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Manifest *CP* violation from Majorana phases

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We hunt for and discuss manifestly *CP*-violating effects which are mediated by Majorana phases. These phases are present if the standard model neutrinos are Majorana particles. We argue that while Majorana phases do affect the strength of neutrinoless double beta decay (a well known fact), they do so in a way that involves no manifest violation of *CP*. The conditions for manifestly *CP*-violating phenomena—differences between the rates for *CP*-mirror-image processes—are presented, and three examples are discussed: (i) neutrino $\leftrightarrow$ antineutrino oscillation; (ii) rare decays of *K* and *B* mesons and their antiparticles; and (iii) the lepton asymmetry generated by the decay of hypothetical very heavy right-handed "seesaw" neutrinos. We also find that, for the case of degenerate light neutrinos, manifestly *CP*-violating effects in neutrino $\leftrightarrow$ antineutrino oscillation vanish, although flavor-changing transitions do not. Finally, we comment on leptogenesis with degenerate right-handed neutrinos, and contrast it with the neutrino $\leftrightarrow$ antineutrino oscillation case.

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I. INTRODUCTION

If neutrinos are Majorana particles, then the leptonic mixing matrix U can contain more *CP*-violating phases than its quark counterpart (for the same number of generations) [1]. The additional phases, known as Majorana phases, have no effect on neutrino oscillation. Indeed, the only current or proposed neutrino experiment that could in principle provide evidence of Majorana phases is the search for neutrinoless double beta decay, $0\nu\beta\beta$ [2]. The rate for this process depends not only on the neutrino masses and mixing angles, but also on *CP*-violating phases, notably including the Majorana phases. However, even if experimental and theoretical uncertainties should permit us to obtain evidence for a nonvanishing Majorana phase from the rate of $0\nu\beta\beta$ [3], the effect of *CP*-violating phases on this reaction is not a manifestly *CP*-violating phenomenon. By the latter, we mean a *CP*-odd effect—a difference between the rate for some physical process and that for its *CP* mirror image. While *CP*-odd phases in the leptonic mixing matrix do affect the rate Γ for $0\nu\beta\beta$, they do so in a *CP*-even way; that is, their effect on the rate Γ for some particular nuclear double beta decay is the same as on the rate $\overline{\Gamma}$ for the *CP*-mirror-image decay (the decay of an antinucleus), so that $\Gamma=\overline{\Gamma}$. Therefore, even if we could study the neutrinoless double beta decay of antinuclei (an impossibility in practice, to say the least), we would be unable to observe a "smoking gun" signal of *CP* violation due to Majorana phases.

In this paper, we ask whether Majorana phases, such as the more familiar *CP*-violating "Dirac" phase in the quark mixing matrix, can lead to *CP*-odd effects. If so, where and under what conditions can these effects occur, and what are they? Are they observable in practice?

An increasingly appealing explanation of the present baryon asymmetry in the Universe rests on early-universe "leptogenesis," resulting from *CP* violation in the decays of so-far hypothetical, very heavy Majorana neutral leptons [4]. The required *CP* violation in this process can come from Majorana phases. Furthermore, it is a *CP*-odd effect—a difference between two *CP*-mirror-image decays, one of which yields a lepton, the other an antilepton. Thus, Majorana phases can, in principle, yield *CP*-odd effects. However, the Majorana phases that act in the early Universe are not those in the mixing matrix U that governs light neutrino mixing [5]. Moreover, the role of these "early-universe phases" depends on the existence of hypothetical heavy Majorana leptons.[1] We therefore ask whether the Majorana phases in the light-neutrino mixing matrix U can lead to *CP*-odd effects that depend only on the (assumed) Majorana nature of the light neutrinos, and not on the existence of any additional Majorana particles.

We find that the answer is yes—Majorana phases in U can induce *CP*-odd effects. In particular, they do so in the process of "neutrino $\leftrightarrow$ antineutrino oscillations" [7]. By that we mean a process in which, for example, a neutrino "beam" is created by incoming positively charged leptons, but is measured in a detector via the production of negatively charged leptons. If Majorana phases are present, the rates for this process and for its *CP* mirror image (where the charges of the charged leptons are reversed) will, in general, differ. We explicitly point out why *CP*-odd effects can occur in neutrino $\leftrightarrow$ antineutrino oscillations but not in $0\nu\beta\beta$. We further discuss under what conditions the neutrino $\leftrightarrow$ antineutrino oscillation process occurs, and when *CP*-odd ef-

[1] The existence of heavy "right-handed neutrinos" is strongly motivated by the seesaw mechanism for generating light neutrino masses [6]. Unfortunately, even if this beautiful theoretical idea is correct, we may never be able to observe direct evidence for the existence of heavy right-handed neutrinos if their masses are indeed many orders of magnitude above the weak scale, as naively indicated by the experimental evidence for neutrino masses.

fects can be observed. For example, we point out that when the neutrino masses are degenerate, *CP*-odd effects disappear, but the neutrino $\leftrightarrow$ antineutrino oscillation process can still take place if the Majorana phases are nontrivial. The rate for this process still depends on the mixing angles in the mixing matrix. Thus, when Majorana phases are present, mixing angles continue to have physical consequences even when the neutrino masses are degenerate. This simple yet remarkable behavior is in marked contrast to the behavior of quark mixing, where flavor mixing would disappear (and the "mixing angles" become unphysical) if all the charge-2/3 quarks or all the charge-($-1/3$) ones were degenerate in mass.

Other processes where leptonic *CP*-odd effects could appear are the rare, lepton-number violating meson decays $K^\pm\to\pi^\mp\mu^\pm\mu^\pm$ and $B^\pm\to\pi^\mp\tau^\pm\tau^\pm$. Here, the rate for the K^+ decay could differ from that for the K^- decay, and similarly for the B^+ and B^- decays. However, we find that new sources of lepton-number violation (on top of the neutrino Majorana masses) are required in this case.

By picturing the neutrino $\leftrightarrow$ antineutrino oscillation process in terms of Feynman diagrams, and rearranging the pieces of these diagrams, we show that the *CP*-odd effect resulting in leptogenesis grows out of the Majorana phases in exactly the same way as the *CP*-odd effect in neutrino $\leftrightarrow$ antineutrino oscillation. This leads us to investigate whether leptogenesis also "disappears" when the masses of all the heavy Majorana leptons are of equal magnitude. We discuss under which conditions this would indeed be the case.

Unfortunately, if neutrinos interact only via standard model left-handed interactions, neutrino $\leftrightarrow$ antineutrino oscillations, while yielding interesting conceptual insights into the possible effects of Majorana phases, are virtually unobservable in practice. This is due to the fact that, because we consider the neutrino Majorana masses to be the source of lepton-number violation, the rate for neutrino $\leftrightarrow$ antineutrino oscillations is suppressed by powers of the neutrino masses (in units of the neutrino total energy). This is also true of the rates for the $\Delta L=2$ decays $K^\pm\to\pi^\mp\mu^\pm\mu^\pm$ and $B^\pm\to\pi^\mp\tau^\pm\tau^\pm$. Since these rates are proportional to positive powers of the neutrino masses, their associated branching ratios are expected to be of $O(10^{-22})$ [the present upper limits on these branching ratios are $O(10^{-9})$ [8]]. However, even though all these processes are unlikely to be observable in the foreseeable future, they provide clear illustrations of how, in principle, Majorana phases can lead to manifestly *CP*-violating effects in low-energy reactions.

Our presentation is the following: First, we define Majorana phases and discuss when they are potentially observable. Second, we discuss in some detail the process of neutrinoless double beta decay, and explain why *CP*-odd effects would not be present even if antinuclear double beta decay could be observed. We then proceed to outline the requirements for observing manifest *CP*-odd effects, and discuss in detail neutrino $\leftrightarrow$ antineutrino oscillations. Next, we show how *CP* phases in the neutrino sector can manifest themselves in differences between the rates for the lepton-number violating decays of K^+ and K^- and B^+ and B^-, and under what conditions. Finally, we comment on the relation between neutrino $\leftrightarrow$ antineutrino oscillations and heavy right-handed Majorana neutrino decays, paying special attention to the dependency of both processes on Majorana phases and the effect of mass-degenerate neutrino states.

II. MAJORANA PHASES

We assume that each neutrino mass eigenstate ν_i, $i=1,2,3,\ldots$ (with mass m_i), is a Majorana fermion. This means that ν_i is its own antiparticle, so that its field is its own charge conjugate, up to a phase factor. Thus,

$$\nu_i=\lambda_i\nu_i^c, \tag{2.1}$$

where the superscript c denotes charge conjugation and λ_i is a phase factor henceforth referred to as the charge conjugation phase factor.

