Remarks on the quark content of the scalar meson $f_0(1370)$

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Based on the measurements of $(D_s^+, D^+) \to f_0(1370)\pi^+$ we determine, in a model independent way, the allowed $s\bar{s}$ content in the scalar meson $f_0(1370)$. We find that, on the one hand, if this isoscalar resonance is a pure $n\bar{n}$ state $[n\bar{n} \equiv (u\bar{u}+d\bar{d})/\sqrt{2}]$, a very large W-annihilation term will be needed to accommodate $D_s^+ \to f_0(1370)\pi^+$. On the other hand, the $s\bar{s}$ component of $f_0(1370)$ should be small enough to avoid excessive $D_s^+ \to f_0(1370)\pi^+$ induced from the external W emission. Measurement of $f_0(1370)$ production in the decay $D_s^+ \to K^+K^-\pi^+$ will be useful to test the above picture. For the decay $D^0 \to f_0(1370)\bar{K}^0$ which is kinematically barely or even not allowed, depending on the mass of $f_0(1370)$, we find that the finite width effect of $f_0(1370)$ plays a crucial role on the resonant three-body decay $D^0 \to f_0(1370)\bar{K}^0 \to \pi^+\pi^-\bar{K}^0$.

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I. INTRODUCTION

It is known that the identification of scalar mesons is difficult experimentally and the underlying structure of scalar mesons is not well established theoretically (for a review, see e.g. [1–3]). It has been suggested that the light scalars below or near 1 GeV—the isoscalars $\sigma(500)$, $f_0(980)$, the isodoublet κ and the isovector $a_0(980)$—form an SU(3) flavor nonet, while scalar mesons above 1 GeV, namely, $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ and $f_0(1500)/f_0(1710)$, form another nonet. A consistent picture [3] provided by the data suggests that the scalar meson states above 1 GeV can be identified as a $q\bar{q}$ nonet with some possible glue content, whereas the light scalar mesons below or near 1 GeV form predominately a $qq\bar{q}\bar{q}$ nonet [4,5] with a possible mixing with 0^+ $q\bar{q}$ and glueball states. This is understandable because, in the $q\bar{q}$ quark model, the 0^+ meson has a unit of orbital angular momentum and hence it should have a higher mass above 1 GeV. On the contrary, four quarks $q^2\bar{q}^2$ can form a 0^+ meson without introducing a unit of orbital angular momentum. Moreover, color and spin dependent interactions favor a flavor nonet configuration with attraction between the qq and $\bar{q}\bar{q}$ pairs. Therefore, the 0^+ $q^2\bar{q}^2$ nonet has a mass near or below 1 GeV.

As the quark content of $a_0(1450)$ and $K_0^*(1430)$ is quite obvious, the internal structure of the isoscalars $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ in the same nonet is controversial and less clear. Though it is generally believed that $f_0(1370)$ is mainly $n\bar{n} \equiv (u\bar{u}+d\bar{d})/\sqrt{2}$, the content of $f_0(1500)$ and $f_0(1710)$ still remains confusing. For example, it has been advocated that $f_0(1710)$ is mainly $s\bar{s}$ and $f_0(1500)$ mostly gluonic (see e.g. [6]), while the analysis in [7] suggests a dominantly $s\bar{s}$ interpretation of $f_0(1500)$. How much is the fraction of glue in each isoscalar meson is another important but unsettled issue, see [8] for a discussion.

Three-body decays of heavy mesons provide a rich laboratory for studying the intermediate state resonances. The Dalitz plot analysis is a powerful technique for this purpose. Many scalar meson production measurements in charm decays are now available from the dedicated experiments conducted at CLEO, E791, FOCUS, and BaBar. The study of three-body decays of charmed mesons not only opens a new avenue to the understanding of the light scalar meson spectroscopy, but also enables us to explore the quark content of scalar resonances. In [9] we have studied the nonleptonic weak decays of charmed mesons into a scalar meson and a pseudoscalar meson. The scalar resonances under consideration there are σ [or $f_0(600)$], κ, $f_0(980)$, $a_0(980)$ and $K_0^*(1430)$.

