

Semileptonic and nonleptonic B decays to three charm quarks: $B \rightarrow J/\psi(\eta_c) D l \nu_l$ and $J/\psi(\eta_c) D \pi$

Gad Eilam, Massimo Ladisa, and Ya-Dong Yang

Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel

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We evaluate the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \pi^-$ nonleptonic decay in a phase space region where the pion factors out. This decay might provide a significant background to the decay process $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^{*0}$, followed by $D^{*0} \rightarrow D^+ \pi^-$, with a slightly off-shell D^{*0} . The relevant form factors describing the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell^- \bar{\nu}_\ell$ semileptonic decay, required for the calculations, have been evaluated within the framework of a QCD relativistic potential model. We estimate the branching ratio for the nonleptonic channels, finding $\mathcal{BR}(\overline{B}^0 \rightarrow J/\psi D^+ \pi^-) = 3.1 \times 10^{-8}$ and $\mathcal{BR}(\overline{B}^0 \rightarrow \eta_c D^+ \pi^-) = 3.5 \times 10^{-8}$. As a by-product of our calculations, we also quote the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \nu_\ell$ semileptonic decay branching ratio: $\mathcal{BR}(\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \nu_\ell) \approx 10^{-13}$.

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In this article we study the \overline{B}^0 meson decays

$$\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell^- \bar{\nu}_\ell \quad (1)$$

$$\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \pi^- \quad (2)$$

These decays which may be classified as “very rare,” at least in the standard model with mesons built solely of quark-antiquark pairs, should be searched for in present and future programs at B factories. As a possible motivation, consider preliminary studies of the inclusive $B \rightarrow J/\psi X$ spectrum, indicating a slow J/ψ hump [1,2], which kinematically corresponds to an invariant mass $m_X \approx 2$ GeV. Some hypotheses have been already suggested in order to account for such a phenomenon: in [3] the $\overline{B}^0 \rightarrow J/\psi \Lambda \bar{p}$ decay is followed by the resonant $\Lambda - \bar{p}$ bound state, whereas in [4] a possible explanation is the intrinsic charm content of the B meson. In the latter case the decay proceeds through the $\overline{B}^0(b\bar{d}(\bar{c}c)_{\text{slow}}) \rightarrow J/\psi_{\text{slow}} D^{*0}$ channel and it accounts for a $\mathcal{BR} \approx 10^{-4}$ provided that the intrinsic charm content fraction in the incoming B meson, is at least 1%.

To corroborate the hypotheses in [3,4], it is worth estimating the mechanisms for these decays in the framework of conventional heavy mesons picture as precisely as we can. In [5] the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^{(*)0}$ decays have been calculated in perturbative QCD; the branching ratios for these decays are estimated around $10^{-7} - 10^{-8}$ and, therefore, too small to account for the Belle and CLEO data. Moreover in [5] the possibility of production of a hybrid $s\bar{d}g$ meson with mass around 2 GeV is briefly discussed and, although the calculation of such a decay is difficult, a decay rate $10^3 \approx 10^4$ larger than the conventional mechanism for the \mathcal{BR} is expected.

The nonresonant decay mode $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \pi^-$ would be interesting to analyze in this context, as it might provide a significant background to the decay process $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^{*0}$, followed by $D^{*0} \rightarrow D^+ \pi^-$ with a slightly off-shell D^{*0} . It is difficult to calculate this background reliably, since precise calculations of the latter one are not available yet. In Ref. [5] an approximate evaluation of the main channel branching ratio has been provided. This leads

