Proposal for a new test of the time independence of the fine structure constant α using orthogonally polarized whispering gallery modes in a single sapphire resonator

Michael Edmund Tobar and John Gideon Hartnett
Frequency Standards and Metrology Research Group, School of Physics, University of Western Australia, Crawley 6009, Australia
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A new experiment to test for the time independence of the fine structure constant α is proposed. The experiment utilizes orthogonally polarized transverse electric and transverse magnetic whispering gallery modes in a single sapphire resonator tuned to similar frequencies. When configured as a dual mode sapphire clock, we show that the anisotropy of sapphire makes it is possible to undertake a sensitive measurement from the beat frequency between the two modes. At infrared frequencies this is possible due to the different effect of the lowest phonon frequency on the two orthogonally polarized modes. At microwave frequencies we show that the phonon effect is too small. We show that the electron spin resonance of paramagnetic impurities (such as Cr^{3+}) in the lattice affects only one polarization with an α^6 dependence. This enables an enhancement of the sensitivity to temporal changes in α at microwave frequencies.

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I. INTRODUCTION

Two major tasks in fundamental physics are the quantization of gravity and the unification of all interactions. All approaches to achieve these tasks lead to deviations from present day physics. For example, violations of Einstein's equivalence principle are possible. Some violations may manifest themselves as spatial and/or time variations of the fundamental physical constants such as the fine structure constant α, and recently attempts to measure these effects have gained considerable attention. One of the most precise tools for testing these theories is the frequency or time standard (clock). The measurement requires the comparison of two clocks with different power law dependence on α [1–3]. Alternative approaches examine astrophysical and geophysical data [4,5].

A novel method using monolithic Fabry-Pérot resonators to test for drift in α was recently proposed [6]. The index of refraction of ionic or molecular crystal depends on α; thus a drift in α will cause a change in the dispersion characteristic of the crystal. In particular, [6] analyzed the dependence of optical phonons and electronic transitions on ionic impurities, and remarked that the technique was applicable down to microwave frequencies. In this paper we show that the proposed technique needs to be modified at lower frequencies incorporating the infrared to microwave range. We show that the first phonon resonance at 10^{13} Hz is inherently low loss at 4 K and may offer the best frequency for this type of measurement. At microwave frequencies we show that the introduction of Cr^{3+} ions to the lattice significantly enhances the dispersion and sensitivity due to an electron spin resonance (ESR) at 11.45 GHz.

Temporal drift in α may be detected by exciting two modes in the same resonator that sample different dispersions. For an isotropic material this may only be achieved at different frequencies. Since sapphire is anisotropic, the dual mode technique [7,8] may be implemented to realize this experiment. Transverse electric (TE) and transverse magnetic (TM) modes may be excited at nearly the same frequency, and will sample different values of dispersion. This effect may be amplified with the presence of paramagnetic impurities. Paramagnetic impurities result in an anisotropic magnetic susceptibility of the lattice. Thus, if both TE and TM modes are excited simultaneously, a drift in α will be measurable from the beat frequency of the two modes. The proposed experiment is to create a 4 K cooled dual mode frequency standard based on a sapphire resonator. The beat frequency will be stabilized and measured. If a positive measurement is made that suggests a drift in α it will be necessary to verify this by constructing two or more systems to distinguish the result from a systematic drift.

II. DIELECTRIC RESONATOR DEPENDENCE ON FINE STRUCTURE CONSTANT: MICROWAVE FREQUENCIES TO THE INFRARED

To calculate the dependence of the fine structure constant on the refractive index at optical frequencies, in Ref. [6] the refractive index was related to the electric susceptibility through the local field model. To model a macroscopic resonator we choose to use the one phonon model, and we relate the permittivity ε_r to the electronic susceptibility χ with the following relation: $\varepsilon_r = 1+\chi$. This model is well known to describe the relative permittivity accurately in sapphire, in the radio frequency to infrared regions, and at temperatures below 50 K [9–11]. At frequencies above the infrared, the susceptibility is summed over more than one lattice vibrational state, but in this case we only consider the region where the lowest phonon frequency influences the results, which avoids the necessity of a summation of states. In this case the complex permittivity is given by

$$\tilde{\varepsilon}_r(f) = 1 + \frac{\chi_{dc}}{1-(f/f_i)^2 - jf/f_iQ_i} \tag{1}$$

where f_i is the phonon frequency, $Q_i = f_i/\Delta f_i$ is the dimensionless quality factor of the phonon transition, and χ_{dc} is the dc electronic susceptibility, which was shown to be indepen-

