Scaling laws in hadronic processes and string theory

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We propose a possible scheme for getting the known QCD scaling laws within string theory. In particular, we consider amplitudes for exclusive scattering of hadrons at large momentum transfer, hadronic form factors, and distribution functions.

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I. INTRODUCTION

As is well known, string theory (the dual-resonance model) was originally invented to describe the physics of hadrons [1]. However, in spite of much effort this idealized theory of hadrons failed and finally was replaced by QCD.

New ideas came in 1997–1999, with Polyakov's proposal for a string theory whose tension is running, Maldacena's conjecture of the AdS conformal field theory (CFT) correspondence, and the Randall-Sundrum proposal for the hierarchy problem [2,3,4]. The key feature of all these is the warped geometry in spacetime; i.e., the spacetime metric is no longer Minkowskian; rather, the normalization of the four-dimensional Minkowski metric is a function of other coordinates. In the simplest case, for example, the world-sheet action (its bosonic part) for the theory with a running tension has the form

$$S=\frac{1}{4\pi}\int d^2z(\partial\varphi\bar{\partial}\varphi+a^2(\varphi)\partial X\cdot\bar{\partial}X), \tag{1.1}$$

where $a^2(\varphi)$ is the running string tension. A natural requirement is that $a(\varphi)\sim e^{\varphi}$ as $\varphi\to\infty$. In other words, spacetime behaves for large φ as AdS_5. The field φ was called the Liouville field in Ref. [2].

Using these ideas, Polchinski and Strassler recently initiated a new attempt to describe the physics of hadrons in the framework of string theory [5].[1] They proposed a scheme for evaluating the high-energy scattering amplitudes of glueballs in terms of vertex operators and found that the amplitudes fall as powers of the momentum. This is the desired result which was found in the physics of hadrons a long time ago [9]. To be more precise, the amplitude for exclusive scattering of $m+2$ glueballs is given by [5]

$$\mathcal{M}(2\to m)\sim\frac{(gN_c)^{(n-2)/4}}{N_c^m\Lambda^{m-2}}\left(\frac{\Lambda}{p}\right)^{n-4}, \tag{1.2}$$

where p is the large momentum scale, g is the string coupling constant, which is the square of the gauge coupling, N_c is the number of colors, and Λ is a scale by the lightest glueball. n denotes the total number of constituents in the glueballs.

In fact, it was known already in the 1970s that dimensional analysis and some simple assumptions (dimensional counting rules) immediately lead to the correct scaling in exclusive hadronic scattering at large momentum transfer:[2]

$$\mathcal{M}(2\to m)\sim p^{-n+4}. \tag{1.3}$$

Indeed, if the total number of constituents in the hadrons is n and the one-particle state $|p\rangle$ is normalized in such a way as to have the dimension of [length], then the amplitude has the dimension $[\text{length}]^{n-4}$. If, moreover, at large momentum transfer, p is the only length scale, then it immediately follows the wanted scaling. Modulo soft violations (logarithms) this scaling is in rather good agreement with experimental results.

It is now clear that the result (1.2) does not completely agree with this dimensional analysis of exclusive processes because it contains two dimensionful parameters. It seems more relevant for inclusive processes where the second parameter is normally interpreted as the missing mass. In this case n represents a subset of the constituents which participate in scattering; the others remain as "spectators." Moreover, in Sec. II we will also see that QCD analysis provides a dependence on the coupling constant which differs from the one in Eq. (1.2).

The purpose of the present paper is twofold. The first is to propose a possible scheme for getting a scaling within string theory which resolves these difficulties. This scheme can be considered as a refinement of [5]. Like old matrix models (2D gravity coupled to conformal matter), where the scaling is obtained via a zero mode of the Liouville field [11], here we also get the scaling via a zero mode. As mentioned earlier, the warped geometry provides a natural candidate for the role of the Liouville field. Note that this significantly simplifies the analysis as a knowledge of the whole dependence of the vertex operators on the Liouville field is not required. As to the zero mode dependence, it is provided by the corresponding Laplace equation in spacetime. Thus our derivation of the scaling seems quite universal. Our second purpose is to apply this scheme to the hadronic form factors and distribution functions for deep inelastic scattering. There are some special limits where these objects can easily be analyzed [12,13]. So we are bound to learn something if we succeed.

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[1] See also [6,7,8].

[2] See, e.g., [10].

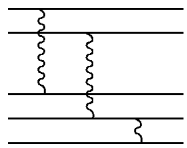

FIG. 1. A typical Born diagram for meson-baryon scattering.