We further assume that the neutrino coupling to charged leptons and the W boson is as prescribed by the standard model (SM). For massive neutrinos, the SM interaction Lagrangian is

$$\mathcal{L}_{\rm int}=-\frac{g}{\sqrt{2}}W_\rho^-\sum_{\alpha,i}\overline{l}_{\alpha L}\gamma^\rho U_{\alpha i}\nu_{iL}-\frac{g}{\sqrt{2}}W_\rho^+\sum_{\alpha,i}\overline{\nu}_{iL}U^*_{\alpha i}\gamma^\rho l_{\alpha L}. \tag{2.2}$$

Here, g is the semiweak coupling constant and α runs over the charged lepton flavors: $\alpha=e,\mu,\tau,\ldots$. The subscript L denotes chiral left-handed projection, and U is the unitary leptonic mixing matrix. It will become useful later to rewrite

$$\overline{l}_{\alpha L}\gamma^\rho U_{\alpha i}\nu_{iL}=-\overline{\nu^c}_{iR}\gamma^\rho U_{\alpha i}l^c_{\alpha R}=-\lambda_i\overline{\nu}_{iR}\gamma^\rho U_{\alpha i}l^c_{\alpha R}, \tag{2.3}$$

where λ_i is the charge conjugation phase factor defined in Eq. (2.1) and R denotes right-handed chiral projection.

As is the case in the quark sector, the leptonic mixing matrix is written in the basis where the charged lepton and (Majorana) neutrino mass matrices are real, positive, and diagonal. Generically, it can be written in the form

$$U=\mathbb{E}^{i\phi_\alpha/2}U'\mathbb{E}^{i\xi_i/2}, \tag{2.4}$$

where $\mathbb{E}^{i\phi_\alpha/2}=\mathrm{diag}(e^{i\phi_e/2},e^{i\phi_\mu/2},\ldots)$, $\mathbb{E}^{i\xi_i/2}=\mathrm{diag}(e^{i\xi_1/2},e^{i\xi_2/2},\ldots)$ are diagonal "phase matrices," and U' is a (non-generic) unitary mixing matrix. Within the SM, the phases contained in $\mathbb{E}^{i\phi_\alpha/2}$ are not physical, as they can be "absorbed" by redefining the right-handed charged lepton fields (which do not feel the charged-current weak interactions). The phases ξ_i are potentially observable, and will henceforth be *defined as Majorana phases.* For example, in the case of two lepton species, U' is real and parametrized by one mixing angle, while there is one potentially observable Majorana phase[2] $\xi\equiv\xi_2-\xi_1$. It is important to stress that if the neutrinos were Dirac particles, all phases contained in

[2]An overall phase, common to all neutrinos, is not physical. One is only sensitive to phase differences.

$\mathbb{E}^{i\xi_i/2}$ would also be unphysical, as they could be "absorbed" by redefining the SM singlet right-handed neutrino fields.

A Majorana phase is therefore characterized as one that is common to all elements of a given column of the leptonic mixing matrix, as defined in Eq. (2.2). That is, it is a phase that affects all $U_{\alpha i}$ equally, irrespective of the flavor α, for a given neutrino mass eigenstate ν_i. Of course, elements of a column of U may contain additional phases that are not common to the whole column [these are the "left-over" phases contained in U', as defined in Eq. (2.4)]. These phases will be *defined as Dirac phases.*[3]

As is well known, if U is not real (i.e., it contains nontrivial phases), the physical processes mediated by Eq. (2.2) need not be *CP* preserving.[4] However, while Dirac phases can lead to *CP* noninvariance irrespective of the nature of the neutrinos, Majorana phases can do so only if the neutrinos are Majorana particles. It is interesting to understand the origin of this fact by comparing a process that can occur regardless of the character of the neutrinos with a related one that can occur only if the neutrinos are Majorana particles.

First, we will analyze the process of "neutrino $\leftrightarrow$ neutrino" oscillation, for which there is ever-increasing experimental evidence. This oscillation may be viewed as the process

$$l_\alpha^- W^+ \to \nu \to l_\beta^- W^+, \tag{2.5}$$

in which the intermediate-state neutrino propagates a macroscopic distance L. This process is depicted in Fig. 1(a). The intermediate-state neutrino can be in any of the mass eigenstates ν_i. Thus, the amplitude A_L for this lepton-number (L) conserving process may be written schematically as

$$A_L = \sum_i \langle l_\beta^- W^+ | H_{\rm int} | \nu_i \rangle \langle \nu_i | H_{\rm int} | l_\alpha^- W^+ \rangle, \tag{2.6}$$

where $H_{\rm int}$ is the interaction Hamiltonian associated with Eq. (2.2). Now, $\langle l_\beta^- W^+ | H_{\rm int} | \nu_i \rangle \langle \nu_i | H_{\rm int} | l_\alpha^- W^+ \rangle \propto U_{\beta i} U_{\alpha i}^*$ [see Eq. (2.2)]. Thus, even if the ith column of U contains the Majorana phase factor $e^{i\xi_i/2}$, it is clear that it will cancel out of $U_{\beta i} U_{\alpha i}^*$ ($\forall\ \beta,\alpha$) and will consequently have no effect on the amplitude A_L.

Next we analyze a qualitatively different process: "neutrino $\leftrightarrow$ antineutrino" oscillation [7]. This is the reaction

$$l_\alpha^+ W^- \to \nu \to l_\beta^- W^+, \tag{2.7}$$

in which, once more, the intermediate neutrino travels a macroscopic distance L. This process is depicted in Fig. 1(b).

[3]The reason for the definition should be clear. If the neutrinos were Dirac fermions, all would-be Majorana phases could be "absorbed" by appropriately redefining the neutrino fields, and the only observable *CP*-odd effects would be parametrized by the Dirac phases.

[4]See [9] for a pedagogical discussion of the conditions which the massive neutrino Lagrangian must satisfy in order to necessarily conserve *CP*.

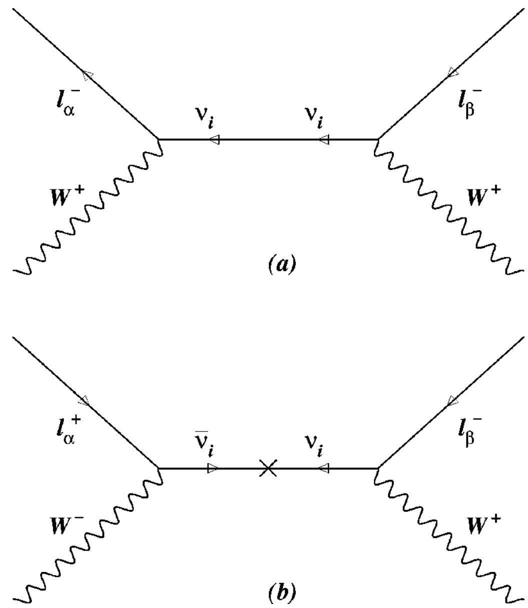

FIG. 1. Feynman diagrams for (a) neutrino $\leftrightarrow$ neutrino oscillation and (b) neutrino $\leftrightarrow$ antineutrino oscillation. Time flows from the left to the right, and the arrows represent the chirality of the various fermions. The $\times$ indicates a chirality flip in the neutrino propagator, which is proportional to the neutrino Majorana mass. Note that $\nu_i = \overline{\nu_i}$ up to a phase factor.

Unlike ordinary flavor oscillations [Eq. (2.5)], the process Eq. (2.7) can only occur if lepton number is no longer a good quantum number. This is exactly the case if the neutrinos have non-vanishing Majorana masses, which also implies that the neutrino mass eigenstates are Majorana particles.[5] As in ordinary flavor oscillations, the intermediate neutrino in Eq. (2.7) can be in any of the mass eigenstates ν_i. Thus, the amplitude $A_{\not L}$ for the lepton-number violating process Eq. (2.7) can be written schematically

$$A_{\not L} = \sum_i \langle l_\beta^- W^+ | H_{\rm int} | \nu_i \rangle \langle \nu_i | H_{\rm int} | l_\alpha^+ W^- \rangle. \tag{2.8}$$

As before, $\langle l_\beta^- W^+ | H_{\rm int} | \nu_i \rangle \propto U_{\beta i}$. However, the second bracket in Eq. (2.8) requires some care. Like the first bracket, $\langle \nu_i | H_{\rm int} | l_\alpha^+ W^- \rangle$ also comes from the first term in Eq. (2.2), but one should use the term as it was rewritten in Eq. (2.3). This is so that the field $\overline{\nu_i}$ in the second term of Eq. (2.8) can be contracted with the field ν_i in the first term in order to make the usual neutrino propagator, $\langle 0 | T(\nu_i \overline{\nu_i}) | 0 \rangle$. From Eq. (2.3), $\langle \nu_i | H_{\rm int} | l_\alpha^+ W^- \rangle \propto \lambda_i U_{\alpha i}$, so that

$$A_{\not L} = \sum_i (\lambda_i U_{\alpha i} U_{\beta i}) K_i, \tag{2.9}$$

where K_i is a kinematical factor.

[5]If *CPT* is also broken, the mass eigenstates are *not* Majorana particles even in the presence of Majorana mass terms [10]. We will assume throughout this paper that *CPT* is a good symmetry.

The phases of the $U_{\alpha i}$ and of λ_i all depend on the phase convention chosen for the state $|\nu_i\rangle$. However, it is readily shown that the product $\lambda_i U_{\alpha i} U_{\beta i}$ is phase-convention-free [11].[6] This means that the interference of the different terms that contribute to $A_{\not L}$ in Eq. (2.9) can lead to convention-free physical effects, which clearly depend on the Majorana phases of U. A Majorana phase factor in the ith column of U should be thought of as the phase factor present in this column for a fixed value of λ_i corresponding to the chosen phase convention for $|\nu_i\rangle$, By phase redefining $|\nu_i\rangle$, we can always remove the Majorana phase from the ith column of U, but this phase would then simply reappear in λ_i, leaving $A_{\not L}$ unchanged.

