Initial state effects on the cosmic microwave background and trans-Planckian physics

Kevin Goldstein* and David A. Lowe†

Department of Physics, Brown University, Providence, Rhode Island 02912

(Received 3 September 2002; published 13 March 2003)

There exists a one complex parameter family of de Sitter invariant vacua, known as α vacua. In the context of slow roll inflation, we show that all but the Bunch-Davies vacuum generates unacceptable production of high energy particles at the end of inflation. As a simple model for the effects of trans-Planckian physics, we go on to consider non–de Sitter invariant vacua obtained by patching modes in the Bunch-Davies vacuum above some momentum scale M_c, with modes in an α vacuum below M_c. Choosing M_c near the Planck scale $M_{\rm Pl}$, we find acceptable levels of hard particle production, and corrections to the cosmic microwave perturbations at the level of $HM_{\rm Pl}/M_c^2$, where H is the Hubble parameter during inflation. More general initial states of this type with $H \ll M_c \ll M_{\rm Pl}$ can give corrections to the spectrum of cosmic microwave background perturbations at order 1. The parameter characterizing the α vacuum during inflation is a new cosmological observable.

DOI: 10.1103/PhysRevD.67.063502 PACS number(s): 98.80.Cq, 04.62.+v

I. INTRODUCTION

Inflation magnifies quantum fluctuations at fundamental length scales to astrophysical scales, where their imprint is left on the formation of structure in the universe. In conventional slow roll inflation, the universe undergoes an expansion of at least 10^{26} during the inflationary phase. With such a huge expansion factor, modes which give rise to observable structures apparently started out with wavelengths much smaller than the Planck length. This is the so-called trans-Planckian problem in inflation [1–5].

In the past year, there has been much debate about whether potential modifications to physics above the Planck scale could actually be observed [6–18]. By considering the local effective action at the Hubble scale H (which we will take to be 10^{13}–10^{14} GeV), [14] has argued that trans-Planckian corrections to the spectrum of cosmic microwave background perturbations could at best be of order $(H/M_{\rm Pl})^2$ which is typically far too small to be observed in conventional inflationary models. However, others [6–13] have obtained a correction of order $H/M_{\rm Pl}$ by considering a variety of methods for modeling trans-Planckian effects. Such a correction is potentially observable in the not too distant future.

In the present work we represent the effect of trans-Planckian physics simply by allowing for nontrivial initial vacuum states for the inflaton field, which we treat as a free scalar field moving in a de Sitter background. The most natural vacuum states to consider are the de Sitter invariant vacuum states constructed in [19,20,21]. The vacuum states are known as α vacua. We find these all lead to infinite energy production at the end of inflation, with the exception of the Bunch-Davies (Euclidean) vacuum state.

We go on to consider non–de Sitter invariant vacuum states constructed by placing modes with comoving wave number $k > M_c a(\eta_f)$ in the Bunch-Davies vacuum, where $a(\eta_f)$ is the expansion factor at the end of inflation. Modes with $k < M_c a(\eta_f)$ are placed in a nontrivial α vacuum. These states have a particularly simple evolution in de Sitter space—the length scale at which the patching occurs simply expands as the scale factor grows. Many more complicated initial states asymptote to such states as the universe expands.

For M_c of order $M_{\rm Pl}$ it is possible to find initial states that do not overproduce hard particles, and produce corrections to the cosmic microwave background spectrum at order H/M_c in agreement with [7]. For $H \ll M_c \ll M_{\rm Pl}$ there are initial states that produce corrections to the spectrum at order 1.

In [11,13] an initial state was constructed by placing modes in their locally Minkowskian vacuum states as the proper wave number passed through the scale of new physics M_c. This turns out to be a special case of the class of initial states we consider. To avoid large back-reaction problems in this case, we show the condition $M_c \ll M_{\rm Pl}$ must hold. This condition is rather easy to satisfy. Our more general initial states may be viewed in a similar way as an initial state that puts modes in a k-independent Bogoliubov transformation of the locally Minkowskian vacuum as the proper wave number passes through the scale M_c.