It is also worth mentioning that while this paper was being written Polchinski and Strassler did new work [14] which has some overlap with what we describe in Secs. II and III.[3]

The outline of the paper is as follows. We start in Sec. II by recalling some basic facts about the QCD analysis of large-momentum-transfer processes. Here we focus on hard subprocesses for hadronic scattering and form factors. In Sec. III we present a scheme for getting the scaling behavior within string theory. We find complete agreement with the Born approximation in QCD. Section IV contains our conclusions and a list of open problems.

II. LARGE-MOMENTUM-TRANSFER PROCESSES IN QCD

In this section we briefly recall some basic facts of QCD analysis of large-momentum-transfer exclusive processes.[4] We do not consider spin dependent effects in what follows.

A. Fixed-angle hadronic scattering

In QCD analysis of a hadronic process $AB\rightarrow CD$ the fixed-angle scattering amplitude related to hard subprocesses is given by the amplitude T_H for the scattering of hadronic constituents, integrated over the possible constituent momenta adding up to the hadron momenta. Explicitly,

$$\mathcal{M}(AB\rightarrow CD)=\prod_{i=A,\ldots,D}\int_0^1[dx^i]\Phi_D^*(x^D,p_\perp^2)\times\Phi_C^*(x^C,p_\perp^2)T_H(x^A,\ldots,x^D,p_\perp^2)\times\Phi_B(x^B,p_\perp^2)\Phi_A(x^A,p_\perp^2), \tag{2.1}$$

where $x^i=\{x_1^i,\ldots,x_{n_i}^i\}$, $[dx^i]=dx_1^i\cdots dx_{n_i}^i\delta(1-\Sigma_{k=1}^{n_i}x_k^i)$, $p_\perp^2=tu/s$, and n_i is the minimal number of constituents (valence quarks) in the ith hadron.[5] The momentum transfer between hadronic constituents occurs via the hard scattering amplitude T_H which, to leading order in the coupling constant, is given by the sum of all Born diagrams with hadrons replaced by their constituents. A typical Born diagram looks like that in Fig. 1. The amplitude Φ_i is the probability amplitude for finding constituents with fractions of longitudinal momenta x_k^i in the ith hadron.

For $s\rightarrow\infty$ at fixed s/t, T_H falls with increasing s as[6]

$$T_H(x^A,\ldots,x^D,p_\perp^2)=\left(\frac{e^2}{4\pi s}\right)^{n/2-2}f(s/t,x^A,\ldots,x^D)\times[1+O(e^2)], \tag{2.2}$$

where $n=n_A+\ldots+n_D$. Since the probability amplitudes in the Born approximation behave as

$$\Phi_i(x^i,p_\perp^2)=\phi_i(x^i), \tag{2.3}$$

the amplitude $\mathcal{M}(AB\rightarrow CD)$ takes the form

$$\mathcal{M}(AB\rightarrow CD)\sim\left(\frac{e^2}{4\pi s}\right)^{n/2-2}. \tag{2.4}$$

At this point a comment is in order. It is well known that radiative corrections in QCD typically contain logarithms that violate the scaling. To next order (one-loop approximation), they are included by replacing $e^2/4\pi\rightarrow\alpha_s(p_\perp^2)$, $\Phi_i(x^i,p_\perp^2)\rightarrow\phi_i(x^i)(\ln p_\perp^2/\Lambda_{\rm QCD}^2)^{-\gamma_i}$, where $\Lambda_{\rm QCD}$ is the QCD parameter and γ_i is some constant. Note that the amplitude can then be rewritten as

$$\mathcal{M}(AB\rightarrow CD)\sim[\alpha_s(p_\perp^2)]^{n/2-2+\Sigma_i\gamma_i}p_\perp^{-n+4}. \tag{2.5}$$

B. Electromagnetic form factors and structure functions

The electromagnetic form factor of a hadron is given by the matrix element of the electromagnetic current between two hadronic states $\langle p+q|J^a(0)|p\rangle$. The form factors are most easily analyzed by using the two invariants $q^2=-Q^2<0$, $\nu_B=p\cdot q$ and then taking the Bjorken limit where Q^2 and ν_B both go to infinity with the ratio $x_B=Q^2/2\nu_B$ fixed [12]. x_B is known as the Bjorken variable. Note that for elastic scattering $x_B=1$. Using Lorentz covariance and gauge invariance, the matrix element can be parametrized in terms of a scalar function (form factor) $F(Q^2)$ as

$$\langle p+q|J^\mu(0)|p\rangle=(2p^\mu+q^\mu)F(Q^2), \tag{2.6}$$

where $J^\mu(0)=\int d^4kJ^\mu(k)$.

To leading order in the coupling constant, the form factor in QCD takes the form

$$F(Q^2)=\int_0^1[dx][dy]\Phi^*(x,Q^2)T_B(x,y,Q^2)\Phi(y,Q^2), \tag{2.7}$$

where $x=\{x_1,\ldots,x_n\}$, $[dx]=dx_1\cdots dx_n\delta(1-\Sigma_{k=1}^n x_k)$, and n is the number of constituents (valence quarks) in the hadron.[7] The amplitude Φ is defined in the same way as in the previous subsection while T_B is now given by the sum of all Born diagrams for the hadron constituents to scatter with

[3]See also [15].