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dent of α [6]. Thus, following the same approach we find that the real part of the permittivity of the material may be expressed as

$$\varepsilon_r(f,\alpha)=1+\frac{\chi_{\rm dc}[1-(f/\alpha^2K_i)^2]}{[1-(f/\alpha^2K_i)^2]^2+(f/\alpha^2K_iQ_i)^2}. \quad (2)$$

Here the phonon frequency is of the form $f_i=\alpha^2K_i$, where K_i is a constant also shown to be independent of α [6]. Also, we assume that the quality factor of the phonon transition is first order independent of α and frequency independent. The frequency independence will be maintained as long as $f<f_c$, where f_c is the cutoff frequency of the validity of the model, which extends to the infrared [9]. Phonon resonances with $f>f_c$ have an exponential frequency dependence on loss, which still satisfies the Kramers-Kronig relation; in this case Eq. (2) takes a more complex form. This led Braxmaier *et al.* [6] to conclude that sampling an ionic crystal near a phonon resonance added too much loss to make use of the high dispersion. On the contrary, in this paper we show that the fundamental phonon resonances in sapphire remains low loss near the high sensitivity region close to the phonon resonance frequency.

By implicitly differentiating Eq. (2) it is straightforward to show the following relation:

$$\frac{d\varepsilon_r}{d\alpha}=-2\frac{f}{\alpha}\frac{d\varepsilon_r}{df}. \quad (3)$$

Thus we can relate a fractional change in permittivity to a fractional change in fine structure constant by

$$\frac{\Delta\varepsilon_r}{\varepsilon_r}=-2\hat{E}\frac{\Delta\alpha}{\alpha} \quad (4)$$

where $\hat{E}$ is the normalized dispersion coefficient of permittivity given by

$$\hat{E}=\frac{d\varepsilon_r}{df}\frac{f}{\varepsilon_r}. \quad (5)$$

Next we must relate the permittivity change to a frequency change. For a dielectric resonator of resonant frequency f_r, with 100% of its energy confined in the dielectric, the following simple relationship holds;

$$\frac{\Delta f_r}{f_r}=-\frac{1}{2}\frac{\Delta\varepsilon_r}{\varepsilon_r}. \quad (6)$$

Thus by combining Eqs. (6) and (4) the relationship between the dielectric resonant frequency and the fine structure constant is given by

$$\frac{\Delta f_r}{f_r}=\hat{E}\frac{\Delta\alpha}{\alpha}. \quad (7)$$

Thus, it is clear that if two frequencies are measured with different values of $\hat{E}$, then the beat frequency will be proportional to changes in the fine structure constant.

III. EXPLOITING THE ANISOTROPIC PERMITTIVITY OF SAPPHIRE

Sapphire is a uniaxial anisotropic material; in general both the real and imaginary parts of the permittivity are anisotropic. The loss tangent $\tan\delta$ of the material is an important parameter for a dielectric resonator, as the quality factor of the resonance Q_r is equal to the inverse of $\tan\delta$ if 100% of the energy is stored in the dielectric. The loss tangent is calculated from the ratio of the imaginary and real permittivity, and is given by

$$\tan\delta(f)=\frac{(\chi_{\rm dc}/Q_i)(f/f_i)}{(f/f_i)^4+(f/f_i)^2(1/Q_i-2-\chi_{\rm dc})+1+\chi_{\rm dc}}. \quad (8)$$

This is related to the absorption coefficient by

$$A(f)=\frac{2\pi f}{c}\sqrt{\varepsilon_r}\tan\delta. \quad (9)$$

The loss tangent or absorption of a low loss material is particularly hard to measure. For example, at microwave frequencies the whispering gallery (WG) mode method was developed and remains the only method capable of measuring loss tangents of order 10^{-6} or less [12,13]. This is particularly true for low temperature measurement near 4 K where the loss tangent is smaller than 10^{-9}. The WG mode method is such a sensitive method that it enabled the first determination of anisotropy in the loss tangent of uniaxial anisotropic crystals [14]. Near 4 K, paramagnetic impurities also affect the result over a broad frequency range even if the transition frequency is as far away as a few THz, which is the case for titanium impurities [15]. This is because the transitions are thermally excited at cryogenic temperatures. Thus, a measurement of the phonon-induced losses is quite difficult unless a very pure sample has been obtained. We have measured many samples in the microwave region, and the losses due to paramagnetic impurities usually limit the Q_r of the resonance around 4 K to a temperature independent value close to 10^9. The exception is the purest grade of Hemex from Crystal Systems. In these samples a Q_r of 10^{10} at 12 GHz has been measured that still exhibits a power law of T^1, which means the Q_r limit is not due to paramagnetic impurities [16]. We believe that this limit is due to the dielectric lattice. This result was used in combination with the low temperature permittivity versus frequency measurements made by Lowenstein *et al.* [17] to calculate the properties of the phonon resonances at 4 K. Fitting these data to Eqs. (1) and (8), the calculated parameters parallel and perpendicular

TABLE I. Fit parameters to the dielectric properties of the first phonon resonances in sapphire.

