Generalized Chaplygin gas and cosmic microwave background radiation constraints

M. C. Bento*
Departamento de Física, Instituto Superior Técnico Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal
and Centro de Física das Interacções Fundamentais, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

O. Bertolami†
Departamento de Física, Instituto Superior Técnico Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

A. A. Sen‡
Centro Multidisciplinar de Astrofísica, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal
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We study the dependence of the location of the cosmic microwave background radiation peaks on the parameters of the generalized Chaplygin gas model, whose equation of state is given by $p=-A/\rho^{\alpha}$, where A is a positive constant and $0<\alpha\leqslant 1$. We find, in particular, that observational data arising from Archeops, BOOMERANG, supernova and high-redshift observations allow constraining significantly the parameter space of the model. Our analysis indicates that the emerging model is clearly distinguishable from the $\alpha=1$ Chaplygin case and the ΛCDM model.

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I. INTRODUCTION

It has been recently suggested that the change of behavior of the so-called dark energy density might be controlled by the change in the equation of state of the background fluid [1] instead of the form of the potential, thereby avoiding the well known fine-tuning problems of quintessence models. This is achieved via the introduction, within the framework of Friedmann-Robertson-Walker cosmology, of an exotic background fluid, the generalized Chaplygin gas, described by the equation of state

$$p_{ch}=-\frac{A}{\rho_{ch}^{\alpha}}, \tag{1}$$

where α is a constant in the range $0<\alpha\leqslant 1$ (the Chaplygin gas corresponds to the case $\alpha=1$) and A a positive constant. Inserting this equation of state into the relativistic energy conservation equation leads to a density evolving as [2]

$$\rho_{ch}=\left(A+\frac{B}{a^{3(1+\alpha)}}\right)^{1/(1+\alpha)}, \tag{2}$$

where a is the scale factor of the Universe and B an integration constant. Remarkably, the model interpolates between a universe dominated by dust and a de Sitter one via a phase described by a ''soft'' matter equation of state, $p=\alpha\rho$ ($\alpha\neq 1$). Notice that even though Eq. (1) admits a wider range of positive α values, the chosen range of values ensures that the sound velocity ($c_s^2=\alpha A/\rho_{ch}^{1+\alpha}$) does not exceed, in the ''soft'' equation of state phase, the velocity of light. Actually, as discussed in Ref. [2], it is only for values in the range $0<\alpha\leqslant 1$ that the analysis of the evolution of energy density fluctuations makes sense.

It was also shown in Ref. [2] that the model can be described by a complex scalar field whose action can be written as a generalized Born-Infeld action corresponding to a ''perturbed'' d-brane in a $(d+1,1)$ spacetime. It is clear that this model has a bearing on the observed accelerated expansion of the Universe [3], as it automatically leads to an asymptotic phase where the equation of state is dominated by a cosmological constant, $8\pi G A^{1/1+\alpha}$. It was also shown that the model admits, under conditions, an inhomogeneous generalization which can be regarded as a unification of dark matter and dark energy [4,2] and that it can be accomodated within the standard structure formation scenarios [2,4,5]. Therefore, the generalized Chaplygin gas model seems to be a viable alternative to models where the accelerated expansion of the Universe is explained through an uncancelled cosmological constant (see [6] and references therein) or through quintessence models with one [7–16] or two scalar fields [17–19].

These promising results have led, quite recently, to a wave of interest aiming to constrain the generalized Chaplygin model using observational data, particularly those arising from SNe Ia [20–25].

In this work, we shall consider the constraints arising from the positions of the first three peaks of the cosmic microwave background radiation (CMBR) power spectrum on the parameter space of the generalized Chaplygin gas, applying the same method that has been used recently to constrain quintessence models (see e.g., Refs. [27–29]).

We find, in particular, that the positions of first and third peaks lead to fairly strong constraints although a sizeable portion of the parameter space of the model is still compatible with BOOMERANG and Archeops data. Further correlating the resulting region with the observations of supernova and high-redshift objects leads to quite tight contraint on the parameter space of the generalized Chaplygin model. It is important to stress that the generalized Chaplygin gas differs,

*Email address: bento@sirius.ist.utl.pt
†Email address: orfeu@cosmos.ist.utl.pt
‡Email address: anjan@x9.ist.utl.pt

as discussed in Ref. [23], from quintessence and tracker models in what concerns the so-called "statefinder" parameters (r,s) [26]:

$$r \equiv \frac{\dddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3(q-1/2)}, \tag{3}$$

where $H=\dot{a}/a$ is the Hubble parameter and $q=-\ddot{a}/aH^2$ is the deceleration parameter. Moreover, only for fairly small values of α the generalized Chaplygin gas becomes indistinguishable from the cold dark matter model with a cosmological constant (ΛCDM). Hence, future SNe Ia surveys for high redshifts may allow a clear discrimination between the generalized Chaplygin gas proposal and quintessence models.