[4]For further discussion, background, and experimental data, see, e.g., [16,17].

[5]Note that adding more constituents (nonvalence quarks) is unimportant to leading order in $p_\perp^2$ at $p_\perp^2\rightarrow\infty$. So n_i every time means the minimal number.

[6]Note that in the center of mass frame $t=-s\sin^2(\theta/2)$, $u=-s\cos^2(\theta/2)$, and $p_\perp^2=(1/4)s\sin^2\theta$. So $p_\perp^2\sim s$ at fixed angle.

[7]The variable y and the integration measure $[dy]$ are defined in the same way.

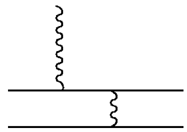

FIG. 2. A typical Born diagram for the meson form factor.

the photon producing the constituents in the final state. A typical Born diagram now looks like that in Fig. 2.

At large Q^2 the Born diagrams give

$$T_B(x,y,Q^2)=\left(\frac{e^2}{4\pi Q^2}\right)^{n-1} t_B(x,y) \tag{2.8}$$

and thus, reasoning as in the previous subsection, the asymptotic behavior of the form factor is given by

$$F(Q^2)\sim\left(\frac{e^2}{4\pi Q^2}\right)^{n-1}. \tag{2.9}$$

A few noteworthy facts are the following. Dimensional analysis and the assumptions of Sec. I which lead to the scaling law for the amplitudes can also be applied to the form factors. This immediately gives the desired scaling law (Bjorken scaling) $F(Q^2)\sim Q^{-2n+2}$. One of the most interesting applications of QCD was the prediction of slow violations of Bjorken scaling by soft $\ln Q^2$'s. To one-loop approximation, the logarithms are included in the same way as in Sec. II A: by replacing $e^2/4\pi\to\alpha_s(p_\perp^2)$, $\Phi_i(x^i,p_\perp^2)\to\phi_i(x^i)(\ln p_\perp^2/\Lambda_{\mathrm{QCD}}^2)^{-\gamma_i}$. Note that the form factor can also be rewritten as

$$F(Q^2)\sim[\alpha_s(p_\perp^2)]^{n-1+2\gamma}(Q^2)^{-n+1}. \tag{2.10}$$

Let us conclude the discussion of QCD by briefly reviewing the hadronic structure functions. These are defined via the hadronic tensor

$$W^{\mu\nu}(Q^2,\nu_B)=\frac{1}{4\pi}\int d^4\xi e^{iq\cdot\xi}\langle p|J^\mu(\xi)J^\nu(0)|p\rangle \tag{2.11}$$

as[8]

$$\begin{aligned}W^{\mu\nu}(Q^2,\nu_B)=&\left(-\eta^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)F_1(Q^2,\nu_B)\\&+\frac{1}{\nu_B}\left(p^\mu-\frac{\nu_B}{q^2}q^\mu\right)\left(p^\mu-\frac{\nu_B}{q^2}q^\nu\right)\\&\times F_2(Q^2,\nu_B).\end{aligned} \tag{2.12}$$

Note that the functions F_1 and F_2 are related to others in common use by $W_1=F_1$ and $W_2=(M^2/\nu_B)F_2$, where M is the hadronic mass.

[8]Since we do not consider spin effects, we omit the antisymmetric part of $W^{\mu\nu}$ which provides two other structure functions g_1 and g_2.

The physical meaning of F_2 is that its x_B dependence probes the longitudinal momentum distribution of the hadron constituents as viewed in the infinite momentum frame of the hadron. In particular, it is expressed in terms of the distribution functions G_i as[9]

$$F_2(x_B)=x_B\sum_{i=1}^{n}\lambda_i^2 G_i(x_B), \tag{2.13}$$

where λ_i is the charge of the ith constituent.

The structure functions are not known completely because they are in general beyond the tools of perturbative theory. However, some asymptotics are available. In particular, the distribution functions which become functions (modulo soft logarithms) only of x_B in the Bjorken limit behave as [13]

$$G_i(x_B)\sim(1-x_B)^{2n-3} \tag{2.14}$$

near the threshold $x_B=1$. As before, n means the total number of constituents in the hadron. It is worth mentioning that this asymptotic behavior can also be determined via a convolution equation for G_i [18].

III. SCALING LAWS VIA STRING THEORY

The aim of this section is to show how the Born approximation for hadronic amplitudes and form factors can be easily obtained in the framework of string theory.