We conclude that when neutrinos are Majorana particles, the rates for lepton-number violating processes depend on the Majorana phases. Processes that do not involve lepton number violation in some form are not, at least at leading order, capable of exploring the leptonic mixing matrix in a way that would reveal the presence of Majorana phases. It should be emphasized, as pointed out by the authors of [12], that the rates for lepton-number conserving processes can show such a presence (which may lead to CP-odd effects [13]), provided that they receive a significant contribution from processes that violate lepton number "$1+(-1)$ times." These contributions, however, are very suppressed and unobservable under most circumstances.

III. NEUTRINOLESS DOUBLE BETA DECAY

The most promising way of probing the Majorana nature of the neutrino is to look for neutrinoless double beta decay ($0\nu\beta\beta$). This is the lepton-number violating nuclear decay process $Z\to(Z+2)+e^-e^-$, where $Z(Z+2)$ is the atomic number of the parent (daughter) nucleus. Assuming CPT invariance, the observation of this process would demonstrate that neutrinos are indeed Majorana particles [14]. If $0\nu\beta\beta$ does occur, it is very likely dominated by a mechanism in which the parent nucleus emits a pair of W^- bosons, turning into the daughter nucleus, and then the W^- bosons exchange one or another neutrino mass eigenstate to produce the two outgoing electrons. The heart of the mechanism is the second step: $W^-W^-\to e^-e^-$ via Majorana neutrino exchange. In the cross channel, this step is simply $e^+W^-\to\nu\to e^-W^+$, i.e., Eq. (2.7) for $l_\alpha=l_\beta=l_e$. Of course, the neutrino now only propagates a very short distance (of the size of a nucleus).

Assuming that Majorana neutrino exchange is indeed the dominant contribution to $0\nu\beta\beta$, its amplitude $A_{\beta\beta}$ should be of the form Eq. (2.9), and, indeed, it is. As is well known [15], when neutrino exchange dominates,

$$A_{\beta\beta}=\sum_i (\lambda_i U_{ei}^2) m_i K, \tag{3.1}$$

[6]Redefining the Majorana phases by rotating the state $|\nu_i\rangle$ so that $U_{\alpha i}\to U_{\alpha i}e^{i\theta_i}$ also leads to $\lambda_i\to\lambda_i e^{-2i\theta_i}$.

where m_i is the mass of the neutrino mass eigenstate ν_i, which is chosen to be real and positive, while K is a kinematical and nuclear factor that does not depend on i. The quantity

$$\left|\sum_i (\lambda_i U_{ei}^2) m_i\right| \equiv m_{\beta\beta} \ \ (\equiv|M_{ee}|) \tag{3.2}$$

is known as the effective neutrino mass for neutrinoless double-beta decay, and is simply the absolute value of the ee element of the Majorana neutrino mass matrix in the basis where the charged lepton mass matrix and the W-boson couplings are diagonal.

Suppose, for the purpose of illustration, that there are three neutrino species, so that U is a 3×3 matrix. Choose the phase convention where $\forall i, \lambda_i=1$, and take

$$U=U'\times\mathrm{diag}(e^{i\xi_1/2},e^{i\xi_2/2},e^{i\xi_3/2}), \tag{3.3}$$

as in Eq. (2.4). It is obvious from Eq. (3.1) that the overall rate for $0\nu\beta\beta$, $\Gamma_{0\nu\beta\beta}=|A_{\beta\beta}|^2$, is affected by (some combination of) the Majorana phases ξ_i. Therefore it is clear (and well known) that Majorana phases lead to physical consequences.

It is interesting to notice that if U contains a Dirac phase—a CP-violating phase that is not common to an entire column of U—as is generically the case if there are at least three neutrino species, then this phase may also influence $0\nu\beta\beta$ in the *same* way that Majorana phases can. The amplitude for $A_{\beta\beta}$ depends on the leptonic mixing matrix through $\lambda_i U_{ei}^2$, and it makes no difference whether some phase factor appears in the entire ith column of U or only in U_{ei}. To be sure, if some U_{ei} is proportional to a Dirac phase factor $e^{-i\delta}$, one can always remove this factor through the ν_i phase redefinition $|\nu_i\rangle\to|e^{i\delta}\nu_i\rangle$. However, this redefinition also results in $\lambda_i\to e^{-2i\delta}\lambda_i$, leaving the phase-convention-independent combination $\lambda_i U_{ei}^2$ unchanged.

Imagine now that one could measure the rate for the CP mirror image of the decay $Z\to(Z+2)+e^-e^-$, namely $\overline{Z}\to\overline{(Z+2)}+e^+e^+$. In contrast to the e^-e^- decay, this e^+e^+ decay involves the second term in Eq. (2.2) and the analogue of Eq. (2.3) for this term. Thus, in the amplitude $A_{\beta\beta}$ the factor $\lambda_i U_{ei}^2$ is replaced by $(\lambda_i U_{ei}^2)^*$ so that the amplitude $\overline{A}_{\beta\beta}$ for $\overline{Z}\to\overline{(Z+2)}+e^+e^+$ is

$$\overline{A}_{\beta\beta}=\sum_i (\lambda_i U_{ei}^2)^* m_i \overline{K}. \tag{3.4}$$

Here, due to the CP invariance of the strong interactions that determine nuclear matrix elements, the kinematical and nuclear factor $\overline{K}$ is identical to K in Eq. (3.1), except for a possible (irrelevant) phase difference. Thus, the rate for "anti-$0\nu\beta\beta$" is identical to the $0\nu\beta\beta$ rate: $|A_{\beta\beta}|^2=|\overline{A}_{\beta\beta}|^2$. That is, while the Majorana phases and, for that matter, the Dirac phases, affect the rate for $0\nu\beta\beta$, they do so in a CP-even way: their effects on a given $0\nu\beta\beta$ process and its CP-mirror-image anti-$0\nu\beta\beta$ process are identical. The only way to determine the effects of CP phases in neutrinoless double beta decay is to determine, through other experiments, the masses m_i and the "mixing angles" $|U_{ei}|^2$ of the leptonic mixing matrix, and compare the results with the obtained measurement of Eq. (3.2).[7] Whether this can be accomplished in practice has recently been explored by several authors [3].

IV. MANIFEST *CP* VIOLATION FROM MAJORANA PHASES IN "LOW-ENERGY" PHENOMENA

A Dirac phase in the quark or lepton mixing matrix can certainly produce *CP*-odd effects. Can a Majorana phase in the lepton mixing matrix do this too? In the rate for $0\nu\beta\beta$, Majorana phases lead only to a *CP*-even effect, as we have just seen.

To try to find a process in which the effects of Majorana phases can include *CP*-odd ones, we begin by asking what it takes to produce a *CP*-odd effect. If *CP* violation comes from phases, we can answer this question in a very general, well known way [9]. Suppose that some physical process P has an amplitude A consisting of two contributions:

$$A = a_1 e^{i\delta_1} e^{i\phi_1} + a_2 e^{i\delta_2} e^{i\phi_2}, \tag{4.1}$$

where $a_{1,2}$ are the magnitudes of the two contributions, $\delta_{1,2}$ are *CP*-odd phases which change sign when one computes the amplitude for the anti-process $\overline{P}$, while $\phi_{1,2}$ are *CP*-even phases that are the same for both P and $\overline{P}$. The amplitude for $\overline{P}$ is, therefore,

$$\overline{A} = a_1 e^{-i\delta_1} e^{i\phi_1} + a_2 e^{-i\delta_2} e^{i\phi_2}. \tag{4.2}$$

Note that the magnitudes $a_{1,2}$ are the same for P and $\overline{P}$ because we are assuming that *CP*-violating effects come from phases.

The *CP*-odd difference $\Delta_{CP} \equiv |\overline{A}|^2 - |A|^2$ is

$$\Delta_{CP} = 4 a_1 a_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2), \tag{4.3}$$

where we used Eqs. (4.1),(4.2). It is clear that in order for there to be a *CP*-odd effect in the rates for a process P and its mirror-image, the amplitude for the process must satisfy the following three requirements:

(i) It must contain at least two distinct contributions.

(ii) The distinct contributions must be proportional to *CP*-odd phase factors with corresponding phases δ_i, satisfying the condition(s) $\delta_i - \delta_j \neq 0 \bmod \pi$ for some i,j.

(iii)These contributions must also be proportional to *CP*-even phase factors with corresponding phases ϕ_i, satisfying the condition(s) $\phi_i - \phi_j \neq 0 \bmod \pi$ for some i,j.

In leptonic processes such as $0\nu\beta\beta$, the necessary *CP*-odd phases may be provided by the leptonic mixing matrix. However, from Eq. (3.1), it is easy to see that the third of the requirements listed above is not satisfied. Note that the *CP*-even phases in Eq. (3.1) are $\phi_i = \arg(K)$, $\forall\ i$. It is, therefore, clear that there are no *CP*-even phase differences. This is why phases in U do not lead to *CP*-odd effects in $0\nu\beta\beta$.

