Interaction of global and local monopoles

Eugênio R. Bezerra de Mello*
Departamento de Física-CCEN, Universidade Federal da Paraíba, 58.059-970, J. Pessoa, Paraíba, Caixa Postal 5.008, Brazil

Yves Brihaye†
Faculté des Sciences, Université de Mons-Hainaut, B-7000 Mons, Belgium

Betti Hartmann‡
Department of Mathematical Sciences, University of Durham, Durham DH1 3LE, United Kingdom

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We study the direct interaction between global and local monopoles. While in two previous papers the coupling between the two sectors was only ''indirect'' through the coupling to gravity, we here introduce a new term in the potential that couples the Goldstone field and the Higgs field directly. We investigate the influence of this term in curved space and compare it to the results obtained previously.

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I. INTRODUCTION

Magnetic monopoles have raised a lot of interest since their first construction by Dirac [1]. While the Dirac monopole has a singularity, the so-called Dirac string, 't Hooft and Polyakov [2] came up with the construction of a particlelike magnetic monopole in the SU(2) Yang-Mills-Higgs (YMH) theory with a triplet Higgs scalar. The magnetic charge of this object results from the topological properties of the solution and is directly proportional to the degree of the map from space-time infinity to the vacuum manifold of the theory. Minimal coupling of the SU(2) YMH model to gravity leads (for suitable choice of the boundary conditions) to globally regular gravitating monopoles [3–6] which exist up to a maximal value of the gravitational coupling. For higher values of that coupling, the Schwarzschild radius of the solution becomes larger than the radius of the monopole core.

Considering only the theory with a scalar Goldstone field leads to a different type of topological defect [7], the so-called global monopole. Like all global defects, this has infinite energy resulting from the $1/r^2$ falloff of the energy density. Coupling to gravity [8,9] leads to the observation that the effective mass of the system becomes negative.

Recently, a self-gravitating magnetic monopole in the space-time of a global monopole has been considered [10,11]. In both papers the potential is the sum of the Higgs potential and the analog Goldstone field potential. Thus, the interaction between the global Goldstone field and the local Higgs field is only indirect, namely, through the coupling to gravity.

Considering the composite topological defect, the effective mass was found to be positive or negative, depending on the coupling constants of the model.

In this paper, we continue the investigation of this system, allowing a direct interaction between the matter fields. This extra interaction is implemented by adding to the potential a gauge invariant term as follows:

$$V_3(\phi^a,\chi^a)=\frac{\lambda_3}{2}(\phi^a\phi^a-\eta_1^2)(\chi^a\chi^a-\eta_2^2). \tag{1}$$

Here, we are mainly interested in the analysis of the critical behavior of the composite system considering now the most general, gauge invariant potential. Because of the high nonlinearity of the set of coupled differential equation, an analytical analysis becomes impossible and only a numerical analysis can provide the results.

This paper is organized as follows. In Sec. II we describe our model and the ansatz. In Sec. III, we give the equations of motion, the boundary conditions and the analysis of the asymptotic behavior of the Goldstone and Higgs field functions. We present our numerical results in Sec. IV and describe how the extra term (1) in the potential provides new results concerning the behavior of the fields near the defect's core as well as concerning the effective mass of the system. We observe, e.g., a strong dependence of the mass on the coupling constant λ_3. We give our conclusions in Sec. V.

II. THE EXTENDED MODEL

This model is described by the following action which is composed of the action for the gravitating global monopole and the action of the gravitating local monopole:

$$S=S_G+S_M=\int \mathcal{L}_G\sqrt{-g}d^4x+\int \mathcal{L}_M\sqrt{-g}d^4x, \tag{2}$$

with the gravity Lagrangian $\mathcal{L}_G$,

$$\mathcal{L}_G=\frac{1}{16\pi G}R \tag{3}$$

and G denotes Newton's constant.