II. GENERAL SETUP

We will conduct our analysis using linearized perturbation theory in a de Sitter (dS) background. Planar coordinates, which cover half of the dS background with flat spatial sections, result in the metric

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2 = dt^2 - a^2(t) d\vec{x}^2. \tag{2.1}$$

It will be more convenient to use conformal coordinates, giving

$$ds^2 = \frac{1}{(\eta H)^2}(d\eta^2 - d\vec{x}^2) = a^2(\eta)(d\eta^2 - d\vec{x}^2) \tag{2.2}$$

where $\eta = \int_t^\infty dt'/a(t') = -\exp(-Ht)/H$. So $t \to -\infty$ as $\eta \to -\infty$, and $t \to \infty$ as $\eta \to 0$.

*Email address: kevin@het.brown.edu

†Email address: lowe@het.brown.edu

The Klein-Gordon equation in curved space is

$$(\Box + m^2 + \zeta R)\phi = 0 \tag{2.3}$$

for a scalar field with mass m and nonminimal coupling to R given by ζ. In momentum space we can solve this equation by defining

$$\phi_k = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}a(\eta)}\chi_k(\eta) \tag{2.4}$$

which leads to

$$\chi_k'' + \left(k^2 + \frac{M^2}{H^2\eta^2}\right)\chi_k = 0 \tag{2.5}$$

with

$$M^2 = m^2 + \left(\zeta - \frac{1}{6}\right)R \tag{2.6}$$

so M^2 is not necessarily positive. The general solution is

$$\chi_k(\eta) = \frac{1}{2}\sqrt{\pi\eta}H_\nu^{(2)}(k\eta) \equiv \chi_{Ek}(\eta) \tag{2.7}$$

together with its complex conjugate, where $\nu = 9/4 - m^2/H^2 - 12\zeta = 1/4 - M^2$.

Such a complete set of orthonormal modes may be used to define a Fock vacuum state by taking the field operator

$$\hat{\chi} = \sum_k \chi_k(\eta)a_k + \chi_k^*(\eta)a_{-k}^\dagger \tag{2.8}$$

and demanding $a_k|0\rangle = 0$. As shown in [19,20,21], the general family of de Sitter invariant vacuum states can be defined using the modes

$$\chi_k = \cosh\alpha\,\chi_{Ek}(\eta) + e^{i\delta}\sinh\alpha\,\chi_{Ek}^*(\eta) \tag{2.9}$$

with $\alpha \in [0,\infty)$ and $\delta \in (-\pi,\pi)$. $\alpha = 0$ is the Bunch-Davies vacuum (otherwise known as the Euclidean vacuum).

For a massless minimally coupled scalar, this solution takes a particularly simple form:

$$\chi_{Ek}(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right). \tag{2.10}$$

As discussed in [20] this case gives rise to difficulties in canonical quantization, and there is no de Sitter invariant Fock vacuum. Nevertheless, we will use this simple example in the following with the understanding that a small mass term could be added to eliminate this problem, and the expressions we derive will not be substantially changed.

We will need to extract two physical quantities from the expression (2.9). The first is the number of particles produced in the mode k defined with respect to the $\alpha = 0$ vacuum. This is simply equal to

$$n_k = \sinh^2\alpha. \tag{2.11}$$

This will be a good approximation to the number of particles produced at the end of inflation, when a transition is made to a much more slowly expanding universe, provided the wavelengths of the modes in question are much smaller than the Hubble radius. This follows simply from the fact that at high wave number the wave equation for χ_k reduces to that of flat space, so we can approximate the final geometry by Minkowski space. We wish to count particles with respect to the Lorentz invariant vacuum state, which corresponds to the $\alpha = 0$ vacuum in this regime.

The second physical quantity of interest is the contribution of this mode to the spectrum of cosmic microwave background radiation (CMBR) perturbations. We compute this by examining $|\phi_k(\eta)|^2$ in the distant future $\eta \to 0$ for the massless scalar (2.10). The contribution is then

$$P_k = \frac{k^3}{2\pi^2a^2}|\chi_k|^2 = \left(\frac{H}{2\pi}\right)^2|\cosh\alpha - e^{i\delta}\sinh\alpha|^2. \tag{2.12}$$

III. INITIAL STATE EFFECTS

We begin by reviewing what happens for the usual Bunch-Davies vacuum $\alpha = 0$. Clearly the particle production at high frequencies (2.11) vanishes. Fluctuations in the scalar field modes mean that different regions of spacetime expand at slightly different rates, which gives rise to density perturbations after inflation has ended. The amplitude of these perturbations is frozen in as these modes expand outside the Hubble radius during inflation, and become density perturbations once they reenter the horizon after the end of inflation. For $\alpha = 0$, $P_k = (H/2\pi)^2$ is independent of k and hence scale invariant. When one allows for the detailed shape of the inflation potential, H becomes effectively k dependent, leading to small deviations from the scale invariant spectrum of perturbations, which in general are highly model dependent.

