

Supersymmetry and branes in M-theory plane waves

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We study brane embeddings in M-theory plane waves and their supersymmetry. The relation with branes in AdS backgrounds via the Penrose limit is also explored. Longitudinal planar branes originate from AdS branes while giant gravitons of AdS spaces become spherical branes which are realized as fuzzy spheres in the massive matrix theory.

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I. INTRODUCTION

The physics of type IIB string theory and M theory in the maximally supersymmetric plane wave backgrounds [1] turns out to be surprisingly rich. In the light-cone gauge the superstring and the supermembrane Green-Schwarz actions both significantly simplify. The string worldsheet theory has free massive bosons and fermions, and the free string light cone spectrum is known exactly [2]. The supermembrane action is already interacting in the flat background, and the gravitational wave adds two new types of terms to the light-cone action: mass terms and bosonic cubic interaction terms [3]. It is well known that the light-cone supermembrane action can be discretized to give Yang-Mills quantum mechanics [4], which is usually called “matrix theory” providing a nonperturbative partonic description of M theory [5]. In relation with type IIA string theory the cubic interaction terms are easily identified as describing the Myers’ dielectric effect [6]: the constituent D0-branes are expanding into fuzzy spheres. Let us quote here the plane-wave solution of eleven-dimensional supergravity which is of utmost interest in this paper:

$$ds^2 = -4dx^+ dx^- - \left[\left(\frac{\mu}{3} \right)^2 y^2 + \left(\frac{\mu}{6} \right)^2 z^2 \right] dx^{+2} + d\vec{y}^2 + d\vec{z}^2,$$

$$F = \mu dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3, \quad (1)$$

where \vec{y}, \vec{z} are vectors in $\mathbb{R}^3, \mathbb{R}^6$, respectively. The matrix theory in this particular background is first given in Ref. [3], and the derivation by discretizing the supermembrane action is demonstrated in Ref. [7]. One notable feature of this solution is that at the level of the metric the symmetry of the nine-dimensional transverse space is already broken to $SO(3) \times SO(6)$. The existence of a dimensionful parameter μ renders the study of matrix model in some sense even more tractable than the flat space counterpart. In the original matrix theory a perturbative approach is hard to achieve first because of continuous moduli and secondly due to the lack of dimensionless parameter. Now with the plane-wave ma-

trix theory the moduli space is a discrete set of fuzzy spheres of different radii, and there exists a dimensionless coupling constant which makes perturbative calculations possible [7]. By exploiting the fact that the symmetry algebra contains a classical Lie superalgebra $SU(2|4)$ and studying its atypical, i.e., short representations, it is shown that there exist protected states whose energies are free from perturbative corrections [8–10].

The aim of this paper is to provide a list of supersymmetric branes in the eleven-dimensional plane waves through supergravity analysis. It can be taken as the M-theory answer to the paper by Skenderis and Taylor [11] who studied supersymmetric D-branes in $AdS_5 \times S^5$ and the plane-wave backgrounds of type IIB string theory. The motivation for such a study is obvious when we recall the importance of D-branes in modern string theory. Especially in terms of the AdS/conformal field theory (CFT) correspondence [12], the branes correspond to several interesting objects such as magnetic monopoles, baryonic vertex [13], giant gravitons [14], and defect conformal field theory [15]. The supergravity analysis of Refs. [11,16] is found to agree with microscopic constructions of D-branes as open string boundary conditions [17] and as squeezed states of closed string sector [18]. These 1/2-Bogomoln’yi-Prasad-Sommerfield (BPS) branes are also constructed as localized supergravity solutions in Ref. [19]. For M theory a comparison can be made with the matrix model constructed in Ref. [3], where 1/2-BPS fuzzy sphere solutions are presented. A systematic search of supersymmetric branes as matrix theory solitons is undertaken in Refs. [20,21], and a new matrix model of fivebranes in plane-wave is constructed in Ref. [22] as $\mathcal{N}=8$ gauge quantum mechanics with extra hypermultiplets. We find our result consistent with the literature, as it should be. For related works on eleven-dimensional plane-wave solutions see Ref. [23].

The particular form of plane-wave makes it natural to classify branes first according to the behavior in terms of x^+ and x^- . We will be interested in the branes which are extended along both x^+ and x^- , and also the branes which are extended along x^+ while localized in x^- . We will call them “longitudinal” and “transverse” branes, respectively. In the matrix theory description the longitudinal M5-branes are realized as four-dimensional objects, while it is the transverse spherical M2-branes which become fuzzy spheres of matrix

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TABLE I. AdS branes and the corresponding planar branes in the plane wave.

Brane	Intersection	AdS embedding	pp -wave embedding
M2	(0 M2⊥M2)	AdS ₂ ×S ₁	(+, −, 1, 0)
M2	(1 M2⊥M5)	AdS ₃	
M5	(1 M5⊥M2)	AdS ₃ ×S ₃	(+, −, 2, 2)
M5	(3 M5⊥M5)	AdS ₅ ×S ₁	(+, −, 0, 4)
M5	(1 M5⊥M5)	AdS ₃ ×S ₃	(+, −, 2, 2)

theory. For completeness we will also present longitudinal M2-branes and transverse M5-branes in the plane wave as well, although they are not immediately related to the solitons of matrix theory.

