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Erratum: Light hadron spectroscopy with two flavors of dynamical quarks on the lattice [Phys. Rev. D 65, 054505 (2002)]

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(i) The definition of the improved axial current in Eq. (24) in Sec. III B should be $A_\mu^{\rm imp}=A_\mu-c_A\nabla_\mu P$ with our definition of the axial current $A_\mu=\bar{q}\gamma_5\gamma_\mu q$. The sign of the $O(a)$ improvement term used in the paper was incorrect. This error affects the numerical results of the AWI quark mass and the pseudoscalar decay constant at finite lattice spacings. Our conclusions about these observables, however, are unchanged as we see below. Results on other quantities, such as the hadron spectrum, are unaffected.

Table XXI shows the corrected results for the AWI quark mass at simulation points in two-flavour QCD. Recalculated parameters of chiral extrapolations are given in Tables VII and XIV. Physical quark masses, m_{ud} and m_s, at finite lattice spacings are summarized in Table XVI. Figures 29–32 show separate and combined continuum extrapolations of the quark masses. Combining the statistical and systematic error listed in Table XVII, the corrected results for the light quark masses are given by

TABLE VII. Parameters of chiral fits to pseudoscalar meson masses as a function of AWI quark mass in two-flavor QCD.

β	$\chi^2/N_{\rm DF}$	b'_v	c'_v	c'_{sv}
1.80	73/33	6.201(28)	−1.21(11)	2.65(18)
1.95	69/33	4.600(32)	−0.18(13)	4.62(29)
2.10	59/33	3.285(26)	1.19(16)	4.11(39)
2.20	35/33	2.712(63)	1.49(40)	3.62(93)

TABLE XIV. Parameters of chiral fits to pseudoscalar meson masses as a function of AWI quark mass in quenched QCD.

β	b'_v	c'_v
2.187	4.898(50)	0.96(35)
2.214	4.782(45)	0.41(28)
2.247	4.551(46)	0.42(30)
2.281	4.322(38)	0.55(29)
2.334	3.987(44)	0.89(30)
2.416	3.485(35)	1.41(32)
2.456	3.305(27)	1.24(31)
2.487	3.170(22)	1.36(25)
2.528	2.995(22)	1.04(25)
2.575	2.836(28)	0.91(31)

TABLE XVI. AWI quark masses (in MeV) in the $\overline{\rm MS}$-scheme at $\mu=2$ GeV at finite lattice spacings in full and quenched QCD. Values in the continuum limit obtained with separate linear fits are also listed. For full QCD data at $\beta=2.2$ were not included in these fits.

β	$m_{ud}^{\rm AWI}$	$m_s^{\rm AWI}$ (K)	$m_s^{\rm AWI}$ (ϕ)
	$N_f=2$ Full QCD		
1.80	3.094(35)	81.75(93)	105.4(2.2)
1.95	3.171(37)	83.13(95)	104.5(1.6)
2.10	3.245(45)	84.3(1.1)	96.9(2.2)
2.20	3.26(15)	84.6(3.8)	99.9(5.0)
$a\to 0$	3.39(9)	86.9(2.3)	90.7(4.7)
$\chi^2/N_{\rm DF}$	0.02	0.001	2.99
	$N_f=0$ Quenched QCD		
2.187	3.704(50)	96.0(1.1)	119.5(2.4)
2.214	3.633(50)	94.7(1.2)	118.4(2.7)
2.247	3.651(51)	95.2(1.2)	116.8(2.6)
2.281	3.767(50)	98.1(1.2)	124.5(3.2)
2.334	3.818(54)	99.1(1.2)	124.7(2.6)
2.416	3.933(59)	101.5(1.3)	128.5(3.3)
2.456	3.853(42)	99.7(1.0)	122.0(2.3)
2.487	3.898(50)	100.7(1.2)	124.4(2.8)
2.528	3.909(46)	101.3(1.1)	125.1(2.4)
2.575	3.900(51)	101.2(1.1)	125.0(2.4)
$a\to 0$	4.290(83)	110.2(1.9)	133.9(4.3)
$\chi^2/N_{\rm DF}$	1.1	1.0	1.1

$$\overline{m}_{ud}^{\overline{\rm MS}}(2\ \text{GeV}) = 3.45^{+0.14}_{-0.20}\ \text{MeV}, \tag{65}$$

$$m_{s}^{\overline{\rm MS}}(2\ \text{GeV}) = 89^{+3}_{-6}\ \text{MeV} \quad M_K\ \text{input}, \tag{66}$$

$$= 90^{+5}_{-10}\ \text{MeV} \quad M_\phi\ \text{input}, \tag{67}$$

in the continuum limit in two-flavor QCD. The corrected results in quenched QCD are given in Table XVIII.