A. Neutrino ↔ antineutrino oscillations

It should be clear that in order to observe *CP*-odd effects due to the Majorana phases in some process, the amplitude for that process must also contain *CP*-even phases which differ from one piece of the amplitude to another. With this requirement in mind, we turn to neutrino ↔ antineutrino oscillations, Eq. (2.7).

Assuming that the neutrinos travel a macroscopic distance, we would like to compute the amplitude for the process Eq. (2.7), which is depicted in Fig. 1(b). As discussed in Sec. II, the amplitude $A_{\not L}$ has the general form Eq. (2.9) and can be written as [7]

$$A_{\not L} = \sum_i (\lambda_i U_{\alpha_i} U_{\beta i}) \frac{m_i}{E} e^{-i m_i^2 L/2E} S, \tag{4.4}$$

where E is the energy of the intermediate-state neutrino mass eigenstate which propagates a macroscopic distance L and S is an additional kinematical factor which depends on the initial and final states. We have used the standard approximations in order to write the neutrino oscillation phase as a function of $m_i^2 L/E$. The phase factor $e^{-i m_i^2 L/2E}$ may be thought of as the neutrino propagator, and, as we will see shortly, will provide the necessary *CP*-even phase discussed above.

It is important to comment that, just as in $0\nu\beta\beta$ [Eq. (3.1)], the amplitude Eq. (4.4) is proportional to the "helicity suppression" factor m_i/E. This factor reflects the fact that for either helicity of the intermediate neutrino ν_i, either the initial vertex $l_\alpha^+ W^- \to \nu_i$, or the final vertex $\nu_i \to l_\beta^- W^+$, is helicity suppressed. In addition, a factor like this must clearly be present in any lepton number violating process, given that the Majorana neutrino masses are (by assumption) the only source for lepton number violation, which consequently should disappear in the limit $m_i \to 0$, $\forall\ i$. As will be commented upon in more detail shortly, it is this helicity suppression factor that renders any observation of neutrino ↔ antineutrino oscillations almost impossible [16].

In order to look for explicitly *CP*-violating effects, we also compute the amplitude $\overline{A_{\not L}}$ for the *CP*-conjugate process $l_\alpha^- W^+ \to l_\beta^+ W^-$, which is given by

$$\overline{A_{\not L}} = \sum_i (\lambda_i U_{\alpha_i} U_{\beta i})^* \frac{m_i}{E} e^{-i m_i^2 L/2E} \overline{S}, \tag{4.5}$$

where $\overline{S}$ is identical to S except, perhaps, for an irrelevant overall phase factor. Not surprisingly, this is very similar to the situation in $0\nu\beta\beta$, discussed in Sec. III [cf. Eqs. (3.1),(3.4)].

Next we restrict ourselves to the two generation case. Working in a phase convention where the charge conjugation phase factors are trivial ($\lambda_1 = \lambda_2 = 1$), we parametrize the leptonic mixing matrix $U_{\alpha i}$, $\alpha = e, \mu$, $i = 1,2$, as

[7] For example, if $m_{\beta\beta} \neq \Sigma m_i |U_{ei}|^2$, we could conclude unambiguously that some of the *CP*-odd phases $\xi_{ij} \equiv \xi_i - \xi_j$, $i,j = 1,2,\ldots$, are different from 0 mod 2π.

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}e^{i\xi_1/2} & 0\\ 0 & e^{i\xi_2/2}\end{pmatrix}. \tag{4.6}$$

Within this parametrization, for $\alpha=e$ and $\beta=\mu$ [7],

$$A_{\not{L}}=\frac{\sin 2\theta}{2}S\left[-e^{i\xi_1}e^{-im_1^2L/2E}\frac{m_1}{E}+e^{i\xi_2}e^{-im_2^2L/2E}\frac{m_2}{E}\right], \tag{4.7}$$

$$\overline{A_{\not{L}}}=\frac{\sin 2\theta}{2}S\left[-e^{-i\xi_1}e^{-im_1^2L/2E}\frac{m_1}{E}+e^{-i\xi_2}e^{-im_2^2L/2E}\frac{m_2}{E}\right], \tag{4.8}$$

while the rates for the process and the anti-process are

$$\Gamma_{\not{L}}=|A_{\not{L}}|^2=\frac{\sin^2 2\theta}{4E^2}|S|^2 \times\left[m_1^2+m_2^2-2m_1m_2\cos\left(\frac{\Delta m^2L}{2E}-\xi\right)\right], \tag{4.9}$$

$$\overline{\Gamma}_{\not{L}}=|\overline{A}_{\not{L}}|^2=\frac{\sin^2 2\theta}{4E^2}|S|^2 \times\left[m_1^2+m_2^2-2m_1m_2\cos\left(\frac{\Delta m^2L}{2E}+\xi\right)\right]. \tag{4.10}$$

Here, $\xi\equiv\xi_2-\xi_1$ and $\Delta m^2\equiv m_2^2-m_1^2$. The *CP*-odd rate difference Δ_{CP} is, therefore,

$$\Delta_{CP}\equiv\overline{\Gamma}_{\not{L}}-\Gamma_{\not{L}}=\frac{\sin^2 2\theta}{E^2}|S|^2m_1m_2\sin\left(\frac{\Delta m^2L}{2E}\right)\sin\xi, \tag{4.11}$$

while the average rate is

$$\frac{\overline{\Gamma}_{\not{L}}+\Gamma_{\not{L}}}{2}=\frac{\sin^2 2\theta}{4E^2}|S|^2\left[m_1^2+m_2^2-2m_1m_2\cos\left(\frac{\Delta m^2L}{2E}\right)\cos\xi\right]. \tag{4.12}$$

While quite simple, the results computed above contain several remarkable properties, which we now make explicit. First of all, answering our original question, a manifestly *CP*-odd effect is present. As outlined in the beginning of this section, the conditions for having such an effect are that (i) there must be at least two interfering contributions to the amplitudes, (ii) these contributions must have a *CP*-odd relative phase, which here is ξ, that is nontrivial (i.e., different from 0 mod π), and (iii) the contributions must also have a *CP*-even relative phase, which here is $\Delta m^2L/2E$, that is nontrivial. It is the crucial presence of this nontrivial *CP*-even phase, coming from the neutrino propagators, that allows neutrino $\leftrightarrow$ antineutrino oscillation to exhibit a *CP*-odd effect while $0\nu\beta\beta$ cannot do so.

Clearly, Δ_{CP} must vanish if either the *CP*-odd phase vanishes (mod π), or the *CP*-even phase vanishes. The latter will occur if $\Delta m^2=0$ (degenerate neutrino masses) or if $L=0$ (vanishing travel distance).[8] From Eq. (4.11), we see that Δ_{CP} does indeed vanish when it must. Furthermore, from Eqs. (4.4) and (4.5), we see that even in the more general case of an arbitrary number of neutrino mass eigenstates, Δ_{CP} still vanishes, as it must, when either the *CP*-odd phases in the factors $\lambda_iU_{\alpha i}U_{\beta i}$ are equal (mod π), or else the neutrino masses m_i are all degenerate or $L=0$, so that the *CP*-even phases in the various terms of the amplitudes are equal.

Before proceeding, we pause to discuss the physical parameter space for these lepton-number violating processes, meaning the range for the values of θ,ξ, and the masses that must be probed in order to describe all the physically distinguishable values of $l_\alpha^+\to l_\beta^-$-transitions. We will *define* the mass eigenstates such that $m_2\geqslant m_1$. Within this definition, the *CP*-odd phase difference ξ yields a potentially different physical observable for each value in the range $[-\pi,\pi]$. It remains to discuss what happens to the mixing angle, θ. As an angular variable, it is certainly constrained to the interval $[-\pi,\pi]$, but the general form for the amplitudes Eqs. (4.4), (4.5) allow for a smaller physical range. Explicitly, $A_{\not{L}}^{\alpha\beta}\propto\sin\theta\cos\theta$ for $\alpha\neq\beta$ or $A_{\not{L}}^{\alpha\alpha}\propto a\sin^2\theta+b\cos^2\theta$, where $a,b\in\mathbb{C}$ (the same applies to $\overline{A}_{\not{L}}^{\alpha\beta},\overline{A}_{\not{L}}^{\alpha\alpha}$). The following two operations leave physical observables ($\propto|A|^2$) unchanged: $\theta\to-\theta$ and $\theta\to\pi-\theta$. This implies that one can choose $\theta\in[0,\pi/2]$ and completely cover the entire physical parameter space. Note that Eqs. (4.11) and (4.12) have an extra symmetry $\theta\to\pi/2-\theta$, such that $\theta\in[0,\pi/4]$ yields the same results as $\theta\in[\pi/2,\pi/4]$. This is not true, in general, for the "diagonal" $A_{\alpha\alpha}$, unless $|a|=|b|$. This situation is different from the standard neutrino $\leftrightarrow$ neutrino oscillations, where $\theta\in[0,\pi/4]$ fully describes two-flavor oscillations in vacuum [17] (as is well known, this degeneracy is lifted if the neutrinos propagate in matter). Here, one can tell whether the electron-type neutrino is predominantly light ([$\theta\in[0,\pi/4]$], the "light side") or heavy ([$\theta\in[\pi/4,\pi/2]$], the "dark side" [18]) even if the neutrinos are propagating exclusively in vacuum. Note that one can always choose other parametrizations, where, for example $\xi\in[0,\pi]$. The different parametrizations are related by a "relabeling invariance," which states that if one redefines the mass eigenstates $1\leftrightarrow2$ ($[\Delta m^2,\xi]\to[-\Delta m^2,-\xi]$) and the mixing angle $\theta\to\pi/2-\theta$, the amplitudes remain unchanged.