Fit parameter (~4 K)	Parallel to c axis	Perpendicular to c axis
$\varepsilon_{\rm dc}=1+\chi_{\rm dc}$	11.2431	9.2129
f_i	1.3×10^{13}	1.6×10^{13}
Q_i	8.1×10^{6}	6.7×10^{6}

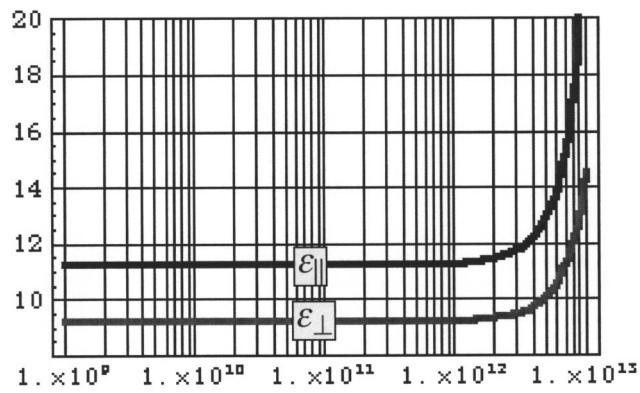

FIG. 1. Real permittivity versus frequency calculated from Eq. (1) with the parameters presented in Table I. Values are plotted from microwave to infrared for the permittivity parallel, $\varepsilon_{\parallel}$, and perpendicular, $\varepsilon_{\perp}$, to the c axis. Permittivity increases markedly around the first phonon resonance.

to the c axis are given in Table I, and the real permittivity, loss tangent, and absorption coefficient are plotted as a function of frequency in Figs. 1–3.

There is a lack of data on the properties of the first phonon resonance at cryogenic temperatures. However, the dc value of permittivity compares well with expected values, and the phonon frequency compares well with other published values at room temperature [18]. It is likely that most loss measurements have not been sensitive enough to measure true absorption coefficients or loss tangents at 4 K (except in the case of the WG method), and we intend to verify the calculated losses shown in Figs. 2 and 3. However, we should note that it is well known that the sapphire loss tangent has $1/f$ dependence as shown.

The idea of this experiment is to exploit the anisotropy of sapphire and construct a dual mode interrogation system or oscillator operating on a TE and TM mode at nearly the same frequency, i.e., $f_{\parallel} \approx f_{\perp} \approx f_r$ [8]. The beat frequency can then be monitored to test for the fine structure constant. For this case the sensitivity of the beat frequency to a drift in fine structure constant is calculated from Eq. (7) to be

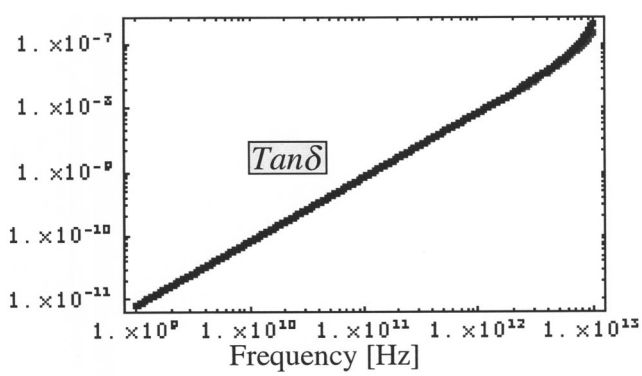

FIG. 2. Loss tangent versus frequency calculated from Eq. (8) with the parameters presented in Table I. Values are plotted from microwave to infrared.

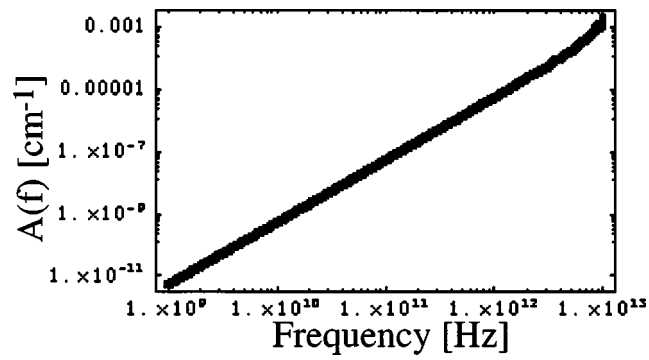

FIG. 3. Absorption coefficient (cm^{-1}) versus frequency calculated from Eq. (9) with the parameters presented in Table I. Values are plotted from microwave to infrared.

$$\frac{|\Delta f_{\parallel} - \Delta f_{\perp}|}{f_r} = \hat{E}_{\mathrm{diff}} \frac{\Delta \alpha}{\alpha} \quad \text{where} \quad \hat{E}_{\mathrm{diff}} = |\hat{E}_{\parallel}(f) - \hat{E}_{\perp}(f)|. \tag{10}$$

Figure 4 shows the sensitivity function $\hat{E}_{\mathrm{diff}}$ as a function of frequency.