The corrected values of m_{ud} and m_s are about 8%, 5%, and 3% smaller for data taken at $\beta=1.8$, 1.95, and 2.1. In the continuum limit the changes are quite small (at most 1%) both in full and quenched QCD, since the $O(a)$ correction to $A_\mu^{\rm imp}$ vanishes toward this limit. Therefore, our conclusions that effects from two flavors of sea quarks reduce m_{ud} and m_s by about 25%, and that including these effects sizably improves the consistency of m_s calculated with M_K and M_ϕ as inputs in two-flavor full QCD, are unchanged.

Chiral extrapolations of the corrected pseudoscalar decay constants are shown in Figs. 33 and 34. Recalculated parameters of the extrapolations are listed in Table XIX, which lead to the corrected values of f_π and f_K in Table XX.

TABLE XVII. Breakdown of contributions to total error of full QCD quark masses in the continuum limit.

	Stat.	Cont. ext.	Chiral	Z factor
m_{ud}	+2.8%	+1.4%	+1.4%	+2.5%
	−2.8%	−1.6%	−2.0%	−4.6%
m_s (M_K input)	+2.7%	+1.0%	+1.1%	+2.1%
	−2.7%	−2.0%	−2.1%	−5.4%
m_s (M_ϕ input)	+4.8%	+0.5%	+0.3%	+1.8%
	−4.8%	−0.3%	−7.8%	−6.6%

TABLE XVIII. Continuum limit quark masses in the $\overline{\rm MS}$ scheme at $\mu=2$ GeV (in MeV).

	Action	m_{ud}	m_s	
			M_K input	M_ϕ input
$N_f=2$	impr.	$3.45^{+0.14}_{-0.20}$	89^{+3}_{-6}	90^{+5}_{-10}
$N_f=0$	impr.	$4.37^{+0.13}_{-0.16}$	111^{+3}_{-4}	132^{+4}_{-5}

TABLE XIX. Parameters of chiral fits to pseudoscalar decay constants.

β	$\chi^2/N_{\rm DF}$	A^F	B_s^F	B_v^F	C_s^F	C_v^F	C_{sv}^F
1.80	49.6/30	0.1942(49)	0.077(13)	0.1210(56)	−0.0205(77)	−0.0122(29)	−0.0069(38)
1.95	14.6/30	0.1150(53)	0.081(21)	0.1432(54)	−0.027(21)	−0.0265(42)	−0.0260(65)
2.10	14.7/30	0.0672(36)	0.159(34)	0.183(12)	−0.208(71)	−0.101(23)	−0.029(29)
2.20	10.8/30	0.050(12)	0.27(15)	0.207(32)	−0.76(44)	−0.127(90)	−0.07(15)

TABLE XX. Decay constants at finite lattice spacings in full and quenched QCD. All decay constants are in GeV units.

β	f_π	f_K (K)	f_K (ϕ)
	$N_f=2$ Full QCD		
1.80	0.1821(47)	0.2114(42)	0.2193(41)
1.95	0.1491(67)	0.1745(63)	0.1805(62)
2.10	0.1266(64)	0.1489(61)	0.1520(61)
2.20	0.117(26)	0.138(26)	0.141(26)
	$N_f=0$ Quenched QCD		
2.187	0.1623(42)	0.1825(34)	0.1876(31)
2.214	0.1555(37)	0.1761(31)	0.1811(29)
2.247	0.1512(41)	0.1724(34)	0.1771(32)
2.281	0.1423(33)	0.1655(28)	0.1717(26)
2.334	0.1462(41)	0.1657(35)	0.1707(33)
2.416	0.1368(39)	0.1561(31)	0.1613(27)
2.456	0.1444(39)	0.1616(33)	0.1654(32)
2.487	0.1358(36)	0.1547(31)	0.1591(28)
2.528	0.1405(47)	0.1588(39)	0.1629(38)
2.575	0.1445(53)	0.1622(42)	0.1662(39)

TABLE XXI. AWI quark masses with degenerate (upper table) and nondegenerate quark combinations (lower table).