The matter Lagrangian $\mathcal{L}_M$ of the extended model with extra direct interaction between the global Goldstone field χ^a and the Higgs field ϕ^a reads ($a=1,2,3$)

*Email address: emello@fisica.ufpb.br
†Email address: Yves.Brihaye@umh.ac.be
‡Email address: Betti.Hartmann@durham.ac.uk

$$\mathcal{L}_M = -\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} - \tfrac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \tfrac{1}{2}(\partial_\mu\chi^a)(\partial^\mu\chi^a) - V(\phi^a,\chi^a), \quad (4)$$

with covariant derivative of the Higgs field

$$D_\mu\phi^a = \partial_\mu\phi^a - e\epsilon_{abc}A^b_\mu\phi^c, \quad (5)$$

field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\epsilon_{abc}A^b_\mu A^c_\nu, \quad (6)$$

and e being the gauge coupling constant. The potential $V(\phi^a,\chi^a)$ is given by

$$V(\phi^a,\chi^a) = \frac{\lambda_1}{4}(\phi^a\phi^a - \eta_1^2)^2 + \frac{\lambda_2}{4}(\chi^a\chi^a - \eta_2^2)^2 + \frac{\lambda_3}{2}(\phi^a\phi^a - \eta_1^2)(\chi^a\chi^a - \eta_2^2), \quad (7)$$

where the third term on the right hand side couples the two sectors directly to each other with coupling constant λ_3. λ_1, λ_2 denote the self-coupling constants of the Higgs and Goldstone field, respectively, while η_1, η_2 are the corresponding vacuum expectation values.

The potential (7) has different properties according to the sign of $\Delta \equiv \lambda_1\lambda_2 - \lambda_3^2$. For $\Delta > 0$, the potential has positive values and its minima are attained for $\phi_a^2 = \eta_1^2$, $\chi_a^2 = \eta_2^2$, for which Eq. (7) is obviously zero. For $\Delta < 0$, these configurations become saddle points and two minima occur for

$$\phi_a^2 = 0, \quad \chi_a^2 = \eta_2^2 + \frac{\lambda_1}{\lambda_3}\eta_1^2 \quad (8)$$

and

$$\chi_a^2 = 0, \quad \phi_a^2 = \eta_2^2 + \frac{\lambda_2}{\lambda_3}\eta_2^2. \quad (9)$$

The potential's values for these extrema are, respectively,

$$V_{min} = \frac{\eta_1^4}{4\lambda_1}\Delta, \quad V_{min} = \frac{\eta_2^4}{4\lambda_2}\Delta \quad (10)$$

which are negative since $\Delta < 0$.

The ansatz

The ansatz for the metric tensor in Schwarzschild-like coordinates reads

$$ds^2 = -A^2(r)N(r)dt^2 + N^{-1}(r)dr^2 + r^2(d\theta^2 + \sin\theta^2 d\phi^2), \quad (11)$$

where we define for later convenience the mass function $m(r)$ as follows:

$$N(r) = 1 - 2\alpha^2 q^2 - \frac{2m(r)}{r}. \quad (12)$$

The ansatz for the global Goldstone field χ^a, the Higgs field ϕ^a and the gauge field A^a_μ in Cartesian coordinates reads

$$\phi^a(x) = \eta_1 h(r)\hat{x}^a, \quad (13)$$

$$\chi^a(x) = \eta_1 f(r)\hat{x}^a, \quad (14)$$

$$A^a_i(x) = \epsilon_{iaj}\hat{x}^j \frac{1-u(r)}{er}, \quad (15)$$

and

$$A^a_0(x) = 0. \quad (16)$$

Substituting the above configurations into the matter Lagrangian density, we obtain

$$\mathcal{L}_M = -4\pi\int_0^\infty dr r^2 A[N\mathcal{K}(f,h,u) + \mathcal{U}(f,h,u)], \quad (17)$$

where

$$\mathcal{K}(f,h,u) = \frac{1}{2}\eta_1^2(f')^2 + \frac{1}{2}\eta_1^2(h')^2 + \frac{(u')^2}{e^2r^2}, \quad (18)$$

and

$$\mathcal{U}(f,h,u) = \frac{(u^2-1)^2}{2e^2r^4} + \frac{\eta_1^2u^2h^2}{r^2} + \frac{\eta_1^2f^2}{r^2} + \frac{\lambda_1\eta_1^4}{4}(h^2-1)^2 + \frac{\lambda_2\eta_1^4}{4}(f^2-q^2)^2 + \frac{\lambda_3\eta_1^4}{2}(h^2-1)(f^2-q^2). \quad (19)$$

The prime denotes the derivative with respect to r.