Because the modes are excited in the same crystal, some form of common mode rejection of noise sources could be expected as discussed in [6]. Also, the difference frequency will be much lower than the resonance frequency. This down-conversion will lower the noise requirements of the readout of the beat frequency. The key point to this analysis is that close to the phonon frequency a sensitivity function of order unity can be obtained with low loss. One would like to excite modes in the sensitive frequency range of 10^{12}–10^{13} Hz (300–30 μm wavelengths). The frequency of a pure WG mode to first approximation is given by

$$f_r = \frac{cm}{2\pi r \sqrt{\varepsilon}}, \tag{11}$$

where c is the speed of light, m is the azimuthal mode number, r is the sapphire radius, and ε is either the parallel permittivity (for a TM mode) or the perpendicular permittivity (for a TE mode). For example, orthogonal WG modes could be excited in a 5 mm diameter sapphire cylinder with m varying from 160 to 2600, and a free spectral range of 4–6 GHz. Single mode sapphire clocks with low fractional frequency instabilities of order 10^{-16}–10^{-15} have already been demonstrated at microwave and optical frequencies [19,20]. It should be possible to adapt this method to the suggested frequencies and has the potential for high sensitivity tests of drifts in the fine structure constant of order 10^{-15}/yr as discussed in [6].

IV. PARAMAGNETICALLY DOPED SAPPHIRE

At microwave frequencies paramagnetic impurities supply transitions between electron spin states. This changes the susceptibility of the sapphire lattice, which is typically an anisotropic effect. The magnitude of the susceptibility and the frequency of the electron spin resonance depends on the fine structure constant. In the following we derive this de-

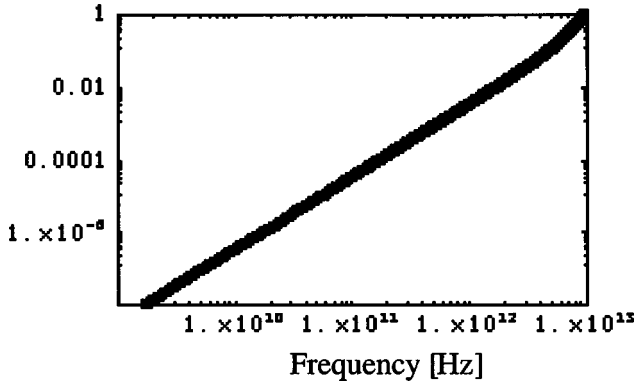

FIG. 4. Calculated $\hat{E}_{\rm diff}$ as a function of frequency calculated from Eqs. (1) and (5) with the parameters presented in Table I.

pendence and calculate the influence on the frequency of a dielectric resonance in the sapphire crystal due to the influence of the Cr^{3+} ESR transition in sapphire.

A. Determination of the dispersion added by Cr^{3+} ions

Paramagnetic Cr^{3+} ions in sapphire change the magnetic susceptibility only perpendicular to the c axis. The form of the complex susceptibility added by the ESR transition may be written as

$$\tilde{\chi}_\perp(f)=\chi'(f)+j\chi''(f)$$
$$=\frac{\chi_\perp^{\rm dc}}{[1-(f/f_0)^2]^2+[(f/f_0)(1/Q_0)]^2}\times\left[\left(1-\left(\frac{f}{f_0}\right)^2\right)+j\left(\frac{f}{f_0}\frac{1}{Q_0}\right)\right]. \quad (12)$$

Here $\chi_\perp^{\rm dc}$ is the dc magnetic susceptibility, f_0 is the frequency of the ESR (11.45 GHz for Cr^{3+} in sapphire [21]), $Q_0=f_0/\Delta f_0=\pi f_0\tau_0$, where Δf_0 is the bandwidth of the ESR and τ_0 is the relaxation time of the ESR. The loss added to the lattice can be characterized by the magnetic loss tangent $\tan\delta_m$ and is given by

$$\tan\delta_m(f)=\frac{\chi''}{1+\chi'}\approx\frac{(\chi_\perp^{\rm dc}/Q_0)(f/f_0)}{[1-(f/f_0)^2][1+\chi_\perp^{\rm dc}-(f/f_0)^2]}. \quad (13)$$

