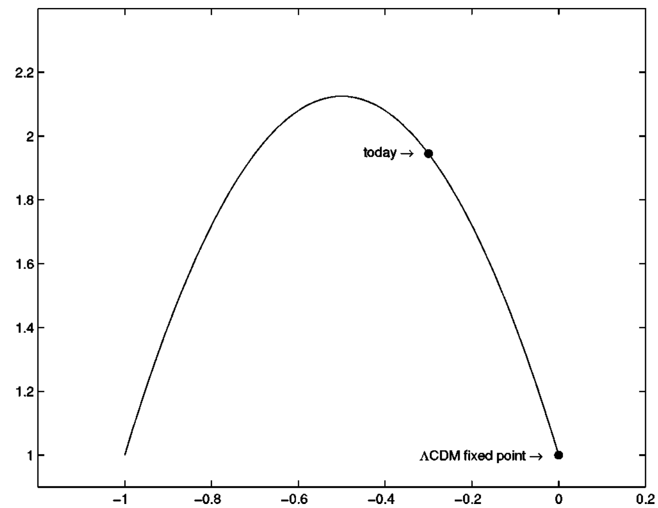

FIG. 1. s-r evolution diagram for the pure Chaplygin gas.

When the cosmological scale factor a changes from 0 to ∞ the velocity of sound varies from 0 to 1 and s varies from -1 to 0. Thus in our model the statefinder s takes negative values; this feature is not shared by the quiessence and tracker models considered in [15].

As s varies in the interval $[-1,0]$, r first increases from $r=1$ to its maximum value and then decreases to the ΛCDM fixed point $s=0$, $r=1$ (see Fig. 1). If $q\approx-0.5$ the current values of the statefinder (within our model) are $s\approx-0.3$, $r\approx1.9$. In [15] an interesting numerical experiment based on 1000 realizations of a SNAP type experiment is reported, probing a fiducial ΛCDM model. Our values of the statefinder lie outside the three-sigma confidence region displayed in [15]. Based on this fact it can be expected that future SNAP experiments should be able to discriminate between the pure Chaplygin gas model and the standard ΛCDM model.

Let us consider now a more "realistic" cosmological model which, in addition to a Chaplygin's component, also contains a dust component. For a two-component fluid Eqs. (7) take the following form:

$$r=1+\frac{9}{2(\rho+\rho_1)}\left[\frac{\partial p}{\partial\rho}(\rho+p)+\frac{\partial p_1}{\partial\rho_1}(\rho_1+p_1)\right], \tag{14}$$

$$s=\frac{1}{p+p_1}\left[\frac{\partial p}{\partial\rho}(\rho+p)+\frac{\partial p_1}{\partial\rho_1}(\rho_1+p_1)\right]. \tag{15}$$

If one of the fluids is dust, i.e., $p_1=p_d=0$, the above formulas become

$$r=1+\frac{9(\rho+p)}{2(\rho+\rho_d)}\frac{\partial p}{\partial\rho}, \quad s=\frac{\rho+p}{p}\frac{\partial p}{\partial\rho}. \tag{16}$$

If the second fluid is the Chaplygin gas, proceeding exactly as before we obtain the following relation:

$$r=1-\frac{9}{2}\frac{s(s+1)}{1+\rho_d/\rho}. \tag{17}$$

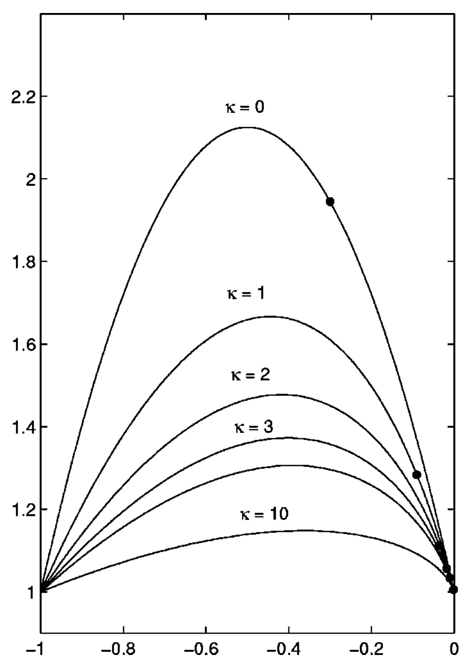

FIG. 2. s-r evolution diagram for the Chaplygin gas mixed with dust. Dots locate the current value of the statefinder.

