# Cosmology with tachyon field as dark energy

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We present a detailed study of cosmological effects of homogeneous tachyon matter coexisting with non-relativistic matter and radiation, concentrating on the inverse square potential and the exponential potential for the tachyonic scalar field. A distinguishing feature of these models (compared to other cosmological models) is that the matter density parameter and the density parameter for tachyons remain comparable even in the matter dominated phase. For the exponential potential, the solutions have an accelerating phase, *followed by* a phase with $a(t) \propto t^{2/3}$ as $t \to \infty$. This eliminates the future event horizon present in cold dark matter models with a cosmological constant ($\Lambda$CDM) and is an attractive feature from the string theory perspective. A comparison with supernova type Ia data shows that for both the potentials there exists a range of models in which the universe undergoes an accelerated expansion at low redshifts which are also consistent with the requirements of structure formation. They do require fine-tuning of parameters but not any more than in the case of $\Lambda$ CDM models or quintessence models.

## I. MOTIVATION

Observations suggest that our universe has entered a phase of accelerated expansion in the recent past. Friedmann equations can be consistent with such an accelerated expansion only if the universe is populated by a medium with negative pressure. One of the possible sources which could provide such a negative pressure will be a scalar field with either of the following two types of Lagrangians:

$$L_{\text{quin}} = \frac{1}{2} \partial_a \phi \partial^a \phi - V(\phi); \tag{1}$$

$$L_{\text{tach}} = -V(\phi)[1 - \partial_a \phi \partial^a \phi]^{1/2}.$$

Both these Lagrangians involve one arbitrary function $V(\phi)$. The first one, $L_{\text{quin}}$, which is a natural generalization of the Lagrangian for a nonrelativistic particle, $L = (1/2)\dot{q}^2 - V(q)$, is usually called quintessence (for a sample of models, see Ref. [1]). When it acts as a source in a Friedmann universe, it is characterized by a time dependent $w(t) \equiv (P/\rho)$ with

$$\rho_q(t) = \frac{1}{2}\dot{\phi}^2 + V; \quad P_q(t) = \frac{1}{2}\dot{\phi}^2 - V; \tag{2}$$

$$w_q = \frac{1 - (2V/\dot{\phi}^2)}{1 + (2V/\dot{\phi}^2)}.$$

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Just as $L_{\text{quin}}$ generalizes the Lagrangian for the nonrelativistic particle, $L_{\text{tach}}$ generalizes the Lagrangian for the relativistic particle [2]. A relativistic particle with a (one dimensional) position $q(t)$ and mass $m$ is described by the Lagrangian $L = -m\sqrt{1 - \dot{q}^2}$. It has the energy $E = m/\sqrt{1 - \dot{q}^2}$ and momentum $p = m\dot{q}/\sqrt{1 - \dot{q}^2}$ which are related by $E^2 = p^2 + m^2$. As is well known, this allows the possibility of having massless particles with finite energy for which $E^2 = p^2$. This is achieved by taking the limit of $m \to 0$ and $\dot{q} \to 1$, while keeping the ratio in $E = m/\sqrt{1 - \dot{q}^2}$ finite. The momentum acquires a life of its own, unconnected with the velocity $\dot{q}$, and the energy is expressed in terms of the momentum (rather than in terms of $\dot{q}$) in the Hamiltonian formulation. We can now construct a field theory by upgrading $q(t)$ to a field $\phi$. Relativistic invariance now requires $\phi$ to depend on both space and time [$\phi = \phi(t, \mathbf{x})$] and $\dot{q}^2$ to be replaced by $\partial_i \phi \partial^i \phi$. It is also possible now to treat the mass parameter $m$ as a function of $\phi$, say, $V(\phi)$ thereby obtaining a field theoretic Lagrangian $L = -V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi}$. The Hamiltonian structure of this theory is algebraically very similar to the special relativistic example we started with. In particular, the theory allows solutions in which $V \to 0$, $\partial_i \phi \partial^i \phi \to 1$ simultaneously, keeping the energy (density) finite. Such solutions will have finite momentum density (analogous to a massless particle with finite momentum $p$) and energy density. Since the solutions can now depend on both space and time (unlike the special relativistic example in which $q$ depended only on time), the momentum density can be an arbitrary function of the spatial coordinate. This form of scalar field arises in string theories [3] and—for technical reasons—is called a tachyonic scalar field. (The structure of this Lagrangian is similar to those analyzed in a wide class of models called $K$ essence; see, for example, Ref. [4].) This provides a rich gamut of possibilities in the context of cosmology [2,4–7].

The stress tensor for the tachyonic scalar field can be written in a perfect fluid form,

$$T^i_k = (\rho + p) u^i u_k - p \, \delta^i_k,$$  (3)

with

$$u_k = \frac{\partial_k \phi}{\sqrt{\partial^i \phi \partial_i \phi}}; \quad \rho = \frac{V(\phi)}{\sqrt{1 - \partial^i \phi \partial_i \phi}};$$  (4)

$$p = -V(\phi) \sqrt{1 - \partial^i \phi \partial_i \phi}.$$

The remarkable feature of this stress tensor is that it could be considered as *the sum of a pressure less the dust component and a cosmological constant* [2]. To show this explicitly, we break up the density $\rho$ and the pressure $p$ and write them in a more suggestive form as

$$\rho = \rho_V + \rho_{DM}; \quad p = p_V + p_{DM},$$  (5)

where

$$\rho_{DM} = \frac{V(\phi) \partial^i \phi \partial_i \phi}{\sqrt{1 - \partial^i \phi \partial_i \phi}}; \quad p_{DM} = 0;$$  (6)

$$\rho_V = V(\phi) \sqrt{1 - \partial^i \phi \partial_i \phi}; \quad p_V = -\rho_V.$$

This means that the stress tensor can be thought of as made up of two components—one behaving like a pressureless fluid, while the other having a negative pressure. In the cosmological context, when $\dot\phi$ is small (compared to $V$ in the case of quintessence or compared to unity in the case of tachyonic field), both these sources have $w \rightarrow -1$ and mimic a cosmological constant. When $\dot\phi \gg V$, the quintessence has $w \approx 1$ leading to $\rho_q \propto (1+z)^6$; the tachyonic field, on the other hand, has $w \approx 0$ for $\dot\phi \rightarrow 1$ and behaves like nonrelativistic matter.

An additional motivation for studying models based on $L_{quin}$ is the following: The standard explanation of the current cosmological observations will require two components of dark matter: (i) The first one is a dust component with the equation of state $p = 0$ contributing $\Omega_m \approx 0.35$. This component clusters gravitationally at small scales ($l \lesssim 500$ Mpc, say) and will be able to explain observations from galactic to supercluster scales. (ii) The second one is a negative pressure component with equation of state like $p = w\rho$ with $-1 < w < -0.5$ contributing about $\Omega_V \approx 0.65$. There is some leeway in the $(p/\rho)$ of the second component but it is certain that $p$ is negative and $(p/\rho)$ is of order unity. The cosmological constant will provide $w = -1$ while several other candidates based on scalar fields with potentials [1] will provide different values for $w$ in the acceptable range. By and large, component (ii) is noticed only in the large scale expansion and it does not cluster gravitationally to a significant extent. Neither of the components (i) or (ii) has laboratory evidence for its existence directly or indirectly. In this sense, cosmology requires invoking wishful thinking twice to explain the current observations. It was suggested recently [2] that one may

be able to explain the observations at all scales using a single scalar field with a particular form of Lagrangian.

In this paper we explore the cosmological scenario in greater detail, concentrating on the background cosmology. Our approach will be based on the above viewpoint and we shall treat the form of the Lagrangian for $L_{tach}$ as our starting point without worrying about its origin. In particular, we shall *not* make any attempt to connect the form of $V(\phi)$ with string theoretic models but will explore different possibilities, guided essentially by their cosmological viability.

We construct cosmological models with homogeneous tachyon matter, assuming that tachyon matter coexists with normal nonrelativistic matter and radiation. Section II presents the equations for the evolution of the scale factor and the tachyon field. In Sec. III we present solutions of these equations and discuss variations introduced by the available parameters in the model. Here we have analyzed two different models of the scalar field potential; one is the exponential potential and the other is the inverse square potential which leads to power law cosmology [7]. In Sec. IV we compare the model with observations and constrain the parameters. Section V discusses structure formation in tachyon models. The results are summarized in a conclusion in Sec. VI.