A. String theory settings

According to our discussion of Sec. I, the metric asymptotically behaves as AdS_5. Since we are interested in the scaling rather than its violation, it is natural to use this metric. So, as in [5] we begin with string theory on the product of AdS_5 with a five-dimensional transverse space K. The spacetime metric is then

$$ds^2=\frac{r^2}{R^2}\eta_{\mu\nu}dX^\mu dX^\nu+\frac{R^2}{r^2}dr^2+R^2ds_{\mathrm{K}}^2, \tag{3.1}$$

where R is the radius of AdS_5 and η is a four-dimensional Minkowski metric. We assume that K does not provide any dimensionful parameter except R. Moreover, ds_{K}^2 does not depend on R.

Before continuing our discussion of string theory settings, let us pause here to stress an important point. Although we use some techniques inspired by the AdS/CFT conjecture, we do not strictly follow this conjecture. The point is that we are interested in the physical processes where perturbation theory is applicable rather than the strong coupling regime. So we postulate the relation

$$g=\frac{e^2}{4\pi}=\frac{R^2}{\alpha'} \tag{3.2}$$

[9]$G_i(x_B)$ is defined as the probability of finding the ith constituent in the hadron with fractional longitudinal momentum x_B (in the infinite momentum frame of the hadron).

between the closed string coupling constant g, the gauge coupling constant e, and the parameters R,α'. This relation is not obviously what is imposed by the AdS/CFT correspondence. It becomes the latter (modulo a numerical factor) by replacing $e^2\to e,\ g\to\sqrt{g}$. It is worth mentioning that some examples where the AdS/CFT results look like the QCD ones after the above replacement are already known in the literature (see, e.g., [19,20]).

In the first-quantized string theory one first introduces a free field action on the worldsheet, then defines physical vertex operators. Finally, scattering amplitudes in spacetime are defined as expectation values of the vertex operators. In general, it is unknown how to implement this program in the case of a curved background like AdS. However, we think that the problem of interest does not require knowledge of the full string theory on AdS. It should be a simple stringy analogue of the dimensional counting rules that results in the scaling laws. Our idea is to relate the scaling to a zero mode of r as is usually done in the context of 2D gravity where the scaling is due to a zero mode of the Liouville field. Then all we need is the dependence of vertex operators on this zero mode. The latter can be found from the Laplace equation on $\mathrm{AdS}_5\times\mathrm{K}$. In fact, our scenario means that nonzero modes of the transverse fields as well as r are not of primary importance for the scaling. To leading order, they contribute a numerical factor. Alternatively, one can say that fluctuations of the transverse fields as well as r are slow, as was assumed in [5].

Under this assumption, it is straightforward to write down the part of the worldsheet action for the remaining nonzero modes that is most appropriate for our purposes:[10]

$$S=\frac{1}{4\pi}\int d^2z\left(\frac{1}{\hat{\alpha}}\,\partial X\cdot\bar{\partial}X+\psi\cdot\bar{\partial}\psi+\bar{\psi}\cdot\partial\bar{\psi}\right),\qquad(3.3)$$

where $\hat{\alpha}=\alpha' R^2/r^2$ and the ψ's have been rescaled as $\psi\to(R/r)\psi$. We use this form of the action for two reasons: (1) It allows us to use the known results for string amplitudes simply by replacing $\alpha'\to\hat{\alpha}$; and (2) it represents a model theory which has the running tension in the sense of Polyakov [see Eq. (1.1)].

To evaluate the correlation functions of vertex operators one needs to define the path integral measure. First let us do so for the zero modes.[11] It is natural to take it in a covariant form $\sqrt{-g}d^{10}\xi$. However, for a reason which will be clear in a moment we need it to be dimensionless. So we define the measure as

$$\frac{1}{\alpha'^2R^6}\sqrt{-g}d^{10}\xi=\frac{1}{\alpha'^2R^4}r^3drd^4xd\Omega_{\mathrm{K}},\qquad(3.4)$$

where $d\Omega_{\mathrm{K}}$ is an invariant measure on K.

[10]Note that this is a conformal invariant action because r does not depend on z.

[11]We only consider the Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector, so there are no zero modes of the ψ's.