us to estimate that the contribution of the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \pi^-$ background channel is around 25%–35%. While a calculation from first principles is not available at the moment, a useful approximation might be the factorization approximation [6] and, within this framework, the decay modes (1) would provide the crucial hadronic matrix elements needed to compute the relevant amplitudes. We stress that no method for calculating the above nonleptonic rates from first principles is nowadays available and one usually employs some approximations to evaluate such decay modes. Among all, the so-called “factorization” has been extensively studied and applied in several papers. In that respect, the evaluation of the relevant form factors for the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \nu_\ell$ semileptonic decay is the first step of the approximation. The $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \nu_\ell$ branching ratio is a by-product of our calculations and we have quoted it for the sake of completeness although it is very small and experimentally irrelevant. From a theoretical standpoint, semileptonic B -meson decays with two hadrons in the final state represent a formidable challenge as they involve hadronic matrix elements of weak currents with three hadrons. They can be studied by pole diagrams, which amounts to a simplification because only two hadrons are involved in the hadronic matrix elements. This is the approach followed in some papers where these decays have been examined in the framework of the chiral perturbation theories for heavy meson decays [7,8]. This method has been successfully applied to decays with light mesons in the final state and it is based on an effective theory implementing both heavy-quark and chiral symmetry [8–11]. This method allows to achieve, for systems comprising of both heavy (Q) and light (q) quarks, rigorous results in the combined $m_Q \rightarrow \infty$, $m_q \rightarrow 0$ limit. However the range of validity of this approach is limited by the requirement of soft pion momenta and it has been never applied, to our knowledge, to a final state with three charm quarks. The aim of this article is to examine the decays (1) in the framework of a QCD relativistic potential model [12] and to extend the kinematical range where theoretical predictions are possible. We shall present a detailed analysis of the form factors relevant for Eq. (1). Subsequently, the decays (2) will be considered. We do not include final state interactions in

our calculation since no consistent way to compute them is presently available. It is clear that they can modify our numerical results [13].

In three recent papers [14–16] an analysis of some semi-leptonic and rare B -meson decays into one and two light hadrons has been presented; it employs the relativistic potential model in an approximation that renders the calculations simpler. We wish to exploit here this approximation in the study of the $B \rightarrow J/\psi(\eta_c) D \ell \nu$ decay.

Let us start with a description of the model (for more details see [12,14–16]). In this approach the mesons are described as bound states of constituent quarks and antiquarks tied by an instantaneous potential $V(r)$, which has a confining linear behavior at large interquark distances r and a Coulombic behavior $\simeq -\alpha_s(r)/r$ at small distances, with $\alpha_s(r)$ the running strong coupling constant (the Richardson's potential [17] is used to interpolate between the two regions). Due to the nature of the interquark forces, the light quarks are relativistic; for this reason one employs for the meson wave function Ψ the Salpeter [18] equation embodying the relativistic kinematics:

$$[\sqrt{-\nabla_1^2 + m_1^2} + \sqrt{-\nabla_2^2 + m_2^2} + V(r)]\Psi(\vec{r}) = M\Psi(\vec{r}), \quad (3)$$

where the index 1 refers to the heavy quark and the index 2 to the light antiquark; M is the heavy meson mass that is obtained by fitting the various parameters of the model, in particular the heavy quark masses which are fitted to the values $m_c = 1452$ MeV, $m_b = 4890$ MeV, and the light quark masses $m_u \simeq m_d = 38$ MeV, $m_s = 115$ MeV. The heavy meson wave function $\Psi_H(\vec{r})$ in its rest frame is obtained by solving Eq. (3) (H =heavy bound state); a useful representation in Fourier momentum space was obtained in [14] and is as follows

$$\psi(k) = 4\pi\sqrt{M\alpha^3} e^{-\alpha k}, \quad (4)$$

with $\alpha = 2.4$ (1.6) GeV^{-1} for $B, D(J/\psi, \eta_c)$ mesons and $k = |\vec{k}|$ the quark momentum in the heavy meson rest frame; this is the first approximation introduced in [14]. At this point we would like to comment that the spectrum obtained depends only weakly on the light quarks masses. We will exploit this fact later when we employ $m_d = \Lambda_{\text{QCD}}$.

The constituent quark picture used in the model is rather crude. There are no propagating gluons in the instantaneous approximation, i.e., the Coulombic interaction is assumed to be static. Moreover, the complex structure of the hadronic vacuum is simplified: confinement can be introduced by the linearly rising potential at large distances, but the chiral symmetry and the Nambu-Goldstone boson nature of the π 's cannot be implemented by the constituent quark picture. For these reasons, while there are good reasons to believe that Eq. (3) may describe the quark distribution inside the heavy meson, one cannot pretend to apply it to light mesons. Therefore pion couplings to the quark degrees of freedom are described by effective vertices (for more details see [14–16]).