In this work we would like to explore the quark content of $f_0(1370)$ from hadronic charm decays. Since $\rho\rho$ and 4π are its dominant decay modes [10], it is clear that $f_0(1370)$ is mostly $n\bar{n}$. However, how much the $s\bar{s}$ component is allowed in the wave function of this isoscalar resonance remains unknown. It turns out that the decay $D_s^+ \to f_0(1370)\pi^+$ is very useful for this purpose. If $f_0(1370)$ is purely an $n\bar{n}$ state, it can proceed only via the W-annihilation diagram. In contrast, if $f_0(1370)$ has an $s\bar{s}$ content, the decay $D_s^+ \to f_0(1370)\pi^+$ will receive an external W-emission contribution. Therefore, this mode is ideal for determining the $s\bar{s}$ component in $f_0(1370)$.

We would work in the model-independent quark-diagram approach in which a least model-independent analysis of heavy meson decays can be carried out. In this diagrammatic scenario, all two-body nonleptonic weak decays of heavy mesons can be expressed in terms of six distinct quark diagrams [11–13]: T, the color-allowed external W-emission tree diagram; C, the color-suppressed internal W-emission diagram; E, the W-exchange diagram; A, the W-annihilation diagram; P, the horizontal W-loop diagram; and V, the vertical W-loop diagram. (The one-gluon exchange approximation of the P graph is the so-called "penguin diagram.") It should be stressed that these quark diagrams are classified according to the topologies of weak interactions with all strong interaction effects included and hence they are *not* Feynman graphs. Therefore, topological graphs can provide information on final-state interactions (FSIs).

Based on SU(3) flavor symmetry, this model-independent analysis enables us to extract the topological quark-graph amplitudes and see the relative importance of different underlying decay mechanisms. For $D\to SP$ decays (S: scalar meson, P: pseudoscalar meson), there are several new features. First, one can have two different external W-emission and internal W-emission diagrams, depending on whether the emission particle is a scalar meson or a pseudoscalar one. We thus denote the prime amplitudes T' and C' for the case when the scalar meson is an emitted particle [9]. Second, because of the smallness of the decay constant of the scalar meson (see, e.g. [14]), it is expected that $|T'|\ll|T|$ and $|C'|\ll|C|$. Moreover, in the flavor SU(3) limit, the primed amplitudes T' and C' diminish under the factorization approximation due to the vanishing decay constants of scalar mesons [9]. Third, since the scalar mesons $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$, $f_0(1500)/f_0(1710)$ and the light ones σ, κ, f_0, a_0 fall into two different nonets, one cannot apply SU(3) symmetry to relate the topological amplitudes in $D^+\to f_0(1370)\pi^+$ to, for example, those in $D^+\to f_0(980)\pi^+$.

The reduced quark-graph amplitudes T,C,E,A for Cabibbo-allowed $D\to PP$ decays have been extracted from the data with the results [15]

$$\begin{aligned} T&=(2.67\pm0.20)\times10^{-6}\ \text{GeV},\\ C&=(2.03\pm0.15)\exp[-i(151\pm4)^\circ]\times10^{-6}\ \text{GeV},\\ E&=(1.67\pm0.13)\exp[i(115\pm5)^\circ]\times10^{-6}\ \text{GeV},\\ A&=(1.05\pm0.52)\exp[-i(65\pm30)^\circ]\times10^{-6}\ \text{GeV}. \end{aligned} \tag{1}$$

These amplitudes will be employed for a guidance when we come to discuss $D\to f_0(1370)P$ decays below.

II. QUARK CONTENT OF $f_0(1370)$

The mass and width of the isoscalar resonance $f_0(1370)$ are far from being well established. The recent study of $f_0(1370)$ production in pp interactions by WA102 [16] yields a mass of order 1310 MeV and width of order 100–250 MeV (see [16] for the detailed values of the mass and width). The E791 experiment by analyzing $D_s^+\to\pi^+\pi^+\pi^-\to f_0(1370)\pi^+$ gives a higher mass of $1434\pm18\pm9$ MeV and width of $172\pm32\pm6$ MeV [17]. The mass and width quoted by the Particle Data Group [10] span a wide range, namely, $m_{f_0(1370)}=1200$–1500 MeV and $\Gamma_{f_0(1370)}=200$–500 MeV.

Since $\rho\rho$ and 4π are the dominant decay modes of $f_0(1370)$ [10], it is clear that this isoscalar resonance is predominately $n\bar{n}$. In the present work we would like to study its content from the three-body decays of charmed mesons to see how much the $s\bar{s}$ component is allowed in $f_0(1370)$.

