Does the QCD plasma contain gluons?

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A comparison of two appropriately chosen screening masses of color singlet operators in the pure glue QCD plasma indicates that at sufficiently high temperature it contains a weakly interacting massive quasiparticle with the quantum numbers of the electric gluon. Still in the deconfined phase, but closer to T_c, the same mass ratio is similar to that at zero temperature, indicating that the propagating modes are more glueball-like, albeit with a lower scale for the masses. We observe a continuity between these two regimes.

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With the BNL Relativistic Heavy Ion Collider (RHIC) fully operational and busy taking data, certain questions about the treatment of the QCD plasma have become urgent. One of the most basic is about the modes of excitation in a plasma: does it contain weakly interacting quark and gluon quasiparticles, or some more complicated collective excitations? We show here that lattice computations yield detailed answers to this question.

Perturbation theory starts with the assumption that gluons are the propagating modes in the plasma, and predicts the Debye screening mass m_D in pure gauge QCD, at temperatures $T>T_c$, as an expansion in the strong coupling g of the form

$$\frac{m_D}{T}=g-\frac{3}{4\pi}g^2\log g+bg^2+cg^3+\mathcal{O}(g^4) \qquad (1)$$

where the leading term g is well known, the second term has been extracted in perturbation theory [1] and the nonperturbative coefficients b and c have been computed in lattice simulations of dimensionally reduced QCD [2]. Since $g=\mathcal{O}(1)$ for $T/T_c\simeq 3$ [3], the error due to the neglect of the g^4 term is about $35d\%$, where d is the coefficient of this term. As a result, it becomes difficult to validate perturbation theory purely by comparing this screening mass to lattice data [4].

We examine the modes of excitations by comparing screening masses m obtained from correlations of two gauge invariant operators with different symmetry properties. They are chosen in such a way that one would be obtained by the exchange of two electric gluons while the other would need three, if indeed such gluons are the lightest excitations in the plasma. As a result, the two screening masses would be roughly in the ratio 3/2. For $T\geqslant 1.5T_c$ we do see such a pattern, which validates perturbation theory, for screening masses. For electric gluons, therefore, the situation is similar to that in the quark sector, where early observations of departures from perturbation theory [5] are now seen to be lattice artifacts, and the continuum limit of the fermion bilinear screening masses are consistent with weakly coupled fermions [6].

Since screening masses involve the transfer matrix in a spatial direction, they are classified by the symmetry group of the lattice sliced perpendicular to a spatial direction [7–9]. For the thermodynamics of a $(3+1)$-dimensional field theory realized on a hypercubic Euclidean lattice, since the Euclidean time direction is distinguished from the spatial directions, this is the group $D_4\times Z_2(T)\times Z_2(C)$, where D_4 is the tetragonal group, $Z_2(C)$ is the charge-conjugation symmetry of the fields, and the $Z_2(T)$ factor arises from the symmetry $t\leftrightarrow -t$ [7,8]. The transfer matrix can be block diagonalized in irreps of this group.

Extensive thermodynamic quantities depend only on the lowest eigenvalue of the transfer matrix, and the phase structure is determined by the degeneracies and symmetries of the corresponding eigenvectors. In high temperature QCD, these are the scalar, A_1^{++} irrep. In the confined phase, the unique ground state is scalar under the Z_3 symmetry of the center of the color $SU(3)$ group. In the deconfined phase the ground state is threefold degenerate and corresponds to the three irreps of Z_3. On any finite lattice the degeneracy is lifted by an exponentially small quantity due to tunnelings between these states. In the A_1^{++} sector, therefore, an unphysical small "tunneling" mass, m_T, may dominate the screening. We show later that we can control this and obtain the correct physical screening mass.

Since the link variable, $U\sim\exp[\int dx\, igA]\sim\exp[iagA]$ (here a is the lattice spacing), one can turn this around and define a lattice gluon field of momentum k by the relation

$$A_\mu(k)=i\sum_x e^{ik\cdot x}[U_\mu(x)-U_\mu^\dagger(x)-\mathrm{Im\,Tr}\,U_\mu(x)], \quad (2)$$

where the sum is over all lattice sites in a slice, the components of k run over the set $2\pi l/N$ where $1\leqslant l\leqslant N$ and N is N_t for the temporal momenta and N_s for the spatial momenta (we assume an $N_t\times N_s^3$ lattice). This definition gives an element of the $SU(N)$ algebra which goes over into the continuum definition of the color octet gluon field in the limit of

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zero lattice spacing. The electric gluon $A_{\hat{t}}(0)$ is in the A_2^{--} irrep, Since $A_2^{--}\otimes A_2^{--}=A_1^{++}$ and $A_2^{--}\otimes A_2^{--}\otimes A_2^{--}=A_2^{--}$, screening correlators in the color singlet A_1^{++} and A_2^{--} sectors would be dominated by two and three electric gluon exchange respectively. We discuss the influence of the magnetic sector later.