For a nontrivial $\alpha \neq 0$ vacuum we immediately see a problem. At the end of inflation there will be a large amount of particle production at wavelengths smaller than the Hubble radius (2.11). Since this production is independent of k [20], this will lead to an infinite energy density, and singular back reaction on the geometry. We conclude then that at wavelengths below some scale the modes must be in a local $\alpha = 0$ vacuum state.[1] Actually, α need not be exactly zero for the high wavenumber modes. We will return to this point at the end of this section.

[1]Reference [22] also concludes that the Euclidean vacuum smoothly patches onto the Lorentz invariant Minkowski vacuum, in the context of two-dimensional de Sitter space. They also point out that for all $\alpha \neq 0$ the vacuum state picks up a nontrivial phase under de Sitter isometries, which cancels in the expectation values.

Nevertheless, we can still consider initial states that involve modes in an $\alpha \neq 0$ state, provided their wavelengths are sufficiently large. Perhaps the simplest such initial state is to place modes at some fixed conformal time η_0 in the $\alpha = 0$ state for $k > M_c a(\eta_f)$ where η_f is the conformal time at the end of inflation, and M_c is some scale at which physics changes, and we have in mind taking $M_c \gg H$. Modes for $k < M_c a(\eta_f)$ can be placed in an $\alpha \neq 0$ state. In the next section we will see such a state arises naturally from time-independent boundary conditions placed at proper wave number M_c, which represents the generic effect of new physics that comes into play above the scale M_c.

In order that the particle production at the end of inflation be irrelevant versus the energy stored in the inflaton, we must have[2]

$$M_c^4 \sinh^2 \alpha \ll \Lambda = \frac{3M_{\rm Pl}^2 H^2}{4\pi} \tag{3.1}$$

where $M_{\rm Pl}$ is the Planck mass.[3] If we saturate this bound, $\sinh \alpha \sim H M_{\rm Pl}/M_c^2$. The correction to the CMBR spectrum P_k (2.12) will then be of order $H M_{\rm Pl}/M_c^2$. This is linear in H in agreement with the estimates of [6,7,11] and is potentially observable. Of course, since we have done the computation in pure de Sitter space, the effect appears as a k-independent modulation of the $\alpha = 0$ result, which on its own would require an independent determination of H to measure directly. However, in inflation H is actually slowly changing, which will translate into k dependence of H, and hence α. This will show up as k-dependent corrections to the cosmic microwave background spectrum P_k which are potentially more easily distinguishable from the $\alpha = 0$ case [6,12].

To obtain an upper bound on the size of the correction to the CMBR spectrum, we can imagine taking M_c to be much smaller than $M_{\rm Pl}$, which is certainly plausible. This allows α to be of order 1 and still consistent with negligible hard particle production (3.1). This limit will lead to corrections to the CMBR spectrum (2.12) at order 1.

Transition at proper energy M_c

Now let us consider a more detailed model for the initial state where we assume the initial condition is fixed due to some change in physics at the proper energy scale M_c. We are primarily interested in the constraints observations give us on this new physics, so we wish to make as few assumptions about the details of this new physics as possible. Let us review the computation of [11,13]. The essential idea was to note that the α vacuum satisfying

$$\cosh \alpha = e^{i(\gamma-\beta)} \frac{2\beta - i}{2\beta},$$

$$e^{i\delta} \sinh \alpha = -e^{i(\gamma+\beta)} \frac{i}{2\beta}, \tag{3.2}$$

with $\beta = M_c/H$ and γ real, can be interpreted as an initial state that places modes in their locally Minkowskian vacuum as the proper wave number k/a passes through the scale M_c. This is seen by noting that at time $\eta = -M_c/Hk$ the field ϕ_k [with χ_k given by Eq. (2.9)] satisfies $\pi_k = -ik\phi_k$ where π_k is the conjugate momentum. Such a relation is satisfied by the Lorentz invariant vacuum in Minkowski space. One may also interpret the state at time $\eta = -M_c/Hk$ as a minimum uncertainty state [11].