It is by now well established that the plane waves are the Penrose limits [24] of AdS solutions. In the Penrose limit the spacetime is blown up around the world line of a chosen null geodesic. For the case of AdS backgrounds, if the massless particle moves in the sphere the limits are plane waves, while for particles moving only in AdS the spacetime becomes Minkowski. Now an interesting question is what happens to the supersymmetric branes in AdS space in the Penrose limit. There are two types of half supersymmetric branes in AdS backgrounds: AdS branes and giant gravitons. In order to get AdS branes it is convenient to start with intersecting branes configurations and take the near horizon limit of one brane. Let us take M5- and M2-branes intersecting on a string as an example. This system preserves eight supersymmetries. When we take the near horizon limit of M2-branes the background geometry becomes AdS₄×S⁷, and likewise the six-dimensional M5-brane world volume occupies AdS₃ subspace of AdS₄ and S³ inside S⁷. The supersymmetry is enhanced to 16, and one can conjecture that this configuration is dual to a two-dimensional superconformal field theory with SO(4) global symmetry. Similar configurations are summarized in the Table I. The giant gravitons are spherical M2 or M5 orbiting at light velocity in S⁴ or S⁷. If we choose null geodesics moving along with the giant gravitons the brane geometry is kept intact. They become transverse spherical branes in plane waves.

In the main part of this paper we use κ symmetric membrane and fivebrane actions to check the equation of motion and supersymmetry of various brane embeddings in plane waves. We choose to use the same notation which is introduced in Ref. [11] to denote different brane configurations. World-volume directions are given in the parenthesis, so, for instance, (+, −, 2, 2) means five-branes extended along x^+, x^- , and two directions in \mathbb{R}^3 and \mathbb{R}^6 each. When one checks the supersymmetry it is essential to have the explicit form of Killing spinors. They can be found in the literature but for completeness and to set the notation we show the derivations in the Appendix.

II. M2 BRANES IN PLANE WAVES

It is a relatively simple matter to check the supersymmetry of membranes with simple geometries in the plane wave

background. The bosonic part of the supermembrane action can be written as [25]

$$S = -T \int d^3\sigma (\sqrt{-\det g} - C), \quad (2)$$

where g and C are the induced metric and the three-form gauge field pulled back on the world volume, respectively. Unbroken supersymmetry requires that the Killing spinors of the background geometry be consistent with the so-called κ -symmetry projections, so

$$\Gamma_\kappa \epsilon = \epsilon, \quad (3)$$

where

$$\Gamma_\kappa = \frac{1}{3!} \epsilon^{mnp} \partial_m X^M \partial_n X^N \partial_p X^P \Gamma_{MNP}. \quad (4)$$

Capital italic letters denote eleven-dimensional indices and lowercase is reserved for world volume indices. The equation of motion is written as

$$\begin{aligned} & \frac{1}{\sqrt{-g}} \partial_m (\sqrt{-g} g^{mn} \partial_n X^N) G_{MN} + g^{mn} \partial_m X^N \partial_n X^P \gamma_{MNP} \\ & = \frac{1}{3!} \epsilon^{mnp} F_{Mnp}, \end{aligned} \quad (5)$$

where G is background metric, γ_{MNP} are Christoffel symbols, and $F = dC$.

A. Longitudinal branes

These branes are one dimensional in the transverse nine-dimensional space, and it is straightforward to see that they satisfy the equation of motion with linear geometries. When lying along the i th direction (hatted indices represent the tangent space)

$$\Gamma_\kappa = \Gamma_{\hat{+} \hat{-} \hat{i}} \quad (6)$$

and in order for the projection to be satisfied at every point of x^+ the conditions

$$\Gamma_{\hat{+} \hat{-} \hat{i}} \epsilon_0 = \epsilon_0, \quad (7)$$

$$\Gamma_{\hat{+} \hat{-} \hat{i}} \Gamma_{\hat{1} \hat{2} \hat{3}} \epsilon_0 = \epsilon_0 \quad (8)$$

have to be satisfied. We find that if the membrane is extended purely in \mathbb{R}^3 it has at least 1/4 supersymmetry and the supersymmetry is enhanced to one half when located at the origin. And the longitudinal membranes lying in \mathbb{R}^6 direction break the supersymmetry completely. It is essentially because Γ_κ anticommutes with $\Gamma_{\hat{1} \hat{2} \hat{3}}$.

B. Transverse branes

Another class of supersymmetric membranes have spherical geometry in \mathbb{R}^3 . The equations of motion for transverse

scalars $z^{a'}$ force them to lie at the origin, and the radius r of the sphere is found to be arbitrary. We further get

$$\Gamma_\kappa = \frac{3}{\mu r} \left[\Gamma_{\hat{+}} - \frac{r^2}{2} \left(\frac{\mu}{3} \right)^2 \Gamma_{\hat{-}} \right] \Gamma_{\hat{1}\hat{2}\hat{3}} \Gamma_{\hat{a}} y^a. \quad (9)$$

It turns out that $\Gamma_\kappa \epsilon = \epsilon$ is satisfied for any x^+ and r , for $\Gamma_{\hat{+}} \epsilon_0 = 0$. This is precisely the projection of Killing spinors which are linearly realized in the supermembrane action in the light-cone gauge or the matrix quantum mechanics. And it agrees with the observation in the matrix theory that the fuzzy sphere solutions in \mathbb{R}^3 preserve the whole linearly realized supersymmetry while breaking the nonlinearly realized supersymmetries completely.