The gravity Lagrangian $\mathcal{L}_G$ is given by

$$\mathcal{L}_G = \frac{1}{2G}\int_0^\infty dr r(N-1)A'. \quad (20)$$

III. EQUATIONS OF MOTION

Varying Eq. (2) with respect to the matter fields and gravitational fields and introducing the dimensionless variable x and dimensionless mass function $\mu(x)$,

$$x = e\eta_1 r, \quad \mu(x) = e\eta_1 m(r), \quad (21)$$

we obtain the following set of differential equations:

$$\frac{d}{dx}\left[x^2AN\frac{df}{dx}\right] = A[2f + x^2\beta_2^2 f(f^2-q^2) + x^2\beta_3^2(h^2-1)f], \quad (22)$$

$$\frac{d}{dx}\left[x^2AN\frac{dh}{dx}\right] = A[2u^2h + x^2\beta_1^2(h^2-1)h + x^2\beta_3^2 h(f^2-q^2)], \quad (23)$$

$$\frac{d}{dx}\left[AN\frac{du}{dx}\right]=A\left[\frac{u(u^2-1)}{x^2}+uh^2\right], \tag{24}$$

$$\frac{d}{dx}(xAN)=[1-2\alpha^2x^2\overline{U}]A, \tag{25}$$

with

$$\overline{U}=\frac{(u^2-1)^2}{2x^4}+\frac{u^2h^2}{x^2}+\frac{f^2}{x^2}+\frac{\beta_1^2}{4}(h^2-1)^2+\frac{\beta_2^2}{4}(f^2-q^2)^2+\frac{\beta_3^2}{2}(h^2-1)(f^2-q^2), \tag{26}$$

and

$$\frac{dA}{dx}=2\alpha^2Ax\overline{K}, \tag{27}$$

with

$$\overline{K}=\frac{1}{2}\left(\frac{df}{dx}\right)^2+\frac{1}{2}\left(\frac{dh}{dx}\right)^2+\frac{1}{x^2}\left(\frac{du}{dx}\right)^2. \tag{28}$$

The equations only depend on the dimensionless coupling constants

$$\alpha^2=4\pi G\eta_1^2, \quad \beta_i^2=\lambda_i/e^2, \quad i=1,2,3, \quad q=\eta_2/\eta_1. \tag{29}$$

With the definition (12), Eq. (25) can be brought to the form

$$\mu'=\alpha^2x^2\left(\overline{K}+\left(\overline{U}-\frac{q^2}{x^2}\right)\right). \tag{30}$$

The *finite* energy of the solution can then be obtained by taking the value of $\mu(x)$ at infinity.

A. Boundary conditions

The boundary conditions at the origin which follow from the requirement of regularity read

$$u(x=0)=1, \quad f(x=0)=0, \quad h(x=0)=0, \quad \mu(x=0)=0. \tag{31}$$

In fact, the behavior of the function $\mu(x)$ near the origin is $\mu(x)\approx-\alpha^2q^2x$. The requirement of finite energy solutions leads to

$$f(x=\infty)=q, \quad h(x=\infty)=1, \quad u(x=\infty)=0,$$
$$N(x=\infty)=1-2\alpha^2q^2. \tag{32}$$

B. Asymptotic behavior

The integration of the equations for generic values of the parameters needs a better understanding of the asymptotic behavior of the functions $f(x)$ and $h(x)$. Two different types of behavior for the functions f and h in flat space seem possible. Either

$$h(x)=1+\frac{A}{x^2}+O\left(\frac{1}{x^3}\right), \quad A=\frac{\beta_3^2}{\beta_2^2\beta_1^2-\beta_3^4},$$
$$f(x)=q+\frac{B}{x^2}+O\left(\frac{1}{x^3}\right), \quad B=\frac{-\beta_1^2}{q(\beta_2^2\beta_1^2-\beta_3^4)}, \tag{33}$$

or

$$h(x)=1+C_1\exp(\rho_1x)+C_2\exp(-\rho_1x)+C_3\exp(\rho_2x)+C_4\exp(-\rho_2x),$$
$$f(x)=q+\tilde{C}_1\exp(\rho_1x)+\tilde{C}_2\exp(-\rho_1x)+\tilde{C}_3\exp(\rho_2x)+\tilde{C}_4\exp(-\rho_2x), \tag{34}$$

where ρ_1^2, ρ_2^2 are the eigenvalues of the matrix

$$\begin{pmatrix} \beta_1^2 & q\beta_3^2 \\ q\beta_3^2 & q^2\beta_2^2 \end{pmatrix}. \tag{35}$$

Our numerical analysis strongly suggests the following results: For $\beta_1^2\beta_2^2-\beta_3^4>0$, the solutions obey the asymptotic behavior (33) for f and h; for $\beta_1^2\beta_2^2-\beta_3^4<0$, the functions f and h have the asymptotic behavior (34). In this case, however, one of the eigenvalues of Eq. (35) becomes negative, consequently its square root is complex and the functions f,h thus oscillate. This behavior is clearly observed when we set $\beta_1=0$, $\beta_2\neq0$ and increase β_3 from 0. For $\beta_3=0$, the solutions exist and f,h increase monotonically, as soon as $\beta_3\neq0$, however, oscillations occur.