To evaluate the amplitude for semileptonic decays, it is useful to follow some simple rules, similar to the Feynman rules by which the amplitudes are computed in perturbative

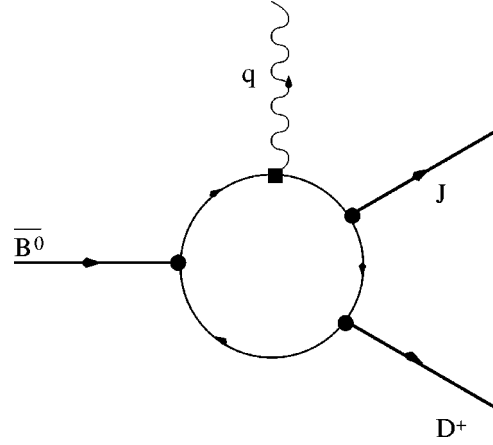


FIG. 1. Feynman diagram for $\bar{B}^0 \rightarrow J D^+$ semileptonic decay. $J \equiv J/\psi, \eta_c$. The curly line represents the weak current; the external legs and the loop line represent, respectively, the on-shell particles and the constituent quarks of the model. The circles and the square represent, respectively, the effective strong and weak couplings.

field theory. The setting of these rules is the main innovation introduced in [14] as compared to [12]. For the decays (1) we draw a quark-meson diagram as in Fig. 1 and evaluate it according to the following rules:

(1) For the heavy meson H in the initial state one introduces the matrix:

$$\mathcal{H} = \frac{1}{\sqrt{3}} \psi_H(k) \sqrt{\frac{m_q m_Q}{m_q m_Q + q_1 \cdot q_2}} \frac{q_1 + m_Q}{2m_Q} \Gamma \frac{-q_2 + m_q}{2m_q} \quad (5)$$

where m_Q and m_q are the heavy and light quark masses, q_1^μ, q_2^μ their 4-momenta and $\Gamma = -i\gamma_5, (\not{\epsilon})$ for a $J^P = 0^-(1^-)$ heavy meson. The normalization factor corresponds to the normalization $\langle H|H \rangle = 2m_H$ and $\int [d^3k/(2\pi)^3] |\psi_H(k)|^2 = 2m_H$ already embodied in Eq. (5). One assumes that the 4-momentum is conserved at the vertex $H\bar{q}Q$, i.e. $q_1^\mu + q_2^\mu = p^\mu = H$ -meson 4-momentum. Therefore $q_1^\mu = (E_Q, \vec{k})$, $q_2^\mu = (E_q, -\vec{k})$ and

$$E_Q + E_q = m_H. \quad (6)$$

(2) For the heavy meson H in the final state one introduces the matrix:

$$-\gamma^0 \mathcal{H}^\dagger \gamma^0, \quad (7)$$

where \mathcal{H} is as defined in Eq. (5).

(3) To take into account the off-shell effects due to the quarks interacting in the meson, one introduces running quark mass $m_Q(k)$, to enforce the condition

$$E = \sqrt{m^2(k) + |\vec{k}|^2} \quad (8)$$

for the constituent quarks.¹

¹By this choice, the average $\langle m_Q(k) \rangle$ does not differ significantly from the value m_Q fitted from the spectrum, see [14] for details.

(4) The condition $m_Q^2 \geq 0$ implies the constraint

$$0 \leq k \leq k_{max}, \quad (9)$$

on the integration over the loop momentum k , where k_{max} actually depends on the kinematics of the process²

$$\int \frac{d^3 k}{(2\pi)^3}. \quad (10)$$

(5) For the weak hadronic current one puts the factor

$$N_q N_{q'} \gamma^\mu (1 - \gamma_5). \quad (11)$$

The normalization factor N_q is as follows:

$$N_q = \begin{cases} \sqrt{\frac{m_q}{E_q}} & (\text{if } q = \text{constituent quark}), \\ 1 & (\text{otherwise}). \end{cases} \quad (12)$$

(6) Finally the amplitude must contain a color factor of 3 and a trace over Dirac matrices.