The production of the resonance $f_0(1370)$ in hadronic decays of charmed mesons has been observed in the decay $D^0\to\bar{K}^0\pi^+\pi^-\to f_0(1370)\bar{K}^0$ by ARGUS [18], E687 [19] and CLEO [20], in $D_s^+\to\pi^+\pi^+\pi^-\to f_0(1370)\pi^+$ by E791 [17], in $D^+\to K^+K^-\pi^+\to f_0(1370)\pi^+$ by FOCUS [21] and in $D^+\to\pi^+\pi^-\pi^+\to f_0(1370)\pi^+$ by E791 [22], respectively, with the results

$$\begin{aligned} &\mathcal{B}(D^0\to f_0(1370)\bar{K}^0)\mathcal{B}(f_0(1370)\to\pi^+\pi^-)=\begin{cases}(4.7\pm1.4)\times10^{-3} & \text{ARGUS,E687}\\ (5.9^{+1.8}_{-2.7})\times10^{-3} & \text{CLEO}\end{cases}\\ &\mathcal{B}(D^+\to f_0(1370)\pi^+)\mathcal{B}(f_0(1370)\to K^+K^-)=(6.2\pm1.1)\times10^{-4}\quad \text{FOCUS}\\ &\mathcal{B}(D^+\to f_0(1370)\pi^+)\mathcal{B}(f_0(1370)\to\pi^+\pi^-)=(7.1\pm6.4)\times10^{-5}\quad \text{E791}\\ &\mathcal{B}(D_s^+\to f_0(1370)\pi^+)\mathcal{B}(f_0(1370)\to\pi^+\pi^-)=(3.3\pm1.2)\times10^{-3}\quad \text{E791.} \end{aligned} \tag{2}$$

However, the E791 measurement of $D^+\to f_0(1370)\pi^+$ does not have enough statistic significance and hence we will ignore it in the ensuing discussion. The branching fractions of $f_0(1370)$ into $\pi^+\pi^-$ and K^+K^- are unknown, though several early attempts have been made (see [10]).

We write the general $f_0(1370)$ flavor wave function as

$$f_0(1370)=n\bar{n}\cos\theta+s\bar{s}\sin\theta. \tag{3}$$

In terms of the quark-diagram amplitudes depicted in Fig. 1, the decay amplitudes of $D\to f_0(1370)P$ have the expressions

$$\begin{aligned} A(D^+\to f_0(1370)\pi^+)&=V_{cd}V_{ud}^*(T_d+A_{u,d})+V_{cs}V_{us}^*C_s',\\ A(D^0\to f_0(1370)\bar{K}^0)&=V_{cs}V_{ud}^*(C_u+E_{d,s}),\\ A(D_s^+\to f_0(1370)\pi^+)&=V_{cs}V_{ud}^*(T_s+A_{u,d}), \end{aligned} \tag{4}$$

where the subscript q of the topological amplitude denotes the $q\bar{q}$ component of $f_0(1370)$ involved in its production. In terms of the mixing angle θ defined in Eq. (3) we have $T_s=\sqrt{2}\,T_d\tan\theta$. We see that if $f_0(1370)$ is an $n\bar{n}$ state in nature, the decay $D_s^+\to f_0(1370)\pi^+$ can only proceed through the topological W-annihilation diagram.

Hadronic charm decays are conventionally studied within the framework of generalized factorization in which the hadronic decay amplitude is expressed in terms of factorizable terms multiplied by the *universal* (i.e. decay process inde-

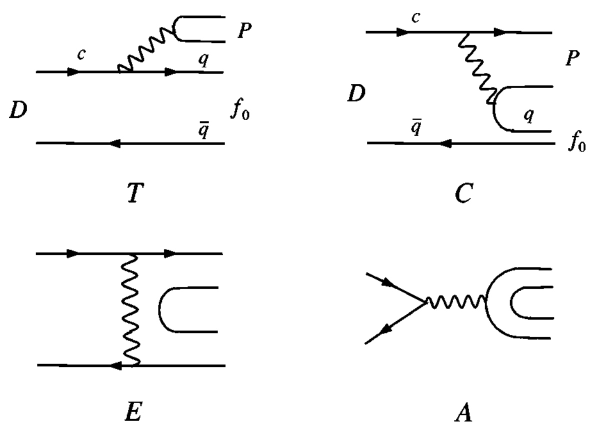

FIG. 1. Topological quark diagrams for $D \to f_0(1370)P$ decays. The diagram C' is the same as the diagram C except for an interchange between P and $f_0(1370)$.

