$B\to\pi$ and $B\to K$ transitions in standard and quenched chiral perturbation theory

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We study the effects of chiral logs on the *heavy→light* pseudoscalar meson transition form factors by using standard and quenched chiral perturbation theory combined with the static heavy quark limit. The resulting expressions are used to indicate the size of uncertainties due to the use of the quenched approximation in the current lattice studies. They may also be used to assess the size of systematic uncertainties induced by missing chiral log terms in extrapolating toward the physical pion mass. We also provide the coefficient multiplying the quenched chiral log, which may be useful if the quenched lattice studies are performed with very light mesons.

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I. INTRODUCTION

Over the past decade, a considerable amount of effort has been put into studying the nonperturbative QCD dynamics of the *heavy→light* decays. The main target was (and still is) the extraction of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$. The prerequisite for its determination from the exclusive $B\to\pi\ell\nu$ decay mode is a precise knowledge of the relevant form factors. Accurate information on the form factors is crucial also when studying the impact of physics beyond the standard model on the exclusive $b\to s\ell^+\ell^-$ modes.

The fact that the kinematically accessible region of momentum transfers is very large (e.g., for $B\to\pi\ell\nu$ it is $0\leqslant q^2\leqslant 26.4\ \mathrm{GeV}^2$) makes QCD-based calculations of form factors ever more difficult. The physical pictures emerging at the two extremities of the q^2 region are quite different and effective field theory approaches, based on the appropriate use of the heavy quark expansion, have been developed to simplify the description of these processes. The heavy quark effective theory (HQET), which is applicable for recoil momenta close to zero ($q^2\to q^2_{\mathrm{max}}$),[1] provides us with valuable scaling laws for the form factors [1]. In the region of large recoils ($q^2\to 0$), instead, the large energy effective theory and its descendants help resolve the heavy quark dependence of the form factors [2,3]. Although these conceptual steps forward are highly beneficial for a better understanding of the underlying dynamics, a model-independent description (calculation) of the form factors in the entire physical region is still missing.

Among the QCD-based approaches employed to compute the *heavy→light* decay form factors, the following two stand apart.

Light cone QCD sum rules (LCSR). This analytic approach contains the least number of assumptions and has the correct heavy quark mass scaling properties. The range of applicability is, however, limited to low q^2's [4].

Lattice QCD. This method allows us to solve the nonperturbative QCD effects numerically. Because of the current insufficient computing power, the $B\to\pi$ transition is reached either (a) by extrapolating the directly computed form factors from the region around the charm to the b quark mass [5,6], or (b) by using a latticized effective theory of the heavy quark, such as nonrelativistic QCD (NRQCD) [7] (see also Ref. [8]), or (c) by reinterpreting lattice QCD in terms of NRQCD when the heavy quark mass becomes larger than the lattice UV cutoff [9]. All these strategies share one feature: the accessible form factors are restricted to the region of small recoils.

It is fair to say that the LCSR and lattice QCD are complementary to each other; it is important to use them both in order to check their consistency and from their comparison perhaps learn more about the underlying nonperturbative QCD dynamics.

Since the lattice studies are expected to provide us with the most accurate predictions about the shapes and absolute values of the form factors, it is important to have good control over the various assumptions that are currently used in lattice simulation and the data analysis. Two sources of systematic uncertainty have so far been ignored: the quenched approximation and possible deviations from the linear or quadratic chiral extrapolation forms.

All the available lattice results for $B\to\pi\ell\nu$ decay form factors are obtained from simulations in the quenched approximation, where the sea quark loop effects in the QCD vacuum fluctuations are neglected ($n_F=0$). To get an idea about the systematic error induced by the quenched approximation, one can confront the expressions for the form factors

[1] Zero recoil refers to the recoil of the daughter meson in the rest frame of the decaying one. In $B\to\pi\ell\nu$, it means that the pion is soft.

derived in the standard and in quenched chiral perturbation theory (ChPT and QChPT, respectively). Such expressions, at leading order in the heavy quark expansion and next-to-leading order (NLO) in the chiral expansion, are provided in the present paper. These expressions are also useful in assessing the systematic uncertainties due to chiral extrapolations. Current lattice studies deal with light mesons of masses $\gtrsim 450$ MeV. The physical pion mass is reached through a linear or quadratic extrapolation in the light quark mass. Although it is not clear for which light quark masses one begins to probe the subtleties of the chiral expansion, it is beyond reasonable doubt that, very close to the chiral limit, chiral log terms of the form $m_\pi^2 \log(m_\pi^2)$ may modify the result of the extrapolation. The coefficients multiplying the chiral logs are predicted by (Q)ChPT and will be presented in this paper. Finally, in the case of $B\to K$ decay the standard lattice strategy is to consider the kaon as a meson consisting of degenerate quark masses. The impact of non-degeneracy in the quenched approximation may also be addressed by using the QChPT expressions for the form factors, as we shall see later on.

It is important to stress a difficulty in getting reliable numerical estimates from this approach. As we just mentioned, it is not clear for what value of the light meson mass the chiral logs become relevant. That ambiguity is important since one extrapolates from the heavier light masses, for which chiral logs have *not* been observed. Another obstacle is the multitude of low energy constants that appear in the Lagrangian and in the transition operators in both quenched and unquenched ChPT. The values of some of those constants are unknown or simply guessed. Furthermore, as we shall see, the appearance of chirally divergent quenched chiral logs obliges us to compare the full and quenched expressions for not-so-light mesons, for which the ChPT is less predictive. For these three reasons, the numerical results inferred from this approach should always be taken with a grain of salt. In other words, rather than true estimates of the quenching errors, our numerical results should be considered as a mere indicator of the size of those errors.

The rest of the paper is organized as follows. In Sec. II we recall the basic elements of (Q)ChPT and its combination with the leading order HQET Lagrangian; in addition to standard definitions of the form factors, we will introduce some that are more convenient for our purposes; in Sec. III we present the one-loop (standard and quenched) chiral corrections for our form factors; in Sec. IV we discuss the values of the low energy constants that we chose for the numerical analysis, which we present in Secs. V and VI; we conclude in Sec. VII.

II. SETTING THE SCENE

In this section we recall some basic features of QChPT, as it was developed in the papers by Sharpe [10] and by Bernard and Golterman [11][2]. Although QChPT resembles the standard ChPT in many aspects there are important differences. The main one is the presence of the light η' state, which in the quenched QCD (QQCD) does decouple from the octet of pseudoscalar mesons. As a result the ‘‘pion’’ propagator exhibits not only a pole structure but also double-pole one, which is the source of the pathology of the quenched approximation.

A. Quenched chiral Lagrangian

In QQCD, in addition to the quarks q_a $(a=1,2,3)$, one also has the bosonic ‘‘ghost’’ quarks $\tilde{q}_a$, of spin $\frac{1}{2}$ and with identical mass $m_{\tilde{q}_a}/m_{q_a}=1$. Their role is to cancel the contributions of the closed quark loops, i.e., they provide quenching. If one assumes that the symmetry breaking pattern of QQCD is similar to that of the full QCD, i.e., the graded $SU(3|3)_L\otimes SU(3|3)_R$ spontaneously breaks down to $SU(3|3)_V$, the following chiral Lagrangian can be written

$$\mathcal{L}_{\rm light}=\frac{f^2}{8}\mathrm{str}(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger)+\frac{f^2\mu_0}{2}\mathrm{str}(\mathcal{M}\Sigma+\mathcal{M}\Sigma^\dagger)+\alpha_0\partial_\mu\Phi_0\partial^\mu\Phi_0-m_0^2\Phi_0^2+\mathcal{L}_4, \tag{1}$$

where we adopt the convention that $f\approx 130$ MeV, and the notation

$$\Sigma=\exp\left(2i\frac{\Phi}{f}\right),\quad \Phi=\begin{pmatrix}\phi & \chi^\dagger\\ \chi & \tilde{\phi}\end{pmatrix}. \tag{2}$$

The following comments are in order.

In addition to the standard $(q\bar{q})$ Goldstone bosons (π,K,η) and the η' meson,[3] organized in the 3×3 matrix

$$\phi=\begin{pmatrix}\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta+\frac{1}{\sqrt{3}}\eta' & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta+\frac{1}{\sqrt{3}}\eta' & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta+\frac{1}{\sqrt{3}}\eta'\end{pmatrix}, \tag{3}$$

[2]For an elegant alternative way to introduce partially quenched ChPT, see Ref. [12].

[3]We will neglect the mixing of η and η' states, as it is irrelevant for the discussion that follows.

the ghost-ghost ($\tilde{q}\overline{\tilde{q}}$) bosons ($\tilde{\phi}$), as well as the pseudoscalar fermions $\overline{\tilde{q}}q$ ($\chi^\dagger$) and $\overline{q}\tilde{q}$ (χ), also appear in Eq. (2).

The global symmetry $\mathrm{SU}(3|3)_L\otimes\mathrm{SU}(3|3)_R$ is graded and in Eq. (1), instead of the familiar trace, one deals with the supertrace $\mathrm{str}(\Phi)=\mathrm{tr}(\phi)-\mathrm{tr}(\tilde{\phi})$. As already stressed, η' does not decouple from the light pseudoscalar octet. Its effect is included in two terms of the Lagrangian (1), each of them multiplied by a new low energy constant, namely, α_0 and m_0. Note that $\Phi_0=\mathrm{str}(\Phi)/\sqrt{6}=(\eta'-\tilde{\eta}')/\sqrt{2}$.

The quark-ghost mass matrix is diagonal $\mathcal{M}=\mathrm{diag}(m_u,m_d,m_s,m_u,m_d,m_s)$. After diagonalizing the mass term in Eq. (1), one gets

$$m_\pi^2=4\mu_0 m_q,\quad m_K^2=2\mu_0(m_q+m_s),\quad m_\eta^2=\frac{4\mu_0}{3}(m_q+2m_s),\tag{4}$$

verifying the familiar Gell-Mann–Okubo relation $4m_K^2-m_\pi^2-3m_\eta^2=0$. Notice that we neglect the isospin symmetry breaking, i.e., we set $m_u=m_d\equiv m_q$.

In Eq. (1) $\mathcal{L}_4$ stands for the terms of $\mathcal{O}(p^4)$, of which we write only those that are relevant to the heavy-to-light form factors, namely,

$$\mathcal{L}_4=4\mu_0\{L_4\mathrm{str}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)\mathrm{str}(\mathcal{M}\Sigma^\dagger+\Sigma\mathcal{M}^\dagger)+L_5\mathrm{str}[\partial_\mu\Sigma^\dagger\partial^\mu\Sigma(\mathcal{M}\Sigma^\dagger+\Sigma\mathcal{M}^\dagger)]+\cdots\}.\tag{5}$$

L_4 and L_5 generate the mass correction to the decay constants (and to the wave function renormalization constant) [13].