To find the term ρ_d/ρ we write down the dependence of the dust density on the cosmological scale factor:

$$\rho_d = \frac{C}{a^3}, \tag{18}$$

where C is a positive constant. Equation (13) gives that $Aa^6 + B = -B/s$ and therefore

$$\frac{\rho_d}{\rho} = \frac{C}{\sqrt{Aa^6+B}} = \kappa\sqrt{-s}, \tag{19}$$

where the constant $\kappa = C/\sqrt{B}$ is the ratio between the energy densities of dust and of the Chaplygin gas at the beginning of the cosmological evolution. Thus

$$r = 1 - \frac{9}{2}\frac{s(s+1)}{1+\kappa\sqrt{-s}}. \tag{20}$$

Graphs of the function (20) for different choices of κ are plotted in Fig. 2.

In this case there are choices of the parameters so that the current values of the statefinder are close to the ΛCDM fixed point. For $\kappa=1$ we have $s=-0.09$ and $r=1.2835$; by increasing κ we get closer and closer to the point (0,1). Already for $\kappa=2$ we get $s=0.035$, $r=1.11$, while for $\kappa \gtrsim 5$ the statefinder essentially coincides with the ΛCDM fixed point (see Fig. 2).

Thus our two-fluid cosmological models (with, κ, say bigger than 5) cannot be discriminated from the ΛCDM model on the basis of the statefinder analysis.

The comparison of the two-fluid (i.e., the Chaplygin gas plus dust) cosmological model with observational data has been studied in [17,18]. In [17], on the basis of an analysis of data coming from 26 supernovae, it has been claimed that the best fitting model seems to be the pure Chaplygin gas without dust. On the other hand, by combining data from 92 supernovae with density perturbation growth constraints, it was pointed out in [18] that the Chaplygin component should mimic closely a standard cosmological constant, assuming that $\Omega_m \sim 0.3$ for dust. When the matter content of the model is entirely baryonic with $\Omega_m = \Omega_b \sim 0.04$ a Λ-like behavior is strongly excluded.

However, even if the Chaplygin component closely mimics the cosmological constant today, this neither spoils the interest of the two-fluid model nor makes it equivalent to ΛCDM; for instance, one advantage of the model is that it may suggest a solution to the cosmic coincidence conundrum: here the initial values of the energies of dust and of the Chaplygin gas can be of the same order of magnitude. In particular, the value $\kappa=1$ is not excluded by current observations. This may be seen by using the results of [18] and taking into account the relation

$$\kappa = \frac{\Omega_m}{(1-\Omega_m)\sqrt{1-v_s^2}}, \tag{21}$$

where $\Omega_m = \rho_d/(\rho_d+\rho)$ and where ρ, ρ_d, and v_s are evaluated at the present epoch.

To get more precise constraints on the parameters of the Chaplygin models one probably needs both new observations and additional diagnostic techniques.

Similar conclusions also hold for a one-fluid model of a generalized Chaplygin gas with a modified equation of state, as introduced in [9]:

$$p = -\frac{A}{\rho^\alpha}, \tag{22}$$

with $0 \leqslant \alpha \leqslant 1$. This model gives a cosmological evolution from an initial dustlike behavior to an asymptotic cosmological constant, with an intermediate epoch that can be seen as a mixture of a cosmological constant with a fluid obeying the state equation $p = \alpha\rho$ ($\alpha=0$ corresponds to the ΛCDM model). This generalized model was studied in some detail in [22]. Equation (22) gives the following dependence of the energy density ρ on the scale factor a:

$$\rho = \left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{1/(\alpha+1)}. \tag{23}$$