This set of rules can now be applied to the evaluation of the hadronic matrix element for the decays (1), corresponding to the diagram in Fig. 1; the result is

$$\begin{aligned} \mathcal{A}^\mu = & \langle D^+(p_D) J | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0(p_B) \rangle = -\sqrt{3} \int \frac{d^3 k}{(2\pi)^3} \mathcal{N} \psi_B(k) \psi_J^*(k) \psi_D^*(k) \theta[k_{max} - k] \\ & \times \text{Tr} \left[\frac{\not{q}_b + m_b}{2} \frac{\not{q}_d + m_d}{2} \frac{\not{q}_b - \not{q} - \not{p}_J + m_{c'}}{2} \Gamma \frac{-\not{q}_b + \not{q} + m_c}{2} \gamma^\mu (1 - \gamma_5) \right] \end{aligned} \quad (13)$$

where $\Gamma = \not{\epsilon}^*$ ($-i\gamma_5$) and $J \equiv J/\psi(p_J, \epsilon^*)[\eta_c(p_\eta)]$ for the outgoing J/ψ (η_c) meson. Note the appearance of $m_{c'} \neq m_c$ resulting from Eq. (8). The factor \mathcal{N} in Eq. (13) accounts for the normalization of the hadronic current and of the heavy mesons:

$$\begin{aligned} \mathcal{N} = & \sqrt{\frac{m_c m_b}{E_b(E_b - q^0)}} \sqrt{\frac{m_d m_b}{E_d m_B + m_d(m_b - m_d)}} \sqrt{\frac{m_d m_{c'}}{m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)}} \\ & \times \sqrt{\frac{m_c m_{c'}}{m_B^2 + q^2 - 2m_c m_{c'} + 2m_d^2 + m_D^2 - m_J^2 - 2(m_B q^0 - E_d(E_J - 2m_B + 2q^0))}}. \end{aligned} \quad (14)$$

We introduce the various form factors for the $B \rightarrow J/\psi D$ semileptonic decay:

$$\begin{aligned} & \langle D^+(p_D) J/\psi(p_J, \epsilon^*) | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0(p_B) \rangle \\ & = i \epsilon_\nu^* \left\{ \left[m_B^2 C_1 g^{\mu\nu} + p_B^\mu (C_2 p_B^\nu + C_3 q^\nu) + q^\mu (C_4 p_B^\nu + C_5 q^\nu) + p_J^\mu (C_6 p_B^\nu + C_7 q^\nu) \right] \right. \\ & \quad + \epsilon^{\nu\mu\alpha\beta} (D_1 p_{J\alpha} q_\beta + D_2 p_{J\alpha} p_{B\beta} + D_3 q_\alpha p_{B\beta}) + \frac{1}{m_B^2} \epsilon^{\mu\sigma\alpha\beta} p_{B\sigma} p_{J\alpha} q_\beta (D_4 p_B^\nu + D_5 q^\nu) \\ & \quad \left. + \frac{1}{m_B^2} \epsilon^{\nu\sigma\alpha\beta} p_{B\sigma} p_{J\alpha} q_\beta (D_6 p_B^\mu + D_7 p_J^\mu + D_8 q^\mu) \right\}, \end{aligned} \quad (15)$$

where $q = p_B - p_D - p_J$. Following [7] we introduce also the form factors for the $B \rightarrow \eta_c D$ semileptonic decay as follows:

$$\langle D^+(p_D) \eta_c(p_\eta) | \bar{c} \gamma^\mu (1 - \gamma_5) b | \bar{B}^0(p_B) \rangle = i w_+ (p_D + p_\eta)^\mu + i w_- (p_\eta - p_D)^\mu + i r q^\mu + 2h \epsilon^{\mu\alpha\beta\delta} p_\alpha p_\eta p_{D\delta}. \quad (16)$$

It is useful to introduce the following variables:

$$s = (p_D + p_J)^2, \quad t = (p - p_J)^2, \quad u = (p - p_D)^2,$$

²For the processes induced by the $b \rightarrow u$ current, for instance, $k_{max} \simeq m_B/2$ (see [14–16]). Here $k_{max} \simeq m_D/2$.