There also exist transverse branes of planar geometry. The equations of motion are satisfied for $(+,1,1)$ and $(+,0,2)$ branes. Due to Wess-Zumino couplings $(+,2,0)$ planar branes do not satisfy the equation of motion without transverse scalar excitations. None of these planar transverse branes are supersymmetric.

III. M5 BRANES IN PLANE WAVES

A. Introduction to PST formulation of five-brane action

M-theory five-branes and the gauge field theory confined on their world volume are certainly one of the most mysterious objects in string theory. The construction of covariant action is a subtlety because of the self-dual three-form field strength. There exist several proposals for M5-brane actions in the literature [26–28]. Among them, covariant field equations from superembedding approach [27] is proven to be equivalent to other approaches [29]. Noncovariant action of Ref. [28] can be obtained from Ref. [26] with gauge fixing of auxiliary field. In this paper we use the Pasti-Sorokin-Tonin (PST) [26,30] formulation which is manifestly covariant. In this section, we review Ref. [30] briefly.

The bosonic part of PST action is

$$S = T_{M5} \int_{\mathcal{M}_6} d^6x \left[-\sqrt{-\det(g_{mn} + \tilde{H}_{mn})} + \frac{1}{4} \sqrt{-g} \tilde{H}^{mn} H_{mn} - \frac{1}{2} l(C_6 + dA_2 \wedge C_3) \right], \quad (10)$$

where g_{mn} is the induced metric on the world volume. C_6 and C_3 are pullback of Ramond-Ramond potentials which are subject to the eleven-dimensional Hodge duality condition

$$F_7 = dC_6 - C_3 \wedge dC_3 = *F_4. \quad (11)$$

A_2 is world volume gauge field, which gives modified field strength on the world volume,

$$H_3 = dA_2 - C_3. \quad (12)$$

There is an auxiliary scalar field $a(x)$ such that

$$H_{mn} = H_{mnp} v^p, \quad \tilde{H}_{mn} = \tilde{H}_{mnp} v^p,$$

with

$$v_p = \frac{\partial_p a}{\sqrt{-g^{mn} \partial_m a \partial_n a}}, \quad (13)$$

where \tilde{H}_3 is Hodge dual to H_3 on the world volume

$$\tilde{H}^{mnp} = \frac{1}{3!} \epsilon^{mnpijk} H_{ijk}. \quad (14)$$

It can be shown that upon double-dimensional reduction, one obtains dual form of D4-brane action and \tilde{H}_{mn} reduces to gauge field strength on the world volume.

The PST action has the following four different gauge symmetries

$$\begin{aligned} (1) \quad & \delta A_2 = d\Lambda, \\ (2) \quad & \delta A_2 = da \wedge \phi, \quad \delta a = 0, \\ (3) \quad & \delta a = \varphi, \quad \delta A_{mn} = \frac{\varphi}{\sqrt{-(\partial a)^2}} \left(\frac{2\delta L_{\text{BI}}}{\delta \tilde{H}^{mn}} - H_{mn} \right), \\ (4) \quad & \delta A_2 = B_2, \quad \delta C_3 = dB_2. \end{aligned} \quad (15)$$

Here $L_{\text{BI}} \equiv \sqrt{\det(\delta_m^n + \tilde{H}_m^n)}$. Note that the first symmetry is the same as the usual gauge symmetry of the Dirac-Born-Infeld action of D-branes and the fourth one is simply a pullback of eleven-dimensional gauge symmetry. Upon gauge fixing of scalar field a , for example, as $a = x^5$, we obtain the noncovariant formulation of Ref. [28]. From the equation of motion of A_2 , self-duality constraint is incorporated automatically :

$$H_{mn} = \frac{\tilde{H}_{mn} - 1/2 \text{tr} \tilde{H}^2 \tilde{H}_{mn} + \tilde{H}_{mn}^3}{L_{\text{BI}}}. \quad (16)$$

The equation of motion for X^M is

$$\begin{aligned} \epsilon^{m_1 \dots m_6} \left(\frac{1}{6!} F^{\hat{m}}_{m_1 \dots m_6} - \frac{1}{(3!)^2} (F^{\hat{m}}_{m_1 m_2 m_3} H_{m_4 m_5 m_6} \right. \\ \left. - \partial_n X^{\hat{m}} F^n_{m_1 m_2 m_3} H_{m_4 m_5 m_6} \right) = -\frac{1}{2} T^{mn} \nabla_m \partial_n X^{\hat{m}}, \end{aligned} \quad (17)$$

where

$$T_{mn} = 2g^{mn} \left(L_{\text{BI}} - \frac{1}{4} \tilde{H}_{mn} H^{mn} \right) - \frac{1}{2} H^{mpq} \tilde{H}_{pq}^n. \quad (18)$$