It is straightforward to verify that after setting str→tr, $\Phi\to\phi$, $\eta'\to 0$, Eq. (1) leads to the standard (full QCD) chiral Lagrangian.[4]

B. Incorporating the heavy quark symmetry

QChPT has been combined with the leading order HQET in the work by Booth [15] and by Sharpe and Zhang [16]. They applied the approach to compute the heavy-light decay constants, the B^0-$\overline{B}^0$ mixing parameter, and the Isgur-Wise function.[5]

To devise a Lagrangian for the heavy-light mesons, it is necessary to include the heavy quark spin symmetry. This is achieved by combining the pseudoscalar (P^a) and vector ($P_\mu^{*\,a}$) heavy-light mesons in one field:

$$H_a(v)=\frac{1+\not{v}}{2}[P_\mu^{*a}(v)\gamma_\mu-P^a(v)\gamma_5],\tag{6}$$

where $(1+\not{v})/2$ projects out the particle component of the heavy quark only. The conjugate field is defined as $\overline{H}_a(v)=\gamma_0 H_a^\dagger(v)\gamma_0$, whereas the covariant derivative and the axial field have the following forms:

$$D_{ba}^\mu H_b=\partial^\mu H_a-H_b\mathbf{V}_{ba}^\mu=\partial^\mu H_a-H_b\frac{1}{2}[\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger]_{ba},$$

$$\mathbf{A}_\mu^{ab}=\frac{i}{2}[\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger]_{ab}.\tag{7}$$

In the above equation, a and b run over the light quark flavors and $\xi=\sqrt{\Sigma}$. With these ingredients in hand, we now write the quenched chiral Lagrangian for the heavy-light mesons as

$$\mathcal{L}_{\mathrm{heavy}}=-\mathrm{str}_a\mathrm{Tr}[\overline{H}_a iv\cdot D_{ba}H_b]+g\,\mathrm{str}_a\mathrm{Tr}[\overline{H}_a H_b\gamma_\mu\mathbf{A}_{ba}^\mu\gamma_5]+g'\mathrm{str}_a\mathrm{Tr}[\overline{H}_a H_a\gamma_\mu\gamma_5]\mathrm{str}(\mathbf{A}^\mu)+\mathcal{L}_3,\tag{8}$$

where g (g') is the coupling of the heavy meson doublet to the Goldstone boson (to η' or $\tilde{\eta}'$). A term with g' is thus an artifact of the quenched theory. The higher order terms in the expansion in $v\cdot p\sim\mathcal{O}(p)$, $\mathcal{O}(p^2)$, and in $m_q\sim\mathcal{O}(p^2)$, denoted as $\mathcal{L}_3$ in Eq. (8), have the following form [15]:

$$\mathcal{L}_3=2\lambda_1\mathrm{str}_a\mathrm{Tr}[\overline{H}_a H_b](\mathcal{M}_+)_{ba}+k_1\mathrm{str}_a\mathrm{Tr}[\overline{H}_a iv\cdot D_{bc}H_b]\times(\mathcal{M}_+)_{ca}+k_2\mathrm{str}_a\mathrm{Tr}[\overline{H}_a iv\cdot D_{ba}H_b]\mathrm{str}_c(\mathcal{M}_+)_{cc}+\cdots,\tag{9}$$

with $\mathcal{M}_+=(1/2)(\xi^\dagger M\xi^\dagger+\xi M\xi)$. We displayed only the terms that contribute to the heavy-to-light form factors. In the above equations, “Tr” stands for the trace over Dirac indices, whereas “str” is the supertrace over the light flavor indices.

As in the previous subsection, one can easily verify that after replacing str→tr, $\Phi\to\phi$, $\eta'\to 0$, and $g'\to 0$, one recovers the standard chiral Lagrangian for heavy-light mesons (for recent reviews see Ref. [18], and for the original papers see Ref. [19]).

C. Form factors

A frequently encountered decomposition of the matrix elements relevant to the leptonic, the semileptonic, and the penguin-induced hadronic matrix elements is

$$\langle 0|\overline{q}\gamma_\mu\gamma_5 b|B(p_B)\rangle=if_B p_{B\mu},$$

$$\langle P(p)|\overline{q}\gamma_\mu b|B(p_B)\rangle=\left((p_B+p)_\mu-q_\mu\frac{m_B^2-m_P^2}{q^2}\right)\times F_+(q^2)+\frac{m_B^2-m_P^2}{q^2}q_\mu F_0(q^2),$$

$$\langle P(p)|\overline{q}\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle=i(q^2(p_B+p)_\mu-(m_B^2-m_P^2)q_\mu)\frac{F_T(q^2)}{m_B+m_P},\tag{10}$$

[4] For a review of the standard ChPT see one of the references listed in [14].

[5] For a recent result on the Isgur-Wise function in partially QChPT, see Ref. [17].

where $q=d$ or s, $|P(p)\rangle$ is the light pseudoscalar meson state (pion or kaon), and $q^\nu=(p_B-p)^\nu$.

We will be working in the static limit, $m_B\to\infty$. The eigenstates of the QCD and HQET Lagrangians are related as

$$\lim_{m_B\to\infty}\frac{1}{\sqrt{m_B}}|B(p_B)\rangle_{\rm QCD}=|B(v)\rangle_{\rm HQET}. \quad (11)$$

In the static limit it is more convenient to use definitions in which the form factors are independent of the heavy meson mass, namely,

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 b_v|B(v)\rangle_{\rm HQET}=i\hat{f}v_\mu,$$
$$\langle P(p)|\bar{q}\gamma_\mu b_v|B(v)\rangle_{\rm HQET}=[p_\mu-(v\cdot p)v_\mu]f_p(v\cdot p)+v_\mu f_v(v\cdot p), \quad (12)$$

where the field b_v does not depend on the heavy quark mass [20]. The form factors $f_{p,v}$ are functions of the variable

$$v\cdot p=\frac{m_B^2+m_P^2-q^2}{2m_B}, \quad (13)$$

which in the heavy meson rest frame is the energy of the light meson E_P. The relation between the quantities defined in Eqs. (10) and (12) is obtained by matching QCD to HQET at the scale $\mu\sim m_b$ [21]:

$$f_B=\frac{C_{\gamma_0\gamma_5}(m_b)}{\sqrt{m_B}}(\hat{f}+\mathcal{O}(1/m_B)),$$
$$F_+(q^2)+\frac{m_B^2-m_P^2}{q^2}[F_+(q^2)-F_0(q^2)]|_{q^2\approx q^2_{\rm max}}=C_{\gamma_1}(m_b)\sqrt{m_B}[f_p(v\cdot p)+\mathcal{O}(1/m_B)],$$
$$(m_B+E_P)F_+(q^2)-(m_B-E_P)\frac{m_B^2-m_P^2}{q^2}[F_+(q^2)-F_0(q^2)]|_{q^2\approx q^2_{\rm max}}=C_{\gamma_0}(m_b)\sqrt{m_B}f_v(E_P)+\mathcal{O}(1/m_B),$$
$$\frac{2m_B}{m_B+m_P}F_T(q^2)=C_{\gamma_1\gamma_0}(m_b)f_p(v\cdot p)\sqrt{m_B}+\mathcal{O}(1/m_B). \quad (14)$$

In the following, we set the matching constants C_{γ_i} to their tree level value ($C_\gamma=1$). By neglecting the subleading terms in the heavy quark expansion, we have[6]

$$F_0(q^2)|_{q^2\approx q^2_{\rm max}}=\frac{1}{\sqrt{m_B}}f_v(v\cdot p),$$
$$F_+(q^2)|_{q^2\approx q^2_{\rm max}}=F_T(q^2)|_{q^2\approx q^2_{\rm max}}=\sqrt{\frac{m_B}{2}}f_p(v\cdot p), \quad (15)$$

which exhibit the usual heavy mass scaling laws for the semileptonic form factors [1]. The equality $F_T(q^2)\equiv F_+(q^2)$ arises from

$$\langle P(p)|\bar{q}\sigma^{i0}(1+\gamma_5)b_v|B(v)\rangle_{\rm HQET}=i\langle P(p)|\bar{q}\gamma^i(1-\gamma_5)b_v|B(v)\rangle_{\rm HQET}, \quad (16)$$

which is easily obtained by using the equation of motion $\gamma_0 b_v=b_v$. In the following our main concern will be the evaluation of the long-distance (chiral) corrections to the form factors $f_{v,p}(v\cdot p)$.

D. Heavy-light current

Having in mind Eq. (16), we only need to consider the $(V-A)$ Dirac structure. In the static heavy quark limit and at next-to-leading order in the chiral expansion, the bosonized *heavy→light* current reads [15]

$$J^\mu\equiv\bar{q}_a\gamma^\mu(1-\gamma_5)Q\to\frac{i\alpha}{2}{\rm Tr}[\gamma^\mu(1-\gamma_5)H_b]\xi^\dagger_{ba}[1+V_L'(0)\Phi_0]+\frac{i\alpha}{2}\varkappa_1{\rm Tr}[\gamma^\mu(1-\gamma_5)H_c]\xi^\dagger_{ba}(\mathcal{M}_+)_{cb}+\frac{i\alpha}{2}\varkappa_2{\rm Tr}[\gamma^\mu(1-\gamma_5)H_b]\xi^\dagger_{ba}{\rm str}_c(\mathcal{M}_+)_{cc}. \quad (17)$$

When bosonizing the current J^μ, one could envisage an arbitrary function $V_L(\Phi_0)$ in the first term of the right of Eq. (17), expanded in a Taylor series. Only the terms linear in Φ_0 are relevant to our purpose and $V_L(0)$ can be normalized to 1 [16]. The appearance of $V_L'(0)$ is yet another artifact of the

[6]Corrections at $\mathcal{O}(1/m_b)$ have been discussed in [22–24].

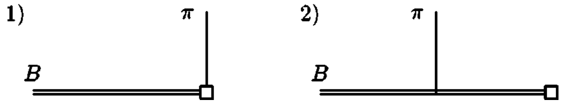

FIG. 1. The point (1) and the pole (2) tree level Feynman diagrams contributing to *heavy→light* transition form factors. The box denotes the weak current insertion.

quenched theory. The phase of the heavy meson can be chosen in such a way that $V_L'(0)$ is completely imaginary, whereas the constants α, $\varkappa_1$, and $\varkappa_2$ are real. Notice that, at leading order in the chiral expansion, the constant α is simply the heavy-light meson decay constant in the static limit $(m_Q\to\infty)$, i.e., $\hat{f}$ in Eq. (12).

III. CHIRAL CORRECTIONS TO $f_p(v\cdot p)$ AND $f_v(v\cdot p)$

In this section we explain the main steps undertaken to compute the chiral logarithmic corrections for $\hat{f}$, $f_p(v\cdot p)$, and $f_v(v\cdot p)$. Our results for the decay constant $\hat{f}$ agree with those of Refs. [15,16]. They will be used in Sec. V C to construct the ratios that are independent of counterterms. In Ref. [25] ChPT was applied to compute the heavy-to-light form factors. We repeat that calculation and extend it to the quenched case.

A. $f_p^{\text{Tree}}(v\cdot p)$ and $f_v^{\text{Tree}}(v\cdot p)$

We start the discussion by writing the tree level expressions

$$f_p(v\cdot p)=\frac{\alpha}{f}\,\frac{g}{v\cdot p+\Delta_i^*},\qquad f_v(v\cdot p)=\frac{\alpha}{f},\qquad \hat{f}=\alpha. \tag{18}$$

The point and the pole diagrams, depicted in Fig. 1, describe f_v and f_p, respectively. The index "i" in $\Delta_i^*=m_{B_i^*}-m_{B_i}$ labels the light quark flavor. When necessary, we will use P_{ij} to denote the light pseudoscalar mesons with valence quark content $q_i\bar{q}_j$.

Although the heavy quark spin symmetry suggests $\Delta_i^*\to 0$, we will keep this term finite because it provides the pole at $m_{B_i^*}^2$ to the form factor f_p, or equivalently to the form factors $F_{+,T}$.[7] The form factor f_v (or F_0), on the other hand, does not depend on $(v\cdot p)$ at the tree level.