Degenerate quark combination ($K_{\mathrm{val}}^{(1)}=K_{\mathrm{val}}^{(2)}=K_{\mathrm{val}}$)					
$\beta=1.80$					
	$K_{\mathrm{val}}=0.1409$	$K_{\mathrm{val}}=0.1430$	$K_{\mathrm{val}}=0.1445$	$K_{\mathrm{val}}=0.1464$	$K_{\mathrm{val}}=0.1474$
$K_{\mathrm{sea}}=0.1409$	0.20503(40)	0.16149(35)	0.13050(33)	0.09114(35)	0.06959(46)
$K_{\mathrm{sea}}=0.1430$	0.19453(67)	0.15059(57)	0.11939(50)	0.08005(51)	0.05870(72)
$K_{\mathrm{sea}}=0.1445$	0.18208(72)	0.13766(57)	0.10610(37)	0.06680(32)	0.04530(44)
$K_{\mathrm{sea}}=0.1464$	0.15806(73)	0.11387(60)	0.08288(41)	0.04470(34)	0.02298(60)
$\beta=1.95$					
	$K_{\mathrm{val}}=0.1375$	$K_{\mathrm{val}}=0.1390$	$K_{\mathrm{val}}=0.1400$	$K_{\mathrm{val}}=0.1410$	$K_{\mathrm{val}}=0.1415$
$K_{\mathrm{sea}}=0.1375$	0.15133(61)	0.11658(51)	0.09353(45)	0.07056(40)	0.05908(39)
$K_{\mathrm{sea}}=0.1390$	0.13969(55)	0.10470(44)	0.08153(36)	0.05843(29)	0.04685(27)
$K_{\mathrm{sea}}=0.1400$	0.13097(54)	0.09543(43)	0.07198(35)	0.04873(29)	0.03701(28)
$K_{\mathrm{sea}}=0.1410$	0.12082(42)	0.08519(38)	0.06148(43)	0.03818(35)	0.02652(32)
$\beta=2.10$					
	$K_{\mathrm{val}}=0.1357$	$K_{\mathrm{val}}=0.1367$	$K_{\mathrm{val}}=0.1374$	$K_{\mathrm{val}}=0.1382$	$K_{\mathrm{val}}=0.1385$
$K_{\mathrm{sea}}=0.1357$	0.10334(50)	0.07730(43)	0.05894(36)	0.03813(28)	0.03029(24)
$K_{\mathrm{sea}}=0.1367$	0.09888(43)	0.07293(36)	0.05471(30)	0.03354(23)	0.02551(18)
$K_{\mathrm{sea}}=0.1374$	0.09551(45)	0.06924(31)	0.05086(21)	0.02986(13)	0.02193(12)
$K_{\mathrm{sea}}=0.1382$	0.09076(50)	0.06461(35)	0.04628(26)	0.02528(18)	0.01745(15)
$\beta=2.20$					
	$K_{\mathrm{val}}=0.1351$	$K_{\mathrm{val}}=0.1358$	$K_{\mathrm{val}}=0.1363$	$K_{\mathrm{val}}=0.1368$	$K_{\mathrm{val}}=0.1372$
$K_{\mathrm{sea}}=0.1351$	0.07981(57)	0.06071(42)	0.04696(30)	0.03318(28)	0.02206(26)
$K_{\mathrm{sea}}=0.1358$	0.07737(97)	0.05804(74)	0.04419(53)	0.03057(43)	0.01948(39)
$K_{\mathrm{sea}}=0.1363$	0.07585(85)	0.05675(65)	0.04308(52)	0.02932(37)	0.01821(26)
$K_{\mathrm{sea}}=0.1368$	0.0753(12)	0.05573(95)	0.04177(68)	0.02779(38)	0.01674(29)
Nondegenerate quark combination ($K_{\mathrm{val}}^{(1)}=K_{\mathrm{sea}}$)					
$\beta=1.80$					
	$K_{\mathrm{val}}^{(2)}=0.1409$	$K_{\mathrm{val}}^{(2)}=0.1430$	$K_{\mathrm{val}}^{(2)}=0.1445$	$K_{\mathrm{val}}^{(2)}=0.1464$	$K_{\mathrm{val}}^{(2)}=0.1474$
$K_{\mathrm{sea}}=0.1409$	—	0.18322(38)	0.16762(36)	0.14783(36)	0.13732(36)
$K_{\mathrm{sea}}=0.1430$	0.17251(62)	—	0.13496(54)	0.11512(49)	0.10453(48)
$K_{\mathrm{sea}}=0.1445$	0.14391(59)	0.12192(52)	—	0.08631(35)	0.07577(36)
$K_{\mathrm{sea}}=0.1464$	0.10068(53)	0.07900(41)	0.06378(37)	—	0.03422(38)
$\beta=1.95$					
	$K_{\mathrm{val}}^{(2)}=0.1375$	$K_{\mathrm{val}}^{(2)}=0.1390$	$K_{\mathrm{val}}^{(2)}=0.1400$	$K_{\mathrm{val}}^{(2)}=0.1410$	$K_{\mathrm{val}}^{(2)}=0.1415$
$K_{\mathrm{sea}}=0.1375$	—	0.13390(56)	0.12228(53)	0.11064(51)	0.10481(50)
$K_{\mathrm{sea}}=0.1390$	0.12213(50)	—	0.09309(40)	0.08146(36)	0.07562(34)
$K_{\mathrm{sea}}=0.1400$	0.10124(45)	0.08366(39)	—	0.06032(32)	0.05446(31)
$K_{\mathrm{sea}}=0.1410$	0.07901(47)	0.06144(43)	0.04979(39)	—	0.03234(33)