that satisfy

$$s + t + u = q^2 + m_B^2 + m_D^2 + m_J^2. \quad (17)$$

The form factors w_{\pm} , r , h , C_i , D_j ($i=1, \dots, 7$; $j=1, \dots, 8$) are functions of three independent variables. One can choose as independent variables s , q^2 , t or, alternatively, s , q^2 , E_J , where E_J is the $J/\psi(\eta_c)$ energy in the B -meson rest frame. The relations between the two sets of invariants are

$$\begin{aligned} t &= m_B^2 + m_J^2 - 2m_B E_J \\ q^2 &= s + m_B^2 - 2m_B(E_D + E_J). \end{aligned} \quad (18)$$

The kinematical range is as follows:

$$\begin{aligned} m_D^2 &\leq t \leq (m_B - m_J)^2 \\ 0 &\leq q^2 \leq (m_B - m_J - \sqrt{t})^2 \\ s_{min} &\leq s \leq s_{max}, \end{aligned} \quad (19)$$

where

$$\begin{aligned} s_{min/max} &= \frac{1}{4t} [(m_B^2 + m_D^2 - m_J^2 - q^2)^2 - (\sqrt{\lambda(t, m_J^2, m_B^2)} \\ &\quad \pm \sqrt{\lambda(m_D^2, t, q^2)})^2], \\ \lambda(x, y, z) &= x^2 + y^2 + z^2 - 2(xy + yz + xz). \end{aligned} \quad (20)$$

From Eq. (13) one can extract the different form factors by multiplying \mathcal{A}^μ by appropriate momenta (see the Appendix for explicit expressions of all the form factors).

The calculation of the trace in Eq. (13) is straightforward and is similar to those performed in [14–16] for similar processes. The evaluation of the integral is even simpler, because, although the three-body decay kinematics is rather involved, all the quarks in the loop of Fig. 1 are constituent and there is no light quark propagating. The integration can be performed numerically but, unlike the semileptonic decays with two pions in the final state [16], here the loop integration domain is not genuinely three-dimensional, due to the smallness of phase space. In fact, the calculation becomes simpler by inserting the wave functions $\psi_H(k)$ ($H = D, J/\psi, \eta_c$) as they are in the H -meson rest frame in the relevant formulas, which is an approximation we perform and is justified by the small value of the outgoing mesons' 4-momenta in the processes (1). This approximation has been already incorporated into the formulas of the Appendix.³

An important point to be stressed is the kinematical range in which the predictions of the present model are reliable. We cannot pretend to extend our analysis to very small meson momenta for the following reasons: first, as discussed in

³The form factors in Eqs. (15), (16) do not differ significantly from their actual value in the kinematical range (19) and within this approximation.

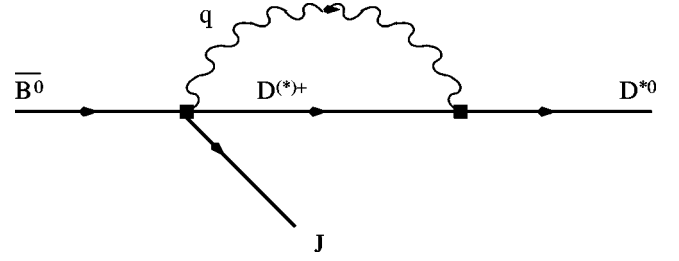


FIG. 2. Feynman diagram for $\bar{B}^0 \rightarrow JD^{*0}$ nonleptonic decay in the QCD relativistic potential model. $J \equiv J/\psi, \eta_c$.

[15,16], when the outgoing meson momenta are small, the results of the model become strongly dependent on a numerical input of our calculation, i.e. the value of the light quark mass m_d . The numerical value of m_d cannot be fixed adequately because the values of the quark masses were fitted from the heavy meson spectrum, which is not very sensitive to the light quark mass (for more details see [12]). Therefore the value of m_d has a theoretical uncertainty and the parameter m_d is the main source of error for the present calculation. We set the value of this parameter to the scale Λ_{QCD} .⁴

Moreover the role of pole diagrams such as those studied in [5] becomes relevant. These diagrams cannot be accounted for by the present scheme, which at most can be used to model a continuum of states, according to the quark-hadron duality ideas. The low-lying resonances, such as those studied in [5] should be added separately.⁵ We expect a large contribution from the D^* (see the discussion in [5]). It is worthwhile to stress that in the present model the resonant production of a pion occurs through the long distance contribution depicted in Fig. 2 with $D^{(*)+}$ mesons as intermediate states; all the diagrams are calculable in the heavy meson chiral Lagrangian [11] and they are found to be of the same order of those calculated in [5].