B. $f_p^{\text{one-loop}}(v\cdot p)$ and $f_v^{\text{one-loop}}(v\cdot p)$

Neglecting the crosses, all the graphs shown in Fig. 2 describe the one-loop chiral contributions to the form factors $f_{v,p}(v\cdot p)$ in the full ChPT. In the quenched theory, in contrast, graphs both with and without crosses appear. The cross denotes the so-called hairpin vertex, i.e., the dipole term in the "pion" propagator:

$$\frac{1}{p^2-m_P^2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}-\frac{\alpha_0 p^2-m_0^2}{(p^2-m_P^2)^2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}. \tag{19}$$

In the computation of the loop integrals, the naive dimensional regularization has been used, with the renormalization prescription modified minimal substraction scheme $(\overline{\text{MS}})+1$, where $\bar{\Delta}=2/\epsilon-\gamma+\ln(4\pi)+1$ is subtracted [13]. We neglect the isospin symmetry breaking $(m_u=m_d\equiv m_q)$, as well as the mass differences between B,B_s,B^*,B_s^* meson states, whenever they appear in the loop. The resulting expressions can be written as follows.

Quenched ChPT:

$$f_p^{B_j\to P_{ij}}(v\cdot p)=\frac{\alpha}{f}\,\frac{g}{v\cdot p+\Delta_i^*}\left[1+\delta f_p^{B_j\to P_{ij}}+\frac{k_1}{2}m_j-4L_5\frac{4\mu_0}{f^2}(m_i+m_j)\right],$$

$$f_v^{B_j\to P_{ij}}(v\cdot p)=\frac{\alpha}{f}\left[1+\delta f_v^{B_j\to P_{ij}}+\left(\frac{k_1}{2}+\varkappa_1\right)m_j-4L_5\frac{4\mu_0}{f^2}(m_i+m_j)\right],$$

$$\hat{f}_j=\alpha\left[1+\delta\hat{f}_j+\left(\frac{k_1}{2}+\varkappa_1\right)m_j\right]. \tag{20}$$

Full ChPT:

$$f_p^{B_j\to P_{ij}}(v\cdot p)=\frac{\alpha}{f}\,\frac{g}{v\cdot p+\Delta_i^*}\left[1+\delta f_p^{B_j\to P_{ij}}+\frac{k_1}{2}m_j-4L_5\frac{4\mu_0}{f^2}(m_i+m_j)+\left(\frac{k_2}{2}-8L_4\frac{4\mu_0}{f^2}\right)(m_u+m_d+m_s)\right],$$

$$f_v^{B_j\to P_{ij}}(v\cdot p)=\frac{\alpha}{f}\left[1+\delta f_v^{B_j\to P_{ij}}+\left(\frac{k_1}{2}+\varkappa_1\right)m_j-4L_5\frac{4\mu_0}{f^2}(m_i+m_j)+\left(\frac{k_2}{2}+\varkappa_2-8L_4\frac{4\mu_0}{f^2}\right)(m_u+m_d+m_s)\right],$$

$$\hat{f}_j=\alpha\left[1+\delta\hat{f}_j+\left(\frac{k_1}{2}+\varkappa_1\right)m_j+\left(\frac{k_2}{2}+\varkappa_2\right)\times(m_u+m_d+m_s)\right]. \tag{21}$$

[7] The pole dominance is easily seen if one rewrites the denominator of $f_p(v\cdot p)$ as $v\cdot p+\Delta^*=(m_{B^*}/2)(1-q^2/m_{B^*}^2)$, where the corrections $(m_{B^*}-m_B)/m_B$ and m_P^2/m_B^2 are neglected.

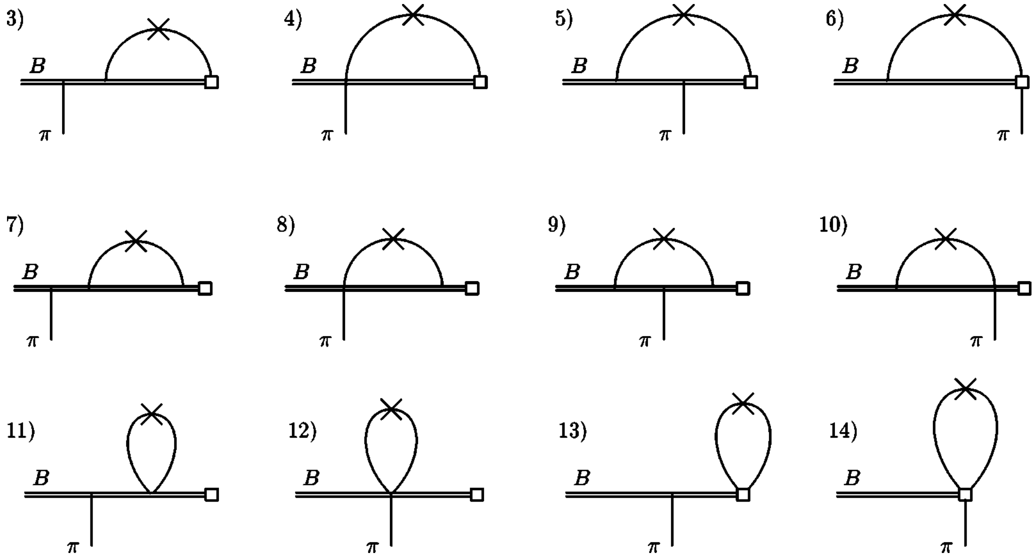

FIG. 2. The one-loop contributions to the $B\to\pi$ transition. Double (single) lines denote the heavy (light) meson, while the weak current insertion is depicted by the empty box. Crosses stand for the m_0 hairpin vertex 1 in Fig. 1. Each graph represents two Feynman diagrams in quenched ChPT: (a) the diagram without the hairpin vertex (cross) and (b) the same diagram with a cross. In full ChPT only the diagrams without the cross are present. The amplitudes corresponding to the diagrams are listed in Appendix B.

We separated the chiral loop corrections (δf) from those involving the counterterms. The loop corrections to the form factors are written as

$$\delta f_{v,p}^{B_j\to P_{ij}}=\sum_I \delta f_{v,p}^{(I)}+\frac{1}{2}\delta Z_{B_j}^{\rm loop}+\frac{1}{2}\delta Z_{P_{ij}}^{\rm loop}, \qquad (22)$$

where the sum runs over all diagrams shown in Fig. 2, and $\delta Z_{B_j(P_{ij})}$ encodes the chiral loop contributions arising from the wave function renormalization of the heavy-light and light-light meson, respectively. The explicit expressions are lengthy and are collected in Appendixes (for the quenched) and (for the full theory). As can be seen from Eqs. (20) and (21), no dependence on $(v\cdot p)$ arises from the counterterms. The modification of the tree-level $(v\cdot p)$ dependence of the form factors $f_{v,p}$ is entirely due to the chiral loop corrections. It is worth noticing that in the quenched approximation the form factor $f_v^{B\to\pi}$ is completely independent of $(v\cdot p)$. This is so because the contribution from diagram 4 in Fig. 2 vanishes in the isospin limit $m_u=m_d$.

An important feature emerging from this calculation is that the quenched form factors $f_{v,p}$ and quenched decay constant $\hat{f}$ suffer from the common quenched pathology, that is, from the presence of the quenched chiral logs $\propto m_0^2\log(m_P^2/\mu^2)$. Such terms are divergent in the limit $m_P\to 0$, suggesting that in the quenched approximation the chiral limit for any of $F_{+,0,T}^{B\to\pi}$ or f_B is not defined. This feature is also present in the light meson sector, e.g., for the chiral condensate, the light meson decay constant consisting of nondegenerate quark flavors, etc. [26,27].

IV. CHOICE OF PARAMETERS

For numerical analysis of the expressions given in Eqs. (20) and (21) we need to make a specific choice of quite a number of low energy constants. To do so we will mainly rely on the existing lattice data. In Table I we collect the parameters whose (range of) values we were able to fix. In the following we briefly discuss each of those values.

α: This constant is equal to the heavy-light decay constant in the static limit of QCD. Its value can be obtained from $f_D\sqrt{m_D}$ and/or $f_B\sqrt{m_B}$, which are then extrapolated (in inverse heavy-light meson mass) to the infinite mass limit as

$$f_H\sqrt{m_H}=\alpha(1+A/m_H+B/m_H^2+\cdots). \qquad (23)$$

From lattice QCD and QCD sum rules, it is known that the slope A is large and negative, whereas the value of the curvature B is small. Therefore, to a good accuracy, one can set $B=0$ and neglect higher terms in $1/m_H$. In this way we obtain the following.

From extensive quenched lattice simulations by the MILC Collaboration, one can deduce $\alpha^{\rm quench}=0.45(5)$ GeV$^{3/2}$ [28]. In the same paper, they also present the results of their unquenched simulations, from which we extract $\alpha^{\rm full}=0.53(7)$ GeV$^{3/2}$. These numbers agree quite well with the

TABLE I. Low energy constants whose determination is discussed in the text.

Coupling	Full theory	Quenched theory
α (GeV$^{3/2}$)	0.56 ± 0.04	0.48 ± 0.03
g	0.50 ± 0.10	0.56 ± 0.12
f(GeV)	0.130	0.124 ± 0.004
μ_0 (GeV)	1.14 ± 0.20	1.13 ± 0.04
$L_4(\times 10^{-3})$	$-(0.5\pm 0.5)$	(0.0 ± 0.5)
$L_5(\times 10^{-3})$	0.8 ± 0.5	0.8 ± 0.2
m_0 (GeV)	–	0.64 ± 0.06
g' (GeV)	–	-0.6 to 0.6

values obtained by CP-PACS Collaboration, namely, $\alpha^{\rm quench}=0.50(4)$ GeV$^{3/2}$, and $\alpha^{\rm full}=0.57(6)$ GeV$^{3/2}$ [29]. Notice that both references use the same treatment of the heavy quark on the lattice (the so-called Fermilab formalism).

The UKQCD [30] and APE [31] Collaborations compute heavy-light meson decay constants using the fully relativistic lattice QCD, but only for the mesons of masses $m_H\in(1.8,2.7)$ GeV. From a linear fit of the form (23), from their quenched data UKQCD obtain[8] $\alpha^{\rm quench}=0.49(4)$ GeV$^{3/2}$, in agreement with the previous result by APE, $\alpha^{\rm quench}=0.48(5)$ GeV$^{3/2}$.