It is interesting that, as Eq. (4.12) shows, lepton-number-violating transitions that also violate flavor ($e^\pm\to\mu^\mp$) already occur when $L=0$. This behavior is in sharp contrast to the ordinary lepton-number-conserving neutrino flavor oscillation. It arises from the fact that when, for example, an incoming e^+ makes a neutrino, at $L=0$ the wrong-helicity (left-handed) component of this neutrino has mass eigenstate composition proportional to

$$\sum_i\frac{m_i}{E}\lambda_iU_{ei}|\nu_i\rangle. \tag{4.13}$$

[8]We disregard the "finely tuned" points where $\Delta m^2L/2E=\pi,\ 2\pi,\ \ldots$, which can only be chosen, in principle, for a monochromatic neutrino beam.

Since the neutrino state that is a pure $|\nu_e\rangle = \Sigma_i U^*_{ei}|\nu_i\rangle$, it is clear that even at $L=0$ the left-handed component of the neutrino made by an e^+ is not a pure ν_e, meaning that it contains other flavors. In the two-generation case being considered here, it contains a ν_μ component, which can instantly produce $\mu^- W^+$. This fact was already noticed by the authors of [19].

An especially interesting property of $\Gamma_{\not L}$ and $\overline{\Gamma_{\not L}}$ is their behavior when the neutrino mass eigenstates are degenerate: $m_1 = m_2 \equiv m$. In this limit

$$\overline{\Gamma_{\not L}} = \Gamma_{\not L} = \frac{m^2 \sin^2 2\theta}{E^2}|S|^2 \sin^2\left(\frac{\xi}{2}\right). \tag{4.14}$$

We see that as long as $\xi \neq 0 \bmod 2\pi$, and the mixing angle $\theta \neq 0, \pi/2$, $e^\pm \to \mu^\mp$ transitions still occur, and their rates depend on the mixing angle. Thus, the mixing angle continues to have physical consequences even when the neutrino masses are degenerate.

When one remembers how quark mixing behaves, this result for Majorana neutrinos is very surprising and puzzling. For, as one will recall, if the masses of all up-type quarks or down-type quarks are degenerate, mixing phenomena are absolutely absent. Indeed, in the presence of this degeneracy, all mixing angles are unphysical. This is easy to show [9]. Assume that the down-type quarks are degenerate in mass. This means that the relevant part of the Lagrangian is given by

$$\mathcal{L} \supset \left(-\frac{g}{\sqrt{2}} W^+_\rho \sum_{i,j} \overline{u}_{iL} \gamma^\rho V_{ij} d_{jL} + \text{H.c.} \right) + (-m\overline{d}_{iL}\delta_{ij} d_{jR} + \text{H.c.}), \tag{4.15}$$

where V is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, m is the hypothetical common down-type quark mass and δ_{ij} is the standard Kronecker-delta symbol. We can now define $d'_{iL} \equiv V_{ij} d_{jL}$, which renders the charged current coupling diagonal, while the mass term is modified to $m\overline{d'}_{iL} V_{ij} d_{jR} \equiv m\overline{d'}_{iL}\delta_{ij} d'_{jR}$ if we redefine $d'_{iR} = V_{ij} d_{jR}$. This final redefinition does not lead to any other physical consequences. Thus, one can choose a basis where both the down quark mass matrix and the charged current weak couplings are diagonal. In this basis, all mixing angles have disappeared. One may summarize the situation in the following heuristic way: the mixing matrix V may be thought of (as one alternative) as describing the relation between the down-type quarks that have definite masses and those that have diagonal couplings to the up-type quarks. When the down-type quark masses become equal, there is nothing left to describe. Any linear combination of down-type quarks has the same mass as any other linear combination, so one may simply choose the down-type quark basis states to be the ones whose couplings to the up-type quarks are diagonal. There is no way to define a physically meaningful mixing matrix.

One can try to repeat the same logic in the neutrino sector [9,20]. The first step is to redefine the neutrino states such that the charged current part of the Lagrangian Eq. (2.2) is diagonal: $\nu_{\alpha L} \equiv U_{\alpha i}\nu_{iL}$ (this is often referred to as the flavor basis). In this basis, the part of the Lagrangian which contains the Majorana neutrino mass term is given by[9]

$$\begin{aligned}\mathcal{L} \supset & -\frac{1}{2}\overline{\nu^c}_{\alpha R} U^*_{\alpha i} M_{ij} U^*_{\beta j} \nu_{\beta L} + \text{H.c.} \\ & = -\frac{1}{2}\overline{\nu^c}_{\alpha R}(U'^* \mathbb{E}^{-i\xi_i/2} M \mathbb{E}^{-i\xi_i/2} U'^\dagger)_{\alpha\beta}\nu_{\beta L} + \text{H.c.},\end{aligned} \tag{4.16}$$

where U' and $\mathbb{E}$ have been defined in Eq. (2.4), and $M \equiv \text{diag}(m_1, m_2, \ldots)$. If $M_{ij} = m\delta_{ij}$, no generic simplification is possible [20]. This means that some of the mixing angles and phases (Dirac and Majorana) which were physical in general are still physical. The issue of counting the number of physical parameters was discussed in detail by the authors of [20], to which we refer readers for more details (see also [9]). One can go one step further, and ask what happens if the model is *CP* invariant. This happens, for example, if U' is real (and therefore an orthogonal matrix), and $\xi_i = 0 \bmod \pi$, so that

$$(U'^* \mathbb{E}^{-i\xi_i/2} M \mathbb{E}^{-i\xi_i/2} U'^\dagger)_{\alpha\beta} = m(U' \mathbb{E}^{i(0,\pi)} U'^\dagger)_{\alpha\beta}, \tag{4.17}$$

where we define $\mathbb{E}^{i(0,\pi)}$ to be a diagonal matrix with elements[10] equal to 1 or -1. If $\mathbb{E}^{i(0,\pi)}$ is proportional to the identity matrix, there are no physical mixing angles. This behavior is clear in Eq. (4.14). In the case $\xi = 0$, such that $\mathbb{E}^{i(0,\pi)}$ is the 2×2 identity matrix, there are no lepton number violating flavor transitions in the mass degenerate case. On the other hand, in the *CP*-conserving but less trivial case of $\xi = \pi$, $\mathbb{E}^{i(0,\pi)} \propto \text{diag}(1,-1)$, and a physical mixing angle can be defined, i.e., there are lepton number violating flavor transitions in the mass degenerate case.

It is interesting to note that, unlike the case of degenerate Majorana neutrino masses, if all *charged lepton* masses were degenerate, there would be no physical mixing angles (or *CP*-odd phases) to speak of. Indeed, if this were the case, one could always choose a basis where the Majorana neutrino mass matrix, the charged current coupling, and the charged lepton mass matrix were all diagonal, so that $l^-_\alpha \to l^+_\beta$ transition processes would be trivially zero for $\alpha \neq \beta$.

We summarize the situation regarding mass-degenerate neutrino eigenstates. Unlike the situation in the quark sector, when all neutrino mass eigenstates have the same mass (some of) the mixing angles and *CP*-odd phases are still meaningful. A well known but under-appreciated (at least by the authors) example of this phenomenon is the effective neutrino mass for $0\nu\beta\beta$, Eq. (3.2). In the three-flavor case using the standard Particle Data Group (PDG) parametriza-

[9]This is the low-energy effective Lagrangian, which ensues after electroweak symmetry breaking. It is independent of the mechanism that generates Majorana neutrino masses.

[10]The diagonal elements of $\mathbb{E}^{i(0,\pi)}$ are also referred to as the relative *CP* parities of the different neutrino mass eigenstates, which are physically meaningful [21].

tion for the mixing angles and assuming that the neutrino masses are degenerate[11] (and $\lambda_i = 1,\ \forall\ i$)

$$m_{\beta\beta} = m|\cos^2\theta_{13}\cos^2\theta_{12} + \cos^2\theta_{13}\sin^2\theta_{12}e^{i\xi} + \sin^2\theta_{13}e^{i\zeta}|, \tag{4.18}$$

where ξ and ζ are the two relevant relative phases and m is the common neutrino mass. Note that despite the degenerate masses the two mixing angles influence $m_{\beta\beta}$, and the same is true of the two *CP*-odd phases. In the *CP*-preserving limit, there are several options. If, for example, $\xi = \zeta = 0 \bmod 2\pi$, $m_{\beta\beta} = m$ (no dependency on mixing angles), while if $\xi = 0 \bmod 2\pi$ and $\zeta = \pi \bmod 2\pi$, $m_{\beta\beta} = m|\cos 2\theta_{13}|$ [dependency on (one) mixing angle]. Even in the "effective-two-generation" case ($\theta_{13} = 0$), $m_{\beta\beta}$ depends on the remaining mixing angle θ_{12} so long as $\xi \neq 0 \bmod 2\pi$.