To evaluate the magnetic susceptibility we use data taken at the University of Western Australia by Mann and analyzed by Krupka [22]. The experiment consisted of analyzing the WG modes in a 5 cm diameter sapphire cylinder with a few parts per million impurity ions of Cr^{3+}, enclosed in an 8 cm diameter copper cavity. The measured real part of the susceptibility is shown in Fig. 5. Mann and Krupka fitted the data with $\chi_\perp^{\rm dc}$ and τ_0 as free parameters and calculated $\chi_\perp^{\rm dc}=(5.1\times10^{-8})\pm2\%$ and $\tau_0=(0.69\times10^{-8})\pm6\%$ s. We point out here that fitting to the real part of the permittivity cannot give an accurate determination of the ESR relaxation rate or bandwidth as a fit to the real part is very insensitive to this parameter. Thus we believe the quoted errors are likely to be wrong, especially so for τ_0. To obtain a more accurate fit the magnetic loss tangent must be calculated. To calculate this effect, WG mode Q factors Q_r must be measured and the magnetic filling factor perpendicular to the c axis, $p_{m\perp}$, calculated. In general the Q_r of a TM dielectric resonance is given by

$$Q_r^{-1}=p_{m\perp}\tan\delta_m+\frac{R_s}{G}. \quad (14)$$

Here R_s is the surface resistance of the copper cavity and G is the geometric factor of the mode. Thus, to calculate $\tan\delta_m$ one needs to measure transverse magnetic modes with $p_{m\perp}$-close to unity and a high G factor. The highest G factor TM modes are the fundamental $WGH_{m,0,0}$ mode family. Only these modes are selected from the measured modes of Mann and Krupka [22] shown in Fig. 5, as all other modes either are limited by the copper cavity or have a low sensitivity to the perpendicular magnetic field. The Q factors and filling factors of the measured $WGH_{m,0,0}$ modes are shown in Table II.

If the data were limited only by the magnetic loss tangent, one would expect an increasing Q factor as we move further from the 11.45 GHz resonance. This is not the case: the cavity losses affect the measurement because of the smaller confinement for lower m numbers. Figure 6 clearly shows this effect; ignoring R_s, the effective $\tan\delta_m$ is plotted as a function of frequency. The two points closest to the 11.45 GHz resonance are identified to be limited by the magnetic loss tangent, while the two low frequency points are limited by the copper surface resistance. Hence by undergoing a simultaneous fit to the real susceptibility and the loss tangent, a more accurate determination of the ESR parameters can be obtained. We calculate $\chi_\perp^{\rm dc}=(5.9\times10^{-8})\pm6\%$ and $\Delta f_0=9\times10^6$, which gives $\tau_0=3.5\times10^{-8}$ s. Note that the dc susceptibility is very close to Mann and Krupka's calculation; however, the spin-spin relaxation rate was calculated to be a factor of 5 longer.

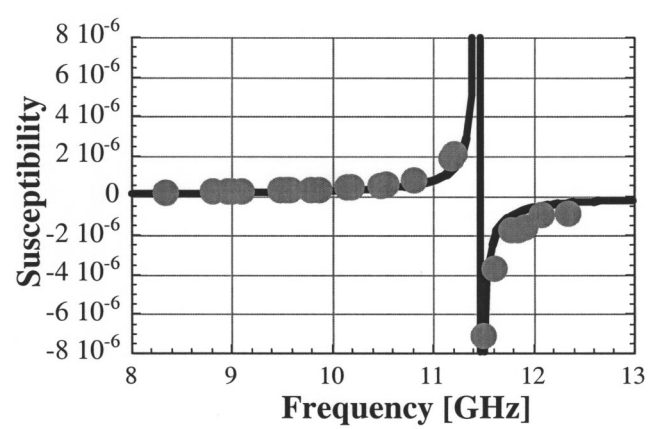

FIG. 5. Magnetic dc susceptibility versus frequency. Dots show calculations due to frequency shifts in WG modes (data taken from Mann and Kruka [22]), while the bold line shows the best fit from Eqs. (12) and (13).

TABLE II. $WGH_{m,0,0}$ mode family data.