pendent) effective parameters a_i that are renormalization scale and scheme independent. In this approach, the quark-graph amplitudes read

$$
\begin{aligned}
T_u &= \frac{G_F}{\sqrt{2}} a_1 f_\pi F_0^{Df_0}(m_\pi^2)(m_D^2 - m_{f_0}^2),\\
T_s &= \frac{G_F}{\sqrt{2}} a_1 f_\pi F_0^{D_s f_0}(m_\pi^2)(m_{D_s}^2 - m_{f_0}^2),\\
C_u &= \frac{G_F}{\sqrt{2}} a_2 f_K F_0^{Df_0}(m_K^2)(m_D^2 - m_{f_0}^2),\\
C_s' &= \frac{G_F}{\sqrt{2}} a_2 f_{f_0} F_0^{D\pi}(m_{f_0}^2)(m_D^2 - m_\pi^2),\\
E_q &= \frac{G_F}{\sqrt{2}} a_2 f_D F_0^{0\to f_0^{q\bar{q}}\bar{K}^0}(m_D^2)(m_{f_0(1370)}^2 - m_K^2),\\
A_q &= \frac{G_F}{\sqrt{2}} a_2 f_D F_0^{0\to f_0^{q\bar{q}}\pi^+}(m_D^2)(m_{f_0(1370)}^2 - m_\pi^2),
\end{aligned}
\tag{5}
$$

where the form factor F_0 is defined in [23] and the typical values of a_i in charm decays are $a_1 = 1.15$ and $a_2 = -0.55$. For $f_0(1370)$, its decay constant $f_{f_0(1370)}$ is zero owing to charge conjugation invariance or conservation of vector current [14]. This means that the amplitude C_s' vanishes under the factorization approximation.

In Eq. (5) the annihilation form factor $F_0^{0\to f_0 P}(m_D^2)$ is expected to be suppressed at large momentum transfer, $q^2 = m_D^2$, corresponding to the conventional helicity suppression. Based on the helicity suppression argument, one may therefore neglect short-distance (hard) W-exchange and W-annihilation contributions. However, as stressed in [24], weak annihilation does receive long-distance contributions from nearby resonances via inelastic final-state interactions from the leading tree or color-suppressed amplitude. The effects of resonance-induced FSIs can be described in a model independent manner and are governed by the masses and decay widths of the nearby resonances. Indeed, the weak annihilation (W-exchange E or W-annihilation A) amplitude for $D \to PP$ decays has a sizable magnitude comparable to the color-suppressed internal W-emission amplitude C with a large phase relative to the tree amplitude T [see Eq. (1)].

In the $q\bar{q}$ description of $f_0(1370)$, it follows from Eq. (3) that

$$
\begin{aligned}
&F_0^{D^0 f_0} = \frac{1}{\sqrt{2}} \cos\theta\, F_0^{D^0 f_0^{u\bar{u}}}, \quad F_0^{D^+ f_0} = \frac{1}{\sqrt{2}} \cos\theta\, F_0^{D^+ f_0^{d\bar{d}}},\\
&F_0^{D_s^+ f_0} = \sin\theta\, F_0^{D_s^+ f_0^{s\bar{s}}},
\end{aligned}
\tag{6}
$$

where the superscript $q\bar{q}$ denotes the quark content of f_0 involved in the transition. In the limit of SU(3) symmetry, $F_0^{D^0 f_0^{u\bar{u}}} = F_0^{D^+ f_0^{d\bar{d}}} = F_0^{D_s^+ f_0^{s\bar{s}}}$ and hence

$$
F_0^{D^0 f_0} = F_0^{D^+ f_0} = \frac{1}{\sqrt{2}} F_0^{D_s^+ f_0} \cot\theta. \tag{7}
$$

Consequently, under the factorization approximation one has $T_s = \sqrt{2}\, T_d \tan\theta$, a relation valid in the more general diagrammatic approach.