IV. NUMERICAL RESULTS

Because of the reasons given previously, we restrict our analysis in the following to the case $\beta_1^2\beta_2^2-\beta_3^4>0$.

The limit $\beta_3=0$ was studied in detail in [10,11]. Here we discuss how the new term influences the behavior of the solutions. One of the main features is that the Higgs function $h(x)$ does not reach its asymptotic value $h(x=\infty)=1$ monotonically. It first reaches a maximum $h_{max}>1$ for $x<\infty$ and then decreases to 1. This phenomenon, which can be expected from the inspection of Eq. (33) since $A>0$ for $\beta_3\neq0$, is illustrated in Fig. 1. This is different from the phenomena observed in [10,11]. There, for all values of the coupling constants, the function $h(x)$ was observed to be monotonically increasing from 0 to 1 as indicated in Fig. 1 for $\beta_3=0$. The Goldstone field function $f(x)$ reaches its asymptotic value q for increasing values of the coordinate x when β_3 is increasing. This again can be explained by Eq. (33) since the value B is a decreasing function of β_3 with a sharp drop at $\beta_3\approx1$. Thus, the function $f(x)$ decays less strong for higher values of β_3.

At the same time, the minimum of the metric function N

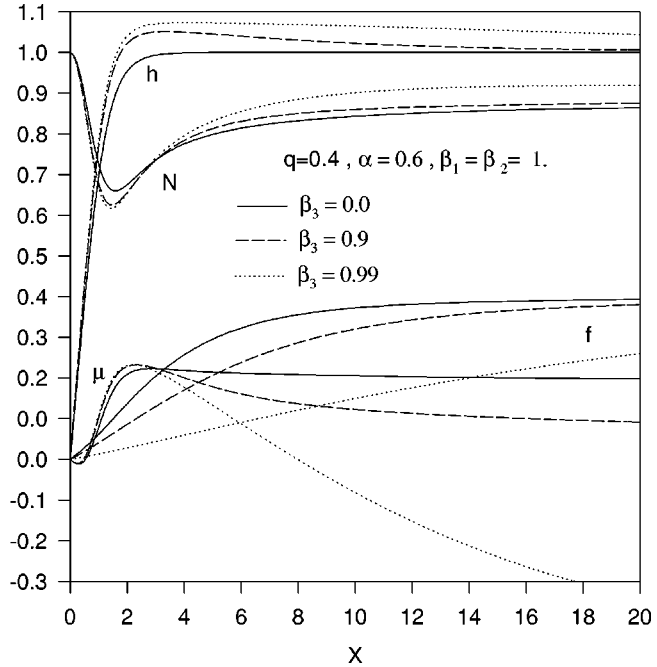

FIG. 1. The metric functions N, μ, the Higgs field function h and the Goldstone field function f are shown as functions of the dimensionless variable x for $q=0.4$, $\alpha=0.6$, $\beta_1=\beta_2=1$, and three different values of β_3.

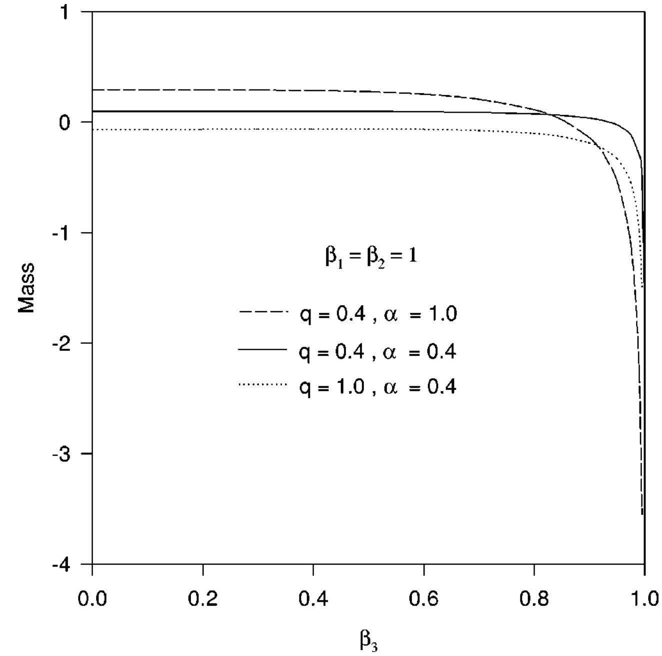

FIG. 2. The mass in units of $4\pi\eta_1/e$ is shown as a function of β_3 for three different combinations of the coupling constants α and q and for $\beta_1=\beta_2=1$.