Mode	Frequency	$p_{m\perp}$	G factor	Q factor
$WGH_{12,0,0}$	8.95045	0.881	1.63×10^7	1.8×10^8
$WGH_{13,0,0}$	9.56922	0.888	3.92×10^7	1.0×10^9
$WGH_{14,0,0}$	10.1862	0.893	9.42×10^7	1.3×10^9
$WGH_{15,0,0}$	10.80142	0.898	2.26×10^8	2.2×10^8

In the case of magnetic spin-spin transitions, we show later that, unlike in the electronic case calculated previously [6], the dc susceptibility depends on α. Thus we take a more general approach to calculating the sensitivity. First we must relate the real magnetic susceptibility change to a frequency change. For a dielectric resonator of resonant frequency f_r with 100% of its energy confined in the dielectric, the simple relationship

$$\frac{df_r}{d\chi'}=-\frac{1}{2}\frac{f_r}{1+\chi'} \tag{15}$$

holds. Using implicit differentiation we can show that the following relationship holds:

$$\frac{\Delta f_r}{f_r}=\hat{M}\frac{\Delta\alpha}{\alpha} \quad \text{where} \quad \hat{M}=\frac{\alpha}{f_r}\frac{df_r}{d\chi'}\frac{d\chi'}{d\alpha}. \tag{16}$$

Here $\hat{M}$ is the sensitivity conversion of the mode frequency with respect to the fine structure constant. By combining Eqs. (15) and (16) it follows that

$$\hat{M}=-\frac{1}{2}\frac{d\chi'}{d\alpha}\frac{\alpha}{1+\chi'}. \tag{17}$$

To calculate the sensitivity conversion Eq. (17) must be evaluated over all parameters that vary with α. In the following analysis we show that the dc susceptibility and ESR frequency are highly dependent on α, and we do not consider any dependence on the spin-spin relaxation time as this will have no first order effect on the frequency of the ESR transition or the dielectric resonance.

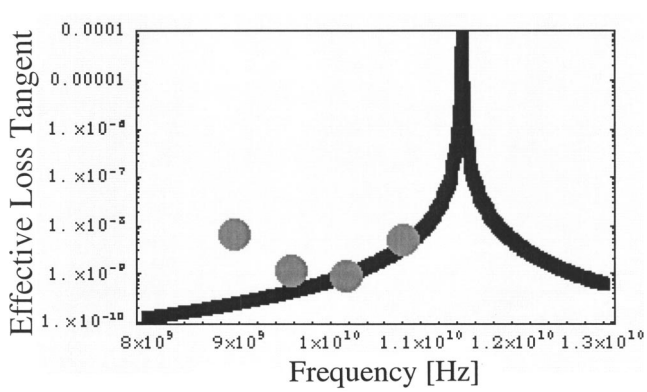

FIG. 6. The magnetic loss tangent versus frequency in hertz. Dots show calculated effective loss tangent due to Q factors of the $WGH_{m,0,0}$ mode family, which is calculated assuming $R_s=0$ in Eq. (14). The bold line shows the best fit from Eqs. (12) and (13). Note that the two lower frequency calculations diverge from the predictions due to the smaller G factors, which cause the measurements to be limited by the surface resistance of the copper cavity. We have calculated from Eq. (14) that a surface resistance of the order of 50 mΩ explains this effect.

B. Cr^{3+} ESR frequency dependence on α

The Cr^{3+} ion has three unpaired electrons, with the free-ion ground state of $^4F_{3/2}$. When the ion is present in a tetragonal lattice (i.e., sapphire), the noncubic symmetry of the field causes a zero applied field splitting of the spin states (or electron spin resonance) due to the crystal field. Orton has used crystal field theory [23] to calculate the energy associated with the transition between spin states using second order perturbation theory. The zero field splitting was calculated to be

$$\Delta E=E_{\pm3/2}-E_{\pm1/2}\approx\frac{8\,\delta\lambda^2}{\Delta^2}. \tag{18}$$

Here λ is the spin-orbit coupling, Δ is the effective isotropic field splitting between the ground ($^4A_2$) and first excited states ($^4T_2$) of the ^4F orbital states, and δ is the splitting of the first excited state due to the anisotropic crystal field. To calculate crystal field effects, the potential experienced by an electron wave function due to the nearest neighbor electrons is calculated [23]. From this calculation one can show the following:

$$\Delta=X_\Delta\frac{e^2}{4\pi\varepsilon_0}\frac{\langle r^4\rangle}{a^5} \quad \text{and} \quad \delta=X_\delta\frac{e^2}{4\pi\varepsilon_0}\frac{(1-b/a)\langle r^4\rangle}{a^5}. \tag{19}$$