Since

$$
\frac{\Gamma(D_s^+ \to f_0(1370)\pi^+)}{\Gamma(D^+ \to f_0(1370)\pi^+)} = \frac{\mathcal{B}(D_s^+ \to f_0(1370)\pi^+)}{\mathcal{B}(D^+ \to f_0(1370)\pi^+)} \frac{\tau(D^+)}{\tau(D_s^+)}, \tag{8}
$$

it follows from Eqs. (2) and (4) that

$$
\left|\frac{T_s + A_{u,d}}{T_d - C_s' + A_{u,d}}\right|_{D\to f_0(1370)P} = (0.76 \pm 0.24) \times \left(\frac{\mathcal{B}(f_0(1370) \to K^+K^-)}{\mathcal{B}(f_0(1370) \to \pi^+\pi^-)}\right)^{1/2}, \tag{9}
$$

where the charmed meson lifetimes are taken from [10]. Let us consider two extreme cases: (i) the W-annihilation term vanishes, and (ii) $f_0(1370)$ is purely a $n\bar{n}$ state so that $T_s = 0$.

To proceed we will take $C_s' = 0$ as suggested by the factorization approach. In the case of a vanishing W-annihilation, $A_{u,d} = 0$. Hence, the left-hand side of Eq. (9) becomes $\sqrt{2}\,|\tan\theta|$. In order to estimate the mixing angle we use the measurement of $R \equiv \Gamma(K\bar{K})/\Gamma(\pi\pi) = 0.46$

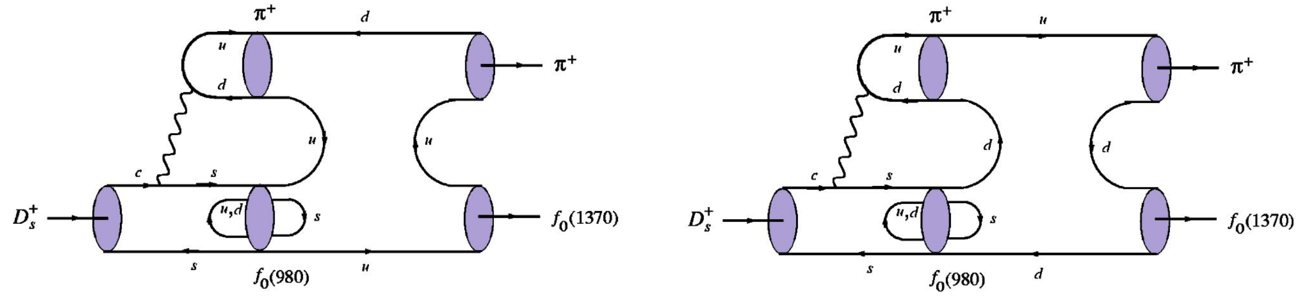

FIG. 2. Contributions to $D_s^+\to f_0(1370)\pi^+$ from the color-allowed weak decay $D_s^+\to f_0(980)\pi^+$ followed by a resonant-like rescattering. This has the same topology as the W-annihilation graph. The flavor wave function of $f_0(980)$ has the symbolic expression $s\bar{s}(u\bar{u}+d\bar{d})/\sqrt{2}$.

$\pm 0.15\pm 0.11$ [16].[1] This leads to

$$\frac{\Gamma(f_0(1370)\to K^+K^-)}{\Gamma(f_0(1370)\to \pi^+\pi^-)}=0.35\pm 0.14. \quad (10)$$

From Eq. (9) we obtain

$$\theta=\pm(17.5^{+6.5}_{-5.9})^\circ. \quad (11)$$

This means that even in the absence of W annihilation, a small amount of the $s\bar{s}$ content in the $f_0(1370)$ wave function will suffice to account for the observed rate of $D_s^+\to f_0(1370)\pi^+$ relative to $D^+\to f_0(1370)\pi^+$.

In the other extreme case where $f_0(1370)$ is a pure $n\bar{n}$ state, $D_s^+\to f_0(1370)\pi^+$ can proceed only via W annihilation which includes both short-distance and long-distance effects. Even the short-distance W annihilation is helicity suppressed, a long-distance contribution to the topological W annihilation in $D_s^+\to f_0(1370)\pi^+$ arises from the color-allowed decay $D_s^+\to f_0(980)\pi^+$ followed by a resonant-like rescattering as depicted in Fig. 2. Note that the flavor wave function of $f_0(980)$ has the symbolic expression $s\bar{s}(u\bar{u}+d\bar{d})/\sqrt{2}$ [4] as the light scalars are favored to be 4-quark states (for a recent discussion, see, e.g. [9]). The decay $D_s^+\to f_0(980)\pi^+$ has a large branching ratio of $(1.8\pm 0.3)\%$ [9]. As discussed in [24], Fig. 2 manifested at the hadron level receives a s-channel resonant contribution from, for example, the 0^- resonance $\pi(1800)$ and a t-channel contribution with one-particle exchange. It follows from Eq. (9) that

$$\left|\frac{A_{u,d}}{T_d+A_{u,d}}\right|_{D\to f_0(1370)P}=0.45\pm 0.18. \quad (12)$$