In order to incorporate fermions and make the action supersymmetric one replaces the fields and coordinates by superforms and supercoordinates. The κ symmetry is more involved than that of membranes because of the gauge field and the auxiliary scalar a :

$$\Gamma_\kappa = -\frac{v_m \Gamma^m}{\sqrt{-\det(g + \tilde{H})}} \left(\frac{1}{5!} \epsilon^{i_1 \dots i_5 n} \Gamma_{i_1 \dots i_5} v_n + \frac{1}{2} \sqrt{-g} \Gamma_{np} \tilde{H}^{np} + \frac{1}{8} \epsilon^{m_1 n_1 p_1 q} \Gamma_m \tilde{H}_{n_1 n_2} \tilde{H}_{p_1 p_2} v_q \right). \quad (19)$$

$\Gamma_m = e_m^{\hat{m}} \Gamma_{\hat{m}}$ is pullback of eleven-dimensional gamma matrices on six-dimensional world volume. One can check that Γ_κ as given above is traceless and squares to identity.

B. Longitudinal branes

1. (+, -, 3, 1) branes

We notice that there is a source term to the world volume flux from the Wess-Zumino coupling to the background four-form fields. This phenomenon is essentially the same as the M5-brane baryonic vertices in $\text{AdS}_7 \times \text{S}^4$ [31], and one could start from the configurations in the AdS background and take the Penrose limit, but here we will derive the general solutions of brane equation of motion. We set the notation for the null fluxes as (we take z^4 to be along the world volume)

$$\tilde{H} = \frac{1}{2} dx^+ \wedge dy^a \wedge dy^b f^c \epsilon_{abc} + dx^+ \wedge dy^a \wedge dz^4 g_a, \quad (20)$$

then from the Bianchi identity we get

$$\begin{aligned} \partial_a f^a &= \mu, \\ \epsilon^{abc} \partial_b g_c &= 0. \end{aligned} \quad (21)$$

Now when we choose $a = z^4$, it is straightforward to get

$$\begin{aligned} H_{+a} &= -i g_a, \\ \tilde{H}_{+a} &= -i f_a \end{aligned} \quad (22)$$

and when they are substituted into the generalized self-duality equation (16) it gives simply

$$f_a = g_a. \quad (23)$$

In fact when we evaluate the nonlinear terms in the equation we find they vanish. This is not unexpected since the fluxes are null and higher order Lorentz invariants constructed by contracting indices typically vanish. One simple solution is given as $f_a = g_a = (\mu/3)x^a$. Now we can check whether these branes are supersymmetric, which means $\Gamma_\kappa \epsilon = \epsilon$ should be satisfied everywhere on the world volume. When we spell out the required conditions we find it is impossible to satisfy especially at every x^+ . Essentially the reason is that $\Gamma_{\hat{+} \hat{1} \hat{2} \hat{3} \hat{4}}$ does not commute with $\Gamma_{\hat{1} \hat{2} \hat{3}}$ which dictates the x^+ dependence of all Killing spinors. Similar objects in IIB plane waves are (+, -, 4, 0) supersymmetric D5-branes with

null fluxes turned on due to Wess-Zumino couplings, so the analogy does not persist here. We give more comments on this issue in Sec. IV.

2. (+, -, 2, 2) branes

For this type of branes the pull back of three-form field vanishes so the world volume gauge field \tilde{H} can be set to zero. For clarity let us choose y^1, y^2, z^4, z^5 to be world volume directions. Using $\Gamma_\kappa = \Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}}$, the projection condition gives the following equations:

$$\Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}} Q \epsilon_0 = Q \epsilon_0, \quad (24)$$

$$\Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}} Q \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} \epsilon_0 = Q \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} \epsilon_0, \quad (25)$$

$$\Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}} Q \Gamma_{\hat{1} \hat{2} \hat{3}} \epsilon_0 = Q \Gamma_{\hat{1} \hat{2} \hat{3}} \epsilon_0, \quad (26)$$

$$\Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}} Q \Gamma_{\hat{+} \hat{1}} \epsilon_0 = Q \Gamma_{\hat{+} \hat{1}} \epsilon_0, \quad (27)$$

where

$$Q \equiv \left(1 + \frac{\mu}{6} y^a \Gamma_a \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} - \frac{\mu}{12} z^a \Gamma_a \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} \right). \quad (28)$$

Now using the commutation properties of matrices involved, it is straightforward to see that Eq. (24) is satisfied if the transverse scalars are set to zero and

$$\Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}} \epsilon_0 = \epsilon_0, \quad (29)$$

implying that (+, -, 2, 2) branes are 1/2-BPS when they sit at the origin. For the branes located away from the origin, they still preserve 1/4 of the supersymmetries for the Killing spinors which are annihilated by $\Gamma_{\hat{+}}$.