Recent results obtained by using the QCD sum rules agree quite well with the above unquenched values. From the compilation of the QCD sum rule estimates in Ref. [32], one has $\alpha^{\rm full}=0.58(9)$ GeV$^{3/2}$ (for the most recent result see [33]).

g: This constant is related to the phenomenological coupling $g_{D^*D\pi}$ as

$$g_{D^*D\pi}=\frac{2\sqrt{m_D m_{D^*}}}{f_\pi}g, \qquad (24)$$

where one can also set $m_{D^*}=m_D$ because the above relation is defined only in the static limit, in which the heavy quark spin symmetry is exact. From the experimentally measured total width of the D^{*+} meson, one gets $g_c^{\rm full}=0.59(7)$ [34]. The subscript "c" warns us that the value is obtained in the charm quark mass sector. On the lattice, that value has been computed recently in the quenched approximation, leading to $g_c^{\rm quench}=0.66(9)$ [35]. To get the value of g one needs to extrapolate to the infinite heavy quark mass limit, i.e., $g_Q=g+\gamma/m_H+\cdots$. The lattice data of Ref. [35] suggest that the slope γ is negligible, and thus $g/g_c\approx 1$. On the other hand, the LCSR calculation suggests that the same slope is significant and positive [36]. By neglecting the terms of $\mathcal{O}(1/m_H^2)$ and higher, Ref. [36] leads to $g/g_c\approx 0.7$. We will take the average of the two (lattice and LCSR), that is, $g/g_c\approx 0.85$, and add the difference in the error bars of both quenched and unquenched values.[9]

f: To get the chiral coupling constant we will use $f_\pi=132$ MeV and $f_K=160$ MeV, and the fact that $m_s/m_q=24.4\pm 1.5$ [38]. After linearly extrapolating to the chiral limit ($m_q\to 0$), we get $f=130$ MeV. The recent extensive quenched lattice study with Wilson fermions gives $f=119\pm 7$ MeV [39], whereas the one with staggered fermions gave $f=125\pm 9$ MeV [40]. The latter result was also obtained on larger lattices with domain wall fermions, namely, $f=125\pm 7$ MeV [41]. As a weighted average, we will take $f=124\pm 4$ MeV.

μ_0: From the Gell-Mann–Oakes–Renner formula [42], it is easy to identify $\mu_0=-\langle\bar{q}q\rangle/f_\pi^2$. Its value (in the full theory) can be fixed by using $\langle\bar{q}q\rangle^{\overline{\rm MS}}(2\ {\rm GeV})=-[267(16)\ {\rm MeV}]^3$ [43]. The quenched value is extracted from the lattice data. Recent results in the $\overline{\rm MS}$ scheme (at the renormalization scale 2 GeV) are $\mu_0=1.13(4)$ GeV [39] and $\mu_0=1.10(11)$ GeV [44]. With these numbers and by using $f=124\pm 4$ MeV, the corresponding numerical values for the chiral condensate in the quenched approximation are $\langle\bar{q}q\rangle^{\overline{\rm MS}}(2\ {\rm GeV})=-[259(6)\ {\rm MeV}]^3$ and $\langle\bar{q}q\rangle^{\overline{\rm MS}}(2\ {\rm GeV})=-[257\ (10)\ {\rm MeV}]^3$.

$L_{4,5}$: These two couplings have been extensively discussed in the literature [14]. Their values in the full theory, at the scale $\mu\simeq 1$ GeV, are given in Table I. In quenched QCD, the coupling L_5 was recently computed in Ref. [45], with the result $\alpha_5=0.99(26)$ and $L_5=\alpha_5/(128\pi^2)=0.8(2)\times 10^{-3}$. On the other hand, the quenched estimate of L_4 is not available. On the basis of the large N_c expansion we only know that L_4 is smaller than L_5 (see Ref. [13]). We will take it to be zero and vary by $\pm 0.5\times 10^{-3}$.

m_0: This mass characterizes the magnitude of the hairpin insertion (crosses in Fig. 2). It enters in the coefficient of the quenched chiral log, which, in the literature, is often referred to as $\delta=m_0^2/(24\pi^2 f_\pi^2)$. The precise value of δ is unknown at present, mainly because in realistic lattice studies it is very difficult (if possible at all) to resolve the finite lattice volume effects from the quenched chiral logs. This is why it is still not completely clear whether the observed deviations from a linear dependence of the pseudoscalar meson squared (m_P^2) on the light quark masses (m_q) is due to the presence of the chiral logs, or if it is an artifact of the lattice geometry (see, e.g., Ref. [46]). With this remark in mind, we now quote recent values for the parameter δ, as obtained from lattice studies of the dependence of m_P^2 on m_q: with Wilson fermions, $\delta=0.10(2)$ [39]; with modified Wilson fermions, $\delta=0.073(20)$ [47];[10] with domain wall fermions, $\delta=0.029(7)$ [48]; with overlap fermions, $\delta=(0.0$–$0.2)$ [49] and $\delta=(0.2$–$0.3)$ [50].

For the pre-1996 results see the review in [27]. The value of δ obtained by the CP-PACS Collaboration stands out because they made an extensive high statistics study on large lattice volumes and extrapolated to the continuum [39]. A worrisome aspect of this value, however, is that the light pseudoscalar mesons that they access directly lie in the range $m_P\in(300,750)$ MeV, for which the significance of the chiral logs may be questionable. Keeping this comment in mind and using their $\delta=0.10(2)$, we get $m_0=0.64(6)$ GeV.

g': The value of this constant can be obtained from $g_{BB^*\eta'}$ and $g_{DD^*\eta'}$, the couplings of the lowest lying heavy-light meson doublet the light quenched η' state. Such

[8] We thank G. Lacagnina for communicating this result to us.

[9] We are aware of the result of Ref. [37], $g=0.42(9)$, where this coupling has been computed for the first time on the lattice, in the static HQET. However, in view of the insignificant statistics, very coarse lattice and only two light quark masses, the value $g=0.42(9)$ should be considered as exploratory. The improved calculation of g, along the lines described in Ref. [37] would be most welcome, though.

[10] The final result of Ref. [47], $\delta=0.065(13)$, is obtained by combining various ways of extracting this quantity from the lattice data. For an easier comparison with other groups, we quote the result given in Eq. (29) of Ref. [47], which is obviously fully consistent with their final value.

a lattice study has not been made so far. To get a rough estimate we may rely on the large N_c limit from which one expects $|g'|<g$.

As for the other parameters appearing in the quenched expressions, in the numerical analysis, we will first set them to zero and then vary their values in the ranges suggested by the large N_c expansion. More specifically, we take $|V_L'|\leqslant 0.5, |\varkappa_1|, |k_1| \leqslant 32\mu_0 L_5/f^2$, and $|\varkappa_2|=|k_2|=0$. The parameter α_0 will be varied in the range $|\alpha_0|<0.1$. It is important to stress that the effect of the variation of these latter parameters is completely negligible for our numerical estimates.

Note that the low energy constants in Table I are extracted from lattice data at the tree level of ChPT (except for the counterterms).[11]

V. QUENCHING ERRORS

In this section we will use the expressions for the form factors (20) and (21), the parameters listed in Table I, to get an estimate of the quenching errors. We reiterate that the results of this section refer to the zeroth order in the $1/m_H$ expansion and to the leading logarithmic chiral corrections, with the specific choice of parameters discussed in the previous section. We also stress that in all the following discussion we will keep the strange quark mass fixed to its quenched value of $m_s^{\overline{\rm MS}}(2\ \text{GeV})=105$ MeV, and $m_s^{\rm full}/m_s^{\rm quench}\simeq 0.85$ [51]. To examine the quenching errors we will study the following ratios:

$$Q_{p,v}(v\cdot p, m_q/m_s)=\frac{f_{p,v}^{\rm full}(v\cdot p)-f_{p,v}^{\rm quench}(v\cdot p)}{f_{p,v}^{\rm full}(v\cdot p)}. \quad (25)$$

The parameter that makes the strongest impact on the results for $Q_{p,v}$ is g'. As stated above, its value is expected to lie in the range $-g\leqslant g'\leqslant g$. Since even its sign is not known we will distinguish between the two "extreme" cases $g'=-g$ and $g'=+g$.

A. $B\to\pi$ transition

To examine the quenching errors for $B\to\pi$ decay we cannot set the pion mass to its physical value, because the spurious quenched chiral logs would become dominant. However, we cannot go far away from the chiral limit either, because the (Q)ChPT approach then becomes inadequate. To those two competing requirements, we should add a third: a desire to be sufficiently close to the region of quark masses (i.e., pseudoscalar meson masses squared) probed by the current quenched lattice studies [5–9]. It is not clear whether or not a mass interval for which all of the above requirements are satisfied exists. We will assume that they are satisfied for $m_P\approx 330$ MeV. From Eq. (4), a "pion" of mass $m_P\approx 330$ MeV is composed of two quarks of mass corresponding to $r=m_q/m_s\simeq 0.25$, with respect to the strange quark mass m_s. In Fig. 3 we plot the ratios $Q_{p,v}(v\cdot p, r=0.25)$.

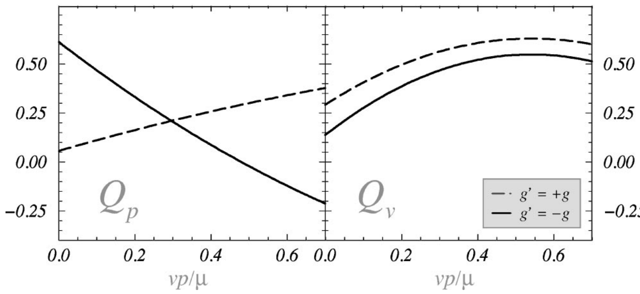

FIG. 3. The ratios $Q_{p,v}(v\cdot p)$ are defined in Eq. (25). The plots refer to mesons consisting of two degenerate (valence) quarks with mass $m_q=rm_s$ and $r=0.25$. The counterterm coefficients $k_{1,2}, \varkappa_{1,2}$, as well as $|V_L'(0)|$ are neglected, while $\mu=1$ GeV$\simeq\Lambda_\chi$. Values of other parameters are given in Table I.

We notice that, regardless of the value chosen for g', the quenching errors in the form factor $f_p(v\cdot p)$ are not excessively large for most of the $v\cdot p$ where ChPT is applicable. This is important since this form factor is the only one entering the $B\to\pi\ell\nu$ decay rate, from which we hope to be able to extract the value for $|V_{ub}|$.

By specifying $g'=+g$, we observe that $Q_p(v\cdot p)>0$, for any $(v\cdot p)/\mu<0.7$ and $r\geqslant 0.2$. In other words, quenched values for f_p are smaller than unquenched ones. If we take $g'=-g$ instead, the ratio $Q_p(v\cdot p)$ has a zero. The point of zero quenching errors in Fig. 3 is found at $(v\cdot p/\mu)=0.47(3)$ for $r=0.25$ and the values of parameters given in Table I. Figure 4 shows the curve of vanishing quenching errors in the $v\cdot p$–r plane, i.e., $Q_p(v\cdot p, r)=0\pm 10\%$. This curve really exists for $g'=-g$, while for $g'=+g$ only the part corresponding to $Q_p(v\cdot p,r)=+10\%$ can be reached for $r<0.4$. In summary, it is possible to find combinations of the pion mass and the pion recoil energy such that the quenching errors in the dominant form factor $f_p(v\cdot p)$ are kept under the 10% level.

From Fig. 3 we also see that there exists a point $(v\cdot p/\mu)\approx 0.3$ such that the ratio Q_p is independent of the value of g'. At that point, we get

$$Q_p(0.3\mu, r\simeq 0.25)\simeq 20\%. \quad (26)$$

This result is a useful estimate of the quenching error for realistic values of $(v\cdot p)$ and r, which are currently probed on the lattice. For the reader's convenience, we have fitted the points corresponding to $(\partial f_p/\partial g')=0$ to a polynomial in $r\in[0.1,1]$ and obtained

$$\frac{v\cdot p}{\mu}=0.17(1)+0.8(1)r-1.3(2)r^2+0.58(2)r^3. \quad (27)$$

[11]The standard procedure would involve "undressing" the chiral loops from the measured low energy constants. This procedure is not applicable when using the lattice data because direct lattice computations are made at $m_\pi\gtrsim 400$ MeV, and the physical results are obtained through a linear or quadratic extrapolation in $m_\pi^2\to(m_\pi^2)^{\rm phys}$. These extrapolations do not include the chiral logs, so "undressing" the chiral loops would lead to unrealistic estimates of the low energy constants. For this reason, we adopted the tree-level approximation in extracting the low energy constants.