In this case the squared velocity of sound

$$v_s^2 = \frac{\partial p}{\partial \rho} = -\frac{\alpha p}{\rho} = \frac{A\alpha}{A + B/(a^{3(\alpha+1)})} \tag{24}$$

varies from 0 to α. From Eq. (9) it follows that $s = v_s^2 - \alpha$ and therefore the admissible values of s are now in the interval between $-\alpha$ and 0. From Eq. (9) we get

$$r=1-\frac{9}{2}\frac{s(s+\alpha)}{\alpha}. \tag{25}$$

Chosing as before the value of the deceleration parameter $q\approx-0.5$ we can get the present values of the statefinder parameters for different values of α:

$$s\approx-0.3\,\alpha, \quad r\approx1+0.9\,\alpha. \tag{26}$$

The present values are situated on a line issuing from the ΛCDM fixed point of the statefinder plane. For small values of α the generalized Chaplygin gas model becomes indistinguishable from the ΛCDM model itself.

For a two-fluid cosmological model including both the generalized Chaplygin gas and dust, by using again Eq. (16) we get the following relation that generalizes Eq. (20):

$$r=1-\frac{9s(s+\alpha)}{2\alpha(1+\kappa(-s)^{1/(\alpha+1)}/\alpha^{1/(\alpha+1)})}. \tag{27}$$

As before, the parameter κ gives the ratio between the energy density of the dust and that of the generalized Chaplygin gas at the beginning of the cosmological evolution. Again, increasing the value of the parameter κ implies a shift of the present value of the statefinder toward the fixed ΛCDM point (see Fig. 3).

The generalization of formula (21) connecting the parameter κ to the parameters Ω_m and v_s^2 is the following:

$$\kappa=\frac{\Omega_m}{1-\Omega_m}\left(1+\frac{\alpha v_s^2}{\alpha-v_s^2}\right)^{1/(\alpha+1)}. \tag{28}$$

In [19] the generalized Chaplygin gas model ($0\leqslant\alpha\leqslant1$) was tested with data coming from 62 supernovae. Results seem to point out that this model is consistent with current data for any value of α in the considered range, although values of $\alpha\sim0.4$ are favored. In [21] the dependence of the cosmic microwave background radiation peaks on the parameters of the cosmological model including the generalized Chaplygin gas and the totally baryonic dustlike matter was studied. It was shown that observational data arising from BOOMERANG for the position of the third peak, combined with the supernova data and the constraints following from age estimates of high-redshift objects [23], allow significant restriction of the range of the parameters of the model. With the hypothesis of a purely baryonic nature of the dustlike matter, the allowed region for values of α is $0.2\leqslant\alpha\leqslant0.6$ [21].

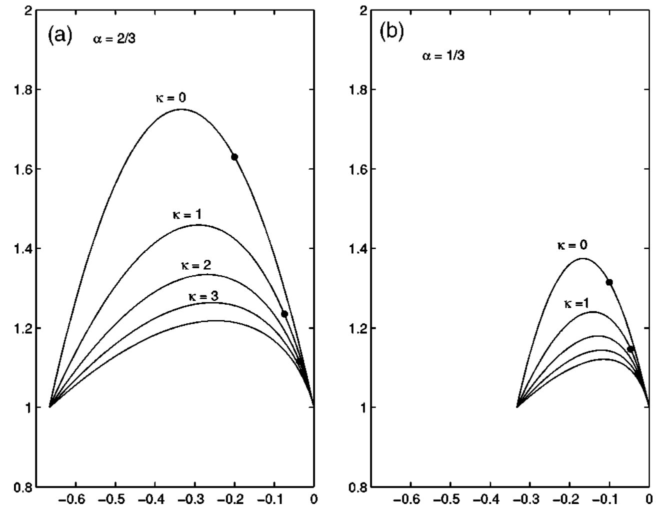

FIG. 3. s-r evolution diagram for the generalized Chaplygin gas mixed with dust. Dots locate the current value of the statefinder.

In conclusion, from the statefinder viewpoint both the pure Chaplygin model and the two-fluid mixture have different behaviors with respect to other commonly studied models. In particular, a future larger amount of data on high-z type Ia supernovae may allow discrimination between the different models. However, from the point of view of the statefinder the mixed two-fluid model becomes practically indistinguishable from ΛCDM for sufficiently large values of the parameter κ ($\kappa\gtrsim5$). On the other hand a mixed two-fluid model with κ of order 1 is attractive from the point of view of a possible solution of the cosmic coincidence problem.

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