In principle the partial widths $\Gamma(\bar{B}^0 \rightarrow JD^+ \ell \bar{\nu}_\ell)$ can be used to extract the relevant $|V_{cb}|$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element: due to the smallness of the phase space, the theoretical predictions are not expected to strongly depend on the specific model employed to achieve the final results. We find $\mathcal{BR}(\bar{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \bar{\nu}_\ell) \simeq 10^{-13}$, which is of course unmeasurable in the foreseeable future.

Instead, we calculate the relevant formulas of the $\bar{B}^0 \rightarrow JD^+ \pi^-$ nonleptonic decays. This channel is a background to $\bar{B}^0 \rightarrow JD^{*0}$ followed by the (almost on-shell) decay D^{*0}

⁴In [16] the low-lying ρ resonance provides the model with a cutoff at small s in the $B \rightarrow \pi\pi$ semileptonic decay form factors; here such a *natural* hadronic scale is absent.

⁵This is the reason why in [15] the B^* pole of the $B \rightarrow \pi$ form factor is not reproduced in the $|\vec{p}_\pi| \rightarrow 0$ region. The same remark holds for [16] about the resonances encountered at small s in the $B \rightarrow \pi\pi$ semileptonic decay form factors, such as the ρ resonance.

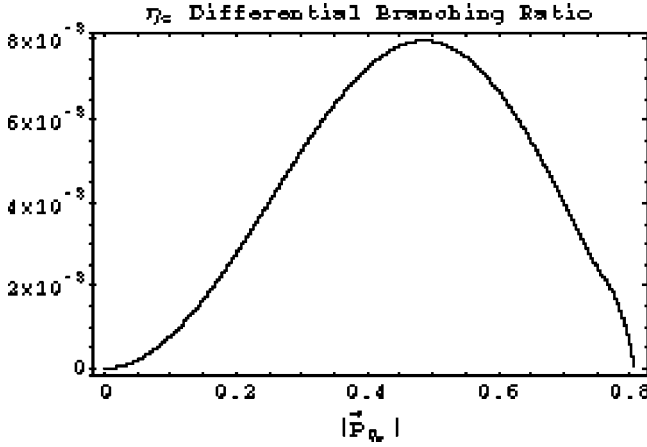


FIG. 3. Differential branching ratio, in GeV^{-1} , for $\overline{B}^0 \rightarrow \eta_c D^+ \pi^-$. The momentum is in GeV.

$D^{*0} \rightarrow D^+ \pi^-$. The relevant amplitudes follow from Eqs. (15) and (16). Numerically we get (for $m_d = 300$ MeV $\simeq \Lambda_{\text{QCD}}$):

$$\begin{aligned} \mathcal{BR}(\overline{B}^0 \rightarrow J/\psi D^+ \pi^-) &= \begin{cases} 1.98 \times 10^{-8} & \text{transverse polarization,} \\ 1.09 \times 10^{-8} & \text{longitudinal polarization,} \end{cases} \\ \mathcal{BR}(\overline{B}^0 \rightarrow \eta_c D^+ \pi^-) &= 3.54 \times 10^{-8}. \end{aligned} \quad (21)$$

It is also interesting to compute the differential branching ratios:

$$\frac{d\mathcal{BR}}{d|\vec{p}_J|} = \frac{\tau_{B^0} f_\pi^2 |V_{cb} V_{ud}^*|^2 G_F^2}{256 \pi^3 m_B^2} \frac{|\vec{p}_J|}{E_J} \int_{s_{\min}}^{s_{\max}} ds |q \mathcal{A}(t, q^2, s)|^2, \quad (22)$$

where $q^2 = m_\pi^2$, $f_\pi = 132$ MeV, $\tau_{B^0} = 1.6$ ps, $V_{cb} = 0.040$, $V_{ud} = 1 - \lambda^2/2$, $\lambda = 0.22$, $m_B = 5.279$ GeV, G_F is the Fermi constant and $s_{\min/\max}$ are as in Eq. (20). \mathcal{A}^μ is the relevant amplitude for the nonleptonic decay $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \pi^-$ given in Eq. (13). The differential branching ratios for the nonleptonic decays we have studied are plotted in Figs. 3 and 4.