The nonzero modes are quantized in an ordinary way as follows from their action (3.3). The only novelty is the appearance of $\hat{\alpha}$ instead of α'. For example, in the case of spherical topology the propagators are given by

$$\langle X^\mu(z,\bar{z})X^\nu(z',\bar{z}')\rangle=-\hat{\alpha}\,\eta^{\mu\nu}\ln|z-z'|,$$

$$\langle\psi^\mu(z)\psi^\nu(z')\rangle=\frac{\eta^{\mu\nu}}{z-z'}.\qquad(3.5)$$

As mentioned earlier, we are interested in the scaling properties of hadron interaction involving transfer of large momenta where all masses are negligible. Therefore the most appropriate string vertex operators to try are the massless ones. In general, one computes such vertex operators in the supergravity approximation (zero mode approximation) by finding solutions of the corresponding Laplace equation on $\mathrm{AdS}_5\times\mathrm{K}$. A general solution looks like $V\sim f(\Omega)\varphi(r)e^{ip\cdot x}$, where φ shows a power-law falloff as $r\to\infty$. As mentioned earlier, the metric is AdS_5 only for large r, so it is pointless to use the exact solution for $\varphi(r)$ as long as we use the AdS_5 metric. The only thing we really need is its power-law falloff $\varphi\sim r^{-n}$. It was suggested in [5] that if one interprets vertex operators as hadronic states, one thinks of the n's as the numbers of hadron constituents. The function f is a solution of the Laplace equation on K which is responsible for internal degrees of freedom. Since we do not take them into account, we will not pay attention to f either.

Now let us extend the analysis to include the nonzero modes. In the approximation we use it can easily be done by using the standard expressions for the vertex operators with α' replaced by $\hat{\alpha}$. Putting all this together, we get

$$V_{n,p}=f(\Omega)r^{-n}e^{ip\cdot x}\int d^2z\,\varepsilon_{\mu\nu}V_i^\mu(p,z)\bar{V}_i^\nu(p,\bar{z}),\qquad(3.6)$$

where i means the superghost charge and ε is the polarization tensor. Note that we extract the zero mode factor $e^{ip\cdot x}$ from the vertex operators V_i and $\bar{V}_i$ here and below. In general, the integrand of Eq. (3.6) is more involved because V_i and $\bar{V}_i$ are dressed by operators constructed from nonzero modes of r and the transverse fields.

Since we do not consider spin dependent effects let us specialize to the dilaton vertex. To fix its normalization, we first make a rescaling $X\to\sqrt{\hat{\alpha}}X$ to bring the integrand into a dimensionless form. Then we insert a factor α'^n. Thus, the vertex takes the form

$$V_{n,p}=f(\Omega)\left(\frac{\alpha'}{r}\right)^n e^{ip\cdot x}\int d^2z\,\varepsilon^{dil}_{\mu\nu}V_i^\mu(\hat{p},z)\bar{V}_i^\nu(\hat{p},\bar{z}).\qquad(3.7)$$

We use $\hat{p}$ as a shorthand notation for $\sqrt{\hat{\alpha}}p$. It is evident that such a vertex has the dimension $[\mathrm{length}]^n$, exactly as needed for the normalization of the n-particle state.

Now that we have the vertex operators for hadronic states, we can focus on the next object of interest, the vertex operator for the electromagnetic current. Since the current is conserved, it obeys

$$q\cdot J(q)=0. \tag{3.8}$$

A natural realization that satisfies such a condition can easily be found in a picture where a string worldsheet admits boundaries.[12] In this case, we have

$$J^{\mu}(q)\sim e^{iq\cdot x}\oint_C dz V_0^{\mu}(\hat{q},z), \tag{3.9}$$

where $V_0^{\mu}=[\partial X^{\mu}+(i/2)(\hat{q}\cdot\psi)\psi^{\mu}]e^{i\hat{q}\cdot X}$, $\hat{q}=\sqrt{\hat{\alpha}}q$. C denotes a worldsheet boundary.

By analogy with the vertex operators of hadrons, we insert the factor $f(\Omega)(\alpha'/r)^n$. Since the current has the dimensionality of [length], we set $n=1$. Thus, the final form of $J^{\mu}(q)$ is given by

$$J^{\mu}(q)=f(\Omega)\,\frac{\alpha'}{r}\,e^{ip\cdot x}\oint_C dz V_0^{\mu}(\hat{q},z). \tag{3.10}$$

B. Evaluation of amplitudes

The calculation of the scattering amplitude for a hadronic process $AB\to CD$ mainly goes along the lines of [5] adjusted to our settings. The amplitude is defined as the expectation value of the product of the vertex operators (3.7):

$$\delta^{(4)}(p_A+\cdots+p_D)\mathcal{M}(AB\to CD)=\frac{1}{g^2}\left\langle\prod_{i=A,\ldots,D}g^{n_i}V_{n_i,p_i}\right\rangle. \tag{3.11}$$

Here the string worldsheet is a sphere as is usual at the tree level in closed string perturbation theory. Some factors of this expression require further explanation: (1) The overall factor g^{-2} comes from the sphere as it should; (2) the factors g^{n_i} are due to our normalization prescription. It differs from the standard one and will be discussed later.