decreases with increasing β_3, while the matter function μ reaches a maximum at roughly the same x at which the function N attains its minimum and then drops down to its asymptotic value which determines the mass of the solution. Clearly, for $\beta_3=0$, this asymptotic value is positive, while for increasing β_3, it decreases and becomes negative for large enough β_3.

Another feature of the new term is that, for all parameters but β_3 fixed, the classical mass of the solution decreases when β_3 increases. This is illustrated by means of Fig. 2 where we have plotted the evolution of the mass as a function of β_3 for three different combinations of the coupling constants α,q. For $q=1.0$, $\alpha=0.4$, the mass of the solution is already negative for $\beta_3=0$ indicating that the influence of the global monopole is already dominating in the limit of vanishing direct interaction of the Higgs field and Goldstone field. For smaller q, the mass becomes negative at some finite value $\beta_3=\beta_3^0$. For fixed q, this value is increasing for decreasing α. Since the local and global monopole are still ''indirectly'' coupled over gravity, a stronger gravitational coupling, of course, couples the two objects in a stronger way. For small gravitational coupling, β_3 thus has to be raised further to make the influence of the global monopole dominating. When the combination $\beta_1^2\beta_2^2-\beta_3^4$ becomes negative, the mass of the solution reaches $-\infty$, independent of the combination of q and α. This again can be related to the fact that we observe that the solutions become oscillating for $\beta_1^2\beta_2^2-\beta_3^4<0$.

We also studied the way the solution bifurcates into a black hole when the parameter α increases while the others are fixed. Fixing $q=0.4$, we have analyzed the critical behavior of the solution and checked that, like for the case $\beta_3=0$, the solution bifurcates into a black hole for a finite value of α, say $\alpha=\alpha_c$. As demonstrated in Fig. 3 for $\beta_1=\beta_2=1$, $q=0.4$ and $\beta_3=0.8$, the function N develops a minimum which becomes deeper while α increases and becomes zero for $\alpha=\alpha_c$. The limiting solution thus represents

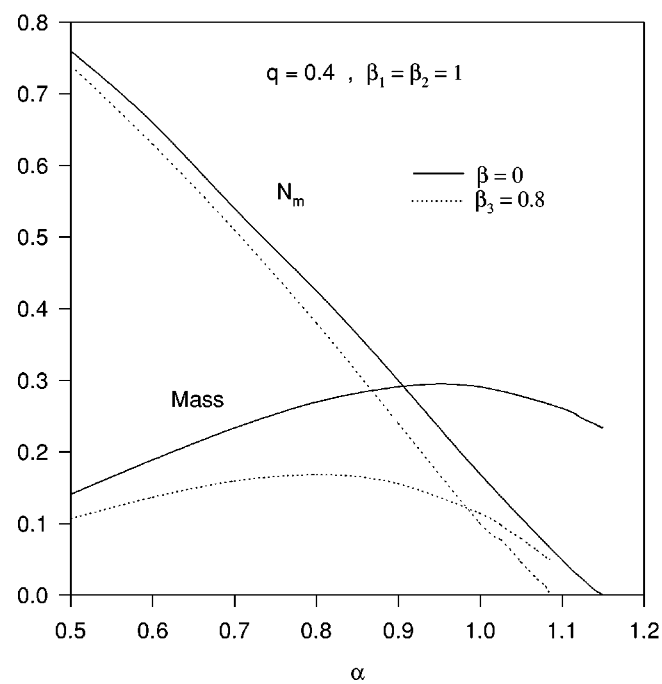

FIG. 3. The mass in units of $4\pi\eta_1/e$ and the minimum N_m of the metric function $N(x)$ are shown as a function of α for $\beta_3=0$ and $\beta_3=0.8$, respectively, and $q=0.4$, $\beta_1=\beta_2=1$.