## II. TACHYON-MATTER COSMOLOGY

For a spatially flat universe, the Friedmann equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$  (7)

where $\rho = \rho_{NR} + \rho_R + \rho_\phi$, with respective terms denoting nonrelativistic, relativistic, and tachyon-matter densities. For the tachyon field $\phi$ we have

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot\phi^2}}, \quad p_\phi = -V(\phi) \sqrt{1 - \dot\phi^2}.$$  (8)

The equation of state for tachyon matter is $p = w\rho$ with $w = \dot\phi^2 - 1$. The scalar field equation of motion is

$$\ddot\phi = -(1 - \dot\phi^2) \left[ 3H\dot\phi + \frac{1}{V(\phi)} \frac{dV}{d\phi} \right].$$  (9)

The structure of this equation suggests that the change in $\dot\phi$ goes to zero as it approaches $\pm 1$. In this case, the equation of state for the tachyon field is dustlike. Thus at any stage if the tachyon field behaves like dust, it will continue to do so for a long time. This behavior persists for a duration that depends on the closeness of $\dot\phi$ to $\pm 1$. Detailed behavior will depend on the form of the potential.

We shall discuss cosmological models with two different $V(\phi)$ in this paper. The first one has the form

$$V(\phi) = \frac{n}{4\pi G} \left(1 - \frac{2}{3n}\right)^{1/2} \phi^{-2}.$$

It was shown in Ref. [7] that the above potential leads to the expansion $a(t) = t^n$ if $\phi$ is the only source. [The form $V(\phi) \propto 1/(\phi - \phi_0)^2$ can be reduced to the form given above by a simple redefinition of the scalar field.] In this case, the term in the square bracket in Eq. (9) is $[3H\dot{\phi} - 2/\phi]$. This term vanishes in the asymptotic limit when tachyons dominate and $\phi \propto t$. This asymptotic solution is stable in the sense that all initial conditions eventually lead to this state. If normal matter or radiation dominates, $\dot{\phi}$ stays close to the transition point $2/(3H\phi)$, unless we start the field very close to $\dot{\phi}^2 = 1$. Thus the presence of nonrelativistic matter and radiation leads to a change in the equation of state for the tachyon field. The change in equation of state for the tachyon field implies that it is *not* a tracker field (for details see Refs. [8,9]). To get a viable model, i.e., matter domination at high redshifts and an accelerating phase at low redshifts, we need to start the tachyon field such that $\dot{\phi}$ is very close to unity, and $\phi$ is very large. We will describe the fine tuning required in greater detail when we discuss numerical solutions of these equations.

The second form of potential which we will consider is the exponential one with $V(\phi) \propto e^{-\phi/\phi_0}$. Then the term in square brackets in Eq. (9) can be written as $[3H\dot{\phi} - 1/\phi_0]$. In a universe dominated by radiation or normal matter, $H(t)$ is a monotonically decreasing function of time while $\phi_0$ is a constant. From this equation, it is clear that $\dot{\phi}$ will increase if $\dot{\phi} < 1/[3H(t)\phi_0]$. As $H(t)$ keeps decreasing, $\dot{\phi}$ will increase slowly and approach unity, asymptotically. However, in the meanwhile, tachyons may begin to dominate in terms of energy density and this changes the behavior. In a tachyon dominated scenario, we get a rapid expansion of the universe and $H(t)$ varies much more slowly than in the matter or radiation dominated era. Thus $\dot{\phi}$ changes at a slower rate but it still approaches unity and hence we—eventually—get a dustlike equation of state for the tachyon field. Whether there is an accelerating phase or not depends on the initial values of $\dot{\phi}$, $\phi_0$, and $\Omega_\phi$. Present values of the density parameter for nonrelativistic matter and tachyons fix the epoch at which tachyons start to dominate the energy density. The parameter $\phi_0$ sets the time when $\dot{\phi}$ approaches unity and the asymptotic dustlike phase for tachyons is reached. The initial value of $\dot{\phi}$ fixes the duration of the accelerating phase. If this value is very close to unity then it departs very little from this value through the entire evolution. On the other hand, if it starts far away from this value, then the equation of state for the tachyon field can lead to a significantly long accelerating phase. It is possible to fine-tune the evolution by choosing a sufficiently large value for the constant $\phi_0$, so that $\dot{\phi}$ is much smaller than unity even at the present epoch, and by requiring that the tachyon field start to dominate the energy density of the universe at the present epoch. In such a case, the accelerating phase is a transient between the matter dominated era and the tachyon dominated era with $a(t) \propto t^{2/3}$ in both of these regimes. This model has been the focus of many studies [10] though most of these have chosen to ignore the role of matter or radiation.

Some of the models discussed below have an accelerating phase for the universe *followed by* a dustlike expansion with $a(t) \propto t^{2/3}$ asymptotically. These models have the attractive feature that they do *not* behave as a de Sitter-like universe in the asymptotic future and thus do not possess a future horizon. String theoretic models have difficulty in incorporating cosmologies which approach the de Sitter-like phase asymptotically; from this point of view, these models can be theoretically attractive.

## III. NUMERICAL SOLUTIONS TO COSMOLOGICAL EQUATIONS

This section presents a detailed study of the background tachyon cosmology. We solve the cosmological equations of motion numerically for the two potentials mentioned in the previous section.

### A. Inverse square potential

We start with the discussion of the potential $V(\phi) \propto \phi^{-2}$. The energy density of the universe is a mix of the tachyon field, radiation, and nonrelativistic matter. The asymptotic solutions for the universe with only the tachyon field as the source are well understood: we get rapid expansion of the universe with the field $\phi$ growing in proportion with time. We wish to see the effect of other species (matter and radiation) on the evolution of the tachyon field, and the stability of the asymptotic solution.

Since only two of the three equations in Eqs. (7) and (9) are independent, we choose to drop the equation with the second derivative of the scale factor. We choose some instant of time as $t_{\text{in}}$ and rescale variables as follows, for numerical convenience:

$$x = tH_{\text{in}}, \quad y = a(t)/a(t_{\text{in}}), \quad y' = dy/dx \quad (10)$$

and $\psi = \phi/\phi_{\text{in}}, \psi' = d\psi/dx$. (We will use the present epoch and Planck epoch for $t_{\text{in}}$, in two different contexts described below.) Then the equations are

$$y' = y \left[ \frac{\Omega_{M_{\text{in}}}}{y^3} + \frac{\Omega_{R_{\text{in}}}}{y^4} + \frac{2n}{3} \left( 1 - \frac{2}{3n} \right)^{1/2} \right.$$

$$\left. \times \frac{1}{\phi_{\text{in}}^2 H_{\text{in}}^2 \psi^2 \sqrt{1 - \phi_{\text{in}}^2 H_{\text{in}}^2 \psi'^2}} \right]^{1/2}, \quad (11)$$

$$\psi'' = (1 - \phi_{\text{in}}^2 H_{\text{in}}^2 \psi'^2) \left( \frac{2}{\phi_{\text{in}}^2 H_{\text{in}}^2 \psi} - 3\frac{y'}{y} \psi'_{\text{in}} \right). \quad (12)$$

The subscript in refers to the initial value, and $\Omega_M$ and $\Omega_R$ refer to the density parameters for matter and radiation, respectively. It is assumed that the sum of all density parameters is unity.