The integration over nonzero modes does not require much work at least in the case of spherical topology where the four-point dilaton amplitude $\mathcal{A}_4$ is well known in the literature [21]. Aside from an irrelevant numerical factor, the amplitude is then given by

$$\mathcal{M}(AB\to CD)=\frac{g^{n-2}}{\alpha'^2R^4}\int_0^{\infty}dr\,r^3\left(\frac{\alpha'}{r}\right)^n \times\mathcal{A}_4(\alpha' R^2 s/r^2,\alpha' R^2 t/r^2,\alpha' R^2 u/r^2). \tag{3.12}$$

Here the integral over Ω_{K} has not disappeared, but was included in the numerical factor. n is the sum of the n_i's. From the above expression it is evident that if we rescale r as $r\to\sqrt{\alpha' s}Rr$ then we get

$$\mathcal{M}(AB\to CD)=F(\theta)\left(\frac{g}{s}\right)^{n/2-2}, \tag{3.13}$$

where $F(\theta)$ is a function of the angle θ defined in the center of mass frame. This is the desired result, and it is identical to the result of QCD, Eq. (2.4).

A couple of comments are in order: (1) The use of the relation (3.2) is crucial for matching with the QCD result. (2) It may appear that the scaling is due to the zero modes only. That is not exactly true. The nonzero modes contribute to the function $F(\theta)$ which contains some important information. We will return to this issue in Sec. IV.

C. Evaluation of form factors and distribution functions

By analogy with the amplitudes, we define the form factor as the expectation value of the product of the vertex operators given by Eqs. (3.7) and (3.10). Explicitly,

$$\langle p+q|J^{\mu}(0)|p\rangle=\int d^4k\,\frac{1}{g}\langle g^nV_{n,p+q}\sqrt{g}J^{\mu}(k)g^nV_{n,p}\rangle. \tag{3.14}$$

The worldsheet is now a disk (upper half plane). So we insert the overall factor g^{-1} as is usual in the case of the disk. Just as before, each closed string vertex carries a factor g^n. From this, it seems natural to accompany each open string vertex by $g^{n/2}$. If so, then J^{μ} is accompanied by $g^{1/2}$.

To evaluate the right hand side of Eq. (3.14), it is convenient to use the worldsheet doubling trick (see, e.g., [22] and references therein). After performing the integration over x and setting the vertex operators at $(z_1,\overline{z_1},z_2,z_3,\overline{z_3})=(iy,-iy,t,i,-i)$, we find (modulo a numerical factor)

$$\langle p+q|J^{\mu}(0)|p\rangle=\frac{g^{2n-1/2}}{\alpha'^2R^4}\int_0^{+\infty}dr\,r^3\left(\frac{\alpha'}{r}\right)^{2n+1}\mathcal{A}^{\mu}(\hat{p},\hat{q}) \tag{3.15}$$

with

$$\mathcal{A}^{\mu}(\hat{p},\hat{q})=\int_{-\infty}^{+\infty}dt\int_0^1 dy\,\varepsilon^{dil}_{\eta\nu}\varepsilon^{dil}_{\lambda\sigma}\langle V^{\eta}_{-1}(iy,-\hat{p}-\hat{q}) \times V^{\nu}_{-1}(-iy,-\hat{p}-\hat{q})V_0^{\mu}(t,2\hat{q}) \times V_0^{\lambda}(i,\hat{p})V_0^{\sigma}(-i,\hat{p})\rangle. \tag{3.16}$$

Here we again include the integral over Ω_{K} in an irrelevant numerical factor.

To keep things as simple as possible, first we choose the infinite momentum frame for the hadron[13]

$$p^{\mu}=(P+M^2/2P,0,0,P),\quad q^{\mu}=(\nu_B/P,q^1,q^2,0). \tag{3.17}$$

Here M^2 is the mass of the hadron. Secondly, we specialize to a convenient current component J^0 or, equivalently, J^3. Then it follows from Lorentz covariance that $\mathcal{A}^0(\hat{p},\hat{q})=\sqrt{\hat{\alpha}}P\mathcal{A}(\hat{\alpha}Q^2)$.

Using Eq. (2.6), we find the following representation for the form factor:

[12] To our knowledge, this is the simplest way of introducing electromagnetic currents in string theory.

[13] Note that $Q^2=-q^2=(q^1)^2+(q^2)^2+O(1/P^2)$.

$$F(Q^2)=\frac{g^{2n-1/2}}{\alpha'^{5/2}R^3}\int_0^{+\infty}drr^3\left(\frac{\alpha'}{r}\right)^{2n+2}\mathcal{A}(\alpha' R^2Q^2/r^2). \tag{3.18}$$

The desired QCD result (2.9) is obtained by rescaling $r\rightarrow\sqrt{\alpha' Q^2}Rr$ and using the relation (3.2).

At this point, it is necessary to make a couple of remarks.