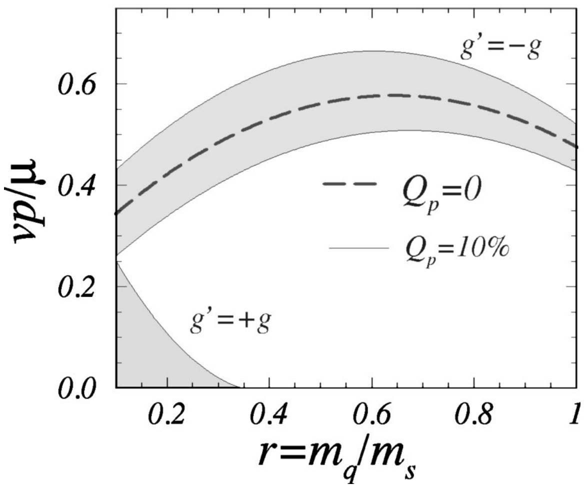

FIG. 4. The thick dashed line corresponds to the zero quenching errors $[Q_p(v\cdot p,r)=0]$ in the case when $g'=-g$ (such a line is not accessible for $g'=+g$). The shaded region corresponds to the variation of $Q_p(v\cdot p,r)$ by 10%. Notice that in the case $g'=-g$, the upper (lower) curve corresponds to $Q_p(v\cdot p,r)$ equal to -10% $(+10\%)$. For the case $g'=+g$, only the region $Q_p(v\cdot p,r)<+10\%$ is shown.

We also checked that for $0.1<r\lesssim 0.35$, with r and $(v\cdot p)$ satisfying Eq. (27), the quenching errors are kept under the 25% level. This concludes our discussion of the form factor f_p.

The situation with the form factor $f_v(v\cdot p)$ is much worse. From Fig. 3, we see that the quenching errors are in the range 30–60 %, and drop below that level only for larger recoils for which the present approach is not appropriate. The quenching errors remain large when varying r in the range $r\in[0.1,1]$. A somewhat less pessimistic situation is present at zero recoil $(v\cdot p=0)$ where, contrary to the case of f_p, the form factor f_v *can* be extracted from the lattice data. For $m_P=330$ MeV, at zero recoil we get

$$Q_v(0,r=0.25)\simeq\begin{cases} 8\% & (g'=+g), \\ 23\% & (g'=-g). \end{cases}$$

Once again, we warn the reader that the numerical estimates made in this section are obtained for the set of low energy constants specified in Table I.

B. $B\to K$ transition

We now turn to the case of a kaon in the final state. In the discussion we shall fix one of the valence quarks to the strange quark mass, and vary the other one $(m_q=rm_s)$ in the range $1/25\lesssim r<0.5$. In this way the "kaon" mass is varied in the range $m_K^{\rm phys}<m_P<600$ MeV. For simplicity we consider $(v\cdot p)/\mu\simeq 0.3$ (reasonably small recoil), and examine the quenching ratios

$$Q_{p,v}^K(0.3\mu,r)=\frac{f_{p,v}^{\rm full}(v\cdot p)-f_{p,v}^{\rm quench}(v\cdot p)}{f_{p,v}^{\rm full}(v\cdot p)}. \qquad (28)$$

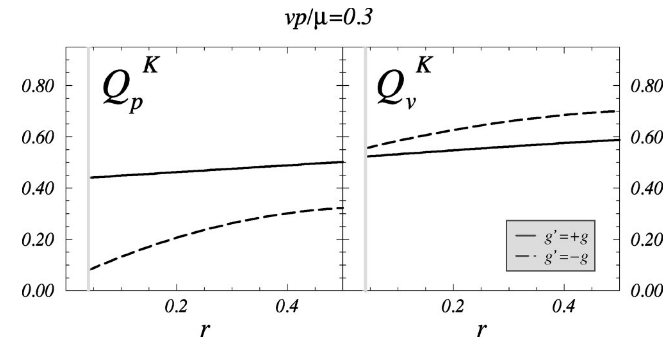

FIG. 5. The ratios $Q_{p,v}(v\cdot p)$ from Eq. (28). The plots refer to mesons whose one valence quark is fixed to the s-quark mass, while the other has the value $m_q=rm_s$. The thick gray vertical line marks $r\simeq 1/25$ for which the physical kaon mass is reached. As before, the counterterm coefficients $k_{1,2},\varkappa_{1,2}$ as well as $V_L'(0)$ are neglected, while $\mu=1$ GeV.

In the quenched expressions for $f_{p,v}$, given in Appendix B, we set for the active quark $M_i^2=4\mu_0 m_s$, while for the spectator one we take $M_j^2=4\mu_0 m_s r$ [see Eqs. (B2), (B3)]. As in the previous subsection, in Fig. 5 we plot $Q_{p,v}^K$ for the two "extreme" scenarios, namely, $g'=+g$ and $g'=-g$. We again observe that, regardless of the value for g', the quenching errors on the form factor $f_v(v\cdot p)$, $Q_v^K>0.5$. The quenching errors Q_p^K, are only moderately large if $g'=-g$, and are large for $g'=+g$.

Therefore, from this and the preceding subsection, we see that in both channels $B\to\pi$ and $B\to K$ our approach suggests that the quenching errors in the form factor $f_v(v\cdot p)$ [i.e., $F_0(q^2)$] are large $\gtrsim 30\%$. The quenching errors in $f_p(v\cdot p)$ [i.e., $F_{+,T}(q^2)$], on the other hand, depend crucially on the value of the low energy constant g': they are large for $g'>0$, and "*not so large*" for $g'<0$.

Before closing this subsection let us mention that in the quenched lattice studies the kaon is usually considered as a composite state of two degenerate quarks. Using $m_P^2=4\mu_0 m_s r$, one varies r in order to reach the physical kaon mass $m_P=m_K^{\rm phys}$, which occurs for $r\simeq 0.5$. One may wonder if the form factors with such a kaon differ from the ones in which the quarks in the kaon are nondegenerate, with one of the quarks fixed to the strange mass and the other varying toward $r\simeq 1/25$ (i.e., the physical kaon mass). To keep the mass of the pseudoscalar meson the same in both situations, we will vary $r_{\rm deg}\in[0.6,0.8]$ and $r_{\rm ndg}=2r_{\rm deg}-1\in[0.2,0.6]$. As before, we take $(v\cdot p/\mu)=0.3$ and examine the following ratio

$$R_{v,p}(r_{\rm deg})=\left.\frac{f_{v,p}^{\rm ndg}(v\cdot p,r_{\rm ndg})-f_{v,p}^{\rm deg}(v\cdot p,r_{\rm deg})}{f_{v,p}^{\rm ndg}(v\cdot p,r_{\rm ndg})}\right|_{r_{\rm ndg}=2r_{\rm deg}-1} \qquad (29)$$

in the quenched theory. From Fig. 6, we again observe the same dichotomy: for $g'=-g$ the situation is quite favorable, i.e., the errors due to degeneracy in the quark masses are very small, whereas for $g'=+g$ the form factor $f_p^{\rm deg}$ is highly overestimated with respect to $f_p^{\rm ndg}$.

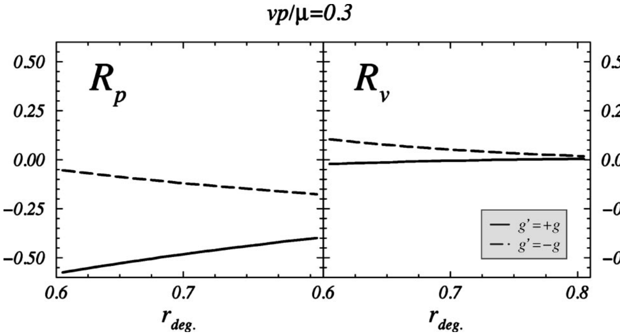

FIG. 6. The ratios $R_{p,v}$ defined in Eq. (29), measuring the errors induced by the degeneracy in the "kaon" quark masses in the quenched calculations.

C. Useful ratios

The double ratio $f_v^{B_s\to K}f_B/f_v^{B\to K}f_{B_s}$ is very gratifying from the ChPT point of view. In a double ratio of the form

$$\frac{f_{v(Q)}^{B_j\to P_{ij}}}{f_{v(Q)}^{B_i\to P_{ji}}}\;\frac{f_{B_i(Q)}}{f_{B_j(Q)}}=1+\delta f_{v(Q)}^{B_j\to P_{ij}}-\delta f_{v(Q)}^{B_i\to P_{ji}}+\delta f_{B_i(Q)}-\delta f_{B_j(Q)}, \tag{30}$$

where $B_j\sim b\bar{q}_j$ and $P_{ij}\sim q_i\bar{q}_j$, the dependence on α cancels completely and so do the counterterms. Quenching errors in this quantity are thus far more reliably predictable in the framework of ChPT than for the separate form factors. This quantity might then prove useful in future lattice simulations with both $B_s\to P$ and $B\to P$ transitions.

A similar (partial) cancellation of dependence on counterterms occurs also for the quenching error in the ratio $f_{p,v}^{B\to K}/f_{p,v}^{B\to\pi}$:

$$\frac{f_{p,v(Q)}^{B_j\to P_{ij}}}{f_{p,v(Q)}^{B_k\to P_{lk}}}-\frac{f_{p,v}^{B_j\to P_{ij}}}{f_{p,v}^{B_k\to P_{lk}}}=\delta f_{p,v(Q)}^{B_j\to P_{ij}}-\delta f_{p,v(Q)}^{B_k\to P_{lk}}-\delta f_{p,v}^{B_j\to P_{ij}}+\delta f_{p,v}^{B_k\to P_{lk}}-\frac{8}{f^2}(L_5^Q-L_5)m_K^2, \tag{31}$$

which does not depend on counterterms if $L_5^{\rm full}=L_5^{\rm quench}$. In Eq. (31) we have neglected counterterms suppressed by m_π^2/m_K^2 and assumed that δf is small (δf is independent of counterterms by construction).