The initial conditions for $y$ and $\psi$ follow from their definition: $y_{\text{in}} = 1, \psi_{\text{in}} = 1$. The initial condition for $\psi'$ can be related to the density parameter of tachyon matter at $t = t_{\text{in}}$,
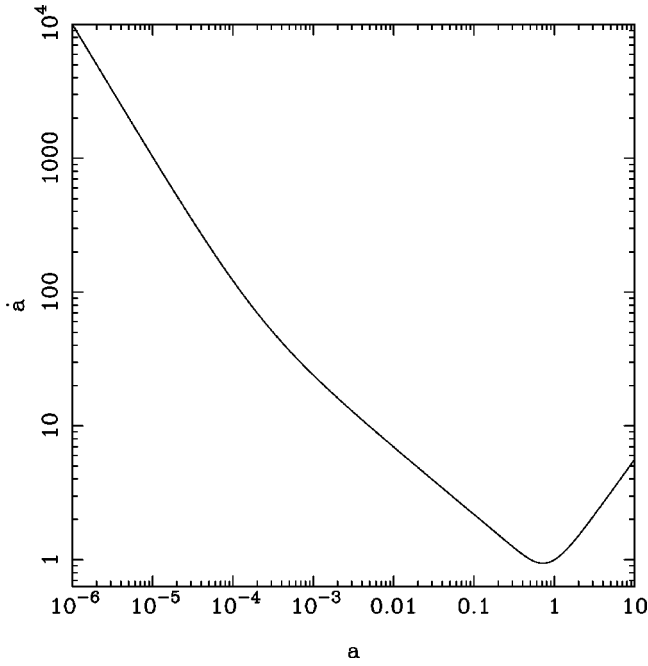
FIG. 1. Phase portrait for the scale factor. This plot is for $\Omega_M$ (present) $=0.3$ and $n=6$. We choose the present value of $\phi_{in}H_{in}=2.4$ where "in" refers to the present epoch ($a=1; z=0$). The transition from a decelerated expansion to an accelerated one is clear in this picture. We have used this model for all the figures pertaining to the inverse square potential.

$$\Omega_{\phi_{in}} = \frac{2n}{3}\left(1 - \frac{2}{3n}\right)^{1/2} \frac{1}{\phi_{in}^2 H_{in}^2 \psi_{in}^2 \sqrt{1 - \phi_{in}^2 H_{in}^2 \psi_{in}'^2}}. \quad (13)$$

There are two branches of solutions, one with positive $\psi'$ and another with negative values of $\psi'$. For each set of $\Omega_{m_{in}}$ and $n$, there is a minimum value of $\phi_{in}H_{in}$ below which we cannot satisfy this equation. We can solve these equations if $\psi'^2 > 0$, or if

$$\phi_{in}^2 H_{in}^2 \geq \frac{2n}{3\Omega_{\phi_{in}}}\left(1 - \frac{2}{3n}\right)^{1/2}. \quad (14)$$

For a given value of $\phi_{in}^2 H_{in}^2$, we get initial values of all the variables from the above equations and using these we evolve these quantities. As shown in the above equations, parameters $n$, $\Omega_{\phi_{in}}$, $\phi_{in}H_{in}$, and $|\dot{\phi}_{in}|$ are interrelated. For a fixed $n$ and $\Omega_{\phi_{in}}$, larger values of the $\phi_{in}^2 H_{in}^2$ imply a larger value of initial $|\dot{\phi}|$, and hence an equation of state that is closer to that of dust.

We study the evolution of such a model in three steps. First we evolve the system from the present day to the future and show that the asymptotic solution [7] is stable, i.e., the entire allowed range of present values leads to the same asymptotic solution. In this context "in" refers to the present values of the parameters. We choose the present value of $\Omega_{M_{in}}=0.3$, and all the results shown here use $n=6$ (results do not change qualitatively for other values of $n$), i.e., the asymptotic solution gives $a(t) \propto t^6$.
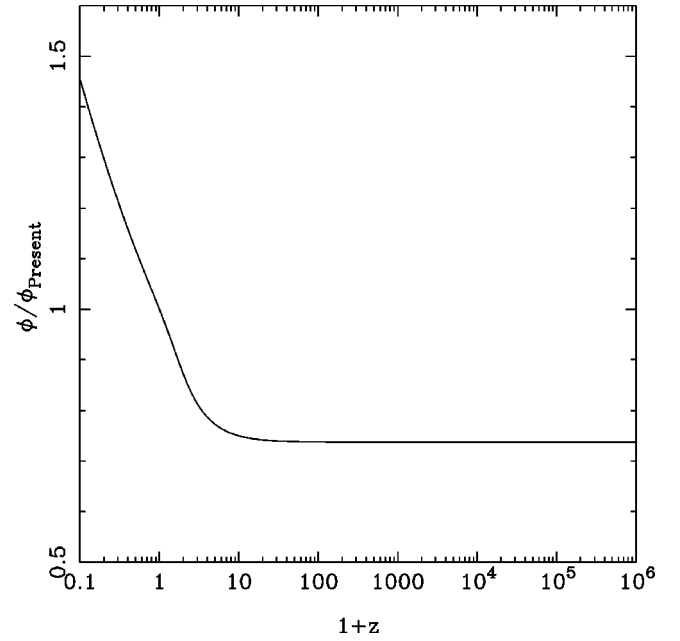


FIG. 2. The tachyon field $\phi/\phi_{present}$ as a function of redshift. The model is the same as in Fig. 1.

We plot the phase portrait for the scale factor in Fig. 1. This figure clearly shows the late time acceleration in a tachyon dominated universe. (The figures exhibit past as well as future evolution; the past evolution is discussed later.) The positive branch is plotted in this and the following figures but the discussion holds true for the negative branch as well. The evolution of field $\phi$ and its time derivative $\dot{\phi}$ are shown in Figs. 2 and 3. These too demonstrate the approach to the asymptotic solution for the two initial conditions. In the asymptotic future, we expect $\dot{\phi}$ to approach a constant value
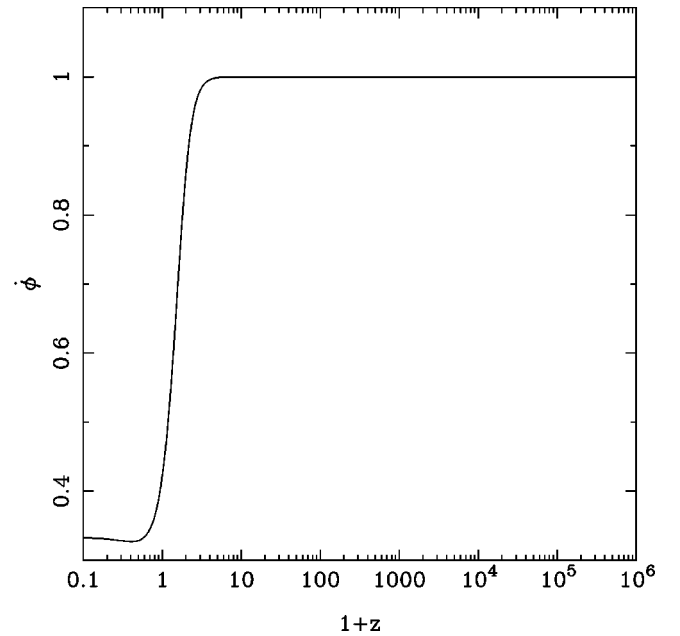


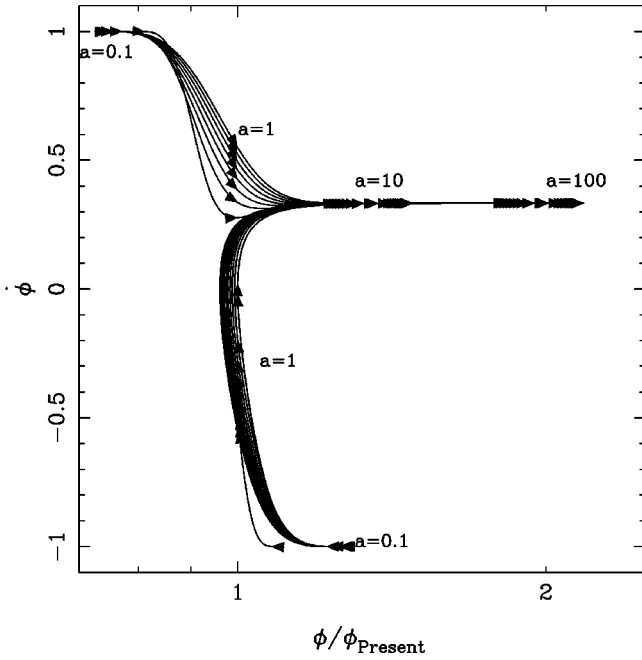FIG. 3. This plot shows $\dot{\phi}$ as a function of redshift for the same model.

FIG. 4. Phase plot for the tachyon field. Here, we started from initial conditions with a fixed value of $\phi$ and $n$. $\dot{\phi}$ was varied and $\Omega_{\phi_{in}}$ was allowed to vary with it. Both the positive branch as well as the negative branch are shown. The scale factor $a(t) = 1/(1 + z)$ is marked along the arrow heads.