(1) Unlike the four-point dilaton amplitude $\mathcal{A}_4$ we used to evaluate the hadronic amplitudes in the previous subsection, the correlator of the five vertex operators in Eq. (3.16) is not well defined in the following sense. As an object of 2D conformal field theory, $\int dtV_0^{\mu}(t,2\hat{q})$ is well defined only at $q^2=0$ while for our purposes we need it at large q^2. In fact, this is the long standing problem of string theory: how to continue correlators of vertex operators defined on shell (at special values of momenta) to off shell (for arbitrary values of momenta). So far, there is no solution to this problem. In the problem of interest it means that $\mathcal{A}^0$ is in general ambiguous.[14] However, it is clear from the above discussion that the explicit form of $\mathcal{A}^0$ is not of principal importance for our purposes. So our results seem rather universal and independent of any special way of going off shell. We will return to this point in Sec. IV.

(2) It is straightforward to evaluate the inelastic form factors by using the same technique. It is clear that the result has the same form as before with n replaced by $(n_1+n_2)/2$. Here n_i means the number of constituents in the ith hadron.

Finally, let us discuss how the asymptotics (2.14) for the distribution functions can be obtained in string theory. In fact, it was realized long ago [13] that this asymptotics is closely related to the asymptotic behavior of the form factors we have just considered. Thus, it seems natural to reproduce it too.

To do so, we first choose a convenient infinite momentum frame defined by Eq. (3.17). Our next task is to evaluate the probability amplitude of finding the ith constituent in the hadron with fractional longitudinal momentum x_B. If $V_{n,p}$ describes a hadronic state with n constituents, then the best that we can use as an approximation to the hadronic state containing the ith constituent with a specific momentum is $V_{1,p'}V_{n-1,p-p'}$. What is important to remark is that, unlike p, all other momenta are not lightlike. Thus, the corresponding vertex operators are off shell. The probability amplitude is simply

$$A_i\sim\langle V_{n,p}V_{1,p'}V_{n-1,p-p'}\rangle. \tag{3.19}$$

To compute the distribution function, we have to integrate $|A_i|^2$ over the momenta of the constituents. In our approximation to the probability amplitude there is no integration over longitudinal momenta as x_B is fixed. As to transverse momenta, it seems natural to parametrize them in terms of q [see Eq. (3.17)]. The lovely thing about the threshold $x_B=1$ is that at leading order in $1-x_B$ we can take $q=p-p'$. Just as before, it is now easy to evaluate the scaling behavior of the amplitude

$$A_i\sim\frac{1}{(Q^2)^{n-2}}. \tag{3.20}$$

Finally, the desired result is obtained after a simple estimation:

$$G_i(x_B)\sim\int_{M^2/(1-x_B)}^{+\infty}dQ^2A_i^2\sim(1-x_B)^{2n-3}. \tag{3.21}$$

At this point, it is worth mentioning that in approaching the threshold one must satisfy the inequality $Q^2(1-x_B)>M^2$ in order to stay in the Bjorken limiting region for x_B. This inequality provides the lower limit of integration.

IV. MANY OPEN PROBLEMS

There is a large number of open problems associated with the circle of ideas explored in this paper. In this section we list a few.

It would be interesting to understand in more detail how string theory reproduces the results of QCD in the Born approximation. The point is that in QCD the calculation of the hadronic scattering amplitudes involves the summation of a huge number of Born diagrams like the one presented in Fig. 1. On the other hand, we saw in Sec. III that in string theory the summation is automatically done and all information is encoded in the function $F(\theta)$. Thus, this function may be considered as a generating function for Born diagrams. If so, it would significantly simplify ordinary QCD calculations. The problem is how to implement this explicitly. Unfortunately, our approximation is invalid for computing the exact form of $F(\theta)$. A possible way to deal with the problem is of course to involve the nonzero modes of r and even the transverse fields. The price for this is a long standing problem: string theory on AdS_5. Although some information that is relevant for deep inelastic scattering has already been extracted from this theory (see, e.g., [7,23]), a complete solution is still missing.

A related problem is understanding more clearly the stringy calculation of the form factors. Even without turning on the nonzero modes of r and the transverse fields, it requires off-shell continuation. In principle, accounting for the nonzero modes might help with off-shell continuation. However, another interesting idea for doing it is to try the original Liouville mode as it comes from a conformal factor of the worldsheet metric [24]. In addition, it would be interesting to compute the matrix element $\langle p|J^{\mu}(\xi)J^{\nu}(0)|p\rangle$ directly by using the vertex operators.