In this work we use two classes of A_1^{++} operators—Wilson loops, specifically several linear combinations of the plaquette and the planar 6-link loop (sometimes called the fenster), and the trace of the real part of the Wilson line. For the A_2^{--} operator we use the imaginary part of the trace of the Wilson line. The zero momentum projection is obtained as usual by summing over all sites in the slice. Since the Wilson lines are nontrivial under Z_3 transformations, they are evaluated in the phase where the expectation value is real, by a global Z_3 rotation if necessary [9,10].

Due to the Z_3 symmetry of the vacua, there are only two tunneling masses of relevance—one in the vicinity of T_c due to tunneling between the disordered state and any of the ordered states, and the other for all $T\geqslant T_c$ due to tunneling between any two of the ordered states (the latter are relevant only to operators which are non-trivial under Z_3). Each mass has very specific dependence on the volume, $\overline{V}=a^3N_s^2N_t$, of a slice orthogonal to the direction of propagation

$$m_T(\overline{V})=\left(\frac{C}{\overline{V}^{\alpha}}\right)\exp(-\sigma\overline{V}), \qquad (3)$$

where C, α and σ are constants. In a d-dimensional scalar theory a one-loop computation gives $\alpha=d/2$ [11]. Tunneling arises when, on a finite system, simulations start exploring the non-Gaussian part of the free energy away from local minima. Clearly, an appropriate correlation function can give m_T only when the order parameter distribution shows multiple peaks. A consequence of Eq. (3) is that one can perform a finite size scaling study to check whether the lowest screening mass obtained is a tunneling mass.

Details of our runs with the pure gauge Wilson $SU(3)$ action are summarized in Table I. The critical coupling, β_c, and its shift on finite lattices, is known for $N_t=4$ with high precision [12,13]. Since the finite-size shift of β_c on the smallest lattice, $N_s=16$, is less than 2 parts in 10^3, the temperature scale is known with high precision at T_c. It is also known at similar precision at $1.5T_c$, $2T_c$ and $3T_c$ from measurements with $N_t=6$, 8 and 12. At other points the temperature scale is interpolated through the QCD beta function and has errors [3], which are indicated in the table. Loop operators are measured at five levels of single link fuzzing [8]. Cross correlations between all loops in the same irrep are measured and the lowest screening mass obtained by a variational procedure. Other details of measurements and analysis remain as in [8].

At $T=T_c$, the statistics collected are large enough that tunnelings between the deconfined and confined phases occur many times, as do those between different deconfined phases. As a result, it is only to be expected that the lowest screening mass that we can extract in the A_1^{++} sector is the tunneling mass. Evidence for this is the good fit to Eq. (3) shown in Fig. 1; the fit gave $\chi^2=1.005$ per degree of freedom.

To stabilize the pure phases we move away from T_c. The distance from criticality, $\Delta\beta=|\beta-\beta_c|$, needed to remain in a single phase for arbitrarily long runs depends on N_s: $\Delta\beta$ can decrease exponentially with N_s. In the confined phase we have estimated $m(T_c)$ by a measurement with $\Delta\beta=0.0125$, corresponding to a $3\pm1\%$ shift of T below T_c. A

TABLE I. For these runs on $N_t=4$ lattices, measurements were taken every 5th sweep, where every sweep was done with a 3 hit pseudo heat-bath (except for the runs at T_c, where measurements were taken every 10th sweep, each with a 6 hit pseudo heat-bath). In each case, an initial 1000–3000 sweeps were discarded for thermalization, and masses were extracted by a jack-knife analysis with 100 bins.