II. LOCATION OF CMBR PEAKS FOR THE GENERALIZED CHAPLYGIN GAS

The CMBR peaks arise from acoustic oscillations of the primeval plasma just before the Universe becomes transparent. The angular momentum scale of the oscillations is set by the acoustic scale l_A which for a flat Universe is given by

$$l_A = \pi \frac{\tau_0 - \tau_{\mathrm{ls}}}{\bar{c}_s \tau_{\mathrm{ls}}}, \tag{4}$$

where τ_0 and τ_{ls} are the conformal time today and at last scattering and $\bar{c}_s$ is the average sound speed before decoupling.

The prior assumptions in our subsequent calculations are as follows: density parameters for radiation and baryons at present $\Omega_{r0}=9.89\times10^{-5}$, $\Omega_{b0}=0.05$, average sound velocity $\bar{c}_s=0.52$, scalar spectral index, $n=1$ and $h=0.65$. Moreover, our conventions for the scale factor are: at present $a_0=1$ and at last scattering $a_{ls}=1100^{-1}$.

We start by computing l_A for the case of the generalized Chaplygin gas. Rewriting Eq. (2) in the form

$$\rho_{ch} = \rho_{ch0}\left(A_s + \frac{(1-A_s)}{a^{3(1+\alpha)}}\right)^{1/1+\alpha}, \tag{5}$$

where $A_s \equiv A/\rho_{ch0}^{1+\alpha}$ and $\rho_{ch0}=(A+B)^{1/1+\alpha}$, the Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3}\left[\frac{\rho_{r0}}{a^4} + \frac{\rho_{b0}}{a^3} + \rho_{ch0}\left(A_s + \frac{(1-A_s)}{a^{3(1+\alpha)}}\right)^{1/1+\alpha}\right], \tag{6}$$

where we have included the contributions of radiation and baryons as these are not accounted for by the generalized Chaplygin gas equation of state.

Several important features of Eq. (5) are worth remarking. First of all, A_s must lie in the interval $0 \leqslant A_s \leqslant 1$ as otherwise p_{ch} will be undefined at some a. Secondly, for $A_s=0$, the Chaplygin gas behaves as dust and, for $A_s=1$, it behaves like a cosmological constant. Notice that only for $\alpha=0$, the Chaplygin gas corresponds to a ΛCDM model. Hence, for the chosen range of α, the generalized Chaplygin gas is clearly different from ΛCDM. Another relevant issue is that the sound velocity of the fluid is given, at present, by αA_s and thus $\alpha A_s \leqslant 1$. Using $\rho_{r0}/\rho_{ch0}=\Omega_{r0}/1-\Omega_{r0}-\Omega_{b0}$ and $\rho_{b0}/\rho_{ch0}=\Omega_{b0}/1-\Omega_{r0}-\Omega_{b0}$, we obtain

$$H^2 = \Omega_{ch0} H_0^2 a^{-4} X^2(a), \tag{7}$$

with

$$X(a) = \frac{\Omega_{r0}}{1-\Omega_{r0}-\Omega_{b0}} + \frac{\Omega_{b0} a}{1-\Omega_{r0}-\Omega_{b0}} + a^4\left(A_s + \frac{(1-A_s)}{a^{3(1+\alpha)}}\right)^{1/1+\alpha}. \tag{8}$$

Using the fact that $H^2=a^{-4}(da/d\tau)^2$, we get

$$d\tau = \frac{da}{\Omega_{ch0}^{1/2} H_0 X(a)}, \tag{9}$$

so that

$$l_A = \frac{\pi}{\bar{c}_s}\left[\int_0^1 \frac{da}{X(a)}\left(\int_0^{a_{ls}} \frac{da}{X(a)}\right)^{-1} - 1\right]. \tag{10}$$

In an idealized model of the primeval plasma, there is a simple relation between the location of the m-th peak and the acoustic scale, namely $l_m \approx m l_A$. However, the location of the peaks is slightly shifted by driving effects and this can be compensated by parametrizing the location of the m-th peak, l_m, as in [30,27]

$$l_m \equiv l_A(m-\varphi_m). \tag{11}$$

It is not in general possible to derive analytically a relationship between the cosmological parameters and the peak shifts, but one can use fitting formulas that describe their dependence on these parameters; in particular, we have for the spectral index of scalar perturbations $n=1$ and for the amount of baryons $\Omega_{b0}h^2=0.02$ [30,27]