Perhaps a more intuitive way of understanding what is going on is to reinterpret the Majorana phases as follows. According to Eq. (4.16), one can rewrite the neutrino mass matrix in the flavor basis as $U'^* M' U'^\dagger$, where $M' = M\mathbb{E}^{-i\xi_i}$ is a diagonal mass matrix whose entries are in general complex. Within this definition, the Majorana phases are interpreted as the phases of the neutrino mass eigenvalues, which are physical if the neutrinos are Majorana particles. Within this language, it is easy to understand what is happening in the mass-degenerate case: while we are setting the absolute values of the masses to be equal, the mass eigenstates are still distinguishable if the phases are different. This may even be true in the *CP*-conserving case, where we can still distinguish two mass eigenstates by the sign of the corresponding mass eigenvalues. With distinguishable mass eigenstates, the mixing matrix still has meaning: it describes the relation between these mass eigenstates and the states with diagonal weak couplings to the charged leptons. This relation has physical consequences.

B. Lepton-number violating meson decay processes

One can look for the effects of Majorana phases in other lepton-number violating processes such as $K^\pm \to \pi^\mp \mu^\pm \mu^\pm$. The present experimental upper limits on this process are at the level of 10^{-8} [8] and further improvements are likely in future. This process is very similar to $0\nu\beta\beta$ with e replaced by μ. The leading order amplitude for the K^+ decay is depicted in Fig. 2(a) and given by

$$A_{\mu\mu} = \sum_i (\lambda_i U_{\mu i}^2)^* m_i K \equiv F e^{i\phi}, \tag{4.19}$$

where K is a kinematical factor. F is the magnitude of $A_{\mu\mu}$, while ϕ is its overall *CP*-odd phase. This definition will become useful shortly. The corresponding equation for the K^- decay amplitude $\overline{A}_{\mu\mu}$ is also given by Eq. (4.19) after replacing $(\lambda_i U_{\mu i}^2)^*$ by $\lambda_i U_{\mu i}^2$ (or $\phi \to -\phi$). As in the case of $0\nu\beta\beta$, since the total decay rate is given by the absolute value of $A_{\mu\mu}$, at the leading order the *CP*-odd effect of Majorana phases will be absent. However, one may consider interference of the lowest order amplitude with the contribution of processes involving physically accessible intermediate states, such as $K^+ \to \mu^+\nu$ followed by $\nu\mu^+ \to \pi^-\mu^+\mu^+$ [depicted in Fig. 2(b)], or $K^+ \to \mu^+\pi^0\nu$ followed by $\pi^0\nu \to \pi^-\mu^+$. Here, the $\Delta L = 2$ interactions (and hence the Majorana phases) play a role in the second step of the process. Let us denote this contribution by $Ge^{i\psi}e^{i\gamma}$ where the origin of $e^{i\gamma}$ (a *CP*-even phase factor) is due to the presence of the physical intermediate state and comes from the absorptive part of the amplitude, while ψ is the *CP*-odd phase of this amplitude and G its magnitude.

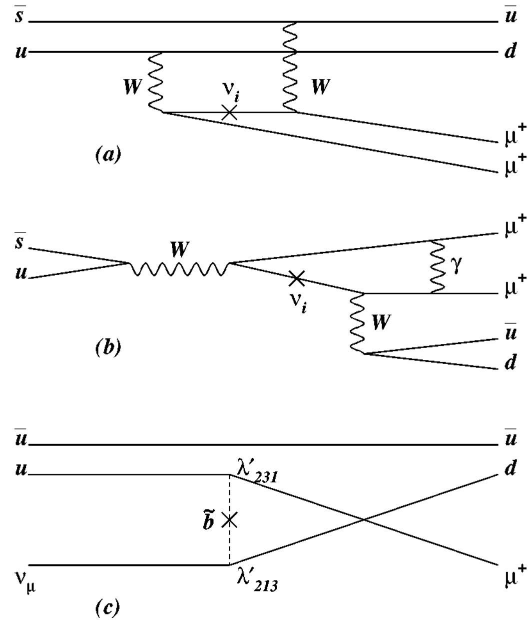

FIG. 2. Feynman diagrams for $K^+ \to \pi^-\mu^+\mu^+$ including (a) the leading tree-level contribution and (b) a one-loop higher-order term (see text for details), plus (c) the Feynman diagram for $\nu_\mu\pi^0 \to \mu^+\pi^-$ scattering via bottom-squark exchange, mediated by supersymmetry (SUSY) *R*-parity violating $\lambda'_{ijk}L_iQ_jD_k^c$ couplings. Time flows from the left to the right. The $\times$ indicates a chirality flip in either the neutrino or the squark propagator.

Here is the crucial point: since the intermediate state $\mu^+\nu$ (or $\pi^0\mu^+\nu$) is a physically accessible state, *CP*-even phases γ will arise from the absorptive parts of these contributions. This renders manifestly *CP*-odd effects potentially observable, for example, when we take the difference of the rates for K^+ and K^- decays. To see this explicitly, one can schematically write the amplitudes for the $K^\pm$ decays as

$$A(K^+ \to \pi^-\mu^+\mu^+) = Fe^{i\phi} + Ge^{i\psi}e^{i\gamma}, \tag{4.20}$$

$$A(K^- \to \pi^+\mu^-\mu^-) = Fe^{-i\phi} + Ge^{-i\psi}e^{i\gamma}. \tag{4.21}$$

The difference Δ_K between the rates for $K^+ \to \pi^-\mu^+\mu^+$ and $K^- \to \pi^+\mu^-\mu^-$ is, therefore, $\Delta_K \propto 4FG\sin(\psi - \phi)\sin\gamma$ (we have not included the phase space factors), as expected from the general form presented in Eq. (4.3). Hence, manifestly

[11]As discussed by the authors of [20], the angles and phases in the standard PDG parametrization are not all independent if the neutrino masses are exactly degenerate.

CP-odd observables can be constructed from lepton-number violating meson decays as long as there are relative *CP*-odd phases ($\phi \neq \psi$).

As far as the generation of a *CP*-even phase is concerned, the situation here is very similar to what happens in the case of leptogenesis, where the presence of physically accessible intermediate states provides the necessary *CP*-even phase which renders manifestly *CP*-odd effects due to the Majorana phases possible. We will discuss certain aspects of leptogenesis in the next section.

Are there nontrivial *CP*-odd phases in order for the *CP*-odd effects to materialize? If the only source of *CP*-odd phases is the leptonic mixing matrix, the answer is, unfortunately, no: $\phi = \psi$. The reason is that both the "direct decay" [Fig. 2(a)] and the processes in which a physically accessible state is produced and later rescatters [Fig. 2(b)] have the *same* *CP*-odd phase [ϕ, see Eq. (4.19)], which is, therefore, unobservable. In order to observe a *CP*-odd effect, new sources of relative *CP*-odd phases are required.

We would like to discuss one new-physics example. Independent $\Delta L = 2$ interactions arise from supersymmetric extensions of the SM with *R*-parity violation. More explicitly, a $\lambda'_{ijk} L_i Q_j D^c_k$ term[12] in the superpotential could lead to the process $\pi^0 \nu \to \pi^- \mu^+$, depicted in Fig. 2(c), but not directly to $K^+ \to \pi^- \mu^+ \mu^+$ decay, therefore introducing a relative *CP*-odd phase. One can make an estimate of this effect using supersymmetry *R*-parity violating effects to generate the absorptive part. In this case, assuming $\psi = 0$, one has

$$\Delta_K \propto G_F^2 \sin\phi \frac{|\lambda'_{231}\lambda'_{213}| m_b}{M^3_{\mathrm{SUSY}}}, \tag{4.22}$$

where G_F is the Fermi constant, m_b is the mass of the bottom-quark and M_{SUSY} is a supersymmetry breaking mass. The observable effect is of course extremely tiny. Nevertheless, this example provides a scenario where in principle *CP* violating Majorana phases might lead to manifestly *CP*-odd effects in $\Delta L = 2$ processes. Of course, one need not restrict oneself to the kaon system, and processes such as $B^\pm \to \pi^\mp \tau^\pm \tau^\pm$ would also work in the same way.

Once new sources of lepton-number violation are introduced, one may wonder whether a manifestly *CP*-odd observable can arise in the case of $0\nu\beta\beta$. The answer is, in principle, yes. It would arise, for example, from $Z \to (Z+2) + e^- e^- \overline{\nu\nu} \to (Z+2) + e^- e^-$, where the second stage is lepton number violating and is mediated by some new kind of interaction. The *CP*-odd observable would be the difference between this process and the decay of the antinucleus. It is important that the second stage be mediated by a new form of lepton number violating interaction (so that "$\phi \neq \psi$") and secondly, the crucial point again is that the intermediate state is a physical state so that we have an absorptive part.