FIG. 5. $\Omega_M$, $\Omega_\phi$, and $\Omega_R$ as functions of redshift. The matter density parameter is almost a constant at $2 \lesssim z \lesssim 10^3$ and then drops to small values as radiation begins to dominate. The present day values of the density parameter for matter and radiation are 0.3 and $10^{-5}$, respectively.

of $\sqrt{2/3n}$. Indeed, both the negative and the positive branches (which, respectively, correspond to positive $\psi'$ and negative $\psi'$) exhibit this behavior. The fact that other initial conditions lead to similar behavior is shown in the phase diagram for field $\phi$ in Fig. 4. We see that for the entire range of initial conditions for the field $\phi$, it quickly approaches the asymptotic solution. It takes longer to reach the asymptotic solution for initial conditions where $|\phi_{in}H_{in}|$ (the value at present) is much larger than its minimum allowed value since in this case $|\dot{\phi}|$ is closer to unity and hence the present equation of state for the tachyon field is like that of a pressureless fluid. In such a case the accelerating phase of the universe starts at late times compared to models where $|\phi_{in}H_{in}|$ is closer to its minimum allowed value. Thus we need to fine-tune the value of this parameter in order to arrange for the accelerating phase of the universe to start at low redshifts, as required by observations.

In future evolution, the density parameter for matter decreases with the start of the accelerating phase. The density parameter for tachyons begins to increase in this phase and quickly approaches unity.

Next we study the behavior of these solutions in the past, i.e., we use the present day conditions and evolve back to see the kind of initial conditions that are required for us to get a viable model today. Figures used to illustrate future evolution also show the past evolution of these quantities. As motivated in Sec. II, we expect matter and radiation to drive $\dot{\phi}$ away from unity towards smaller values in forward evolution. So, as we evolve the equations back, we expect all present day conditions to lead towards $\dot{\phi} = 1$. This is indeed
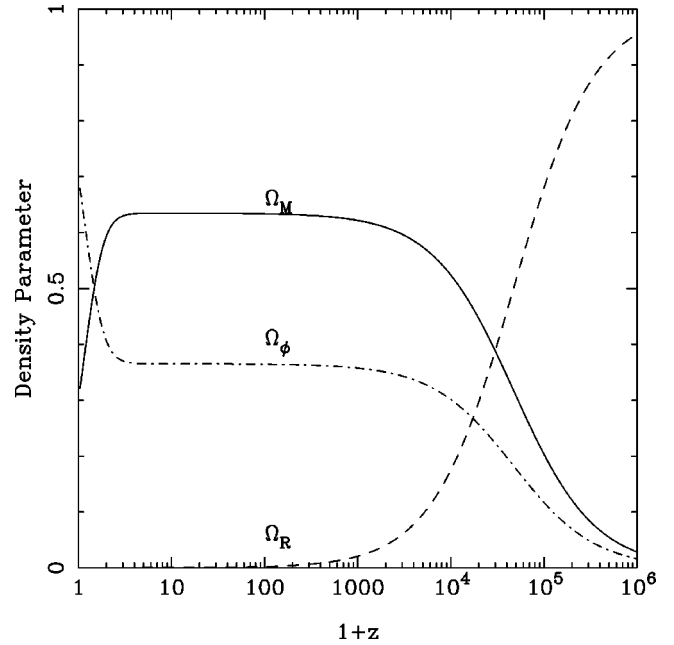
what we find in numerical solutions. The equation of state for the tachyon field approaches that of a pressureless fluid, and the universe expands with $a(t) \propto t^{2/3}$ at high redshifts. At even earlier times, radiation takes over and both matter and tachyons become subdominant components.

The time dependent matter density parameter $\Omega_M(t) = 8\pi G \rho_M(t)/3H^2(t)$ is plotted as a function of redshift in Fig. 5. Here nonrelativistic matter dominates in the sense that $\Omega_M > \Omega_\phi$ but evolution does not drive $\Omega_\phi$ towards zero or $\Omega_M$ close to unity. Instead the two density parameters remain comparable during much of the "matter dominated" era (from $z \approx 10^3$ to $z \approx 2$). This is a unique feature not seen in other cosmological models. In the phase where the equation of state of the tachyon field approaches $w \simeq 0$, the ratio of the density parameter for nonrelativistic matter and tachyon field becomes a constant. For models of interest, i.e., where the accelerating phase starts at low redshifts, this ratio is of order unity for most choices of parameters and the relative importance of tachyons and matter does not change up to the time when the accelerating phase begins. (This has a significant effect on gravitational instability and growth of perturbations; see Sec. V.)

The tachyon field $\phi$ approaches a nearly constant value in the past and changes very little from its value in the early universe to the time when the accelerating phase begins. In the preacceleration phase, $\dot{\phi}$ is close to unity. This means that the field $\phi$ starts from large values in the models that satisfy basic observational constraints. This is the fine tuning required in constructing viable tachyon models. In Fig. 6 we show the evolution of $\phi(t)H(t)$ as a function of redshift. As expected, this is a constant in the asymptotic regime where
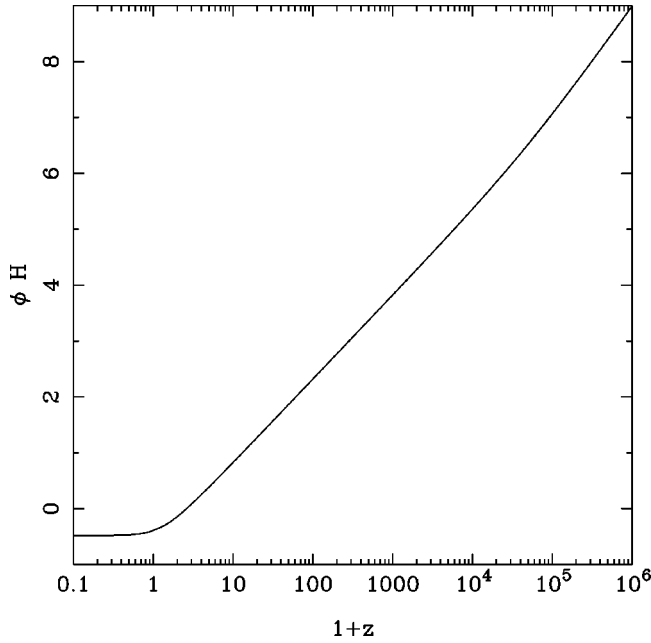
FIG. 6. Plot of $\phi H$ as a function of redshift.



FIG. 7. Plot of $w_\phi = \dot{\phi}^2 - 1$ as a function of the scale factor. The curve on the left has the lowest value of the initial $\phi H$ and increases by an order of magnitude for respective curves as we approach the right.

tachyons dominate. However, this function evolves rapidly in the dustlike regime. Since $\phi$ remains a constant in this regime, the change is mainly due to change in $H(t)$. The asymptotic value of this product is $\phi H(t) \rightarrow \sqrt{2n/3}$, so we expect $\phi H(t)$ to decrease from its initial value and make a transition to the constant asymptotic value. We need to choose the initial value of this product so that the transition from dustlike to accelerating phase happens around now. Since the asymptotic value is $\phi H(t) \rightarrow \sqrt{2n/3}$, and this is of order unity for small $n$, the value of $\phi$ in the early universe has to be large. The product $\phi(t_{Pl})H(t_{Pl})$ is approximately $\phi(t_0)H(t_{Pl})$ since $\phi$ does not change by much during its evolution. Since the present day $\phi(t_0)H_0$ is of order unity, $\phi(t_{Pl})H_{Pl} \approx H_{Pl}/H_0$. (Here $t_0$ is the present time and $H_0$ is the present value of the Hubble constant.) Thus the initial value of $\phi$ sets the time scale when the transition to asymptotic behavior takes place. This fine-tuning is similar to that needed in models with the cosmological constant since the value of $\Lambda$ has to be tuned to arrange for the accelerating phase to start at low redshifts.