Here a is the atomic spacing perpendicular to the c axis, b is the atomic spacing parallel to the c axis, $\langle r^4\rangle$ is the mean radius to the power of four of the electron wave function, and X_Δ and X_δ are dimensionless constants of order unity. Now we use similar reasoning to that of Braxmaier *et al.* [6], that the interatomic spacing and the wave function size are proportional to the Bohr radius a_0 of the electron, and by substituting Eq. (19) into Eq. (18) we obtain

$$\Delta E\propto\lambda^2\frac{a_0 4\pi\varepsilon_0}{e^2}. \tag{20}$$

The spin-orbit coupling and Bohr radius may be written in terms of the fine structure constant as

$$\lambda=m_e c^2 Z^4\alpha^4 \quad \text{and} \quad a_0=\frac{\hbar}{\alpha m_e c}. \tag{21}$$

By substituting Eq. (21) into Eq. (20) one may show that the frequency of the transition is given by

$$f_0\propto\left(\frac{c}{\Lambda}\right)Z^8\alpha^6, \quad \Lambda=\frac{h}{m_e c}. \tag{22}$$

Here Λ is the Compton wavelength. Thus the frequency of the paramagnetic resonance is proportional to the sixth

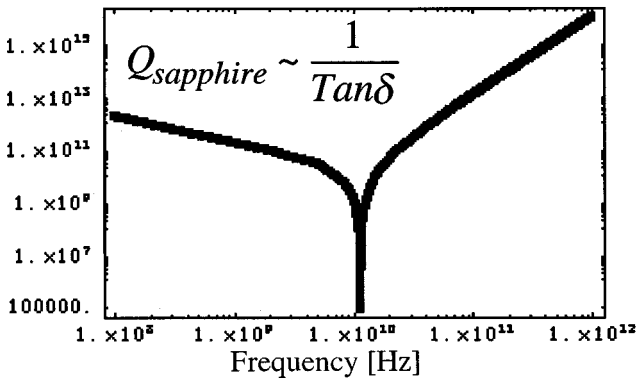

FIG. 7. Calculated magnetic Q-factor limit versus frequency for a TM mode in the doped sapphire presented in this paper.

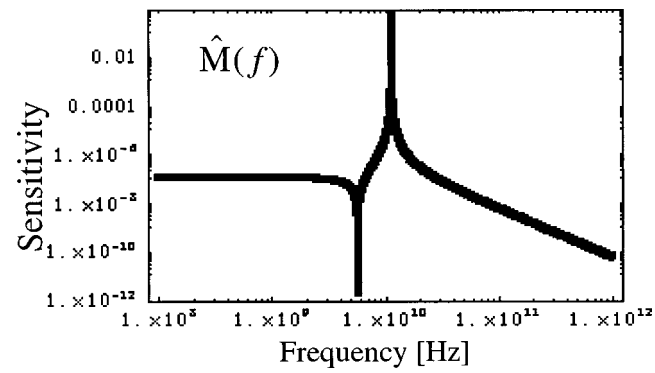

FIG. 8. Calculated sensitivity $\hat{M}$ as a function of frequency using Eqs. (25) and (12) for the doped sapphire presented in this paper.

power of α. Experimentally the resonance has been confirmed to be at 11.45 GHz, and thus a microwave sapphire clock operating close to this frequency is suited for this type of experiment.

C. Cr^{3+} ESR susceptibility dependence on α

The susceptibility of the transition is proportional to the dc susceptibility given by [21]

$$\chi_{\perp}^{dc} \propto N\mu_B^2, \quad N \sim \frac{1}{a_0^3}, \quad \mu_B \sim \frac{e\hbar}{2m_e c}. \tag{23}$$

Here N is the number density of ions, k is Boltzmann's constant, and μ_B is the Bohr magneton. Thus, it is straightforward to show that

$$\chi_{\perp}^{dc} \propto \alpha^4. \tag{24}$$

Unlike the case of a phonon resonance, the dc susceptibility due to the spin-spin interaction is dependent on the fine structure constant.

D. Sensitivity of Cr^{3+}-doped sapphire to α

Braxmaier *et al.* [6] considered electronic transitions in doped ionic crystals to increase the electronic dispersion in the crystal. They assumed that the normalized sensitivity (in our case $\hat{M}$) was unity. In the following we do some similar analysis for doped sapphire at microwave frequencies. By combining Eqs. (22) and (24) with Eq. (17) the normalized sensitivity of a dielectric mode in doped sapphire is given by

$$\hat{M}(f) = \frac{3f\partial\chi'/\partial f - 2\chi'}{1+\chi'}. \tag{25}$$

For the doped sapphire investigated in this paper, the sensitivity and inverse magnetic loss tangent as a function of frequency are plotted in Figs. 7 and 8. The sensitivity is unity at the resonance frequency with a loss tangent of order 10^{-5}. The sensitivity could be further enhance with the deliberate addition of more Cr^{3+} ions (ruby). However, the enhancement in sensitivity compromises the loss tangent of the material. This means a better lock to the resonance is needed for the experiment to work. Another option is to find a transition in sapphire with a longer relaxation time.

