

Analytic properties of thermal corrected boson propagators

H. C. G. Caldas^{1,*} and M. Hott^{2,†}

¹*Departamento de Ciências Naturais, DCNAT, Universidade Federal de São João del Rei, UFSJ, Praça Dom Helvécio, 74. 36300-000, São João del Rei, MG, Brazil*

²*UNESP, Campus de Guaratinguetá, P.O. Box 205. 12516-410, Guaratinguetá, SP, Brazil*

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We investigate the analytic properties of finite-temperature self-energies of bosons interacting with fermions at one-loop order. A simple boson-fermion model was chosen due to its interesting features of having two distinct couplings of bosons with fermions. This leads to a quite different analytic behavior of the bosons self-energies as the external momentum $K^\mu = (k^0, \vec{k})$ approaches zero in the two possible limits. It is shown that the plasmon and Debye masses are consistently obtained at the pole of the corrected propagator even when the self-energy is analytic at the origin in the frequency-momentum space.

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I. INTRODUCTION

It is well known that the analytic properties of self-energies at finite temperature are extremely important, since they are related to physical processes such as the *pole physics*.¹ Nevertheless, they have attracted some attention only in the past decade, as pointed out by Weldon in Ref. [1]. The existence of a unique limit as the external momentum $K_\mu \rightarrow 0$ has been admitted only if the internal lines of the loop have propagators with different masses [2]. We show by using a simple theory that (i) even if the self-energy is analytic at the origin in the frequency-momentum space (although the limits need not commute [3]), it still leads to the plasmon and Debye masses which arise from the consistent calculation at the pole of the corrected propagators and (ii) the analyticity, at the origin, exhibited by one of the one-loop graphs is due to its very peculiar dependence on the external momenta.

The paper is organized as follows. In Sec. II, we discuss the one-loop self-energy of the bosons due their interactions with fermions. In Sec. III, we study the pole physics which lead to the correct determination of the plasmon and Debye bosons masses. In this section we also obtain the high temperature limit for these masses. In Sec. IV we analyze the dispersion relation which relates the real and imaginary parts of a one-loop self-energy. We conclude in Sec. V.

II. THE BASIC INTERACTIONS

Let us consider the boson-fermion interaction (we have ignored, for simplicity, isospin) described by the Lagrangian density which is part of the Gell-Mann and Levy model in its broken chiral symmetry phase [4]

$$\mathcal{L}(\bar{\psi}, \psi, \phi_i) = \bar{\psi}[i\gamma^\mu \partial_\mu - m - g(\sigma + i\pi\gamma^5)]\psi + \mathcal{L}_0(\phi_i), \quad (1)$$

*Email address: hcaldas@funrei.br

†Email address: hott@feg.unesp.br

¹The dispersion relation, the plasmon and Debye masses, the damping and decay rates, etc.

where g is a nondimensional positive coupling constant and $\mathcal{L}_0(\phi_i)$ is the free Klein-Gordon Lagrangian for the bosons.

More recently the Gell-Mann and Levy model has been considered to obtain the thermal masses of the fermions and bosons at high temperature by using a modified self-consistent resummation (MSCR) [5], to study the renormalization of the effective action at finite temperature [6] and to study the issue of analyticity of bubble diagrams [7].

To one loop order, the zero temperature retarded self-energies for the pion and sigma fields in the Minkowski space read

$$\Pi(K) = ig^2 \int \frac{d^4P}{(2\pi)^4} \text{Tr} \left[(a\gamma_5 - b) \times \frac{1}{\not{P} + \not{K} - m} (a\gamma_5 - b) \frac{1}{\not{P} - m} \right], \quad (2)$$

where for the pion one has $a=1$ and $b=0$ and for the sigma $a=0$ and $b=i$. As is well known, the γ_5 matrix will be responsible for a minus sign in pion self-energy which will bring the differences between the corrections for the two bosons. After performing the trace, this expression gives

$$\Pi(K) = -4ig^2 \int \frac{d^4P}{(2\pi)^4} \frac{P^2 + P^\mu K_\mu + cm^2}{[(P+K)^2 - m^2][P^2 - m^2]}, \quad (3)$$

where $c=-1$ for the pion and $c=1$ for the sigma. This allows us to write

$$\Pi(K)_\sigma = \Pi(K)_\pi + \tilde{\Pi}(k_0, k), \quad (4)$$

where $k \equiv |\vec{k}|$ and

$$\Pi(K)_\pi = 4g^2[\mathcal{F} + \mathcal{G}], \quad (5)$$

$$\tilde{\Pi}(k_0, k) = 4g^2\mathcal{H}, \quad (6)$$

with

$$\mathcal{F}(m, T) = -i \int \frac{d^4 P}{(2\pi)^4} \frac{1}{[(P+K)^2 - m^2]}, \quad (7)$$

$$\mathcal{G}(m, T, K) = -i \int \frac{d^4 P}{(2\pi)^4} \frac{P^\mu K_\mu}{[(P+K)^2 - m^2][P^2 - m^2]}, \quad (8)$$

and

$$\mathcal{H}(m, T, k_0, k) = -i \int \frac{d^4 P}{(2\pi)^4} \frac{2m^2}{[(P+K)^2 - m^2][P^2 - m^2]}. \quad (9)$$