N_s	β	T/T_c	Statistics
24	5.6500	0.89 (1)	11370
24	5.6800	0.97 (1)	12080
16	5.6908	1.00	20000
20	5.6918	1.00	20000
24	5.6920	1.00	20000
16	5.7010	1.02 (1)	10000
20	5.7010	1.02 (1)	10000
24	5.7010	1.02 (1)	20020
24	5.7100	1.04 (1)	10160
24	5.7200	1.07 (1)	18010
24	5.8000	1.27 (2)	32220
24	5.8941	1.50	15130
24	6.0625	2.00	31350
24	6.3500	3.00	31630

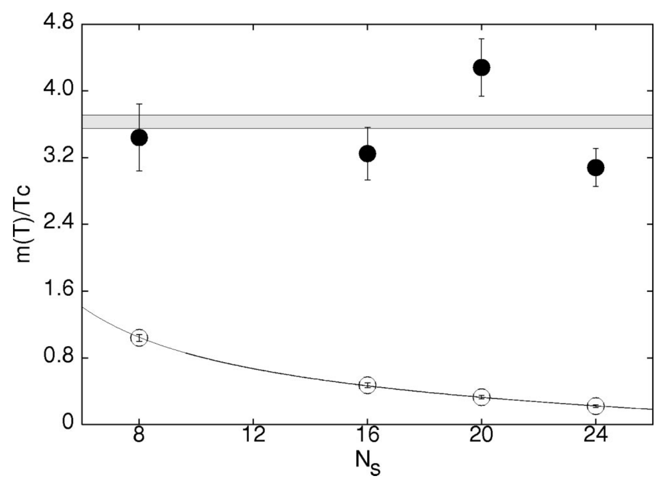

FIG. 1. A_1^{++} screening masses at T_c and their dependence on the lattice size. The lowest mass (unfilled circles) is a tunneling mass, as evidenced by the good fit to the form in Eq. (3). Also shown are estimates of the second variational mass (filled circles) and a comparison with a measurement of the lowest mass at $T/T_c=0.97$ on a 4×24^3 lattice (horizontal band). Data for $N_s=8$ is from [8].

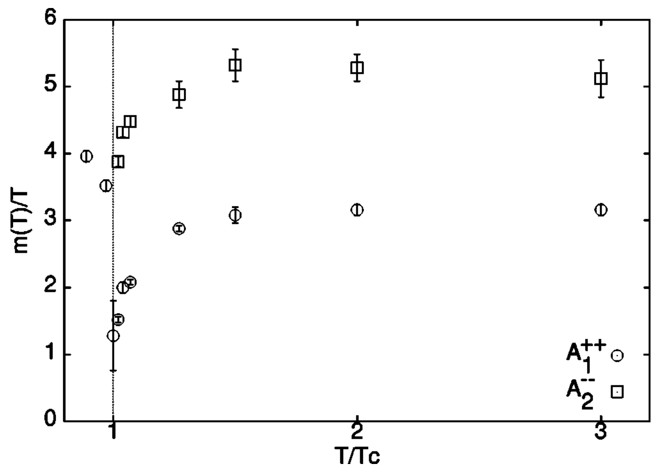

FIG. 2. A_1^{++} and A_2^{--} screening masses as a function of temperature on 4×24^3 lattices. Note the discontinuity in the A_1^{++} screening mass at T_c.

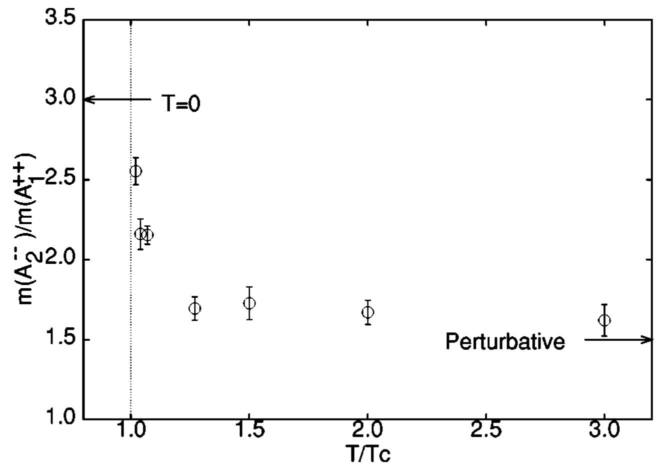

FIG. 3. Ratio of the screening masses in the A_2^{--} and A_1^{++} sectors as a function of temperature.