We can also consider nonzero gauge fields on the world volume. We will see that turning on null fluxes $H_{+45} = H_{+12}$ does not break the supersymmetry provided the brane is accordingly moved away from the origin. The origin of such world volume fields in AdS backgrounds is not hard to find. (+, -, 2, 2) branes are Penrose limits of $\text{AdS}_3 \times \text{S}^3$ branes, where nonzero three-form flux can be turned on through AdS_3 and S^3 . When the Penrose limit is taken the flux becomes null just like the background four-form field. This example is similar to $\text{AdS}_4 \times \text{S}^2$ D5-branes with nonzero flux through S^2 which was explicitly studied in Ref. [11].

We have $\Gamma_\kappa = \Gamma_{\hat{+} \hat{1} \hat{2} \hat{4} \hat{5}} - H_{+45} \Gamma_{\hat{+} \hat{1} \hat{2}}$, and the branes are supersymmetric if

$$\Gamma_\kappa Q \epsilon_0 = Q \epsilon_0, \quad (30)$$

$$\Gamma_\kappa Q \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} \epsilon_0 = Q \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} \epsilon_0, \quad (31)$$

$$\Gamma_\kappa Q \Gamma_{\hat{+} \hat{1} \hat{2} \hat{3}} \epsilon_0 = Q \Gamma_{\hat{1} \hat{2} \hat{3}} \epsilon_0 \quad (32)$$

are satisfied. There are at least 1/4 supersymmetries with $\Gamma_{\hat{+}} \epsilon_0 = 0$, and the supersymmetry is enhanced to 1/2, with Eq. (29) as the projection rule and

$$H_{+45} = \frac{\mu}{3} y^3, \quad (33)$$

with other transverse scalars set to zero.

3. (+, −, 1, 3) branes

For this orientation the branes are not supersymmetric irrespective of positions.

4. (+, −, 0, 4) branes

The analysis is similar to that of (+, −, 2, 2) branes. The branes are 1/2-BPS at the origin and 1/4-BPS away from it. One might ask whether it is also possible to turn on world volume flux and move the branes away from the origin, similar to (+, −, 2, 2) branes. When one proceeds, for instance, with nonzero $H_{+67} = H_{+45}$ one finds that there is no relation between the flux and the position of the brane similar to (+, −, 2, 2) branes. So it is not possible to compensate the harmonic potential with null world volume fluxes in this case.

C. Transverse branes

The consideration is analogous to the spherical M2-branes in \mathbb{R}^3 . The effective harmonic potential of light-cone gauge action puts the five-dimensional sphere of arbitrary radius at the origin of \mathbb{R}^3 , and we have

$$\Gamma_\kappa = \frac{6}{\mu r} \left[\Gamma_{\hat{+}} - \frac{r^2}{2} \left(\frac{\mu}{6} \right)^2 \Gamma_{\hat{-}} \right] \Gamma_{\hat{4}\hat{5}\hat{6}\hat{7}\hat{8}\hat{9}} \Gamma_{\hat{a}'z^{a'}}. \quad (34)$$

The projection condition is again satisfied provided $\Gamma_{\hat{+}} \epsilon_0 = 0$. They can be traced back to $\text{AdS}_4 \times S^7$ backgrounds in the same way as giant gravitons or M5-branes orbiting S^7 . Unlike spherical membranes, these solutions are not realized as solitons of the massive matrix model in Ref. [3]. This is not unrelated to the well-known difficulty of constructing odd-dimensional objects in matrix models.

The study of transverse planar M5-branes is again similar to that of transverse planar M2-branes. Because of the Wess-Zumino couplings (+, 3, 2) should have gauge fields while transverse scalar field has to be turned on in (+, 0, 5) branes. (+, 2, 3) and (+, 1, 4) branes satisfy the equations of motion without field excitations. They are all non-supersymmetric.

IV. DISCUSSIONS

In this paper we have employed κ symmetric membrane and five-brane actions to find supersymmetric branes in eleven-dimensional plane waves. The result is consistent with the predictions based on known supersymmetric brane configurations in AdS backgrounds, and the next step is naturally to compare with the branes found in the matrix theory. From the matrix equation of motion one readily sees that the mass terms invalidate the planar membrane solutions of ordinary matrix theory in flat space, let alone supersymmetry. Membranes with rather nontrivial geometries such as hyperbolic surfaces can be found instead [20]. The non-supersymmetric transverse planar membranes reported in this paper

should not be taken as contradictory with matrix theory results. The matrix theory is obtained after light-cone gauge fixing, and x^- is not at our disposal but determined by the Virasoro constraints. In this work x^- is always set to a constant for transverse branes. Usually the constraint equation does not allow us to set x^- to a constant, but for transverse spherical membranes the constraint equation becomes trivial and that is why two approaches coincide.

An alternative to soliton description is possible with five-branes in matrix theory. The open string modes between D0- and D4-branes in type-IIA string picture [32] give rise to hypermultiplets in the matrix quantum mechanics. The plane wave deformation of this matrix theory is presented in Ref. [22] for (+, −, 2, 2) branes in our notation, and certainly it will be interesting to construct the matrix theory of (+, −, 0, 4) branes which are also supersymmetric.

By and large our result goes hand in hand with type-IIB branes in $\text{AdS}_5 \times S^5$ and plane waves. Especially with AdS branes and giant gravitons we find perfect analogy, so we look for other pairs of supersymmetric branes in ten- and eleven-dimensional plane waves. We are especially interested in two types of type-IIB branes in plane waves which have the peculiarity that supersymmetries do not depend where they are located. Curiously we have not found similar objects in M-theory plane waves.