VI. CHIRAL EXTRAPOLATION IN $B\to\pi$ DECAY

A. Quenched case

So far the quenched lattice studies of the $B\to\pi$ transition matrix element were confined to the region of not very light pseudoscalar meson masses which (from the point of view of QChPT) is almost fortunate since one stays away from the region in which the spurious quenched chiral logs dominate over the other (physical) contributions. Supposing that one manages to push the lattice studies toward ever lighter "pions," the quenched chiral log will become a more important effect. From the expressions presented in Sec. III and in Appendix B, for the very light meson $m_P<(v\cdot p)$, we obtain

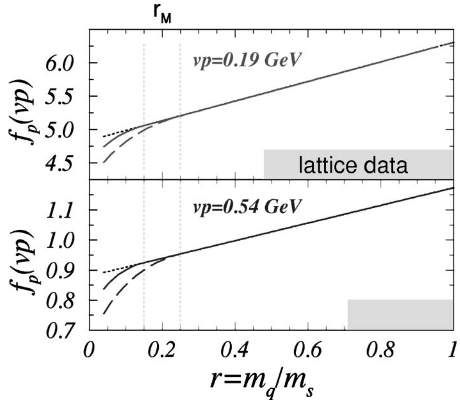

FIG. 7. Illustration of the effect of inclusion of the chiral log in extrapolating to the physical pion from the masses accessed from the lattice simulations (marked by the shaded boxes). The full (dashed) curves are obtained by including the chiral logs in extrapolation starting from $r_M=0.15$ and from 0.25 (gray vertical lines) using full (unquenched) ChPT. For comparison we also show the (dotted) line corresponding to the linear extrapolation.

$$f_p(v\cdot p)=\frac{\alpha g}{f(v\cdot p+\Delta^*)}\left[1+(c_\ell^Q+c_\ell^p m_P^2)\ln\left(\frac{m_P^2}{\mu^2}\right)+c_0^p+c_1^p m_P+c_2^p m_P^2+\cdots\right],$$

$$f_v(v\cdot p)=\frac{\alpha}{f}\left[1+(c_\ell^Q+c_\ell^v m_P^2)\ln\left(\frac{m_P^2}{\mu^2}\right)+c_0^v+c_2^v m_P^2+\cdots\right]. \tag{32}$$

The quenched chiral log term is proportional to

$$c_\ell^Q=-\frac{m_0^2(1+3g^2)}{6(4\pi f)^2}, \tag{33}$$

and it is a quenched artifact that diverges in the chiral limit. The presence of this term has an important consequence, that by approaching ever lighter m_P, the form factors $f_p^{\rm quench}$ and $f_v^{\rm quench}$ *increase*. This is in contrast to the full (dynamical) theory in which the effect is the opposite, i.e., the chiral logs lower the form factors for small masses, as we shall see in the next subsection (see Fig. 7). The form factor $f_p(v\cdot p)$ also picks a term linear in m_P in this limit, which is yet another artifact of the quenched approximation. The accompanying coefficient reads

$$c_1^p=-\frac{g^2m_0^2}{(4\pi f)^2}\,\frac{4\pi}{3v\cdot p}. \tag{34}$$

B. Unquenched case: An illustration

The unquenched equivalents to the expressions (32) are not straightforward to derive. The main obstacle comes

TABLE II. The parameters describing the linear chiral extrapolation for the $B\to\pi$ form factors $f^{\rm latt}_{v,p}(v\cdot p)$ at fixed $(v\cdot p)$, as indicated in Eq. (36). The values are those obtained in the quenched lattice studies in Refs. [6,7].

$(v\cdot p)$	$a_v^{(0)}$ (GeV$^{1/2}$)	$a_v^{(1)}$ (GeV$^{-3/2}$)	$a_p^{(0)}$ (GeV$^{-1/2}$)	$a_p^{(1)}$ (GeV$^{-5/2}$)	Reference
0.55 GeV	$2.5(2)^{+0.0}_{-0.6}$	$1.1(2)^{+0.0}_{-0.2}$	$0.9(1)^{+0.0}_{-0.2}$	$0.7(1)^{+0.0}_{-0.1}$	[6]
0.19 GeV	0.8(3)	0.3(3)	4.8(4)	3.6(4.5)	[7]

from the fact that, instead of one mass in the integrals $I_2(M,v\cdot p)$ and $J_1(M,v\cdot p)$, one now has three masses: $M\in\{m_\pi,\ m_K,\ \text{and}\ m_\eta\}$. The behavior of those integrals depends on the sign of $1-(v\cdot p)/M$ (see Appendix A). The variation of the pion mass entails a change of m_K and m_η, which straddle $(v\cdot p)$. This then changes the behavior of the integrals I_2 and J_1. For the $B\to\pi$ transition, we have

$$f_p(v\cdot p)=\frac{\alpha g}{f(v\cdot p+\Delta^*)}\Bigg\{1+\frac{1}{(4\pi f)^2}\Bigg[g^2\Bigg(4J_1(m_\pi,v\cdot p)+3J_1(m_K,v\cdot p)+\frac{2}{3}J_1(m_\eta,v\cdot p)\Bigg)-\frac{1+3g^2}{12}(9m_\pi^2\log(m_\pi^2)+6m_K^2\log(m_K^2)+m_\eta^2\log(m_\eta^2))\Bigg]+d_0^p+\cdots\Bigg\},$$

$$f_v(v\cdot p)=\frac{\alpha}{f}\Bigg\{1+\frac{1}{(4\pi f)^2}\Bigg[\frac{15-27g^2}{12}m_\pi^2\log(m_\pi^2)+\frac{1-3g^2}{2}m_K^2\log(m_K^2)-\frac{1+3g^2}{12}m_\eta^2\log(m_\eta^2)+2I_2(m_\pi,v\cdot p)+I_2(m_K,v\cdot p)\Bigg]+d_0^v+\cdots\Bigg\}, \tag{35}$$

where $d_0^{v,p}$ is a constant and the ellipses stand for the higher order terms in the $m^2_{\pi,K,\eta}$ expansion.

To exemplify the impact of the chiral logs, we will now use the existing (quenched) lattice results for the $B\to\pi$ form factors presented in Refs. [6] and [7], in which the chiral extrapolation has been made linearly, i.e.,

$$f^{latt}_{v,p}(v\cdot p)=a^{(0)}_{v,p}(v\cdot p)+a^{(1)}_{v,p}(v\cdot p)m_P^2. \tag{36}$$

The parameters $a^{(0,1)}_{v,p}$ of Ref. [7] are obtained by fitting in the region of light pseudoscalar mesons that corresponds to $m_P\in(450,800)$ MeV. Those of Ref. [6] are obtained from a fit in $m_P\in(540,840)$ MeV. The numerical values are given in Table II.[12]

[12]We are particularly indebted to Tetsuya Onogi for communicating these results to us.

We will now assume that (a) these results are the same in the full (unquenched) theory;[13] (b) the form (36) holds true down to a point $m_M\approx 250$ MeV or $m_M\approx 330$ MeV, where we smoothly match the lattice and ChPT results. The full ChPT form factor, given in Appendix B, is used from the matching point m_M down to the physical pion mass. In other words, for a fixed value of $(v\cdot p)$, we take

$$f_{p,v}(m_P^2)=\theta(m_P^2-m_M^2)f^{\rm latt}_{p,v}(m_P^2)+\theta(m_M^2-m_P^2)[f^{\rm ChPT}_{p,v}(m_P^2)-[f^{\rm ChPT}_{p,v}(m_M^2)-f^{\rm latt}_{p,v}(m_M^2)]-(\partial f^{\rm ChPT}_{p,v}/\partial m_P^2|_{m_P^2=m_M^2}-a^{(1)}_{v,p})(m_P^2-m_M^2)]. \tag{37}$$

In Fig. 7 we show the effect for the form factor $f_p(v\cdot p)$. We observe that the form factor obtained by including the chiral logs in the extrapolation to $m_P^2=m_\pi^2$ is smaller than the one obtained by extrapolating linearly (dotted lines in the plot). The amount of that suppression obviously depends on the value of $m_M^2=4\mu_0 m_s r_M$: the effect of the chiral log is smaller for smaller r_M. In our example we took $r_M=0.15$ ($m_M\simeq 250$ MeV) and $r_M=0.25$ ($m_M\simeq 330$ MeV). Using

$$\mathrm{err}(f_{p,v})=\frac{f^{\rm Eq.\ (37)}_{p,v}-f^{\rm Eq.\ (36)}_{p,v}}{f^{\rm Eq.\ (36)}_{p,v}} \tag{38}$$

to measure the error due to chiral logs that were not included in the extrapolation to the physical pion mass, we obtain the following results.

r_M=0.15:

$$\mathrm{err}[f_p(v\cdot p=0.19\ \mathrm{GeV})]\simeq -2\%,$$

$$\mathrm{err}[f_p(v\cdot p=0.54\ \mathrm{GeV})]\simeq -5\%.$$

r_M=0.25:

$$\mathrm{err}[f_p(v\cdot p=0.19\mathrm{GeV})]\simeq -7\%,$$

$$\mathrm{err}[f_p(v\cdot p=0.54\mathrm{GeV})]\simeq -15\%.$$

The above analysis applied to the form factor f_v leads to even smaller uncertainties: $\mathrm{err}(f_v)_{r_M=0.15}<3\%$ and $\mathrm{err}(f_v)_{r_M=0.25}<6\%$, for both values of $(v\cdot p)$.

[13]Since the purpose of the discussion in this section is to illustrate the impact of the chiral logs on the result of an extrapolation to the physical pion mass, this assumption should not worry the reader.

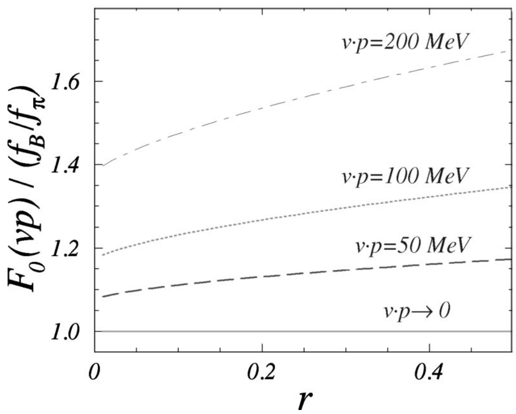

FIG. 8. The ratio $F_0(v\cdot p)/(f_B/f_\pi)$ or $f_v/(\hat{f}/f)$, which satisfies the soft pion theorem at zero recoil $(v\cdot p\to 0)$. The figure shows the ratio in the dynamical theory at which the violation of the soft pion theorem grows fast with the recoil momentum. The illustration is provided for three momenta and for masses corresponding to $r\in(0,0.5)$, i.e., $m_\pi\in(0,0.5)$ GeV. Notice that in the quenched theory this ratio does not depend on $(v\cdot p)$ and is equal to 1.

This exercise is made just to illustrate how one can proceed in order to get an estimate of the systematic uncertainties due to the chiral extrapolation. As we saw, the amount of estimated uncertainty is highly sensitive to the choice of the point r_M, which is why the outcome of this exercise should be considered only as a rough estimate.

It is important to stress again that had we used the quenched expressions 32 to guide the chiral extrapolation, the result would stay above the result of the linear extrapolation, precisely the opposite to what happens in the full theory (which we show in Fig. 7).

Finally, we would like to make a comment on the ratio $f_v/(\hat{f}/f)$ or equivalenty $F_0(v\cdot p)/(f_B/f_\pi)$, which, according to the soft pion theorem, should be equal to 1 at the zero recoil point $v\cdot p\to 0$. In the quenched theory the form factor F_0 is independent of $v\cdot p$ and its chiral corrections are exactly the same as in f_B/f_π. Therefore the ratio is indeed 1 for all combinations of $(r,v\cdot p)$. In the full (unquenched) theory, in contrast, the chiral corrections cancel only at $v\cdot p\to 0$ and the soft pion theorem is satisfied. It is worth noticing, however, that when a small recoil is introduced the violation of the soft pion theorem is quite large in the full theory (see Fig. 8).