These values are about 7%, 5%, and 3% smaller at $\beta=1.8$, 1.95, and 2.1 compared to the original values. The correction-does not change the conclusions, however; as Fig. 35 shows the decay constants in full QCD exhibit a large violation of scaling, which becomes much reduced in ratios of decay constants as seen in Figs. 36 and 40.

(ii) In Table XV for quenched QCD, the values for m_{K*} with ϕ-input are incorrect. With our choice of the fitting function to vector meson masses, Eq. (59), m_{K*} with ϕ-input has a trivial value $(m_\rho+m_\phi)/2=893.9$ MeV at each lattice spacing and in the continuum limit. The change of m_{K*} is quite small, namely at most 0.7%. Therefore our conclusions on the hadron spectrum are unchanged.

TABLE XXI. (*Continued*).

Degenerate quark combination ($K_{\mathrm{val}}^{(1)}=K_{\mathrm{val}}^{(2)}=K_{\mathrm{val}}$)					
$\beta=2.10$					
	$K_{\mathrm{val}}^{(2)}=0.1357$	$K_{\mathrm{val}}^{(2)}=0.1367$	$K_{\mathrm{val}}^{(2)}=0.1374$	$K_{\mathrm{val}}^{(2)}=0.1382$	$K_{\mathrm{val}}^{(2)}=0.1385$
$K_{\mathrm{sea}}=0.1357$	—	0.09030(47)	0.08091(43)	0.07039(42)	0.06642(42)
$K_{\mathrm{sea}}=0.1367$	0.08588(39)	—	0.06381(33)	0.05330(30)	0.04933(27)
$K_{\mathrm{sea}}=0.1374$	0.07308(32)	0.06003(26)	—	0.04034(17)	0.03637(15)
$K_{\mathrm{sea}}=0.1382$	0.05794(29)	0.04489(25)	0.03573(22)	—	0.02136(16)
$\beta=2.20$					
	$K_{\mathrm{val}}^{(2)}=0.1351$	$K_{\mathrm{val}}^{(2)}=0.1358$	$K_{\mathrm{val}}^{(2)}=0.1363$	$K_{\mathrm{val}}^{(2)}=0.1368$	$K_{\mathrm{val}}^{(2)}=0.1372$
$K_{\mathrm{sea}}=0.1351$	—	0.07026(50)	0.06337(44)	0.05643(37)	0.05083(32)
$K_{\mathrm{sea}}=0.1358$	0.06776(83)	—	0.05128(65)	0.04435(59)	0.03879(57)
$K_{\mathrm{sea}}=0.1363$	0.05943(68)	0.04991(58)	—	0.03621(45)	0.03068(38)
$K_{\mathrm{sea}}=0.1368$	0.05126(86)	0.04166(67)	0.03476(55)	—	0.02223(38)