First there exist D-strings from unstable D-strings in $\text{AdS}_5 \times S^5$ which are wrapped on a great circle of S^5 [33]. The supersymmetry is enhanced under Penrose limit and these D-strings have eight supersymmetries in plane waves everywhere in \mathbb{R}^8 . For these 1/4-BPS D-strings, the analogy might be membranes wrapping S^2 of S^4 in $\text{AdS}_7 \times S^4$. This configuration satisfies the equation of motion, but surely this is not supersymmetric.¹ If we take the Penrose limit, the result should be (+, −, 1, 0) membranes with enhanced symmetries. Just up to this point, situations seem to be the similar to type-IIB case, but this type of membranes are 1/2-BPS at the origin differently from D-strings. And in fact (+, −, 1, 0) can be obtained from $\text{AdS}_2 \times S_1$ membranes as presented in Table I.

We are also interested in (+, −, 4, 0) D5-branes with null world-volume flux turned on by Wess-Zumino couplings. Once the gauge field is turned on they have 16 supersymmetries irrespective of positions. It is (+, −, 3, 1) M5-branes which can be matched with (+, −, 4, 0) D5-branes but according to our analysis these M5-branes are not supersymmetric. In our opinion this does not contradict the known baryonic M5-branes wrapping S^4 in $\text{AdS}_7 \times S^4$ which are supersymmetric [31]. The Penrose limits of baryonic D5-branes are studied in Ref. [34], where it is illustrated that resulting configuration is localized in x^+ which originates from the affine parameter of the null geodesic. It is because the null geodesics have a nonvanishing component along the radial direction of Poincare coordinate system, so they intersect with the brane world volume at a point. Static configurations in global coordinates could give longitudinal branes

¹We can check explicitly using the Killing spinors in global coordinates presented in the Appendix.

because massless particles moving purely in S^5 can be chosen,² but the baryonic branes in the literature are all constructed in Poincare coordinates. Unfortunately finding supersymmetric baryonic branes which are static in global coordinates does not seem promising. In global coordinates the Killing spinors depend on all coordinates,³ so satisfying Γ_κ projection everywhere on the world volume is more difficult. In fact it is not even clear how to put background D3-, M2-, or M5-branes supersymmetrically.

We thus conclude that the analogy between type-IIB and M-theory is not extended beyond AdS branes and giant gravitons. It might simply mean that the Penrose limit acts differently with different AdS solutions, but one cannot rule out the possibility that D-strings and $(+, -, 4, 0)$ D5-branes are in fact spurious and unphysical. We think it is an important matter to check their consistency for instance following the approach advocated in Ref. [36].

Note added. After this paper was completed we received an interesting paper by Skenderis and Taylor [38], where open string boundary conditions for light-cone world sheet action in type-IIB plane waves is carefully reinvestigated. It is argued that one can restore some of the broken spacetime supersymmetries by using world sheet symmetries. It will be very exciting to check whether such additional symmetries can be found also for branes in M-theory plane waves.

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APPENDIX: THE DERIVATION OF KILLING SPINORS

1. Plane waves

An explicit derivation of the Killing spinors in eleven dimensional plane wave can be also found in Ref. [37]. The pp -wave solution of interest in this paper is given as

$$ds^2 = -4dx^+ dx^- - \left[\left(\frac{\mu}{3} \right)^2 y^2 + \left(\frac{\mu}{6} \right)^2 z^2 \right] dx^{+2} + d\vec{y}^2 + d\vec{z}^2,$$

$$F_{+123} = \mu, \quad (\text{A1})$$

where indices of y^a and $z^{a'}$ are $a=1,2,3$ and $a'=4,5,6,7,8,9$ for each. We define the tangent space as follows:

²For further details of choosing null geodesics when one takes the Penrose limits in different coordinate systems we refer the readers to Ref. [35].

³We could not find the explicit form of Killing spinors in global coordinates from the literature, so we decided to present the solutions in an appendix.

$$\eta_{\hat{+}\hat{-}} = \eta_{\hat{-}\hat{+}} = 1, \quad \eta_{\hat{+}\hat{+}} = \eta_{\hat{-}\hat{-}} = 0, \quad \eta_{\hat{i}\hat{j}} = \delta_{ij},$$

$$e_{-\hat{-}} = 2, e_{+\hat{-}} = \frac{1}{2} \left[\left(\frac{\mu}{3} \right)^2 y^2 + \left(\frac{\mu}{6} \right)^2 z^2 \right], e_{+\hat{+}} = -1, e_{\hat{i}\hat{j}} = \delta_{ij}. \quad (\text{A2})$$

Thus,

$$\Gamma^- = \frac{1}{2} \Gamma_{\hat{+}} + \frac{1}{4} \left[\left(\frac{\mu}{3} \right)^2 y^2 + \left(\frac{\mu}{6} \right)^2 z^2 \right] \Gamma_{\hat{-}},$$

$$\Gamma^+ = -\Gamma_{\hat{-}},$$

$$\Gamma_- = 2\Gamma_{\hat{-}},$$

$$\Gamma_+ = -\Gamma_{\hat{+}} + \frac{1}{2} \left[\left(\frac{\mu}{3} \right)^2 y^2 + \left(\frac{\mu}{6} \right)^2 z^2 \right] \Gamma_{\hat{-}}. \quad (\text{A3})$$