VII. CONCLUSIONS

In this paper we explored an approach that contains at its core the leading order in the heavy quark expansion and the next-to-leading order in ChPT, to derive expressions for the heavy-to-light pseudoscalar semileptonic form factors. The expressions are worked out in both standard and quenched ChPT. In the latter case we observe a familiar feature of quenched calculations, namely, the appearance of divergent chiral logarithms (quenched chiral logs), which makes it harder to compare the results with the formulas obtained in standard ChPT for $B\to\pi$ form factors: one should find a fiducial window in which the masses are not light so that the quenched chiral logs do not dominate the QChPT expressions, yet small enough for the chiral expansion to be meaningful. Furthermore, for the numerical estimates a number of low energy constants must be fixed by using both the available experimental data and the results of quenched lattice simulations. Our numerical estimates in which we use the QChPT expressions are highly sensitive to the value of the parameter g' (the coupling of the doublet of heavy-light mesons to a light η' meson). Until a lattice computation of that parameter is made, we are not able to make firm quantitative statements. Even information about the sign of g' would be welcome. It turns out that, for $g'=-g$ (which, on the basis of the large N_c expansion, is the limiting value), we get a less pessimistic scenario: one can even find combinations of $(v\cdot p)$ and the light meson mass m_P such that the quenching errors in $F_{+,T}(q^2)$ vanish. We were also able to find combinations of $(v\cdot p)$ and m_P such that the quenching errors do not depend on the value of g'; at these points and for small recoils the quenching errors are between 15% and 25%. We reiterate that the numerical estimates do depend on the specific choice of the low energy constants.

As for the form factor $F_0(q^2)$, the present approach suggests that the quenching errors are large regardless of the value of the coupling g', the quenched value being larger than the unquenched one. Only at the point corresponding to zero recoil are those errors reasonably small. Away from that point, they are large.

The same observations apply also when the final meson is a kaon. In that case, by using the QChPT expressions, we were able to verify that the form factors obtained for the kaon consisting of degenerate and nondegenerate quarks are effectively indistinguishable, provided the value of the coupling is $g'=-g$. For $g'=+g$ these uncertainties also become large.

Finally, the formulas presented in this work may be useful in assessing the systematic uncertainty due to chiral extrapolations. We showed how to include the chiral logs to extrapolate from the region in which the pion is heavier than 400 MeV. As could have been anticipated, the estimated uncertainty of the chiral extrapolation depends on the mass from which the chiral logs are included in the extrapolation.

We also provided the quenched chiral log coefficient, which might be useful if lattice calculations are performed with very light mesons.

We verified that the ratio $F_0^{B\to\pi}/(f_B/f_\pi)$ satisfies the soft pion theorem, i.e., it is equal to 1 at zero recoil, in both theories. In the quenched theory that value remains unchanged even when a small recoil momentum is introduced. In the dynamical theory, instead, this ratio is significantly larger than 1 away from zero recoil.

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APPENDIX A: CHIRAL LOOP INTEGRALS

In this appendix we list the dimensionally regularized integrals encountered in the course of the calculation. For more details, see [52] and references therein:

$$i\mu^{\epsilon}\int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{1}{q^2-m^2}=\frac{1}{16\pi^2}I_1(m),$$

$$i\mu^{\epsilon}\int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{1}{(q^2-m^2)(q\cdot v-\Delta)}=\frac{1}{16\pi^2}\frac{1}{\Delta}I_2(m,\Delta), \tag{A1}$$

where

$$I_1(m)=m^2\ln\left(\frac{m^2}{\mu^2}\right)-m^2\bar{\Delta},$$

$$I_2(m,\Delta)=-2\Delta^2\ln\left(\frac{m^2}{\mu^2}\right)-4\Delta^2F\left(\frac{m}{\Delta}\right)+2\Delta^2(1+\bar{\Delta}), \tag{A2}$$

where $\bar{\Delta}=2/\epsilon-\gamma+\ln(4\pi)+1$. The function $F(x)$ was calculated in Ref. [53], for both negative and positive values of the argument:

$$F\left(\frac{1}{x}\right)=\begin{cases}-\dfrac{\sqrt{1-x^2}}{x}\left[\dfrac{\pi}{2}-\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)\right], & |x|\leqslant 1,\\ \dfrac{\sqrt{x^2-1}}{x}\ln(x+\sqrt{x^2-1}), & |x|\geqslant 1.\end{cases} \tag{A3}$$

In addition to the integrals (A1), one also needs the following two:

$$i\mu^{\epsilon}\int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^\mu}{(q^2-m^2)(q\cdot v-\Delta)}=\frac{v^\mu}{16\pi^2}[I_2(m,\Delta)+I_1(m)], \tag{A4}$$

$$i\mu^{\epsilon}\int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}}\frac{q^\mu q^\nu}{(q^2-m^2)(q\cdot v-\Delta)}=\frac{1}{16\pi^2}\Delta[J_1(m,\Delta)\eta^{\mu\nu}+J_2(m,\Delta)v^\mu v^\nu],$$

with

$$J_1(m,\Delta)=\left(-m^2+\frac{2}{3}\Delta^2\right)\ln\left(\frac{m^2}{\mu^2}\right)+\frac{4}{3}(\Delta^2-m^2)F\left(\frac{m}{\Delta}\right)-\frac{2}{3}\Delta^2(1+\bar{\Delta})+\frac{1}{3}m^2(2+3\bar{\Delta})+\frac{2}{3}m^2-\frac{4}{9}\Delta^2, \tag{A5a}$$

$$J_2(m,\Delta)=\left(2m^2-\frac{8}{3}\Delta^2\right)\ln\left(\frac{m^2}{\mu^2}\right)-\frac{4}{3}(4\Delta^2-m^2)F\left(\frac{m}{\Delta}\right)+\frac{8}{3}\Delta^2(1+\bar{\Delta})-\frac{2}{3}m^2(1+3\bar{\Delta})-\frac{2}{3}m^2+\frac{4}{9}\Delta^2. \tag{A5b}$$

The functions $J_1(m,\Delta),J_2(m,\Delta)$ differ from the ones in Ref. [22] by the last two terms in Eq. (A5), which are of $\mathcal{O}(m^2,\Delta^2)$. These additional (finite) terms originate from the fact that $\eta^{\mu\nu}$ is the $(4-\epsilon)$-dimensional metric tensor.

APPENDIX B: EXPLICIT EXPRESSIONS FOR THE ONE LOOP CHIRAL CORRECTIONS

As already mentioned in the body of the paper, the chiral loop corrections to the form factors can be written in the form

$$\delta f_{p,v}=\sum_I \delta f_{p,v}^{(I)}+\frac{1}{2}\delta Z_B^{\rm loop}+\frac{1}{2}\delta Z_P^{\rm loop}, \tag{B1}$$

where the sum goes over all the graphs depicted in Fig. 2. In what follows we give the explicit expressions for both form factors graph by graph and in both chiral theories (quenched and standard).

1. Quenched theory

In the calculation of one-loop contributions, we made several approximations in order to simplify the final expressions. We make use of the fact that $v\cdot p>\Delta^*$ for the $B\to P$ transition and thus consistently neglect the mass differences between B, B^*, B_s, and B_s^* mesons in the loops. This neglect induces a spurious singularity in the expression for the diagrams 7a and 7b in Fig. 2, at $v\cdot p\to 0$. To handle those singularities we follow the proposal by Falk and Grinstein [25] and resum the corresponding diagrams and then subtract the term that would renormalize the B^* meson mass. We recall that a (b) superscripts distinguish the diagrams without (with) hairpin insertion. The formulas for the $B_j\to P_{ij}$ transition (with the valence quark content of mesons $B_j\sim b\bar{q}_j$ and $P_{ij}\sim q_i\bar{q}_j$) are expressed in terms of the pseudoscalar meson mass $M_i^2=4\mu_0 m_i$:

$$\delta f_p^{(7a)}=\frac{6gg'}{(4\pi f)^2}\left[J_1(M_i,v\cdot p)-\frac{1}{v\cdot p}\frac{2\pi}{3}M_i^3\right],$$

$$\delta f_p^{(7b)}=-\frac{g^2}{(4\pi f)^2}\left[\alpha_0+(\alpha_0M_i^2-m_0^2)\frac{\partial}{\partial M_i^2}\right]\times\left(J_1(M_i,v\cdot p)-\frac{1}{v\cdot p}\frac{2\pi}{3}M_i^3\right),$$

$$\delta f_p^{(9a)}=\frac{gg'}{(4\pi f)^2}\bigg[J_1(M_i,v\cdot p)+J_1(M_j,v\cdot p)-\frac{1}{v\cdot p}\,\frac{2\pi}{3}(M_i^3+M_j^3)\bigg],$$

$$\delta f_p^{(9b)}=\frac{g^2}{3(4\pi f)^2}\bigg\{\frac{\alpha_0M_j^2-m_0^2}{M_j^2-M_i^2}\bigg[\frac{1}{v\cdot p}\,\frac{2\pi}{3}M_j^3-J_1(M_j,v\cdot p)\bigg]-\frac{\alpha_0M_i^2-m_0^2}{M_j^2-M_i^2}\bigg[\frac{1}{v\cdot p}\,\frac{2\pi}{3}M_i^3-J_1(M_i,v\cdot p)\bigg]\bigg\},$$

$$\delta f_p^{(12b)}=\frac{T}{18(4\pi f)^2}\bigg\{\frac{2m_0^2-\alpha_0(M_i^2+M_j^2)}{M_j^2-M_i^2}[I_1(M_j)-I_1(M_i)]+(\alpha_0M_i^2-m_0^2)\frac{\partial I_1(M_i)}{\partial M_i^2}+(\alpha_0M_j^2-m_0^2)\frac{\partial I_1(M_j)}{\partial M_j^2}\bigg\},$$

$$\delta f_p^{(13a)}=-\frac{iV_L'(0)f}{\sqrt{6}(4\pi f)^2}I_1(M_i),$$

$$\delta f_p^{(13b)}=\frac{1}{6(4\pi f)^2}\bigg[\alpha_0I_1(M_i)+(\alpha_0M_i^2-m_0^2)\frac{\partial I_1(M_i)}{\partial M_i^2}\bigg],\tag{B2}$$

where $T=0$ for $i=j$, and $T=1$ otherwise. The functions $I_1(m),J_1(m,\Delta)$ are given in Appendix A. As for the form factor $f_v(v\cdot p)$, the nonzero chiral loop corrections are

$$\delta f_v^{(4a)}=-\frac{iTV_L'(0)f}{\sqrt{6}(4\pi f)^2}\bigg[I_2(M_j,v\cdot p)-I_2(M_i,v\cdot p)+\frac{1}{2}[I_1(M_j)-I_1(M_i)]\bigg],$$

$$\delta f_v^{(4b)}=\frac{T}{6(4\pi f)^2}\bigg\{\frac{1}{M_j^2-M_i^2}\{(\alpha_0M_j^2-m_0^2)[I_1(M_j)+2I_2(M_j,v\cdot p)]-(\alpha_0M_i^2-m_0^2)[I_1(M_i)+2I_2(M_i,v\cdot p)]\}-\bigg(\alpha_0+(\alpha_0M_i^2-m_0^2)\frac{\partial}{\partial M_i^2}\bigg)\times[I_1(M_i)+2I_2(M_i,v\cdot p)]\bigg\}$$

$$\delta f_v^{(14a)}=-\frac{iV_L'(0)f}{2\sqrt{6}(4\pi f)^2}[I_1(M_i)+I_1(M_j)],$$

$$\delta f_v^{(14b)}=\frac{1}{18(4\pi f)^2}\bigg\{(\alpha_0M_j^2-m_0^2)\bigg[\frac{I_1(M_j)}{M_j^2-M_i^2}+\frac{\partial I_1(M_j)}{\partial M_j^2}\bigg]+(\alpha_0M_i^2-m_0^2)\bigg[\frac{I_1(M_i)}{M_i^2-M_j^2}+\frac{\partial I_1(M_i)}{\partial M_i^2}\bigg]+\alpha_0(I_1(M_i)+I_1(M_j))\bigg\}.\tag{B3}$$