We can therefore conclude that from an experimental point of view the B -meson nonleptonic decay channel with three charm quarks in the final state represents a (barely) interesting process. We have investigated the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \nu_\ell$ semileptonic decays and the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \pi^-$ nonresonant nonleptonic decay channels by using the factorization approximation and the $\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+$ semileptonic decay form factors. The calculation has been performed in the framework of a QCD relativistic potential model. The branching ratios for these decays are: $\mathcal{BR}(\overline{B}^0 \rightarrow J/\psi(\eta_c) D^+ \ell \nu_\ell) \simeq 10^{-13}$, $\mathcal{BR}(\overline{B}^0 \rightarrow J/\psi D^+ \pi^-) = 3.07 \times 10^{-8}$ and $\mathcal{BR}(\overline{B}^0 \rightarrow \eta_c D^+ \pi^-) = 3.54 \times 10^{-8}$. We can compare our previous paper (Ref. [5])

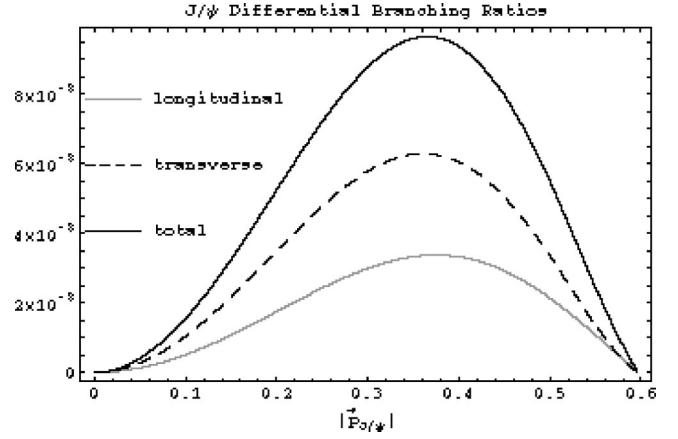


FIG. 4. Differential branching ratio, in GeV^{-1} , for $\overline{B}^0 \rightarrow J/\psi D^+ \pi^-$. The momentum is in GeV.

which discusses, among other reactions, $\overline{B}^0 \rightarrow J/\psi D^0$ employing a completely different method of analysis (basically, perturbative QCD). There, we obtained: $\mathcal{BR}(\overline{B}^0 \rightarrow J/\psi D^{(*)0}) \simeq 7 - 8 \times 10^{-8}$, to be compared with $\mathcal{BR}(\overline{B}^0 \rightarrow J/\psi D^+ \pi^-) \simeq 3 \times 10^{-8}$. Thus adding a pion reduces the rate significantly, in agreement with the effect expected from severe phase space suppression. We therefore believe that any reasonable method of analysis will lead to branching ratios of $\mathcal{O} = 1 \times 10^{-8}$ for the three body three charm quarks process. These theoretical findings provide us with an order of magnitude for those processes and, in that respect, they do not seem to account for the slow J/ψ hump as indicated by the preliminary results of CLEO and Belle Collaborations. Since the charmonium spectrum will be extensively studied at the B factories in the near future, it is important to confirm whether the slow J/ψ hump exists. In that respect, a refined measurement is needed. If the hump persists, it will be hard to find a consistent explanation within the conventional models. Thus a new scenario, like those discussed in [3–5], could be applicable.

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APPENDIX

From Eq. (13) one can extract the different form factors by multiplying \mathcal{A}^μ by appropriate momenta. One gets⁶

⁶ $D_j = 0$ ($j = 4, \dots, 8$) is a consequence of the $H\bar{q}Q$ ($H = B, D, J/\psi, \eta_c$) couplings introduced in our model and of the nature of the quarks involved (all constituents): there is no way to generate higher powers in the meson 4-momenta in the loop of Fig. 1.