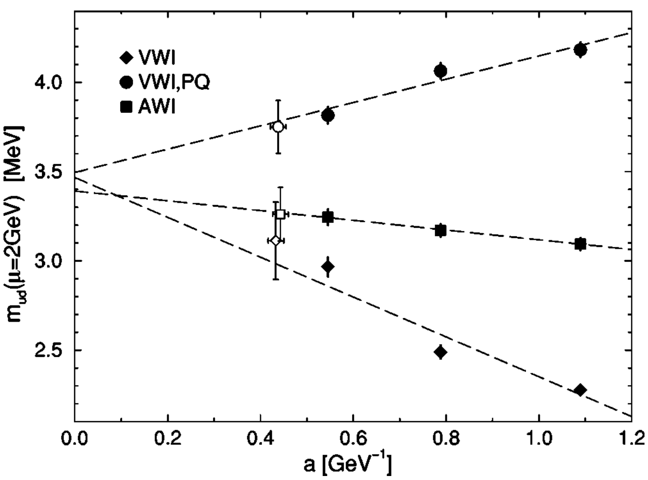

FIG. 29. Average up and down quark mass for three different definitions in full QCD. Lines are from linear extrapolations to the continuum limit made separately for each definition.

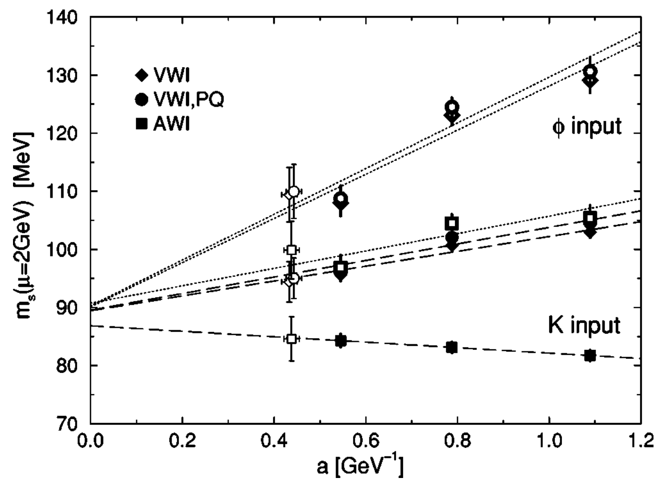

FIG. 30. Strange quark mass for three different definitions and two different experimental inputs in full QCD. Lines are from linear extrapolations to the continuum limit made separately for each definition.

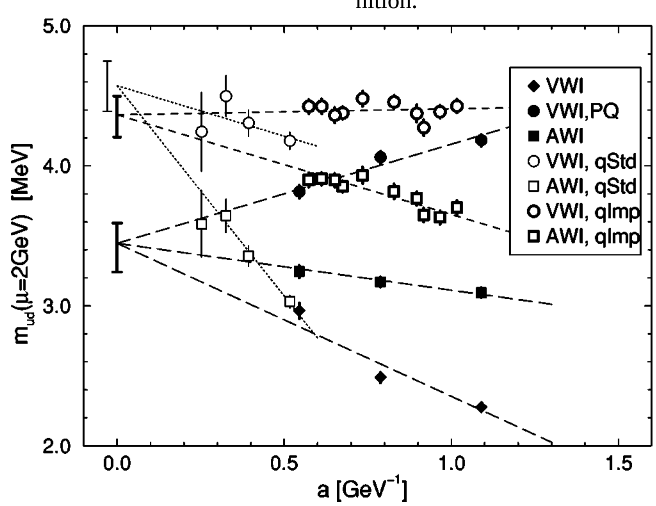

FIG. 31. Comparison of average up and down quark mass in quenched and full QCD. Lines are from combined linear continuum extrapolations.