$$\varphi_1 \approx 0.267\left(\frac{r_{ls}}{0.3}\right)^{0.1}, \tag{12}$$

where $r_{ls}=\rho_r(z_{ls})/\rho_m(z_{ls})$ is the ratio of radiation to matter at last scattering. Since, according to our dark matter-energy unification hypothesis, ρ_{ch} will behave as dust or nonrelativistic matter at last scattering

$$\rho_{ch} \approx \frac{\rho_{ch0}}{a^3}(1-A_s)^{1/1+\alpha}, \tag{13}$$

we get

$$r_{ls} = \frac{\Omega_{r0}}{\Omega_{ch0}} \frac{a_{ls}^{-1}}{(1-A_s)^{1/1+\alpha}} \simeq \frac{\Omega_{r0} a_{ls}^{-1}}{(1-\Omega_{r0}-\Omega_{b0})(1-A_s)^{1/1+\alpha}}. \tag{14}$$

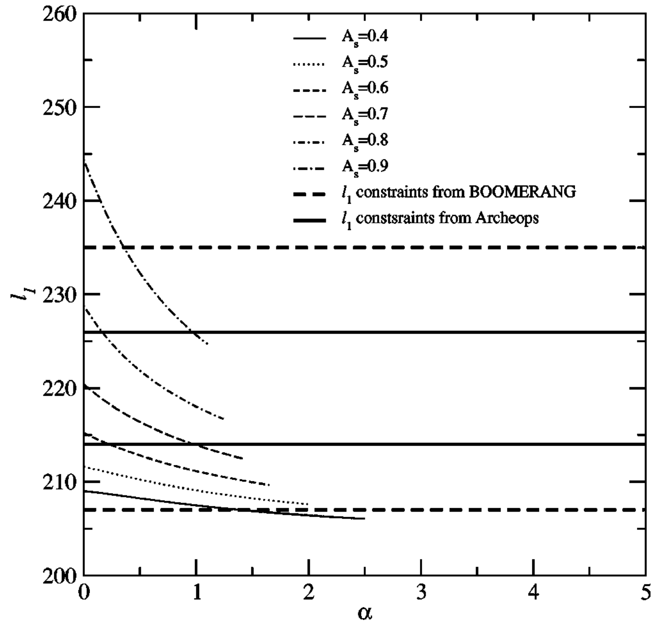

FIG. 1. Dependence of the position of the CMBR first peak, l_1, as a function of α for different values of A_S. Also shown are the observational bounds on l_1 from BOOMERANG (dashed lines), [see Eq. (15)] and Archeops (full lines) [see Eq. (16)].

Using Eqs. (10) and (11)–(14), we have plotted in Fig. 1, l_1 as a function of α for different values of A_s, where we have also drawn lines corresponding to the observational bounds on l_1 as derived from BOOMERANG [31] (dashed lines)

$$l_1 = 221 \pm 14 \tag{15}$$

and Archeops data [32] (full lines)

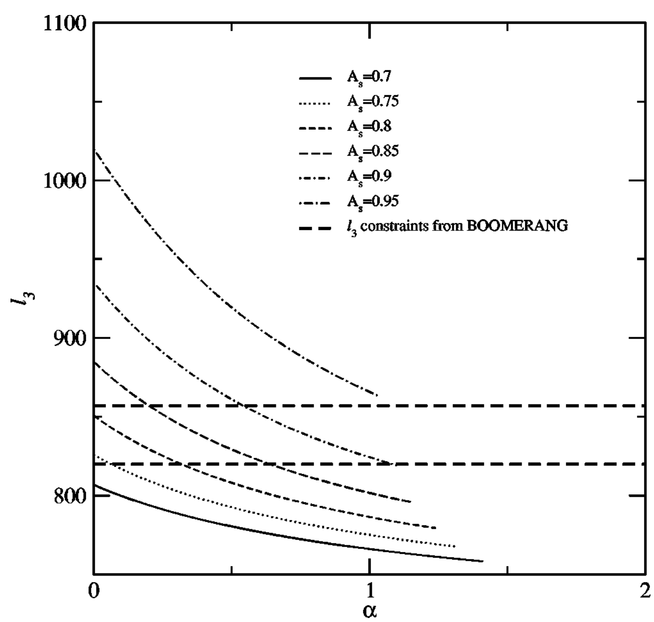

FIG. 2. Dependence of the position of the CMBR third peak, l_3, as a function of α for different values of A_S. Also shown are the observational bounds on l_3 (dashed lines) [see Eq. (18)].