In the wave function renormalization factors $Z_{B,P}$, we separate the one-loop chiral contribution $\delta Z_{B,P}^{\rm loop}$ from the pieces coming from the counterterms $\delta Z_{B,P}^{\rm ct}$, i.e.,

$$Z_{B,P}=1+\delta Z_{B,P}=1+\delta Z_{B,P}^{\rm loop}+\delta Z_{B,P}^{\rm ct}.\tag{B4}$$

More specifically,

$$\delta Z_{B_j}^{\rm loop}=\frac{1}{(4\pi f)^2}\bigg[(2g^2\alpha_0M_j^2-6gg'M_j^2-g^2m_0^2)\ln\bigg(\frac{M_j^2}{\mu^2}\bigg)+\alpha_0g^2M_j^2-m_0^2g^2+(-2g^2M_j^2\alpha_0+6gg'M_j^2+g^2m_0^2)\bar{\Delta}\bigg],\tag{B5a}$$

$$\delta Z_{P_{ij}}^{\rm loop}=\frac{1}{9(4\pi f)^2}\bigg\{\frac{\ln(M_j^2/M_i^2)}{M_j^2-M_i^2}[2\alpha_0M_j^2M_i^2-m_0^2(M_j^2+M_i^2)]+2m_0^2-\alpha_0(M_i^2+M_j^2)\bigg\},\tag{B5b}$$

while the counterterms contribute as

$$\delta Z_{B_j}^{\rm ct}=k_1m_j,\qquad \delta Z_{P_{ij}}^{\rm ct}=-8L_5\frac{4\mu_0}{f^2}(m_i+m_j).\tag{B6}$$

Thus the wave function renormalization factor for π and K mesons reads

$$Z_\pi=1-8L_5\frac{4\mu_0}{f^2}\;2m_q,\tag{B7a}$$

$$Z_K=1-\frac{1}{9(4\pi f)^2}\bigg\{\frac{\ln(m_s/m_q)}{(m_K^2-m_\pi^2)}(\alpha_0m_\pi^4-\alpha_02m_K^2m_\pi^2+m_0^2m_K^2)+2\alpha_0m_K^2-2m_0^2\bigg\}-8L_5\frac{4\mu_0}{f^2}(m_q+m_s),\tag{B7b}$$

where we ignore the isospin-breaking effects and set $m_u=m_d=m_q$.

Finally, we also display the expression for the heavy-light meson decay constant:

$$f_{B_i}=\frac{\alpha}{\sqrt{m_B}}\left\{1+\frac{1}{6(4\pi f)^2}\left[[\alpha_0-if\sqrt{6}V_L'(0)]I_1(M_i)+(\alpha_0M_i^2-m_0^2)\frac{\partial I_1(M_i)}{\partial M_i^2}\right]+\varkappa_1 m_i+\frac{1}{2}\delta Z_{B_i}\right\},\quad \text{(B8)}$$

in agreement with Refs. [15,16].

2. Full (unquenched) theory

In this subsection we present the expressions for the form factors in the full theory. The nonanalytic contributions to the form factors in this theory have already been calculated in Ref. [25]. Our results also include the analytic terms. As in the quenched case, we work in the isospin limit $m_u=m_d=m_q$ and neglect the differences of heavy meson masses in the loops. We now list the results for $\delta f_{v,p}^{(I)}$ in $B_j\to P_{ij}$ mediated by the $(V-A)$ operator, where P_{ij} stands for the light pseudoscalar meson with the valence quark content $\overline{q}_i q_j$.

The form factor $f_p(v\cdot p)$ receives the following one-loop corrections:

$$\delta f_p^{(7a)}=\frac{3g^2}{(4\pi f)^2}\left\{\sum_{P'}C^{(7a)}_{B_jP'P_{ij}}\left[J_1(m_{P'},v\cdot p)-\frac{1}{v\cdot p}\frac{2\pi}{3}m_{P'}^3\right]\right\},$$

$$\delta f_p^{(9a)}=-\frac{g^2}{(4\pi f)^2}\left\{\sum_{P'}C^{(9a)}_{B_jP'P_{ij}}\left[J_1(m_{P'},v\cdot p)-\frac{1}{v\cdot p}\frac{2\pi}{3}m_{P'}^3\right]\right\},$$

$$\delta f_p^{(12a)}=\frac{1}{(4\pi f)^2}\left[\sum_{P'}C^{(12a)}_{B_jP'P_{ij}}I_1(m_{P'})\right],$$

$$\delta f_p^{(13a)}=\frac{1}{(4\pi f)^2}\left[\sum_{P'}C^{(13a)}_{B_jP'P_{ij}}I_1(m_{P'})\right],\quad \text{(B9)}$$

where the coefficients $C_{B_jP'P_{ij}}$ depend on the final and initial states. A detailed list of coefficients includes the following.

For the $B\to K$ transition,

$$C^{(7a)}_{B\pi K}=0,\ C^{(7a)}_{BKK}=2,\ C^{(7a)}_{B\eta K}=2/3;\quad C^{(9a)}_{B\pi K}=0,$$

$$C^{(9a)}_{BKK}=0,\ C^{(9a)}_{B\eta K}=\frac{1}{3};$$

$$C^{(12a)}_{B\pi K}=-\frac{1}{4},\ C^{(12a)}_{BKK}=-\frac{1}{2},\ C^{(12a)}_{B\eta K}=-\frac{1}{4};$$

$$C^{(13a)}_{B\pi K}=0,\ C^{(13a)}_{BKK}=-1,\ C^{(13a)}_{B\eta K}=-\frac{1}{3}.$$

For the $B\to\pi$ transition,

$$C^{(7a)}_{B\pi\pi}=\frac{3}{2},\ C^{(7a)}_{BK\pi}=1,\ C^{(7a)}_{B\eta\pi}=\frac{1}{6};\quad C^{(9a)}_{B\pi\pi}=\frac{1}{2},$$

$$C^{(9a)}_{BK\pi}=0,\ C^{(9a)}_{B\eta\pi}=-\frac{1}{6};$$

$$C^{(12a)}_{B\pi\pi}=-\frac{2}{3},\ C^{(12a)}_{BK\pi}=-\frac{1}{3},\ C^{(12a)}_{B\eta\pi}=0;$$

$$C^{(13a)}_{B\pi\pi}=-\frac{3}{4},\ C^{(13a)}_{BK\pi}=-\frac{1}{2},\ C^{(13a)}_{B\eta\pi}=-\frac{1}{12}.$$

For the $B_s\to K$ transition,

$$C^{(7a)}_{B_s\pi K}=\frac{3}{2},\ C^{(7a)}_{B_sKK}=1,\ C^{(7a)}_{B_s\eta K}=\frac{1}{6};\quad C^{(9a)}_{B_s\pi K}=0,$$

$$C^{(9a)}_{B_sKK}=0,\ C^{(9a)}_{B_s\eta K}=\frac{1}{3};$$

$$C^{(12a)}_{B_s\pi K}=-\frac{1}{4},\ C^{(12a)}_{B_sKK}=-\frac{1}{2},\ C^{(12a)}_{B_s\eta K}=-\frac{1}{4};$$

$$C^{(13a)}_{B_s\pi K}=-\frac{3}{4},\ C^{(13a)}_{B_sKK}=-\frac{1}{2},\ C^{(13a)}_{B_s\eta K}=-\frac{1}{12}.$$

The nonvanishing one-loop chiral corrections to the $f_v(v\cdot p)$ form factor are

$$\delta f_v^{(4a)}=\frac{1}{(4\pi f)^2}\left\{\sum_{P'}D^{(4a)}_{B_jP'P_{ij}}\left[I_2(m_{P'},v\cdot p)+\frac{1}{2}I_1(m_{P'})\right]\right\},$$

$$\delta f_v^{(14a)}=\frac{1}{(4\pi f)^2}\left[\sum_{P'}D^{(14a)}_{B_jP'P_{ij}}I_1(m_{P'})\right],\quad \text{(B10)}$$

where the process-dependent coefficients $D_{B_jP'P_{ij}}$ are as follows.

For the $B\to K$ transition,

$$D^{(4a)}_{B\pi K}=0,\ D^{(4a)}_{BKK}=2,\ D^{(4a)}_{B\eta K}=1;\quad D^{(14a)}_{B\pi K}=-1/4,$$

$$D^{(14a)}_{BKK}=-1/2,\ D^{(14a)}_{B\eta K}=-1/12.$$

For the $B\to\pi$ transition,

$$D^{(4a)}_{B\pi\pi}=2,\ D^{(4a)}_{BK\pi}=1,\ D^{(4a)}_{B\eta\pi}=0;\quad D^{(14a)}_{B\pi\pi}=-5/12,$$

$$D^{(14a)}_{BK\pi}=-1/3,\ D^{(14a)}_{B\eta\pi}=-1/12.$$

For the $B_s\to K$ transition,

$$D^{(4a)}_{B_s\pi K}=3/2,\ D^{(4a)}_{B_sKK}=1,\ D^{(4a)}_{B_s\eta K}=1/2;$$

$$D^{(14a)}_{B_s\pi K}=-1/4,\ D^{(14a)}_{B_sKK}=-1/2,\quad D^{(14a)}_{B_s\eta K}=-1/12.$$

The wave function renormalization factors Z for B mesons in the full theory are

$$Z_{B_{u,d}}=1-\frac{3g^2}{(4\pi f)^2}\left[\frac{3}{2}I_1(m_\pi)+I_1(m_K)+\frac{1}{6}I_1(m_\eta)\right]+k_1 m_q+k_2(m_u+m_d+m_s),$$

$$Z_{B_s}=1-\frac{3g^2}{(4\pi f)^2}\left[2I_1(m_K)+\frac{2}{3}I_1(m_\eta)\right]+k_1 m_s+k_2(m_u+m_d+m_s). \tag{B11}$$

while for light mesons we have

$$Z_K=1+\frac{1}{(4\pi f)^2}\left[\frac{1}{2}I_1(m_\pi)+I_1(m_K)+\frac{1}{2}I_1(m_\eta)\right]-8L_5\frac{4\mu_0}{f^2}(m_q+m_s)-16L_4\frac{4\mu_0}{f^2}(m_u+m_d+m_s),$$

$$Z_\pi=1+\frac{2}{3(4\pi f)^2}[2I_1(m_\pi)+I_1(m_K)]-8L_5\frac{4\mu_0}{f^2}2m_q-16L_4\frac{4\mu_0}{f^2}(m_u+m_d+m_s). \tag{B12}$$

As in the previous subsection we close the list of results by showing also the corresponding formulas for the heavy-light decay constants. We have

$$f_{B_s}=\frac{\alpha}{\sqrt{m_B}}\left(1-\frac{1}{(4\pi f)^2}\left[I_1(m_K)+\frac{1}{3}I_1(m_\eta)\right]+\varkappa_1 m_s+\varkappa_2(m_u+m_d+m_s)+\frac{1}{2}\delta Z_{B_s}\right),$$

$$f_{B_{u,d}}=\frac{\alpha}{\sqrt{m_B}}\left(1-\frac{1}{(4\pi f)^2}\left[\frac{3}{4}I_1(m_\pi)+\frac{1}{2}I_1(m_K)+\frac{1}{12}I_1(m_\eta)\right]+\varkappa_1 m_q+\varkappa_2(m_u+m_d+m_s)+\frac{1}{2}\delta Z_{B_{u,d}}\right), \tag{B13}$$

in agreement with the results of Refs. [15,16].

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