The magnitude of A/T depends on the its phase. Since W annihilation is expected to be dominated by the imaginary part, we will have $|A_{u,d}/T_d|=0.50^{+0.36}_{-0.17}$ if the relative phase between A and T is 90°, for example. This means that if $f_0(1370)$ is composed of only $n\bar{n}$, then one will need a very sizable W annihilation to account for the observed $D_s^+\to f_0(1370)\pi^+$ decay. However, recall that in Cabibbo-allowed $D\to PP$ decays, the topological amplitudes given in Eq. (1) lead to

$$\left.\frac{A}{T}\right|_{D\to PP}=(0.39\pm 0.20)e^{-i(65\pm 30)^\circ}. \quad (13)$$

This indicates that although the W-annihilation term induced from nearby resonances via FSIs is sizable, it is probably unlikely that it can be big enough to satisfy the constraint (12). In reality, both external W emission and W annihilation contribute to the decay and the $s\bar{s}$ component in $f_0(1370)$ is smaller than that implied by Eq. (11).

III. $D^0\to f_0(1370)\bar{K}^0$ AND THE FINITE WIDTH EFFECT

We next turn to the decay $D^0\to f_0(1370)\bar{K}^0$ relative to $D^+\to f_0(1370)\pi^+$. From Eqs. (2) and (4) we have

$$\left|\frac{C_u+E_{d,s}}{T_d-C_s'+A_{u,d}}\right|_{D\to f_0(1370)P}=(0.58\pm 0.15)\frac{1}{\sqrt{r}}, \quad (14)$$

where $r=p_c(D^0\to f_0\bar{K}^0)/p_c(D^+\to f_0\pi^+)$, and p_c is the c.m. momentum of the final-state particles in the rest frame of the charmed meson. However, the momentum p_c in the decay $D^0\to f_0(1370)\bar{K}^0$ is very sensitive to the $f_0(1370)$ mass. For example, $p_c=0$, 34, 214 MeV and hence $r=0$, 0.083, 0.47 for $m_{f_0}=1400$, 1370, 1310 MeV, respectively. Therefore, when $m_{f_0}=1370$ MeV, one needs $C/T\sim 7$ to account for the observed decay rate of $D^0\to f_0(1370)\bar{K}^0$ relative to $D^+\to f_0(1370)\pi^+$, which is certainly very unlikely. The difficulty has something to do with the decay width of the scalar resonance which we have neglected so far. As the decay $D^0\to f_0(1370)\bar{K}^0$ is marginally or even not allowed kinematically, depending on the $f_0(1370)$ mass, it is important to take into account the finite width effect of the resonance. That is, one should evaluate

[1] A reanalysis of the old data on the reactions $\pi^-p\to\pi^-\pi^+n$ and $\pi^+\pi^-\to K\bar{K}$ yields $R=1.33\pm 0.67$ [25]. This is inconsistent with naive expectation. First, the $\pi\pi$ phase space is larger than the $K\bar{K}$ one by a factor of 1.8. Second, the $g_{f_0\pi\pi}$ coupling is larger than $g_{f_0K\bar{K}}$ if $f_0(1370)$ is mostly $n\bar{n}$.

the two-step process $\Gamma(D^0\to f_0(1370)\overline{K}^0\to\pi^+\pi^-\overline{K}^0)$ and compare the resonant three-body rate with experiment.

The decay rate of the resonant three-body decay is given by

$$\Gamma(D\to SP\to P_1P_2P) = \frac{1}{2m_D}\int_{(m_1+m_2)^2}^{(m_D-m_P)^2}\frac{dq^2}{2\pi}|\langle SP|\mathcal{H}_W|D\rangle|^2\frac{\lambda^{1/2}(m_D^2,q^2,m_P^2)}{8\pi m_D^2} \times\frac{1}{(q^2-m_S^2)^2+(\Gamma_{12}(q^2)m_S)^2}g_{SP_1P_2}^2\frac{\lambda^{1/2}(q^2,m_1^2,m_2^2)}{8\pi q^2}, \tag{15}$$

where λ is the usual triangular function $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$, m_1 (m_2) is the mass of P_1 (P_2), and the "running" or "comoving" width $\Gamma_{12}(q^2)$ is a function of the invariant mass $m_{12}=\sqrt{q^2}$ of the P_1P_2 system and it has the expression [26]