$$\begin{aligned}
C_1 = & \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{6}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] \{2E_d^2m_B(m_B + q^0) \\
& + E_d[-m_B^3 - 2m_B^2q^0 + (m_b - m_d)(m_c - m_{c'})q^0 + E_J(-(m_b - m_d)(m_c + m_d) + m_B(m_B + q^0)) + m_B(m_b(m_c - m_{c'})) \\
& + m_cm_{c'} + 2m_d(m_b - m_c + m_{c'}) - 3m_d^2 - q^2] + m_d[m_B^2(m_c - m_{c'} + m_d) + E_Jm_B(m_b - m_c - 2m_d) \\
& + m_d(m_d + m_c)(m_d - m_{c'}) + m_Bq^0(m_c - m_{c'} + 2m_d) + E_J(m_b - m_d)q^0 + m_dq^2 - m_b((m_d - m_{c'})(m_d + m_c) \\
& + m_B(m_B + 2q^0) + q^2)]\},
\end{aligned}$$

$$\begin{aligned}
C_2 = & -2 \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{6}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)](E_d - m_B) \\
& \times [m_Bm_d(m_b - m_c - 2m_d) + E_d(m_B^2 - (m_b - m_d)(m_c + m_d))],
\end{aligned}$$

$$\begin{aligned}
C_3 = & m_B \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] \\
& \times [m_Bm_d(2m_b - m_c - m_{c'} - 4m_d) + E_d(2m_B^2 - (m_b - m_d)(m_c + m_{c'} + 2m_d))],
\end{aligned}$$

$$\begin{aligned}
C_4 = & -m_B \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] \\
& \times [-2E_dE_b m_B + m_Bm_d(-2m_b + m_c - m_{c'} + 2m_d) + E_d((m_b - m_d)(m_c - m_{c'} + 2m_d))],
\end{aligned}$$

$$C_5 = 2m_B^2 \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)][E_d m_B + m_d(m_b - m_d)],$$

$$C_7 = C_6 = -\frac{1}{2}C_5,$$

$$D_1 = m_B^2 \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)][E_d m_B + m_d(m_b - m_d)],$$

$$\begin{aligned}
D_2 = & m_B \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] \\
& \times [m_Bm_d(m_b - m_c - 2m_d) + E_d(m_B^2 - (m_b - m_d)(m_c + m_d))],
\end{aligned}$$

$$D_3 = -m_B \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)][(m_c - m_{c'})(E_d(m_b - m_d) + m_B m_d)],$$

$$D_4 = D_5 = D_6 = D_7 = D_8 = 0,$$

$$\begin{aligned}
w_+ = & -\frac{m_B}{2} \int_0^{k_{\max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_cm_{c'}m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] \{-4E_d^2(m_b - m_d)(m_B - q^0) \\
& + m_Bm_d[m_B^2 - 2m_{c'}(m_c + 2m_d) + m_d(3m_c + m_d) + m_b(-m_c + 2m_{c'} + m_d) + 2(E_Jq^0 + q^2)] \\
& + E_d[-m_B^2(m_c - 2m_{c'} + 4m_d) - 2m_d(-m_cm_{c'} + m_cm_d + m_d^2 + E_Jq^0 + q^2) + 7m_Bm_dq^0 + m_b(m_B^2 - 3m_Bq^0 \\
& + 2(-m_cm_{c'} + m_cm_d + m_d^2 + E_Jq^0 + q^2))]\},
\end{aligned}$$

$$\begin{aligned}
w_- &= -\frac{m_B^2}{2} \int_0^{k_{max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_c^2m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] \\
&\quad \times [(m_b - m_d)(E_d(q^0 - m_B) + m_d(m_d + m_c)) + m_B(m_d(q^0 - m_B) + E_d(m_c + m_d))], \\
&= m_B \int_0^{k_{max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_c^2m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)](E_d m_b + E_b m_d) \\
&\quad \times [m_B q^0 + (m_B - q^0)(E_J + 2E_d - m_B) - q^2 + (m_c + m_d)(m_{c'} - m_d)], \\
h &= \frac{m_B}{2} \int_0^{k_{max}} \frac{dkk^2}{2\pi^2} \mathcal{N} \frac{\psi_B(k)\psi_J^*(k)\psi_D^*(k)}{8\sqrt{3}m_B^2m_d^2m_c^2m_b} [m_d(m_d - m_{c'}) + E_d(E_J - m_B + q^0)] [E_d(m_b - m_d) + m_B m_d].
\end{aligned}$$

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