[12] L_i, Q_i are, respectively, lepton and quark doublet chiral superfields, while D^c_i is the down-antiquark singlet chiral superfield. The λ'_{ijk} are dimensionless couplings, and $i,j,k = 1,2,3$ are family indices.

V. COMMENTS ON LEPTOGENESIS

Perhaps the best known case of a manifestly *CP*-violating effect in a lepton-number violating process is leptogenesis [4]. The central idea is the following: if neutrino masses are generated via the seesaw mechanism, there are extra singlet fermions (right-handed neutrinos) which possess a (very heavy) Majorana mass and couple to the lepton left-handed doublet and the Higgs-boson doublet via a Yukawa interaction. In the early Universe, these right-handed neutrinos will be present in the primordial thermal bath, and will eventually decay into leptons and scalars as soon as the Universe is cold enough. Since the decays of such states violate lepton number, if such decays take place out of thermal equilibrium, a net lepton number for the Universe can be generated as long as *CP* is also violated. Later, the net lepton number is converted in part to a net baryon number by nonperturbative sphaleron processes [22]. For detailed reviews, we refer readers to, for example, [23,24].

Here, we would like to concentrate on physical effects, in particular the *CP*-odd ones, which are related to the "Majorananess" of the processes that lead to leptogenesis. We would like to compare how *CP* is violated (and under what conditions) during the decay of the right-handed neutrinos to how it was violated in the neutrino $\leftrightarrow$ antineutrino oscillation process analyzed in Sec. IV A. For that reason, we will concentrate on a much simpler setup, which captures all the features we are interested in, while leaving out several unnecessary complications.

We will consider the following interaction Lagrangian added to the SM one [which contains Eq. (2.2) and the Yukawa coupling between the Higgs boson, the lepton doublet and the right-handed charged lepton field]:

$$\mathcal{L} = -\frac{M_{ij}}{2}\overline{N^c}_{iL} N_{jR} - y_{\alpha i}[\overline{\nu}_{\alpha L}\overline{\varphi}^0 - \overline{l}_{\alpha L}\varphi^-] N_{iR} + \mathrm{H.c.} \tag{5.1}$$

Here $\phi = (\varphi^+, \varphi^0)$ is the Higgs-boson weak isodoublet, $\ell_{\alpha L} = (\nu_L, l_L)_\alpha$ are the left-handed lepton doublets, and N_{iR} are the right-handed neutrinos. We assume that the scalar doublet is massless [and that $SU(2)_L$ is not broken]. We will choose a basis where M is diagonal, and where its eigenvalues are real and positive. In this basis, y is a generic complex matrix of Yukawa couplings.

We will study the decay of the heavy right-handed neutrinos, and, in particular, address whether *CP* is violated; namely, whether the branching ratio for $N_i \to \ell\phi$ differs from the branching ratio for $N_i \to \overline{\ell\phi}$, where we sum over the final state flavors of ℓ_α. One may picture the following gedanken experiment: place inside a box a certain amount of right-handed neutrinos (of a certain "species"). Wait until they all have decayed, and count the total lepton number inside the box. If the total lepton number is not zero, *CP* has been violated (and lepton number has been "created").

At the tree level, of course, the branching ratios are identical. At one loop one has to compute, on top of the tree-level contribution [Fig. 3(a)], the "vertex-correction" one-loop

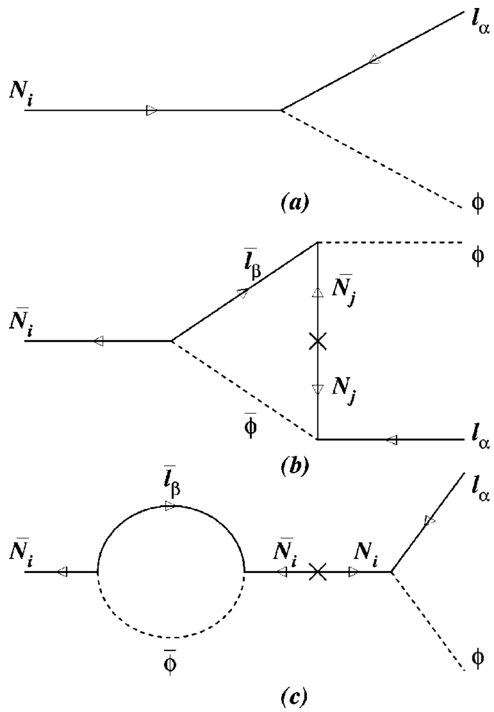

FIG. 3. Feynman diagrams for the decay of a right-handed neutrino: (a) tree-level contribution, (b) one-loop "vertex-correction" contribution and (c) one-loop, "propagator-correction" contribution. Time flows from the left to the right, and the arrows represent the chirality of the various fermions. The $\times$ indicates a chirality flip in the neutrino propagator, which is proportional to the neutrino Majorana mass. We only include the diagrams that will lead to a CP-odd contribution to the decay rate. Recall that $N_i = \overline{N_i}$ up to a phase factor.

diagram [Fig. 3(b)] and the "propagator-correction" one-loop diagram [Fig. 3(c)]. The amplitudes for $N_i \to \ell_\alpha \phi$ and $N_i \to \overline{\ell}_\alpha \overline{\phi}$ are, respectively,

$$A_{\alpha i} = \left[y_{\alpha i} + \sum_{j,\beta} f(i,j) y^*_{\beta i} \Lambda^*_i y_{\beta j} \Lambda_j y_{\alpha j} \right] K_i, \tag{5.2}$$

$$\overline{A}_{\alpha i} = \left[y^*_{\alpha i} \Lambda^*_i + \sum_{j,\beta} f(i,j) y_{\beta i} y^*_{\beta j} \Lambda^*_j y^*_{\alpha j} \right] K_i. \tag{5.3}$$

Here, Λ_i is the charge conjugation phase factor for the heavy Majorana mass eigenstate N_i:

$$N_i = \Lambda_i N_i^c. \tag{5.4}$$

The quantity K_i is a kinematical factor, and $f(i,j)$ is a loop function, which depends on the mass of the decaying right-handed neutrino and the mass of the right-handed neutrino in the loop (see, for example, [23,24]). We have neglected terms which only serve as trivial corrections to the tree-level coupling, and assumed the scalar and the left-handed leptons to be massless.

The expressions for $A_{\alpha i}$ and $\overline{A}_{\alpha i}$ above may be simplified by introducing the modified Yukawa coupling matrix Y, defined by

$$Y_{\alpha i} = y_{\alpha i} \omega_i, \tag{5.5}$$

where [12]

$$\omega_i = (\Lambda_i)^{1/2}. \tag{5.6}$$

It is easy to show that Y is unchanged by phase redefinitions of the N_i. Thus, rewriting $A_{\alpha i}$ and $\overline{A}_{\alpha i}$ in terms of Y makes them manifestly phase-redefinition invariant.

The CP-odd asymmetry in the N_i decay rate, summed over all the left-handed lepton flavors α, is given by

$$\Delta_i \equiv \sum_\alpha (|\overline{A}_{\alpha i}|^2 - |A_{\alpha i}|^2) \Big/ \sum_\alpha (|\overline{A}_{\alpha i}|^2 + |A_{\alpha i}|^2) \tag{5.7}$$

$$\simeq \frac{2}{(Y^\dagger Y)_{ii}} \sum_j \mathrm{Im}\{(Y^\dagger Y)^2_{ij}\} \times \mathrm{Im}\{f(i,j)\}. \tag{5.8}$$

As expected, the CP-odd asymmetry is proportional to the sine of CP-odd phases (which are present in the Yukawa coefficients $y_{\alpha i}$) *and* the sine of CP-even phases; namely, the phases of the $f(i,j)$. Do the $f(i,j)$ have complex phases? Fortunately, the answer is yes: the complex phase of $f(i,j)$ comes from the absorptive parts of both one-loop diagrams. These are indeed present since the virtual $\ell \phi$ states in the loop can be produced on mass shell and rescatter.

How does this compare with CP-odd effects in neutrino $\leftrightarrow$ antineutrino oscillation, discussed in Sec. IV A? From Eq. (5.8), we see that in the decay of the right-handed neutrino N_i, the interference between the tree level diagram Fig. 3(a) and the one-loop diagrams Figs. 3(b) and 3(c) involving an intermediate state N_j and ℓ_β leads to $\mathrm{Im}[(\Lambda_i y_{\alpha i} y_{\beta i})^* (\Lambda_j y_{\alpha j} y_{\beta j})]$. From Eq. (4.4), on the other hand, we see that in the "oscillation" process $l^+_\alpha W^- \to l^-_\beta W^+$, the interference between two diagrams of the type Fig. 1(b) involving, respectively, ν_i and ν_j as intermediate particles leads to a CP-odd contribution to the oscillation probability proportional to $\mathrm{Im}[(\lambda_i U_{\alpha i} U_{\beta i})^* (\lambda_j U_{\alpha j} U_{\beta j})]$. Obviously, the imaginary parts occurring in N_i decay and neutrino $\leftrightarrow$ antineutrino oscillation are identical *in structure*. When we "transform" from neutrino $\leftrightarrow$ antineutrino oscillation to N_i decay, the light neutrino mixing matrix U, which acts as an effective coupling matrix, is simply replaced by the heavy N_i Yukawa coupling matrix y. Similarly, the light neutrino charge conjugation phase factors λ_i are replaced by the heavy N_i counterparts, the Λ_i.