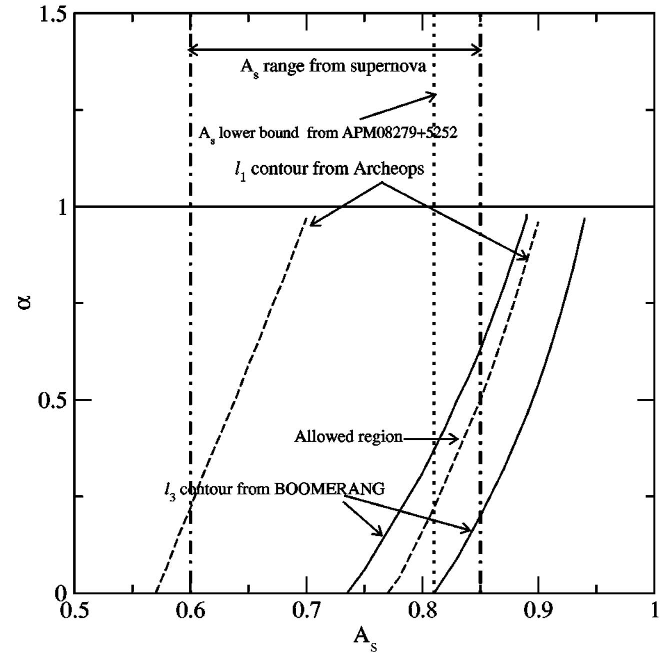

FIG. 3. Contours in the (α, A_S) plane arising from Archeops constraints on l_1 (full contour) and BOOMERANG constraints on l_3 (dashed contour), supernova and APM 08279+5255 object. The allowed region of the model parameters lies in the intersection between these regions.

$$l_1 = 220 \pm 6. \tag{16}$$

Notice that, since $\alpha A_s \leqslant 1$, for a specific value of A_s curves end where this relation gets saturated, $\alpha A_s = 1$.

It is very difficult to extract any constraints from the position of the second peak since it depends on too many parameters, hence we shall disregard it hereafter.

As for the shift of the third peak, it turns out to be a relatively insensitive quantity [28]

$$\varphi_3 \approx 0.341. \tag{17}$$

Figure 2 shows l_3 as a function of α for different values of A_s, where the dashed lines are the current lower and upper bounds on l_3 as derived from BOOMERANG data [31]

$$l_3 = 845^{+12}_{-25}. \tag{18}$$

We see that l_3 puts rather tight constraints on the parameters of the model, α and A_s.

Figure 3 shows the constraints on the parameter space of the generalized Chaplygin gas model, the (A_s, α) plane, that are obtained from the observational bounds on the location of the first (full contour) and third (dashed contour) CMBR peaks. Hence, from the CMBR point of view, the allowed region of the model parameters lies in the intersection between these two contours.

III. DISCUSSION AND CONCLUSIONS

In this paper we have shown that the location of the CMBR peaks, as determined via Archeops and BOOMER-

ANG data, allows constraining a sizeable portion of the parameter space of the generalized Chaplygin gas model. We should stress, however, that our analysis relies on the priors explained at the beginning of Sec. II.

Our results indicate that the constraints arising from the position of the first peak, as recently announced by the Archeops Collaboration, imply, for $\alpha \lesssim 1$, that $0.57 \lesssim A_s \lesssim 0.91$.

On the other hand, the location of the third acoustic peak arising from the BOOMERANG Collaboration provides strong constraints on the parameter space of the model, as indicated in Fig. 3 (dashed contour region). Notice that compatibility with data requires that only the fairly small intersecting region is allowed, that is $0.74 \lesssim A_s \lesssim 0.90$. Furthermore, consistency with SNe Ia data suggests that $0.6 \lesssim A_s \lesssim 0.85$ [24] and this, together with the bound arising from the age of the APM 08279+5255 quasar, $A_s \gtrsim 0.81$ [25], finally lead us to the following constraints: A_s, $0.81 \lesssim A_s \lesssim 0.85$, and $0.2 \lesssim \alpha \lesssim 0.6$. We stress that the allowed region in Fig. 3 is clearly distinct from the Chalpygin gas ($\alpha = 1$) and the ΛCDM model.

Clearly, with future high precision measurements of the MAP and PLANCK satellites, we expect that the position of the first three peaks will be determined with very high accuracy, thus allowing further constraints on the parameter space of the generalized Chaplygin gas model. Correlating the resulting constraints with SNe Ia, redshift objects and, for instance, gravitational lensing data may uniquely determine these parameters.

Note added in proof. After submitting this article, we became aware of three papers related to this work. In Ref. [33], the authors claim that we overestimate the peak positions; however, it should be noted that our priors differ from theirs. Reference [34] concludes that the generalized Chaplygin gas model is a feasible unification model for dark energy and dark matter only if $\alpha = 0$; however, the authors do not account for the effect of baryons, which could be important, and they use a linear approximation in a regime where it is unlikely to work. Finally, in Ref. [35], the results of [34] were used and the generalized Chaplygin gas model was analyzed in a regime where it is clearly indistinguishable from a cosmological constant.

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