It should be stressed that the string theory construction we are dealing with has an essential difference from the standard one. Usually each external leg of a Feynman diagram corresponds to a vertex operator in the corresponding string correlator representing the amplitude. For example, one has at tree level in closed string perturbation theory

[14]Because of this, it seems pointless to give an explicit calculation of $\mathcal{A}^0$. It will suffer from ambiguity like any off-shell continuation.

$$g^{m-2}\langle V_{n_1,p_1}\cdots V_{n_m,p_m}\rangle, \tag{4.1}$$

where each operator is accompanied by g. It is transparent from the diagrams of Figs. 1 and 2 that in the problem of interest we assigned a vertex operator to a number of external legs also. This is as it should be because hadrons are composite objects. As a consequence, our normalization prescription for the vertex operators is different from Eq. (4.1). It is clear that the standard prescription fails if it is blindly applied to recover the QCD results. To see what happens consider our normalization in more detail. We begin with a modification of Eq. (4.1) via replacing $gV_{n_i,p_i}\to g^{g_i}V_{n_i,p_i}$. This gives the overall factor g^{n-2}, where $n=n_1+\cdots+n_m$. However, this is not the whole story. The point is that the expectation value of the product of the vertex operators provides an additional factor $g^{-n/2}$. This effect is unknown in the case of Minkowski spacetime because it is due to the warped geometry. Thus, we end up with the desired answer. Modulo g^{-2}, the effect of the warped geometry is in fact the transformation of the closed string coupling constant to the open string coupling constant $g\to\sqrt{g}$. It would be interesting to see whether the warped geometry also transforms the open string coupling e to $\sqrt{e}$. If so, then it might help to explain the known effect $e^2N_c\to\sqrt{e^2N_c}$ observed in AdS/CFT calculations (see, e.g., [19,20]).

Another interesting problem involves computing the quantum corrections. Our discussion here was entirely classical. At first glance, the QCD results of Sec. II formally assume a slight modification of the Born approximation at one-loop level that on the string theory side can be implemented by just replacing $g^{n_i}\to g^{n_i+\gamma_i}$. But the real situation is much more involved. The point is that the coupling constant is now running. So, if we indeed wish to recover the QCD results, we need to provide a mechanism which makes the coupling run. One possible way is to deform the string background to get the desired running. How to implement this and what will happen remain to be seen.

As we mentioned, we do not strictly follow the prescriptions of the AdS/CFT correspondence. So we have postulated the relation (3.2). Our motivation is that, first of all, we would like to describe the known results of QCD to see whether the ideas work or not. Note also that one can exploit this relation to make a simple estimation of the size of the internal compact space K in terms of α':

$$R^2=0.1\alpha'. \tag{4.2}$$

Here we used the fact that the typical value of the coupling constant obtained from deep inelastic scattering experiments is of order 0.1. The next step would be to see what happens in the strong coupling regime where it is believed that the AdS/CFT correspondence holds. At present we lack the solution of string theory on $AdS_5\times S^5$ or on its nonconformal deformations that could help us. However, let us nonetheless see what information about high-energy scattering can be found by using our approximation. Assuming as in [5] that at small r the geometry given by Eq. (3.1) is truncated at $r=r_0$, the amplitude (3.12) then becomes

$$\mathcal{M}(AB\to CD)\sim\frac{g^{(3/4)n-2}}{N_c^{n/4}}\left(\frac{1}{\sqrt{s}}\right)^{n-4}\int_{r_0/\sqrt{\alpha' s}R}^{\infty}dr\,r^{3-n}\times\mathcal{A}_4\left(\frac{1}{r^2},-\frac{\sin^2\theta/2}{r^2},-\frac{\cos^2\theta/2}{r^2}\right), \tag{4.3}$$

where we also rescaled r as $r\to\sqrt{\alpha' s}Rr$. It is at least somewhat plausible in the hard scattering limit ($s\to\infty$) that $r_0/\sqrt{\alpha' s}R\ll 1$.[15] If so, then the leading behavior of the amplitude has the same power-law falloff as in [5].[16] It is of some interest to evaluate corrections to the scaling. To do so, first we note that at small r the four-point amplitude $\mathcal{A}_4$ behaves as $\mathcal{A}_4\sim e^{-(1/r^2)f(\theta)}$. Next we estimate the correction as

$$\frac{g^{(3/4)n-2}}{N_c^{n/4}}\left(\frac{1}{\sqrt{s}}\right)^{n-4}\int_0^{r_0/\sqrt{\alpha' s}R}dr\,r^{3-n}e^{-(1/r^2)f(\theta)}\sim e^{-(\alpha' R^2/r_0^2)sf(\theta)}. \tag{4.4}$$

Unlike in Sec. II, where the radiative QCD logarithms violate the scaling, there is now an exponential correction which already violates the scaling at the tree level.

There is a list of other interesting issues including spin, flavor, color, soft subprocesses, the evolution of the distribution functions, and many others, which certainly deserve to be addressed.

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[15] In the model described in [5], $r_0=\Lambda R^2$, where Λ is the scale of the lightest hadron. So it means that $\Lambda g^{1/4}\ll\sqrt{s}$.

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