To illustrate this point further, we start the integration at Planck time for a range of initial conditions and show that indeed we obtain this behavior. The age of the universe at the time of transition from pressureless behavior for the tachyon field to an effective cosmological constant behavior is indeed proportional to the initial value of $\phi$. To illustrate this, we first plot $w$ for the tachyon field as a function of the scale factor in Fig. 7. This is plotted for many values of $\phi(t_{Planck})H(t_{Planck})$. An order of magnitude increase in the initial value delays the onset of the accelerating phase by roughly an order. Hence to achieve the accelerating phase at
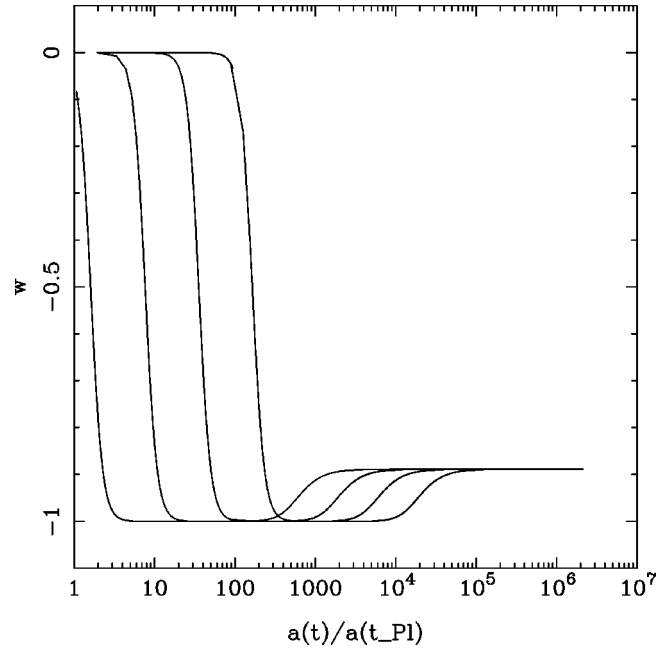
late times, one requires the fine-tuned value mentioned above. The same behavior is summarized in Fig. 8, where we plot the scale factor when $w_\phi = w_{\phi_{asym}}/2 = [2/(3n) - 1]/2$ as a function of $\phi(t_{Planck})H(t_{Planck})$.
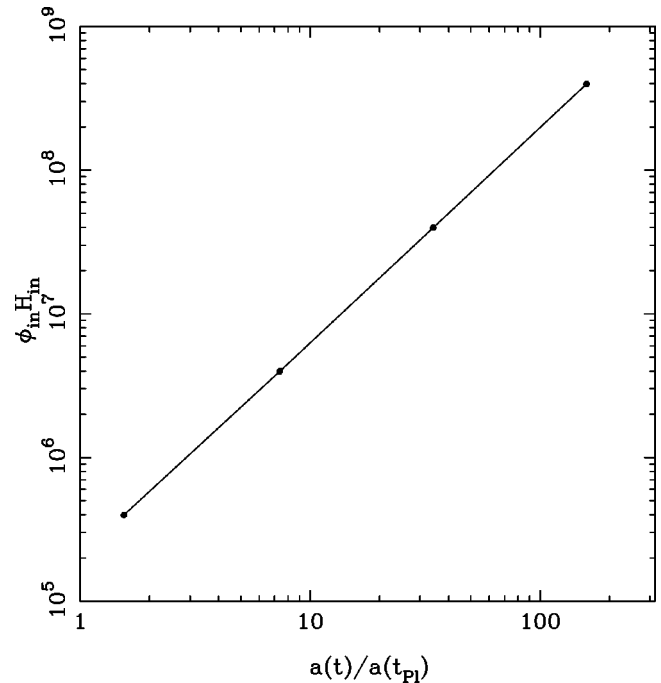


FIG. 8. Plot of $\phi_{in}H_{in}$ (at Planck time) as a function of the scale factor value at which $w_\phi = \dot{\phi}^2 - 1$ becomes half its asymptotic value.
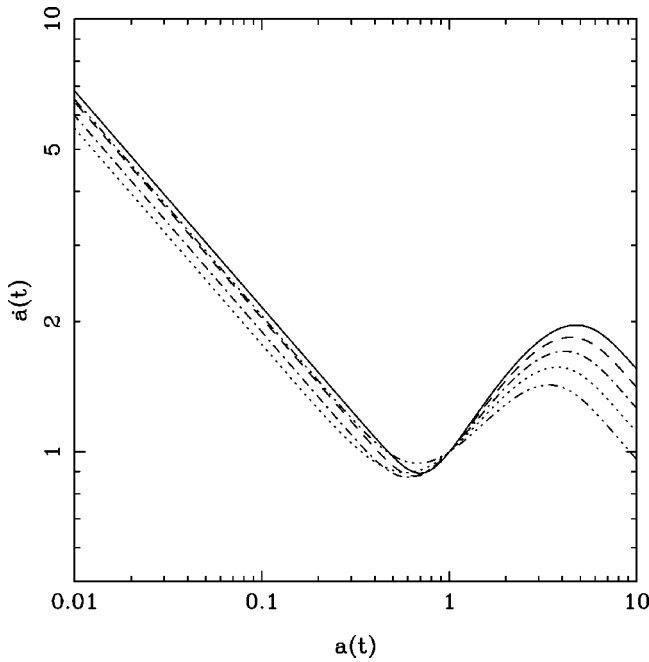
FIG. 9. Phase plot for the exponential potential. The duration of the accelerated phase depends on the initial value of $\dot{\phi}$. The accelerated phase ends into a dustlike phase of tachyons. The solid line is for $\dot{\phi}_{in}=0.1$, the dashed line for $\dot{\phi}_{in}=0.3$, and the dot-dashed, dashed, and dot-dot-dashed curves for $\dot{\phi}_{in}=0.5$, 0.7, and 0.9, respectively.



FIG. 10. Plot of density parameters for matter and tachyon as functions of redshift. The models are the same as those in the previous figure.

### B. Exponential potential

We now repeat the above analysis for the exponential potential. As before, we consider a mix of nonrelativistic matter, radiation/relativistic matter, and the tachyon field. We fix $\Omega_\phi=0.7$ at the present epoch. [In the literature, it has been suggested that the exponential potential does arise in some of the string theoretic models. However, in our approach, we think of $V(\phi)$ as an arbitrary function, just as in quintessence models.] For the purpose of numerical work, we have taken the value of $\phi/\phi_0$ to be $10^2$ at the present epoch. The choice of $\phi$ fixes the amplitude of the potential since we have already fixed $\Omega_\phi$ and $\phi/\phi_0$. The remaining parameter is $\phi_0$, and since we have already fixed $\phi_{in}/\phi_0$, fixing $\phi_0$ is equivalent to fixing $\phi_{in}$. We use the combination $\phi_{in}H_{in}$ to construct a dimensionless parameter which indicates the value of $\phi_0$. $\phi_{in}H_{in}$ should be of order unity if $\phi_{in}$ is comparable to the age of the Universe today. We shall study the variation of the quantities of interest with time for a range of values of $\dot{\phi}$ and $\phi_{in}H_{in}$.

In the case of the exponential potential, the equation of state for the tachyon field approaches that of dust as $t\rightarrow\infty$. Thus any accelerating phase that we might get will be followed by a dustlike phase. The accelerating phase occurs when the tachyon field dominates the energy density of the universe since it is only in this case that we can get $\rho+3p<0$. For an accelerating phase to exist at all, the tachyon field should begin to dominate *before* it enters the dustlike phase. Thus the duration of the accelerating phase will de-
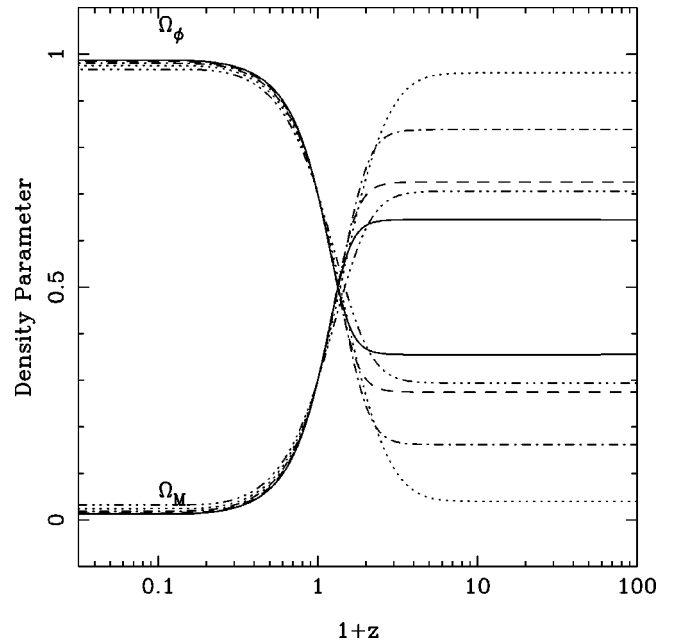
pend on how different $\dot{\phi}$ is from unity at the start of the tachyon dominated phase.