By setting the variation of gravitino to zero we get the Killing spinor equations

$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon - \frac{1}{288} (\Gamma_{MNPQR} F^{NPQR} - 8F_{MNPQ} \Gamma^{NPQ}) \epsilon = 0, \quad (\text{A4})$$

where

$$\nabla_\mu = \partial_\mu + \frac{1}{4} \omega_{\hat{m}\hat{n}}^{\hat{+}} \Gamma_{\hat{m}\hat{n}}. \quad (\text{A5})$$

For each components they become

$$\nabla_- \epsilon = 0, \quad (\text{A6})$$

$$\nabla_a \epsilon - \frac{\mu}{6} \Gamma_a \Gamma_{\hat{-}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} \epsilon = 0, \quad (\text{A7})$$

$$\nabla_{a'} \epsilon + \frac{\mu}{12} \Gamma_{a'} \Gamma_{\hat{-}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} \epsilon = 0, \quad (\text{A8})$$

$$\nabla_+ \epsilon + \frac{\mu}{12} (-\Gamma_{\hat{+}\hat{-}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} + 2\Gamma_{\hat{+}\hat{+}}) \epsilon = 0. \quad (\text{A9})$$

Spin connections can be calculated as

$$\omega_{+\hat{+}}^{\hat{-}} = -\left(\frac{\mu}{3} \right)^2 y^a,$$

$$\omega_{+\hat{+}}^{a'} = -\left(\frac{\mu}{6} \right)^2 z^{a'}. \quad (\text{A10})$$

From Eq. (A6), we know that ϵ is independent of $-$. From Eqs. (A7), (A8), we get

$$\epsilon = \left(1 + \frac{\mu}{6} y^a \Gamma_a \Gamma_{\hat{-}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} - \frac{\mu}{12} z^{a'} \Gamma_{a'} \Gamma_{\hat{-}} \Gamma_{\hat{+}} \Gamma_{\hat{+}} \right) \chi, \quad (\text{A11})$$

where $\chi = \chi(+)$. Inserting this into Eq. (A9), we get

$$\partial_+ \chi + \frac{\mu}{12} (-\Gamma_{\hat{+}\hat{-}\hat{1}\hat{2}\hat{3}} + 2\Gamma_{\hat{1}\hat{2}\hat{3}}) \chi = 0. \quad (\text{A12})$$

The solution is

$$\chi = e^{(\mu/12)x^+ \Gamma_{\hat{+}\hat{-}\hat{1}\hat{2}\hat{3}}} e^{-(\mu/6)x^+ \Gamma_{\hat{1}\hat{2}\hat{3}}} \epsilon_0, \quad (\text{A13})$$

where ϵ_0 is a 32 components constant spinor.

2. AdS \times S in global coordinates

Global metric for $\text{AdS}_7 \times S^4$ is

$$ds^2 = R_{\text{AdS}}^2 (-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_5^2) + R_S^2 dS_4^2, \\ F_4 = 3R_S^3 \text{vol}(S^4), \quad (\text{A14})$$

where

$$dS_n^2 = d\theta_n^2 + \sin^2 \theta_n dS_{n-1}^2. \quad (\text{A15})$$

Here $R_{\text{AdS}} = 2R_S$. The Penrose limit is taken, with $R_S \rightarrow \infty$,

$$\theta_i = \frac{\pi}{2} - \frac{y_i}{R_S} \quad (i=1,2,3), \\ \rho = \frac{z}{R_{\text{AdS}}}, \\ \tau = \frac{\mu x^+}{6} + \frac{6x^-}{R_{\text{AdS}}^2 \mu}, \\ \theta = \frac{\mu x^+}{3} - \frac{3x^-}{R_S^2 \mu}, \quad (\text{A16})$$

where $\theta = \theta_4$. We get

$$ds^2 = -4dx^+ dx^- - \left[\left(\frac{\mu}{3} \right)^2 y^2 + \left(\frac{\mu}{6} \right)^2 z^2 \right] dx^{+2} + d\vec{y}^2 + d\vec{z}^2, \\ F_4 = \mu dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3. \quad (\text{A17})$$

Killing spinor equations for global $\text{AdS}_7 \times S^4$ space are

$$\partial_\tau \epsilon + \frac{1}{2} \sinh \rho \Gamma_{\hat{\tau}\hat{\rho}} \epsilon - \frac{1}{2} \cosh \rho \Gamma_{\hat{\tau}} \Gamma_{\hat{*}} \epsilon = 0, \\ \partial_\rho \epsilon - \frac{1}{2} \Gamma_{\hat{\rho}} \Gamma_{\hat{*}} \epsilon = 0, \\ \nabla_a \epsilon - \frac{1}{2} \sinh \rho e_a^{\hat{a}} \Gamma_{\hat{a}} \Gamma_{\hat{*}} \epsilon = 0, \\ \nabla_{a'} \epsilon + \frac{1}{2} e_{a'}^{\hat{a}'} \Gamma_{\hat{a}'} \Gamma_{\hat{*}} \epsilon = 0, \quad (\text{A18})$$

where

$$\Gamma_{\hat{*}} = \Gamma_{\hat{\theta}_1 \hat{\theta}_2 \hat{\theta}_3 \hat{\theta}_4}. \quad (\text{A19})$$