$$\Gamma_{12}(q^2)=\Gamma_S\frac{m_S}{m_{12}}\frac{p'(q^2)}{p'(m_S^2)}, \tag{16}$$

where $p'(q^2)=\lambda^{1/2}(q^2,m_1^2,m_2^2)/(2\sqrt{q^2})$ is the c.m. momentum of P_1 or P_2 in the P_1P_2 rest frame and $p'(m_S^2)$ is the c.m. momentum of either daughter in the resonance rest frame. The propagator of the resonance is assumed to be of the Breit-Wigner form.

When the resonance width Γ_S is narrow, the expression of the resonant decay rate can be simplified by applying the so-called narrow width approximation

$$\frac{1}{(q^2-m_S^2)^2+m_S^2\Gamma_{12}^2(q^2)}\approx\frac{\pi}{m_S\Gamma_S}\delta(q^2-m_S^2). \tag{17}$$

Noting

$$\Gamma(D\to SP)=|\langle SP|\mathcal{H}_W|D\rangle|^2\frac{p}{8\pi m_D^2}, \quad \Gamma(S\to P_1P_2)=g_{SP_1P_2}^2\frac{p'(m_S^2)}{8\pi m_S^2}, \tag{18}$$

where $p=\lambda^{1/2}(m_D^2,m_S^2,m_P^2)/(2m_D)$ is the c.m. three-momentum of final-state particles in the D rest frame, we are led to the "factorization" relation

$$\Gamma(D\to SP\to P_1P_2P)=\Gamma(D\to SP)\mathcal{B}(S\to P_1P_2) \tag{19}$$

for the resonant three-body decay rate.

In practice, this factorization relation works reasonably well as long as the two-body decay $D\to SP$ is kinematically allowed and the resonance is narrow. However, when $D\to SP$ is kinematically barely or even not allowed, the off resonance peak effect of the intermediate resonant state will become important. For example, the fit fractions of $D^0\to\rho(1700)^+K^-\to\pi^+\pi^0K^-$, $D^0\to K_0^*(1480)\overline{K}^0\to K^+\pi^-\overline{K}^0$ have been measured by the CLEO [20] and BaBar [27] Collaborations, respectively. It is clear that the on-shell decays $D^0\to\rho(1700)^+K^-$ and $D^0\to K_0^*(1480)\overline{K}^0$ are kinematically not allowed and it is necessary to take into account the finite width effect.

Since $f_0(1370)$ is broad with a width ranging from 200 to 500 MeV, *a priori* there is no reason to neglect its finite width effect. For simplicity in practical calculations, we shall fix the weak matrix element $\langle SP|\mathcal{H}_W|D\rangle$ and the strong coupling $g_{SP_1P_2}$ at $q^2=m_S^2$ and assume that they are insensitive to the q^2 dependence when the resonance is off its mass shell. Let us define the parameter η:

$$\eta\equiv\frac{\Gamma(D\to SP\to P_1P_2P)}{\Gamma(D\to SP)\mathcal{B}(S\to P_1P_2)}. \tag{20}$$

The deviation of η from unity will give a measure of the violation of the factorization relation (19). Then it has the expression

$$\eta=\frac{m_S^2}{4\pi m_D}\frac{\Gamma_S}{pp'(m_S^2)}\int_{(m_1+m_2)^2}^{(m_D-m_P)^2}\frac{dq^2}{q^2}\lambda^{1/2}(m_D^2,q^2,m_P^2) \times\lambda^{1/2}(q^2,m_1^2,m_2^2)\frac{1}{(q^2-m_S^2)^2+(\Gamma_{12}(q^2)m_S)^2}. \tag{21}$$

For $m_{f_0(1370)}=1370$ MeV and $\Gamma_{f_0(1370)}=200$ MeV (500 MeV), we find $\eta=3.8$ (4.3), 0.83 (0.67), 0.89 (0.74) for the decays $D^0\to f_0(1370)\overline{K}^0\to\pi^+\pi^-\overline{K}^0$, $D^+\to f_0(1370)\pi^+\to\pi^+\pi^-\pi^+$ and $D_s^+\to f_0(1370)\pi^+\to\pi^+\pi^-\pi^+$, respectively. It is evident that the finite width effect of $f_0(1370)$ is very crucial for $D^0\to f_0(1370)\overline{K}^0$. This also indicates that the measured branching ratios shown in Eq. (2) are actually for resonant three-body decays.