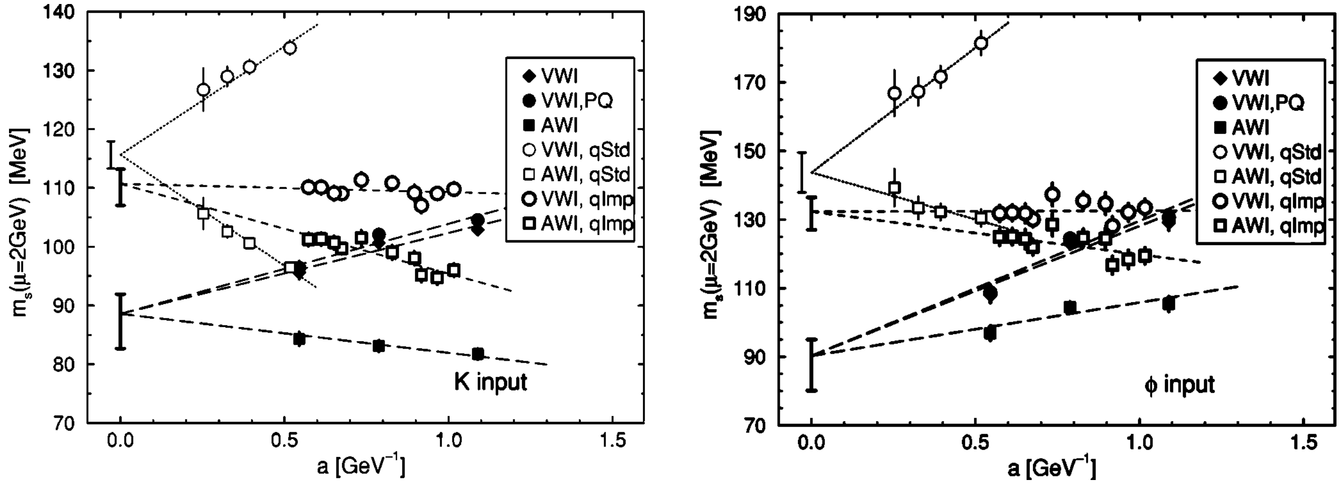

FIG. 32. Comparison of strange quark mass in quenched and full QCD using as experimental input the K meson mass (left figure) or the ϕ meson mass (right figure). Lines are from combined linear continuum extrapolations.