Here $a = \phi_1, \dots, \phi_5$ and $a' = \theta_1, \dots, \theta_4$. Vielbeins are defined such that $e_i^{\hat{i}} e_j^{\hat{j}} \eta_{\hat{i}\hat{j}} = \bar{g}_{ij}$, where \bar{g}_{ij} is metric of unit sphere. The solution is

$$\epsilon = e^{(\rho/2) \Gamma_{\hat{\rho}} \Gamma_{\hat{*}}} e^{(\phi_1/2) \Gamma_{\hat{\rho}} \hat{\phi}_1} \left(\prod_{k=1}^3 e^{[(\phi_{k+1})/2] \Gamma_{\hat{\phi}_k \hat{\phi}_{k+1}}} \right) \\ \times e^{-(\theta_1/2) \Gamma_{\hat{\theta}_1} \Gamma_{\hat{*}}} \left(\prod_{k=1}^3 e^{[(\theta_{k+1})/2] \Gamma_{\hat{\theta}_k \hat{\theta}_{k+1}}} \right) e^{(\tau/2) \Gamma_{\hat{\tau}} \Gamma_{\hat{*}}}. \quad (\text{A20})$$

The global metric for $\text{AdS}_4 \times S^7$ is

$$ds^2 = R_{\text{AdS}}^2 (-\cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_2^2) + R_S^2 dS_7^2, \\ F_4 = 3R_{\text{AdS}}^3 \cosh \rho \sinh^2 \rho \sin \phi_1 d\tau \wedge d\rho \wedge d\phi_1 \wedge d\phi_2, \quad (\text{A21})$$

where

$$dS_n^2 = d\theta_n^2 + \sin^2 \theta_n dS_{n-1}^2. \quad (\text{A22})$$

Here $R_{\text{AdS}} = 1/2 R_S$. The Penrose limit is taken, with $R_S \rightarrow \infty$,

$$\theta_i = \frac{\pi}{2} - \frac{z_i}{R_S} \quad (i=1,2,3), \\ \rho = \frac{y}{R_{\text{AdS}}}, \\ \tau = \frac{\mu x^+}{3} + \frac{3x^-}{R_{\text{AdS}}^2 \mu}, \\ \theta = \frac{\mu x^+}{6} - \frac{6x^-}{R_S^2 \mu}, \quad (\text{A23})$$

where $\theta = \theta_7$. We get the same metric as Eq. (A17).

Killing spinor equations for global $\text{AdS}_4 \times S^7$ space are

$$\partial_\tau \epsilon + \frac{1}{2} \sinh \rho \Gamma_{\hat{\tau}\hat{\rho}} \epsilon - \frac{1}{2} \cosh \rho \Gamma_{\hat{\tau}} \Gamma_{\hat{*}} \epsilon = 0, \\ \partial_\rho \epsilon - \frac{1}{2} \Gamma_{\hat{\rho}} \Gamma_{\hat{*}} \epsilon = 0, \\ \nabla_a \epsilon - \frac{1}{2} \sinh \rho e_a^{\hat{a}} \Gamma_{\hat{a}} \Gamma_{\hat{*}} \epsilon = 0, \\ \nabla_{a'} \epsilon + \frac{1}{2} e_{a'}^{\hat{a}'} \Gamma_{\hat{a}'} \Gamma_{\hat{*}} \epsilon = 0, \quad (\text{A24})$$

where

$$\Gamma_{\hat{*}} = \Gamma_{\hat{\tau}\hat{\rho}\hat{\phi}_1\hat{\phi}_2}. \quad (\text{A25})$$

Here $a = \phi_1, \dots, \phi_2$ and $a' = \theta_1, \dots, \theta_7$. Vielbeins are defined such that $e_i^{\hat{i}} e_j^{\hat{j}} \eta_{\hat{i}\hat{j}} = \bar{g}_{ij}$, where \bar{g}_{ij} is metric of unit sphere. The solution of the Killing spinor equation is

$$\epsilon = e^{(\rho/2)\Gamma_{\hat{\rho}}\hat{\Gamma}_{*}} e^{(\phi_1/2)\Gamma_{\hat{\rho}}\hat{\phi}_1} e^{(\phi_2/2)\Gamma_{\hat{\phi}_1}\hat{\phi}_2} e^{-(\theta_1/2)\Gamma_{\hat{\theta}_1}\hat{\Gamma}_{*}} \left(\prod_{k=1}^6 e^{[(\theta_{k+1})/2]\Gamma_{\hat{\theta}_k}\hat{\theta}_{k+1}} \right) e^{(\tau/2)\Gamma_{\hat{\tau}}\hat{\Gamma}_{*}}. \quad (\text{A26})$$

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