Let us return back to Eq. (14). The parameter r there should be replaced by $r=I_1/I_2$ with

$$I_1=\int_{4m_K^2}^{(m_D-m_\pi)^2}\frac{dq^2}{q^2}\lambda^{1/2}(m_D^2,q^2,m_\pi^2)\lambda^{1/2}(q^2,m_\pi^2,m_\pi^2) \times\frac{1}{(q^2-m_{f_0}^2)^2+(\Gamma_{12}(q^2)m_{f_0})^2},$$
$$I_2=\int_{4m_\pi^2}^{(m_D-m_K)^2}\frac{dq^2}{q^2}\lambda^{1/2}(m_D^2,q^2,m_K^2)\lambda^{1/2}(q^2,m_\pi^2,m_\pi^2) \times\frac{1}{(q^2-m_{f_0}^2)^2+(\Gamma_{12}(q^2)m_{f_0})^2}. \tag{22}$$

Note that the lower bound of the integral I_1 is $4m_K^2$ rather than $4m_\pi^2$ in order to have a real $p'(q^2)$. For the representative values of $m_{f_0(1370)}=1370$ MeV and $\Gamma_{f_0(1370)}=250$ MeV, we find $r=0.36$ and hence

$$\left|\frac{C_u+E_{d,s}}{T_d-C'_s+A_{u,d}}\right|_{D\to f_0(1370)P}=0.97\pm 0.25, \tag{23}$$

which is to be compared with

$$\left|\frac{C+E}{T+A}\right|_{D\to PP}\sim 0.78 \tag{24}$$

in $D\to PP$ decays [see Eq. (1)]. Therefore, the decay $D^+\to f_0(1370)\pi^+\to\pi^+\pi^-\pi^+$ can be explained once the finite width effect of $f_0(1370)$ is taken into account.

The comparison of $D^0\to f_0(1370)\bar{K}^0$ with $D_s^+\to f_0(1370)\pi^+$ in principle allows one to obtain some information on the mixing angle. However, since the relation between the amplitudes C_u and T_s is unknown, it does not allow a model-independent extraction. Finally, it should be remarked that owing to the finite width effect, Eqs. (11) and (12) are slightly modified to

$$\theta=\pm(18.8^{+6.8}_{-7.4})^\circ, \quad \left|\frac{A_{u,d}}{T_d+A_{u,d}}\right|_{D\to PP}=0.48\pm 0.20. \tag{25}$$

IV. DISCUSSION AND CONCLUSION

The decay $D^+\to f_0(1370)\pi^+$ receives the main contribution from the external W-emission diagram via the $n\bar{n}$ component of $f_0(1370)$, while $D_s^+\to f_0(1370)\pi^+$ proceeds via the external W emission through the $s\bar{s}$ content; both channels receive W annihilation. Assuming the absence of W annihilation, we showed in a model independent way that both modes can be accommodated provided that $\theta=\pm(17.5^{+6.5}_{-5.9})^\circ$. That is, even a small $s\bar{s}$ component in $f_0(1370)$ can induce adequate $D_s^+\to f_0(1370)\pi^+$ via the external W emission. In the other extreme case where $f_0(1370)$ is a pure $n\bar{n}$ state, it is found that one needs a very large W annihilation to explain the decay $D_s^+\to f_0(1370)\pi^+$. Therefore, we conclude that $f_0(1370)$ is unlikely a pure $n\bar{n}$ state. In reality, both external W emission and W annihilation contribute to the decay and the mixing angle is smaller than the above-mentioned value.

To extract the upper limit on the mixing angle we have employed the experimental value of $\Gamma(K\bar{K})/\Gamma(\pi\pi)$. The uncertainty with the branching fractions of $f_0(1370)$ can be circumvented if $D_s^+\to f_0(1370)\pi^+\to K^+K^-\pi^+$ is measured and compared with $D^+\to f_0(1370)\pi^+\to\pi^+\pi^-\pi^+$.

For the decay $D^0\to f_0(1370)\bar{K}^0$ which is barely or even not allowed kinematically, depending on the mass of $f_0(1370)$, it is important to take into account the finite width effect of $f_0(1370)$. We find that it plays a crucial role on the resonant three-body decay $D^0\to f_0(1370)\bar{K}^0\to\pi^+\pi^-\bar{K}^0$.

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