For sufficiently large k, the above prescription does not apply, because the relevant time η will be after the end of inflation. These modes may safely be placed in the Bunch-Davies vacuum.

This initial state is a special case of the type described above. High frequency particle creation at the end of inflation gives an energy density of order $M_c^4 \sinh^2 \alpha$. Since here $\sinh \alpha \sim H/M_c$, we require

$$M_c^2 H^2 \ll M_{\rm Pl}^2 H^2. \tag{3.3}$$

This will hold whenever $M_c \ll M_{\rm Pl}$, which is easy to satisfy. This condition was also obtained in [6].

Note that the general class of initial states described above may be reinterpreted in the same way as states arising from a boundary condition placed at a fixed proper energy scale. Rather than imposing the condition that the initial state corresponds to a locally Minkowski vacuum as the wave number k/a passes through M_c, one instead demands that the mode be in a general k-independent Bogoliubov transformation of the locally Minkowski vacuum. This corresponds to a generic boundary condition at the scale M_c that is independent of time. In this way, modes are placed in a nontrivial α vacuum when $k/a(\eta_f)$ at the end of inflation is below the scale M_c. Higher k modes will remain in the Bunch-Davies vacuum. This is precisely the type of state we described above.

As an aside, note that the α vacua may be regarded as squeezed states on top of the Bunch-Davies vacuum [21]. Such squeezed states arise naturally in nonlinear quantum systems [23]. The states described above might then be thought of as arising due to nonlinear effects above the scale M_c placing modes in an α vacuum as they descend below the scale M_c. However, we emphasize that our goal is to find a generic way to parametrize the effects of trans-Planckian physics and is not tied to any particular model of these effects.

It is interesting to view this boundary condition in the context of the nice slice argument of [24] used to define effective field theory in a curved background. The conformal time slicing (2.2) satisfies the criteria for a "nice slicing." This means we may define fields with, for example, a spatial lattice cutoff on proper wavelengths below $1/M_c$. As one

[2]Reference [5] previously considered constraints on trans-Planckian physics which are required to prevent excessive particle production.

[3]This condition is necessary to avoid large back reaction on the geometry. It would also be interesting to consider the limit when this energy is not irrelevant, and to use this particle production as a source for reheating.

moves forward on these time slices, the proper wavelength of a given mode (2.7) expands, so new modes descend from above the cutoff scale, and we assume these are placed in their ground state. Note that prior to passing below the cutoff scale the modes are simply fictitious from the effective field theory viewpoint. The main difference from asymptotically flat space is that we now have the option of placing these modes in one of the nontrivial de Sitter invariant α vacua. Any other choice would lead to continuous creation of particles at the cutoff scale which would cause drastic back reaction on the geometry.[4]

Provided the interacting quantum field theory in de Sitter space is consistent in a general α vacuum, there seems to be no dynamics that prefers one value of α over another. In the context of slow roll inflation, we should therefore regard the value of α during inflation as a new cosmological observable which encodes information about trans-Planckian physics.

At the end of inflation we make a transition from the de Sitter geometry to a standard cosmological geometry. To describe the UV cutoff in this more general context, we need to replace the simple α vacuum suitable for de Sitter space, by a boundary condition fixed by some more general dynamical condition such as the locally Minkowskian boundary condition of [11,13] described above [Eq. (3.2)]. The effective value of α will then change as the effective value of H changes. Note that for us H is not in general the Hubble parameter, but is determined by the vacuum energy density $V(\phi)$. In the limit that the cosmological constant becomes very small (the effective H decreases by a factor of 10^{-56} or so to match with today's vacuum energy density), we make a smooth transition to a $\alpha \sim 10^{-56} H/M_c$ boundary condition at the cutoff scale after the end of inflation. Previously, one might have objected that the comoving wave number at which the α vacua was patched was fine-tuned to the duration of inflation. However, now we see this can naturally arise as a consequence of new physics above the scale M_c and a relation between α and $V(\phi)$.