4×24^3 lattice was observed to stay in the confined phase throughout the run. The second variational level was seen to correspond to the screening mass in this phase. Our measurement of the A_1^{++} screening mass at $T/T_c=0.97$ gives $m(T)/T_c=3.41\pm 0.08$ [at $T/T_c=0.89$ we have $m(T)/T_c=3.52\pm 0.07$]. In comparison, a zero temperature measurement of the scalar glueball mass at a similar lattice spacing gives $m(T=0)/T_c=3.93\pm 0.05$ [14]. At $T=0$, glueball masses depend strongly on the lattice spacing, whereas mass ratios are less sensitive. In this phase, we may then expect that the ratio $m(T)/m(T=0)$ may scale better than the ratio $m(T)/T$ as the lattice spacing goes to zero. Our measurements show that $m(T)/m(T=0)\sim 0.8$ in the scalar channel near T_c, in agreement with recent measurements from temporal correlators [15].

Another method of extracting the physical correlation length is to pick out the configurations in which the whole lattice is in a single phase. We do this through the distribution of the action density, which has peaks corresponding to each of the phases. By restricting measurements of correlation functions to configurations with some $S>S_{cut}$, suitably chosen, we can isolate the deconfined phase and measure the physical screening mass. This procedure on the largest lattice gave $m(T_c)/T_c\approx 1$ in the deconfined phase. However with increasing S_{cut} statistics become poorer; consequently, errors increase rapidly, and it is hard to quote a more precise value.

We worked in the deconfined phase by taking $\Delta\beta \geqslant 0.0085$ for $N_s=16$, 20 and 24, corresponding to moving off from T_c by $2\pm 1\%$ or more. None of our $T>T_c$ runs showed any tunnelings between the different ordered states except the run at $T/T_c=1.02$ on the 4×16^3 lattice, where we found one tunneling event between two deconfined vacua (which forced us to use only the correlations of loop operators here). Within the precision of our measurement, the screening masses at $T/T_c=1.02$ are independent of the lattice size: an indication that they are not tunneling masses. For all our measurements of the A_1^{++}, shown in Fig. 2, the masses extracted from the Wilson loops and those from the Wilson line agree at the 95% confidence level.

The rough agreement between the measurement at $1.02T_c$ and that at T_c in the deconfined phase, displayed in Fig. 2, should also be noted. We also draw attention to the feature that the screening masses, when expressed in units of T, dip near T_c. If the dip persists, then it cannot be understood in the context of the perturbation theory leading to Eq. (1). An attempt to capture this effect in a model has been made in [17]. We plan a more detailed study in both phases of the region near T_c.

In [4] screening masses have been extracted from the free energy change due to the addition of a static quark pair in the plasma, i.e., the logarithm of the point-to-point Wilson-line correlation. Our measurements of $m(T)/T$ for the scalar in the deconfined phase are completely compatible with their results. This agreement is nontrivial since we extract the screening mass from zero-momentum screening correlators.

Our results for the A_2^{--} screening mass are also shown in Fig. 2 [16]. These are extracted from effective mass plateaus over the range $1\leqslant az\leqslant 5$. In Fig. 3 we have displayed the same data in the form of the ratio of the two screening masses. Clearly, already at temperatures a little above $1.25T_c$, the ratio is close to the perturbative value of 3/2, approaching it from above. At $3T_c$ perturbation theory is supported at a confidence level of better than 95%. It has been shown earlier [8] that no mass assignment for magnetic gluons simultaneously satisfies the B_1^{++} and B_2^{++} screening masses, both of which should arise due to the exchange of two such gluons. If these color singlet channels are treated as collective excitations, then their large screening mass prevents them from contributing through a pair exchange to the A_1^{++} correlators. Even if we choose to disregard this argument, and insist on the existence of magnetic gluons, their screening mass is very high (in this temperature range) simply because of the large screening mass of the B_1^{++}. The simple fact that the A_1^{++} has the smallest screening mass protects the argument of electric gluon exchange when the ratio $m(A_2^{--})/m(A_1^{++})$ is seen to be 3/2 as in Fig. 3.

We have not extended our measurement of the A_2^{--} screening mass below T_c since previous studies have shown that in the confined phase the degeneracies of the screening mass are those expected from the $T=0$ symmetries of the transfer matrix [7]. At $T=0$ the A_2^{--} is just one component of a very heavy vector glueball (T_1^{--} in the usual $T=0$ notation), and the measurement of its screening mass would require large numbers of operators and a significantly larger data set. It is interesting, though, that $T=0$ measurements, made over a broad range of lattice spacings, show that the ratio of this vector mass to the scalar is roughly 3 [18]. The ratio $m(A_1^{++})/m(A_2^{--})$ for $T>T_c$ seems to interpolate smoothly between the $T=0$ and the infinite temperature values, crossing over rapidly from one regime to another in the temperature range between T_c and $1.25T_c$.