V. DISCUSSION

The simple requirement for this technique requires a low loss crystal resonator with large dispersion. The two requirements are in fact contradictory due to the Kramers-Kronig relations between the real and imaginary parts of a resonance system. We have shown that the first phonon resonance at 10^{13} Hz is inherently low loss at 4 K and may offer the best frequency for this type of measurement. At microwave frequencies zero field splitting of the ESR due to Cr^{3+} ions in the sapphire lattice significantly enhances the dispersion and hence the sensitivity. However, we show that due to the Kramers-Kronig relation Cr^{3+} adds magnetic losses because of the short spin-spin relaxation time of 3.5×10^{-8} s. Improvement of this technique requires the addition of paramagnetic impurities with longer spin-spin relaxation times.

Braxmaier *et al.* [6] showed that a similar technique at optical frequencies is capable of measuring drifts in the fine structure constant of the order of 10^{-15}/yr. We have proposed two new experiments that can achieve a similar result at lower frequencies.

ACKNOWLEDGMENTS

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[1] J. D. Prestage, R. L. Tjoelker, and L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995).
[2] A. Godone, C. Novero, P. Tavella, and K. Rahimullah, Phys. Rev. Lett. **71**, 2364 (1993).
[3] J. P. Turneaure, C. M. Will, B. F. Farrell, E. M. Mattison, and R. F. C. Vessot, Phys. Rev. D **27**, 1705 (1983).
[4] T. Damour and F. Dyson, Nucl. Phys. **B480**, 37 (1996).
[5] J. K. Webb, M. T. Murphy, V. V. Flambaum, V. A. Dzuba, J. D. Barrow, C. W. Churchill, J. X. Prochaska, and A. M. Wolfe, Phys. Rev. Lett. **87**, 091301 (2001).
[6] C. Braxmaier, O. Pradl, H. Müller, A. Peters, J. Mlynek, V. Loriette, and S. Schiller, Phys. Rev. D **64**, 042001 (2001).
[7] M. E. Tobar, E. N. Ivanov, C. R. Locke, and J. G. Hartnett, Electron. Lett. **38**, 948 (2002).
[8] M. E. Tobar, E. N. Ivanov, J. G. Hartnett, D. Cros, and P. Bilski, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **49**, 1349 (2002).
[9] M. E. Thomas, in *Handbook of Optical Constants of Solids II*, edited by E. D. Palik (Academic, New York, 1991), pp. 177–201.
[10] V. Gurevich and A. Tagantskv, Adv. Phys. **40**, 719 (1993).
[11] V. B. Braginsky, V. S. Ilchenko, and K. S. Bagdassarov, Phys. Lett. A **120**, 300 (1987).
[12] J. Krupka, K. Derzakowski, M. E. Tobar, J. G. Hartnett, and R. G. Geyer, Meas. Sci. Technol. **10**, 387 (1999).
[13] J. Krupka, K. Derzakowski, A. Abramowicz, M. E. Tobar, and R. Geyer, IEEE Trans. Microwave Theory Tech. **47**, 752 (1999).
[14] M. Tobar, J. Krupka, E. Ivanov, and R. Woode, J. Appl. Phys. **83**, 1604 (1998).
[15] J. G. Hartnett, M. E. Tobar, A. G. Mann, E. N. Ivanov, J. Krupka, and R. Geyer, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **46**, 993 (1999).
[16] S. Chang, Ph.D. thesis, University of Western Australia, 2000.
[17] E. V. Lowenstein, D. R. Smith, and R. L. Morgan, Appl. Opt. **12**, 398 (1973).
[18] W. J. Tropf and M. E. Thomas, in *Handbook of Optical Constants of Solids III*, edited by E. D. Palik (Academic, New York, 1998), pp. 653–682.
[19] S. Chang, A. Mann, and A. Luiten, Electron. Lett. **36**, 480 (2000).
[20] R. Storz, C. Braxmaier, K. Jäck, O. Pradl, and S. Schiller, Opt. Lett. **23**, 1031 (1998).
[21] A. E. Siegman, *Microwave Solid-State Masers* (McGraw-Hill, New York, 1964).
[22] A. G. Mann and J. Krupka, in *Proceedings of MIKON-2000, 13th International Conference on Microwaves, Radar and Wireless Communications* (IEEE, Warsaw, 2000), Vol. 2, pp. 421–424.
[23] J. W. Orton, *Electron Paramagnetic Resonance: An Introduction to Transition Group Ions in Crystals* (Iliffe, London, 1968).