The value of α during inflation may be selected by local physics at Planckian energies, but in general α may also be influenced by the initial state of the universe. This initial state is not necessarily completely determined by physics at Planckian energies. For example, the initial state may emerge as a special state of very high symmetry as a result of dynamics on much higher energy scales, which will leave their imprint on the value of α in the de Sitter phase.

[4]Since the state preserves de Sitter invariance for time-independent α the expectation value of the stress-energy tensor (when properly defined in the interacting theory) is proportional to the metric, and hence covariantly constant. In this sense the α vacua are compatible with energy-momentum conservation.

IV. CONCLUSIONS

We have constructed a very simple class of initial states for the inflaton field which can be used to model effects of trans-Planckian physics. A new cosmological observable emerges from this analysis in the context of slow roll inflation, namely, the α parameter characterizing the vacuum state during inflation.

Other previous approaches have typically assumed some definite model for the trans-Planckian physics which led to particular states of this type at momenta much below the Planck scale. We have found for certain ranges of parameters, the initial states do not lead to excess particle production at the end of inflation, and lead to potentially observable corrections to the cosmic microwave background spectrum.

ACKNOWLEDGMENTS

We thank R. Brandenberger, R. Easther, B. Greene, and B. Lehmann for helpful discussions and comments. D.L. thanks the Aspen Center for Physics for hospitality during the completion of this work, and the participants at the Center for stimulating discussions. This research is supported in part by DOE Grant DE-FE0291ER40688-Task A and by BSF Grant 2000359.

[1] R. H. Brandenberger, ''Inflationary Cosmology: Progress and Problems,'' hep-ph/9910410.

[2] R. H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001).

[3] J. Martin and R. H. Brandenberger, Phys. Rev. D **63**, 123501 (2001).

[4] J. Martin and R. H. Brandenberger, Phys. Rev. D **65**, 103514 (2002).

[5] A. A. Starobinsky, Pisma Zh. Eksp. Teor. Fiz. **73**, 415 (2001) [JETP Lett. **73**, 371 (2001)].

[6] R. Easther, B. R. Greene, W. H. Kinney, and G. Shiu, Phys. Rev. D **64**, 103502 (2001).

[7] R. Easther, B. R. Greene, W. H. Kinney, and G. Shiu, Phys. Rev. D (to be published), hep-th/0110226.

[8] R. H. Brandenberger and J. Martin, Int. J. Mod. Phys. A **17**, 3663 (2002).

[9] R. Brandenberger and P. M. Ho, Phys. Rev. D **66**, 023517 (2002).

[10] F. Lizzi, G. Mangano, G. Miele, and M. Peloso, J. High Energy Phys. **06**, 049 (2002).

[11] U. H. Danielsson, Phys. Rev. D **66**, 023511 (2002).

[12] R. Easther, B. R. Greene, W. H. Kinney, and G. Shiu, Phys. Rev. D **66**, 023518 (2002).

[13] U. H. Danielsson, J. High Energy Phys. **07**, 040 (2002).

[14] N. Kaloper, M. Kleban, A. E. Lawrence, and S. Shenker, Phys. Rev. D **66**, 123510 (2002).

[15] J. C. Niemeyer, R. Parentani, and D. Campo, Phys. Rev. D **66**, 083510 (2002).

[16] A. Kempf and J. C. Niemeyer, Phys. Rev. D **64**, 103501 (2001).

[17] L. Hui and W. H. Kinney, Phys. Rev. D **65**, 103507 (2002).

[18] A. A. Starobinsky and I. I. Tkachev, Pisma Zh. Eksp. Teor. Fiz. **76**, 291 (2002) [JETP Lett. **76**, 235 (2002)].
[19] N. A. Chernikov and E. A. Tagirov, Ann. Inst. Henri Poincare, Sect. A **9**, 109 (1968); E. A. Tagirov, Ann. Phys. (N.Y.) **76**, 561 (1973).
[20] B. Allen, Phys. Rev. D **32**, 3136 (1985).
[21] E. Mottola, Phys. Rev. D **31**, 754 (1985).
[22] R. Floreanini, C. T. Hill, and R. Jackiw, Ann. Phys. (N.Y.) **175**, 345 (1987).
[23] D. F. Walls, Nature (London) **306**, 141 (1983).
[24] D. A. Lowe, J. Polchinski, L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **52**, 6997 (1995).