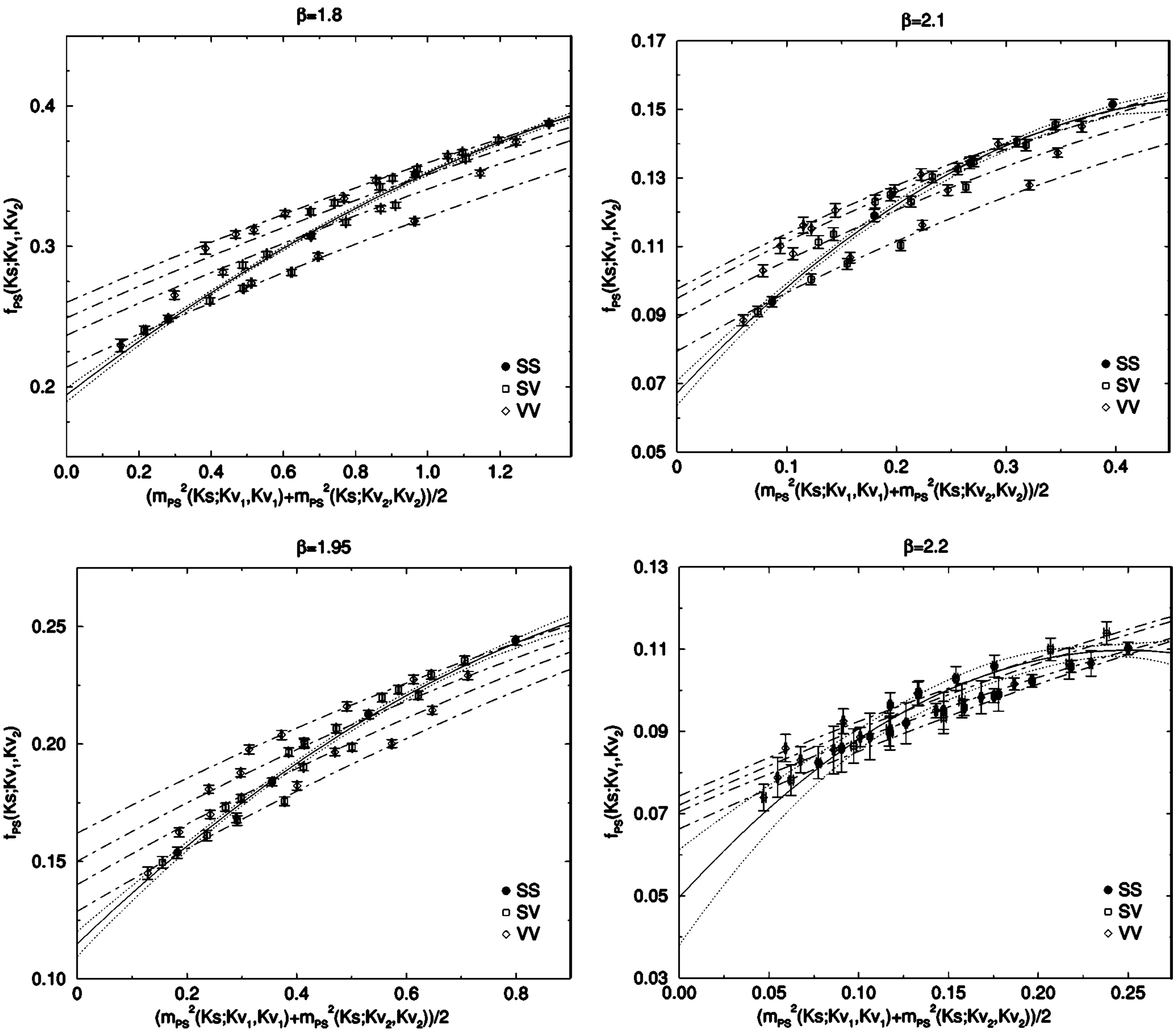

FIG. 33. Chiral extrapolations of pseudoscalar decay constants in two-flavor QCD.

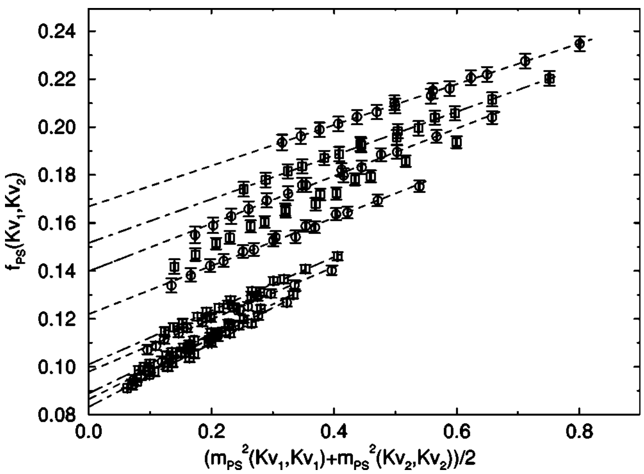

FIG. 34. Chiral extrapolations of pseudoscalar decay constants in quenched QCD.