It is interesting to recall that the entropy density of the plasma is relatively low for $T/T_c<1.1$, and begins to saturate only for $T/T_c>1.25$ [13]. While thermodynamics is more sensitive to short distance modes and screening to long-distance modes in the plasma, the two observations can nevertheless be due to a single cause, if the partition function for $T/T_c<1.1$ were essentially saturated by the color singlet modes that we have seen. At sufficiently high temperatures, the propagating electric gluons can contribute significantly to the entropy [19].

In summary, we have shown that the screening in the scalar (A_1^{++}) sector is discontinuous across the pure QCD transition at T_c and falls abruptly by a factor of nearly 3 in going from the confined to the deconfined phase. At larger temperatures, this screening mass rises faster than linearly in T. Above T_c, the ratio of the A_2^{--} to the scalar screening mass interpolates smoothly from a value close to the $T=0$ ratio to the value 3/2 expected when these correlations are saturated by electric gluon exchange. The agreement with the latter value for $T>1.25T_c$ indicates the presence of weakly coupled, massive quasiparticles with the quantum numbers of the electric gluon as the lightest excitations of the QCD plasma. Closer to T_c the mass ratio is more similar to its $T=0$ value. Taken together with the smallness of the entropy density, this indicates that excitations are more nearly glueball-like.

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[1] A.K. Rebhan, Nucl. Phys. **B430**, 319 (1994).

[2] K. Kajantie *et al.*, Phys. Rev. Lett. **79**, 3130 (1997); M. Laine and O. Philipsen, Phys. Lett. B **459**, 259 (1999).

[3] S. Gupta, Phys. Rev. D **64**, 034507 (2001).

[4] O. Kaczmarek *et al.*, Phys. Rev. D **62**, 034021 (2000).

[5] C. DeTar and J.B. Kogut, Phys. Rev. Lett. **59**, 399 (1987); A. Gocksch *et al.*, Phys. Lett. B **205**, 334 (1988); K.D. Born *et al.*, Phys. Rev. Lett. **67**, 302 (1991); R.V. Gavai *et al.*, Phys. Rev. D **65**, 054506 (2002); M. Asakawa *et al.*, Prog. Part. Nucl. Phys. **46**, 459 (2001); I. Wetzorke *et al.*, Nucl. Phys. B (Proc. Suppl.) **106**, 510 (2002).

[6] R. V. Gavai and S. Gupta, Phys. Rev. D **67**, 034501 (2003).

[7] B. Grossman *et al.*, Nucl. Phys. **B417**, 289 (1994).

[8] S. Datta and S. Gupta, Nucl. Phys. **B534**, 392 (1998); Phys. Lett. B **471**, 382 (2000).

[9] P. Arnold and L. Yaffe, Phys. Rev. D **52**, 7208 (1995).

[10] We thank Sanatan Digal for this remark.

[11] E. Brezin and J. Zinn-Justin, Nucl. Phys. **B257**, 867 (1985).

[12] Y. Iwasaki *et al.*, Phys. Rev. Lett. **67**, 3343 (1991).

[13] G. Boyd *et al.*, Nucl. Phys. **B469**, 419 (1996).

[14] H. Chen *et al.*, Nucl. Phys. B (Proc. Suppl.) **34**, 357 (1994).

[15] N. Ishii *et al.*, Phys. Rev. D **66**, 014507 (2002).

[16] At $2T_c$ we have $m(A_2^{--})/T=5.28\pm0.20$, which compares well with the value $(5.0\pm0.1)T$ quoted for the same irrep (0_-^{+-} in the continuum notation) computed in a dimensionally reduced theory at the same temperature in A. Hart *et al.*, Nucl. Phys. **B586**, 443 (2000). The results for $m(A_1^{++})$ also agree.

[17] R.D. Pisarski, Phys. Rev. D **62**, 111501 (2000); A. Dumitru and R.D. Pisarski, Phys. Lett. B **525**, 95 (2002).

[18] C.J. Morningstar and M.J. Peardon, Phys. Rev. D **60**, 034509 (1999).

[19] See for example, J.O. Andersen *et al.*, Phys. Rev. Lett. **83**, 2139 (1999); J.-P. Blaizot *et al.*, *ibid.* **83**, 2906 (1999).