The phase plot for the scale factor is shown in Fig. 9. We have kept $\phi_{in}H_{in}$ fixed at the present day and varied $\dot{\phi}$ for a range of values. The unique feature of this model is that we regain the dustlike phase in the future. It is clear from this plot that the duration of the accelerating phase varies considerably across models shown here. The supernova observations require the universe to be in an accelerating phase at $z<0.25$, so for models that do not have an accelerating phase at all or are too small an accelerating phase can be ruled out easily.

The fact that these cosmological models have two decelerating phases, with an accelerating phase sandwiched in between, is noteworthy. In such models, one can accommodate the current acceleration of the universe without the model "getting stuck" in the accelerating phase for eternity. Since the universe has $a(t)\propto t^{2/3}$ asymptotically, it follows that there will be no future horizon (for some other attempts to eliminate the future horizon, see [16]). String theoretic models have difficulty in accommodating an asymptotically de Sitter-like universe and our model could help in this context.

The plot in Fig. 10 shows $\Omega_m, \Omega_\phi$ as functions of redshift and shows that $\Omega_\phi$ increases monotonically with time.

A plot of $\dot{\phi}$ as a function of $\Omega_\phi$ in Fig. 11 shows that all initial conditions lead to $\dot{\phi}=1$. The function $\phi H$ increases with $1+z$. It is clear that all initial conditions lead to a narrow range of asymptotic values for this product.

If we keep the value of $\dot{\phi}$ fixed and increase $\phi_{in}H_{in}$, the duration of the accelerating phase increases. This is illustrated in Fig. 12. A plot of the density parameters is shown in Fig. 13.
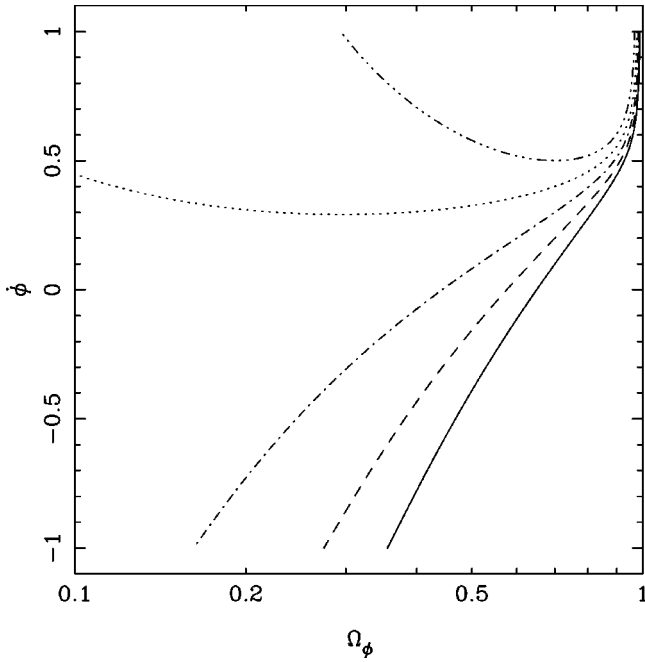
FIG. 11. Plot of $\dot{\phi}$ vs $\Omega_\phi$. The asymptotic value $\dot{\phi}=1$ is reached irrespective of the initial conditions. The models are the same as these in the previous figure.

## IV. COMPARISON WITH SUPERNOVA IA OBSERVATIONS

We now compare the results with supernova type Ia observations by Perlmutter *et al.* [12,13]. The data from the
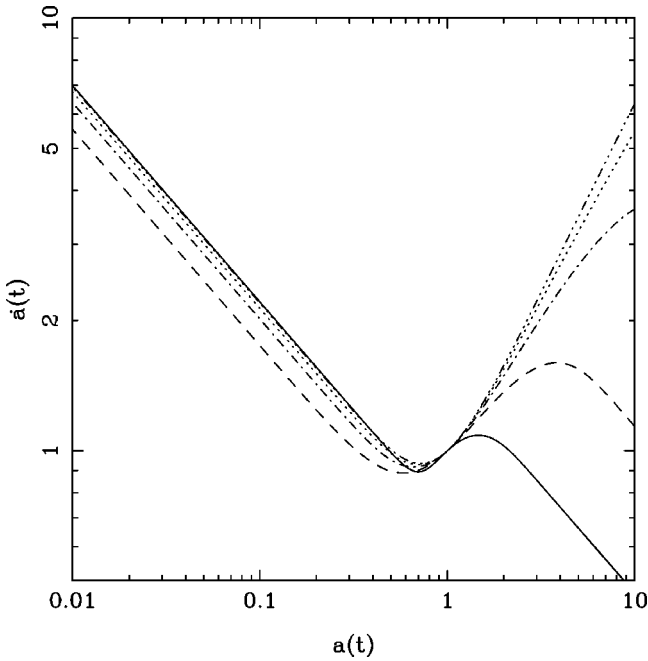


FIG. 12. Phase plot for a fixed value of $\dot{\phi}=0.38$ and varying $\phi_{in}H_{in}$ (present day). The solid line is for $\phi_{in}H_{in}=1/3$, and the dashed, dot-dashed, dotted, and dot-dot-dashed lines for $\phi_{in}H_{in}=5/3$, $7/3$, $3$, and $11/3$, respectively. As we increase the value of $\phi_{in}H_{in}$ the duration of accelerated phase increases.
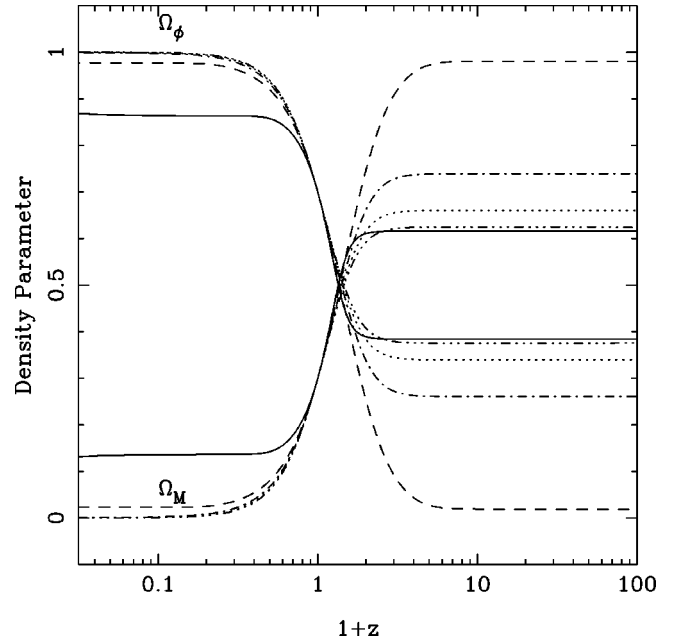


FIG. 13. This plot shows density parameters for matter and tachyon for a fixed value of $\dot{\phi}=0.38$ and varying $\phi_{in}H_{in}$. The models are same as these in Fig. 12.

redshift and luminosity distance observations of these supernovae, collected by the Supernova Cosmology Project, conclude that the universe is currently going through an accelerating phase of expansion. This is a powerful constraint on cosmological models, e.g., it rules out the Einstein–de Sitter model at a high confidence level. The obvious contender is a positive cosmological constant, though more exotic matter driving the accelerated expansion is also a possibility. Since currently the energy density of the universe is dominated by a form of matter with negative pressure, the presence of a tachyon source is thus expected to be consistent with these observations.

The results of theoretical model and observations are compared for luminosity measured in logarithmic units, i.e., magnitudes defined by

$$m_B(z) = \mathcal{M} + 5\log_{10}(D_L), \quad (15)$$

where $\mathcal{M}=M-5\log_{10}(H_0)$ and $D_L=H_0 d_L$, the factor $M$ being the absolute magnitude of the object and $d_L$ is the luminosity distance,

$$d_L=(1+z)a(t_0)r(z); \quad r(z)=c\int\frac{dt}{a(t)} \quad (16)$$

where $z$ is the redshift.