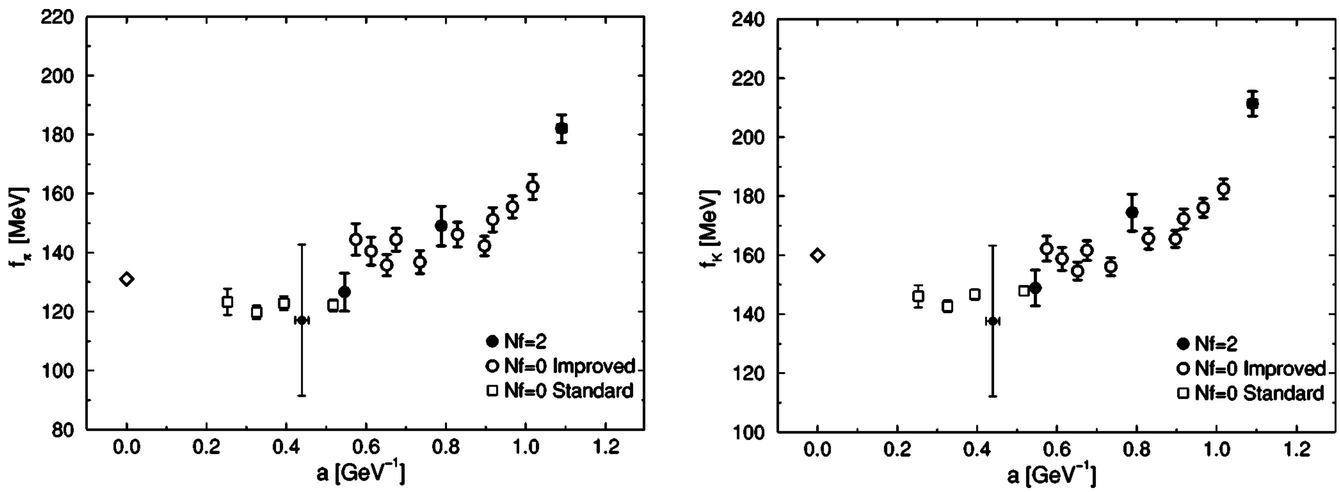

FIG. 35. Lattice spacing dependence of pseudoscalar decay constants f_{π} and f_K in full QCD (filled circles) and quenched QCD with improved actions (large open circles) or standard action (small open squares). The strange quark mass used in the calculation of f_K is fixed with the K meson mass as input.

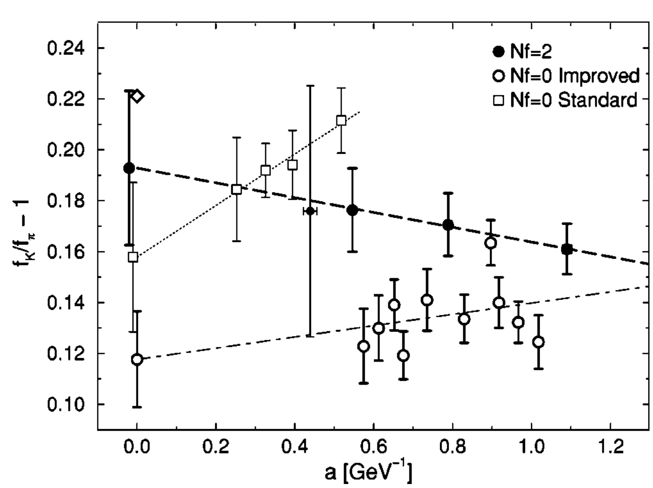

FIG. 36. Comparison of $f_K/f_{\pi}-1$ in full and quenched QCD. Fit lines are linear for all data.

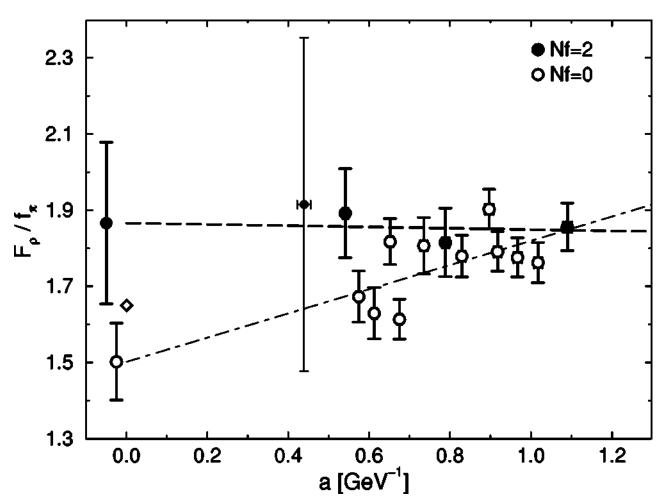

FIG. 40. Ratio of pseudoscalar and vector decay constants in full and quenched QCD.