In neutrino $\leftrightarrow$ antineutrino oscillation, each interfering diagram contains two vertices. In N_i decay, the tree-level diagram contains only one vertex, while the one-loop diagrams with which the tree level interferes each contain three vertices. What has happened is that in the translation between neutrino $\leftrightarrow$ antineutrino oscillation and N_i decay, one of the vertices in one of the interfering diagrams has not only been replaced by its heavy N_i counterpart, but has also "left" its original diagram, to be replaced by its complex conjugate in the other diagram. Of course, this "migration" from one diagram to the other does not change the interference that yields the *CP*-odd effect.

Next, as we did in Sec. IV A, we examine under what conditions *CP* is restored. This happens, of course, in the trivial case of only one right-handed neutrino, since $(Y^\dagger Y)_{ii}^2$ is real [note that the $j=i$ term of the sum in Eq. (5.8) does not contribute to Δ_i]. This means that in order to violate *CP*, there must be nontrivial "mixing angles" (as is always the case). What do these mixing angles relate? They relate two different bases for the right-handed neutrino states: the mass basis, and the "decaying basis," i.e., the basis in which $Y^\dagger Y$ is diagonal.[13]

Under what conditions does $Y^\dagger Y$ contain no nontrivial phase factors? One such case is when $Y^\dagger Y$ is diagonal in the same basis where the right-handed neutrino mass is diagonal. In this case, the decaying right-handed neutrino states coincide with the right-handed neutrino mass eigenstates, and one can do away with any "mixing." This would happen, for example, if the eigenvalues x of Y had the same magnitude: $Y=V\,\mathrm{diag}(x,x,\ldots)U^\dagger, U,V$ unitary matrices. In this case, $Y^\dagger Y=U\,\mathrm{diag}(|x|^2,|x|^2,\ldots)U^\dagger=\mathrm{diag}(|x|^2,|x|^2,\ldots)$. If the eigenvalues of Y have the same magnitude, one can always choose a basis where the decaying state and the mass eigenstate are the same. Perhaps the closest analogue of this situation in Sec. IV A is the case when the charged leptons had the same mass. There, one could redefine the charged lepton states such that there were no $l_\alpha^\pm \to l_\beta^\mp$-transitions for $\alpha\neq\beta$.

Finally, we discuss the curious case of right-handed neutrinos with degenerate masses. Here, as in Sec. IV A when the light neutrinos all have the same mass, no significant simplification can be performed. In particular, similar to the situation in Sec. IV A, mixing angles are still generically present because of the presence of the Majorana phases. As we argued in Sec. IV A, one can reinterpret the Majorana phases as the phases of the mass eigenvalues. In this case, even if the right-handed neutrino masses have the same magnitude, one can still distinguish the states by the phase factors. This implies that the decaying basis can still differ from the mass basis, and that mixing angles may still be defined.

This means that in the case of mass-degenerate right-handed neutrinos, there is no general reason to believe that $\Delta_i=0$. In this case, however, Δ_i (for some fixed i) might not be the relevant quantity to compute if one wants to explain the excess of matter over antimatter in the Universe. One may, perhaps, have to compute the total lepton number created by the simultaneous decay of all right-handed neutrinos (this is naively expected, as they all have the same mass anyway).

In our gedanken setup, we proceed to analyze what happens if the "box" contains identical amounts of all the mass-eigenstate right-handed neutrinos N_i. In this case, the total lepton number produced by the ensuing decay of all right-handed neutrino species is [see Eq. (5.8)]

$$\Delta\equiv\sum_i \Delta_i \propto \mathrm{Im}(f)\sum_{i,j}\frac{\mathrm{Im}\{(Y^\dagger Y)_{ij}^2\}}{(Y^\dagger Y)_{ii}}. \tag{5.9}$$

Here $f\equiv f(i,j),\forall\ i,j$ is the loop factor, which has become independent[14] of i and j. In general, Δ does not vanish, which means that, in our gedanken setup, a global lepton number is generated through the decay of equal numbers of degenerate right-handed neutrinos.

The situation here is dramatically different from the one in Sec. IV A. There, no *CP*-odd effects were present in the mass-degenerate case because the *CP*-even phase in the case of oscillations ($\propto\Delta m^2$) vanished exactly. Here, the *CP*-even phase does not vanish in the case of mass-degenerate states. It is curious to note, however, that the combination $\Sigma_i\Delta_i(Y^\dagger Y)_{ii}$ does vanish exactly when all right-handed neutrinos are degenerate in mass.

Our result is independent of the number of right-handed neutrinos and left-handed leptons. For example, the same situation occurs if instead of three right-handed neutrinos there are two right-handed neutrinos leading to a 3×2 seesaw, which has been explored in several recent papers [27] in order to try to establish a connection between potentially measurable "low-energy" phases, and the *CP*-odd phases which are present in the leptogenesis process.

In "thermal equilibrium leptogenesis" [23,24] the right-handed neutrinos are assumed to be in thermal equilibrium with the SM fields at some very large temperature. Under these conditions, the abundance of different degenerate-mass right-handed neutrinos is guaranteed to be the same. In order to generate a net lepton number which will later be converted to a net baryon number, the right-handed neutrinos must not only decay in a *CP*-odd fashion, but must do so out of thermal equilibrium. Do degenerate-mass right-handed neutrinos that decay out of thermal equilibrium do so in such a way that the net lepton number generated is nonzero? The answer to this question is rather academic,[15] and beyond the intentions of this paper.

VI. SUMMARY

In this paper we have discussed *CP*-violating leptonic and semileptonic processes that can probe the *CP*-odd phases in

[13] $YY^\dagger$ can be chosen diagonal by redefining the ℓ_α fields. This redefinition would "resurface," for example, in the charged lepton Yukawa coupling, which does not concern us for this discussion.

[14] As was discussed by several authors, the case of leptogenesis with mass-degenerate right-handed neutrinos is rather subtle [25,26]. In particular, the contribution to the decay coming from the "bubble diagram" Fig. 3(b) is divergent unless one considers the decay width of the propagating right-handed neutrino. If the calculation is correctly performed, however, it has been shown [26] that the contribution of Fig. 3(b) to $f(i,j)$ exactly vanishes in the mass-degenerate limit, while the vertex correction contribution Fig. 3(c) does not. It does satisfy $f(i,j)=f$, when the masses are all degenerate.

[15] One should worry about the definition of "degenerate" right-handed neutrinos. If the tree-level right-handed neutrino masses are all the same, quantum corrections are bound to make them distinct, unless, for example, all right-handed neutrinos have the same decay widths (i.e., couplings). If this is the case, as was discussed earlier, there are no nontrivial mixing angles.

the leptonic mixing matrix, especially the so-called Majorana phases. It is nontrivial for this probing to reveal that the Majorana phases are genuinely *CP*-violating quantities. This nontriviality may be seen by looking at neutrinoless double beta decay, which is often discussed as a way to get information on the Majorana phases. Even though this process does depend on these phases, in the leading order there is no difference between the rate for the neutrinoless double beta decay of a given nucleus and that of the corresponding antinucleus (say ^{76}Ge and anti-^{76}Ge). Therefore, these processes do not involve any *manifest* violation of *CP*.

There are, however, processes which do exhibit manifestly *CP*-violating effects. We have outlined the conditions under which such effects can occur and discussed three examples: (i) neutrino $\leftrightarrow$ antineutrino oscillation, (ii) rare leptonic decays of K and B mesons, such as $K^{\pm}\to\pi^{\mp}l^{\pm}l^{\pm}$ and similar modes for the B meson, and (iii) leptogenesis in the early Universe, which may be responsible for the present matter-antimatter asymmetry.

We have discussed some limiting cases where the *CP* violation disappears. A particularly interesting case, encountered in neutrino $\leftrightarrow$ antineutrino oscillation, is when the light neutrinos are degenerate. We have explained why manifest *CP* violation is absent there. However, we have noted that, while the quark mixing matrix loses its meaning when all the masses of the quarks of a given charge are of equal size, the leptonic mixing matrix continues to have physical consequences even when all the masses of the neutrinos are of equal size. This is true so long as the neutrinos are Majorana particles and the relative Majorana phases are not zero. The origin of this distinction between the behavior of quark and lepton mixing matrices was identified.

In the case of the *CP*-violation present in the decay of hypothetical right-handed neutrinos, we also discussed under what conditions *CP*-violating effects would disappear. In particular, we investigated briefly the limit of right-handed neutrinos with degenerate masses. We comment that, in contrast to the neutrino $\leftrightarrow$ antineutrino oscillation with degenerate neutrino masses, *CP*-odd effects need not vanish.

Admittedly, none of the ''low-energy'' processes we have considered seem to be observable in practical laboratory experiments. However, they illustrate with concrete examples the important point that Majorana phases, like the more familiar ''Dirac'' phase in the quark mixing matrix, can produce manifestly *CP*-violating effects.

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