We perform the $\chi^2$ test of goodness of fit on the model with $V(\phi)\propto\phi^{-2}$. For our analysis, we have used all the 60 supernovae quoted in Ref. [12]. Of these 42 are high redshift supernovae reported by the supernova cosmology project [12,13] and 18 low redshift supernovae of the Callan-Tollolo survey [14,15]. We have three parameters: $\Omega_M(in)$, $n$, and $\phi_{in}H_{in}$ (with "in" referring to the present epoch). An additional freedom is the choice of sign of $\dot{\phi}(x=t_{in}H_{in})$. We

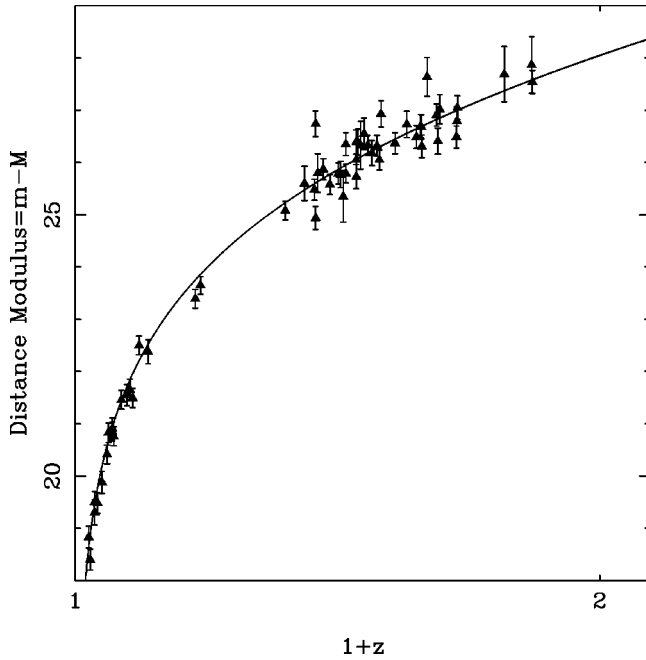FIG. 14. Comparison of the model with $n = 6$ and the present $\phi H = 2.56$ with supernova type Ia data.



FIG. 15. Contours of $\Omega_M$ at $z = 10$ (dashed lines) and reduced $\chi^2$ (solid lines) for $V(\phi) \propto 1/\phi^2$. There is a small region where a small $\chi^2$ overlaps $\Omega_m > 0.9$. This region is near $n = 1.2$ and $\log(\phi_{in}H_{in}/\phi_{in1}H_{in1}) = 0.05$. The values along the $x$ axis are ratios of $\phi_{in}H_{in}$ to the minimum value of $\phi_{in}H_{in}$ for a particular value of power $n$.

freeze $\Omega_M(in)$ at 0.3 and we find that for any choice of $n$, we can get a reasonably low value for reduced $\chi^2$ by choosing $\phi_0 H_0$ judiciously. In addition, the value of $\chi^2$ does not vary much over the range of parameter values that we have studied. The minimum value of $\chi^2$ per degree of freedom that we encountered is around 1.93. This test does not isolate any particular region in parameter space so we shall refrain from quoting any particular values of parameters as our best fit. Suffice it to say that a large range of each of these parameters is allowed by the supernova observations. This discussion holds for both the potentials discussed above.

We have plotted the distance modulus $dm(z) = m - M$ as a function of redshift for one of the models in Fig. 14. The data points for the 60 supernovae are plotted as well. The contours of reduced $\chi^2$ and the value of $\Omega_m$ at redshift $z = 10$ are illustrated in Fig. 15. This is of interest while studying structure formation in these models.

We also did a $\chi^2$ analysis of models with a range of parameters for the exponential potential. Here again the theoretical models satisfy the supernova constraints and we cannot rule out a specific model by this analysis. We plot the contours of $\Omega_M$ at $z = 10$ and $\chi^2$ in Fig. 16. We obtain results similar to those for $1/\phi^2$ potential.

## V. STRUCTURE FORMATION IN TACHYONIC MODELS

Cosmological models with tachyons and nonrelativistic matter have a significantly different behavior as compared to quintessence or the cosmological constant models. The most important difference here is that the source of acceleration in the Universe makes an insignificant contribution to the energy density of the universe beyond $z > 1$ in quintessence or cosmological constant models, whereas in tachyon models the density paramete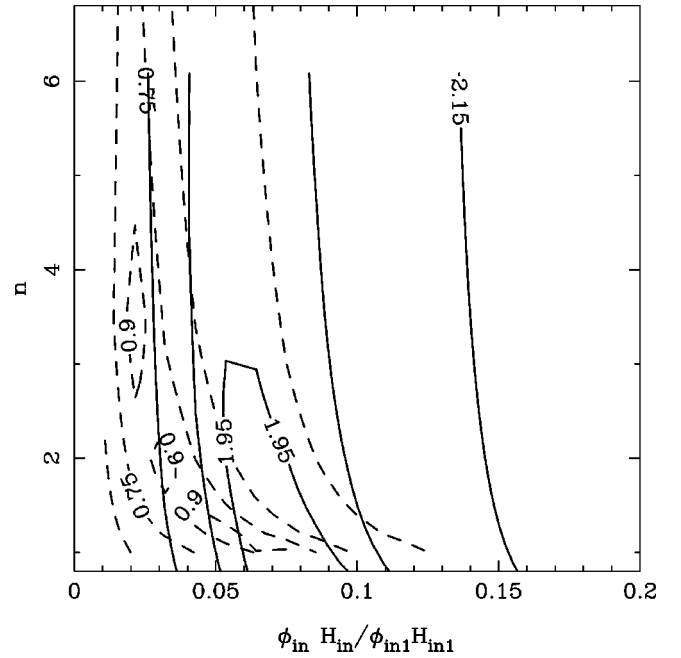r for the scalar field does not become insignificant in comparison with the density parameter for matter in most models. This has important implications for structure formation in these models since the density parameter of the matter that clusters is always smaller than unity, and the rate at which perturbations grow will be smaller than in standard models. The exception to this rule is a small subset of models where $\Omega_\phi$ approaches values much smaller than unity beyond $z \approx 1$. As can be seen in the contour plots in the previous section, these are a small subset of the models that satisfy the constraints set by supernova observations.

Given that the density parameter for matter is almost a constant at high redshifts ($3 < z < 10^3$), we can solve for the rate of growth for density contrast in the linear limit. The equation for the density contrast is given by [11]

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \rho \delta, \tag{17}$$

where $\delta = (\rho - \bar{\rho})/\bar{\rho}$, the factor $\bar{\rho}$ being the average density. Rescaling in the same manner as the cosmological equations we have

$$\delta'' + 2\frac{H}{H_0}\delta' = \frac{3}{2}\Omega_{M_0}\frac{a_0^3}{a^3}\delta. \tag{18}$$

Here $\Omega_{M_0}$ is the density parameter for nonrelativistic matter at the epoch when the scale factor is $a_0$ and the Hubble parameter is $H_0$. Since $\Omega_M$ is nearly constant at high redshift, we get $\delta \propto t^m$, and $m = (1/6)(\sqrt{1 + 24\Omega_M} - 1)$ for the
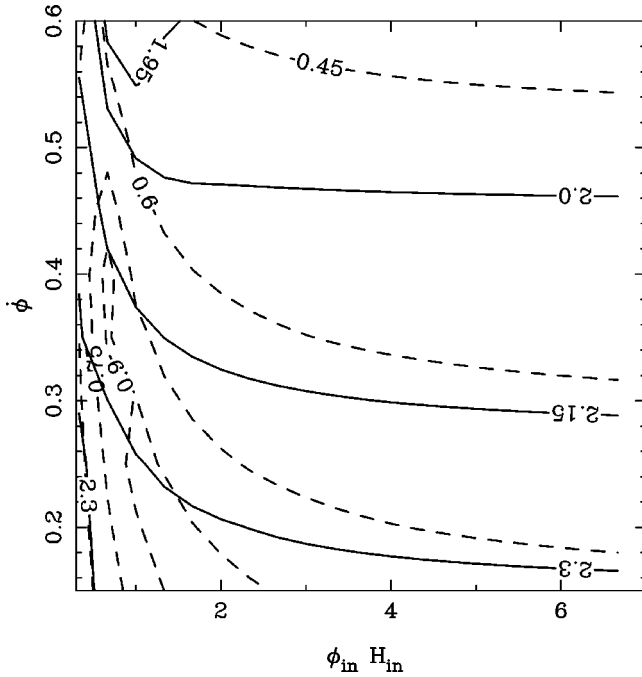
FIG. 16. Contours of $\Omega_M$ at $z=10$ (dashed lines) and reduced $\chi^2$ (solid lines). The favored region is around $\phi_{in}H_{in}=1$ and $0.25 \leqslant \dot{\phi} \leqslant 0.4$.