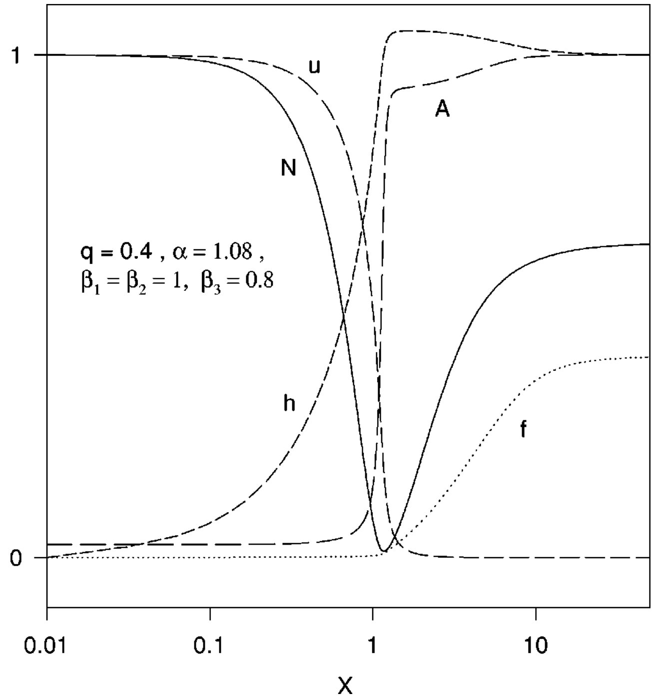

FIG. 4. The profiles of the functions N,A,u,f,h are shown for $\alpha\approx\alpha_c=1.08$, $\beta_1=\beta_2=1$, $\beta_3=0.8$, and $q=0.4$.

an extremal black hole solution with horizon x_h. In Fig. 3, we also show the evolution of the mass with α. Finally, the critical solution corresponding to $\beta_3=0.8$ and $\alpha_c\approx1.085$ is displayed in Fig. 4. This figure clearly suggests that the limiting solution is not an Abelian black hole for $x>x_h$, like, e.g., in the case of the pure local monopole [5], where the solutions bifurcate with the branch of Reissner-Nordström (RN) solutions and consequently the functions reach their RN values for $x>x_h$. Here, the function f is equal to zero for $0\leqslant x\leqslant x_h$ and nontrivial for $x_h\leqslant x\leqslant x_0$. Similarly, the functions A and h are nontrivial for $x\leqslant x_0$ and are equal to their asymptotic values for $x>x_0$, while the gauge field function u reaches its asymptotic value for $x\approx x_h$. This solution thus represents a ''black hole inside a global monopole'' as was observed previously for the $\beta_3=0$ limit [11].

V. CONCLUSIONS

In this paper we have analyzed the composite system of a global and a local gravitating monopole considering the most general gauge-invariant potential. This potential contains a direct interaction between the Goldstone and the Higgs field. This term leads to important consequences concerning the local behavior of the fields themselves as well as concerning the global properties of the system. One of the most relevant consequences is related to the effective mass associated with the composite topological defect. The numerical results show a strong dependence of this mass on the coupling constant λ_3. Although the Goldstone and Higgs fields are indirectly coupled through gravity, the extra direct interaction is more effective. Increasing the parameter λ_3, the mass becomes negative, indicating the dominance of the global sector over the local one. Compared to the results of the $\lambda_3=0$ case [10,11], we observe the modulus of the negative mass to become very large in our system. Another point which deserves to be mentioned is that the extra direct interaction term is not positive definite. Denoting by $x_{h=1}$ the value of x for which the Higgs field function h is equal to one, h becomes bigger than one for $x>x_{h=1}$ and the new term in the potential becomes negative. However, for a particular choice of the self-coupling constants such that they fulfill $\Delta\equiv\lambda_1\lambda_2-\lambda_3^2>0$, the total potential is positive, vanishing only at the minima $\phi_a^2=\eta_1^2$, $\chi_a^2=\eta_2^2$.

As possible extensions of the model studied here, let us mention the coupling to a scalar dilaton which arises naturally in low energy effective actions of string theory. The gravitating local monopole was studied recently coupled to a dilaton [12] and it was found that in the limit of critical gravitational coupling, the solutions bifurcate with the branch of extremal Einstein-Maxwell-dilaton solutions which are associated with naked singularities. It would be interesting to see what sort of critical solution the composite system of a global and local monopole reaches since our analysis indicates that the behavior of the functions close to the core of the local monopole is strongly influenced by the global monopole.

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