FIG. 17. Evolution of the density contrast with redshift. The curves correspond to $\Omega_{in}=0.3$ (present day value). The solid curve is for the $\Lambda$CDM model given for comparison. The density parameter $\Omega_M \approx 0.98$ at $z=10$ for the dashed line and $\Omega_M \approx 0.56$ at $z=10$ for the dotted line.

growing mode. The unique feature here is that at high redshift, the matter density parameter does not saturate at unity for all the models. This is true for both exponential and $1/\phi^2$ potentials. For models in which the matter density parameter does not reach unity, the growth of perturbations is slow, as can be seen from the above equation. For models in which the density parameter is unity at high redshifts, the growth of perturbations is closer to that in the cold dark matter model with a cosmological constant ($\Lambda$CDM) model. The slower growth of perturbation implies that rms fluctuations in mass distribution were larger at the time of recombination as compared to conventional models. This will have an impact on the temperature anisotropies in the microwave background in these models. Since the latter is tightly constrained from cosmic microwave background radiation (CMBR) measurements, models with slow growth of perturbations can be ruled out.

We use the solution outlined above to set the initial conditions for Eq. (18) and evolve forward through the regime where tachyons begin to dominate and matter becomes irrelevant. As the universe begins to accelerate, at late times we anticipate that the growing mode would slow down and eventually saturate. This is indeed true. The rate of growth for $\delta$ slows down once the universe begins to accelerate. It comes to a halt around the epoch where the accelerating phase begins. This late time behavior is similar to what happens in most models in which the universe begins to accelerate at late times. The evolution is illustrated in Fig. 17. Here we have plotted two different models, one in which the density parameter saturates at a small value and the other in which it approaches unity.
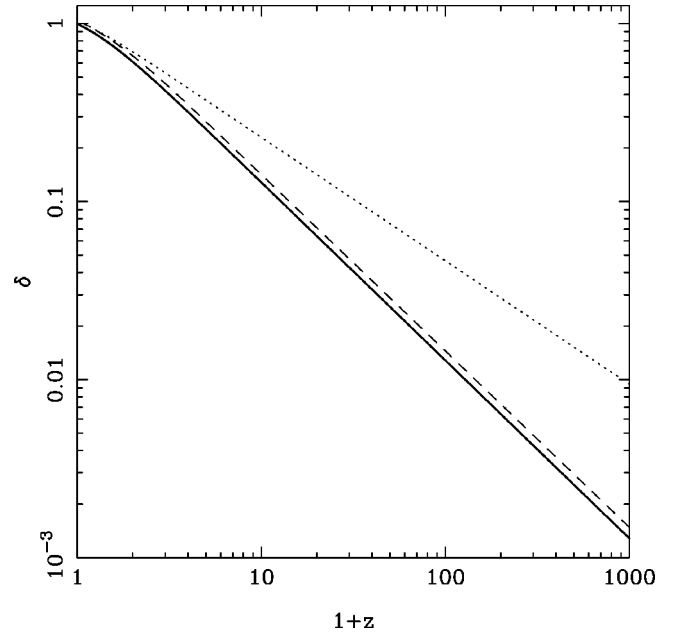
It is clear from the figure that one can indeed construct models in which the growth of perturbation is very similar to that $\Lambda$CDM models. Such models are clearly viable. It is also obvious that these models are confined to a narrow range of parameters, as described in the figure captions; if one moves out of this range, then the perturbations grow more slowly and should have higher amplitude in the past in order to maintain a given amplitude today. These are ruled out by CMBR observations. The following caveat, however, needs to be kept in mind when ruling out such models. It was suggested in Ref. [2] that one can construct models with a tachyonic scalar field in which the equation of state is different at small scales and large scales. In such models, our conclusions will apply only at large scales, and growth of structure at small scales will still be possible, i.e., inhomogeneities in the tachyon field will play a role and may offset the conclusions about the growth of perturbations at small scales, whereas our results for the expansion of the universe will remain valid at sufficiently large scales.

## VI. CONCLUSIONS

We have shown that it is possible to construct viable models with tachyons contributing significantly to the energy density of the universe. In these models, matter, radiation and tachyons coexist. We show that a subset of these models satisfies the constraints on the accelerating expansion of the universe. For the accelerating phase to occur at the present epoch, it is necessary to fine-tune the initial conditions.

We have further demonstrated that the density parameter for tachyons does not become negligible at high redshifts, hence the growth of perturbations in nonrelativistic matter is

slower for most models than, e.g., in the $\Lambda$CDM model. This problem does not effect a small subset of models. However, given that the density parameter of tachyons cannot be ignored in the "matter dominated era," it is essential to study the fate of fluctuations in the tachyon field.

[1] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); C. Wetterich, Nucl. Phys. **B302**, 668 (1988); P. G. Ferreira and M. Joyce, Phys. Rev. D **58**, 023503 (1998); J. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995); P. Brax and J. Martin, Phys. Rev. D **61**, 103502 (2000); L. A. Ureña-López and T. Matos, *ibid.* **62**, 081302(R) (2000); T. Barreiro, E. J. Copeland, and N. J. Nunes, *ibid.* **61**, 127301 (2000); I. Zlatev, L. Wang, and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000); A. Feinstein, Phys. Rev. D **66**, 063511 (2002).

[2] T. Padmanabhan and T. Roy Choudhury, Phys. Rev. D **66**, 081301(R) (2002).

[3] A. Sen, J. High Energy Phys. **04**, 048 (2002); **07**, 065 (2002); Mod. Phys. Lett. A **17**, 1797 (2002).

[4] A. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B **458**, 209 (1999).

[5] G. W. Gibbons, Phys. Lett. B **537**, 1 (2002); M. Fairbairn and M. H. Tytgat, *ibid.* **546**, 1 (2002); S. Mukohyama, Phys. Rev. D **66**, 024009 (2002); A. Frolov, L. Kofman, and A. A. Starobinsky, Phys. Lett. B **545**, 8 (2002); D. Choudhury, D. Ghoshal, D. P. Jatkar, and S. Panda, *ibid.* **544**, 231 (2002); X. Li, J. Hao, and D. Liu, Chin. Phys. Lett. **19**, 1584 (2002); G. Shiu and I. Wasserman, Phys. Lett. B **541**, 6 (2002); L. Kofman and A. Linde, J. High Energy Phys. **07**, 004 (2002); H. P. Benaoum, hep-th/0205140; M. Sami, P. Chingangbam, and T. Qureshi, Phys. Rev. D **66**, 043530 (2002); T. Mehen and B. Wecht, hep-th/0206212; Y. S. Piao, R. G. Cai, X. M. Zhang, and Y. Z. Zhang, Phys. Rev. D **66**, 121301(R) (2002); X. Z. Li, D. J. Liu, and J. G. Hao, hep-th/0207146; J. M. Cline, H. Firouzjahi, and P. Martineau, J. High Energy Phys. **11**, 041 (2002); A. Buchel, P. Langfelder, and J. Walcher, Ann. Phys. (N.Y.) **302**, 78 (2002); B. Wang, E. Abdalla, and R. K. Su, hep-th/0208023; S. Mukohyama, Phys. Rev. D **66**, 123512 (2002); M. C. Bento, O. Bertolami, and A. A. Sen, hep-th/0208124.

[6] C. Acatrinei and C. Sochichiu, hep-th/0104263; S. H. Alexander, Phys. Rev. D **65**, 023507 (2002); A. Mazumdar, S. Panda, and A. Perez-Lorenzana, Nucl. Phys. **B614**, 101 (2001); S. Sarangi and S. H. Tye, Phys. Lett. B **536**, 155 (2002).

[7] T. Padmanabhan, Phys. Rev. D **66**, 021301(R) (2002).

[8] T. Chiba, T. Okabe, and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000).

[9] T. Chiba, Phys. Rev. D **66**, 063514 (2002).

[10] M. Sami, hep-th/0205146; X. Li, J. Hao, and D. Liu, Chin. Phys. Lett. **19**, 1584 (2002); X. Li, D. Liu, and J. Hao, hep-th/0207146.

[11] T. Padmanabhan, *Cosmology and Astrophysics Through Problems* (Cambridge University Press, Cambridge, England, 1996).

[12] S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).

[13] S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998).

[14] M. Hamuy *et al.*, Astron. J. **112**, 2391 (1996).

[15] M. Hamuy *et al.*, Astron. J. **109**, 1 (1995).

[16] J. Hao and X. Li, Phys. Rev. D **66**, 087301 (2002); V. Sahni and Y. Shtanov, astro-ph/0202346; M. Sami and T. Padmanabhan, Phys. Rev. D (to be published), hep-th/0212317.