Since we are interested in studying the thermal effects on the analytic structure of the self-energy, we shall take only the nonzero temperature parts of the integrals above. Applying the usual finite temperature techniques in the imaginary-time formalism from Eqs. (7) to (9), we find the following expressions:

$$\mathcal{F}_\beta(m, T) = 2 \int_0^\infty \frac{p^2 dp}{(2\pi)^2} \frac{n_\psi(\omega)}{\omega}, \quad (10)$$

where n_ψ is the fermion distribution function $n_\psi(\omega) = 1/[\exp(\omega/T) + 1]$, $\omega \equiv \sqrt{p^2 + m^2}$, and

$$\mathcal{G}_\beta(m, T, K) = K^2 \int_0^\infty \frac{p^2 dp}{(2\pi)^2} \frac{n_\psi(\omega)}{\omega} \frac{1}{4pk} \times \ln \left[\frac{(2pk + k^2 - k_0^2)^2 - 4k_0^2 \omega^2}{(-2pk + k^2 - k_0^2)^2 - 4k_0^2 \omega^2} \right], \quad (11)$$

$$\mathcal{H}_\beta(m, T, k_0, k) = -2m^2 \int_0^\infty \frac{p^2 dp}{(2\pi)^2} \frac{n_\psi(\omega)}{\omega} \frac{1}{2pk} \times \ln \left[\frac{(2pk + k^2 - k_0^2)^2 - 4k_0^2 \omega^2}{(-2pk + k^2 - k_0^2)^2 - 4k_0^2 \omega^2} \right], \quad (12)$$

where $K^2 = K^\mu K_\mu = k_0^2 - k^2$.

From here on, for the sake of simplicity of the notation, we drop the subscript β in the self-energies, whose limits are

$$\Pi(k_0=0, k \rightarrow 0)_\pi = \Pi(k_0 \rightarrow 0, k=0)_\pi = 4g^2 \mathcal{F}(m, T), \quad (13)$$

$$\Pi(k_0 \rightarrow 0, k=0)_\sigma = \mathcal{F}(m, T) - 8g^2 m^2 \int_0^\infty \frac{p^2 dp}{(2\pi)^2} \frac{n_\psi(\omega)}{\omega^3}, \quad (14)$$

$$\Pi(k_0=0, k \rightarrow 0)_\sigma = \mathcal{F}(m, T) - 8g^2 m^2 \int_0^\infty \frac{dp}{(2\pi)^2} \frac{n_\psi(\omega)}{\omega} \quad (15)$$

which shows that the successive limits do not coincide at the origin of the external four-momentum only in the sigma self-energy, as has already been shown recently in a slightly different manner [7].

III. THE POLE PHYSICS

Although the pion self-energy by itself is analytic at the origin of the frequency-momentum space, it is the pole of the corrected propagator which has physical relevance. In order to investigate the effect of the thermal corrections due to the interactions with the fermions, let us consider the zero-temperature bosons as massless. Below we calculate the effective boson masses induced by the thermal medium.

A. The thermal corrected pion propagator

The thermal corrected boson propagator is given by

$$\begin{aligned} \mathcal{D}_{\sigma, \pi}(\omega_n, \vec{k})^{-1} &= \mathcal{D}_{0\sigma, \pi}(\omega_n, \vec{k})^{-1} + \Pi(\omega_n, \vec{k})_{\sigma, \pi} \\ &= \omega_n^2 + \vec{k}^2 + m_{\sigma, \pi}^2 + \Pi(\omega_n, \vec{k})_{\sigma, \pi}, \end{aligned} \quad (16)$$

where $\mathcal{D}_{0\sigma, \pi}(\omega_n, \vec{k})$ is the tree-level boson propagator.

1. The pion plasmon mass

It is well known that particles immersed in a hot medium have their properties modified. As they propagate in this plasma, they become dressed by the interactions. Examples of immediate consequences are the appearance of an effective thermal mass and the damping rate of collective excitations [8,9]. As we are considering massless bosons ($m_{\sigma, \pi}^2 = 0$), at the pole of the pion corrected propagator at zero momentum ($k=0$), we have

$$k_0^2 = \Pi(k_0, k=0)_\pi = 4g^2 \mathcal{F}(m, T) \left[1 + \frac{k_0^2}{4\omega^2 - k_0^2} \right]. \quad (17)$$

Since the right-hand side (RHS) of Eq. (17) has singularities, we write $k_{0, \pi} = M_\pi - i\gamma_\pi$, where M_π and γ_π are real. Let us now define $\mathcal{I}_\pi = \Pi(k_{0, \pi} = M_\pi - i\gamma_\pi, k=0)_\pi$. This allows us to get the leading contribution for the plasmon thermal mass as well as the (weak) damping rate, $\gamma \ll M$, respectively, of the pion:

$$M_\pi^2 = \mathcal{P} \text{Re } \mathcal{I}_\pi = 4g^2 \mathcal{F}(m, T), \quad (18)$$

$$\begin{aligned} \gamma_\pi &= -\frac{1}{2M_\pi} \mathcal{P} \text{Im } \mathcal{I}_\pi = \frac{g^2}{4\pi} M_\pi \left(1 - \frac{4m^2}{M_\pi^2} \right)^{1/2} n_\psi(M_\pi/2) \\ &\rightarrow \frac{g^2}{8\pi} M_\pi, \end{aligned} \quad (19)$$

where \mathcal{P} is the principal part of the integral. The arrow in Eq. (19) refers to the limit of vanishing fermion mass.

2. The pion Debye mass

Another example of a fundamental property of a plasma is the Debye mass, M_D , whose inverse is the screening length

for electric fields in the plasma [10]. We adopt the definition of the Debye mass in terms of the location of the pole in the static propagator for complex k . It was shown by Rebhan [11] that, both for QED and QCD, this definition is the correct one,²

$$\begin{aligned} k_\pi^2 &= -\Pi(k_0=0, k_\pi^2 = -M_{\pi,D}^2)_{\pi \rightarrow M_{\pi,D}^2} \\ &= 4g^2 \mathcal{F}(m, T) \left[1 + \frac{M_{\pi,D}^2}{4p^2} \right], \end{aligned} \quad (20)$$

where in the last equation, before the identification $k_\pi^2 = -M_{\pi,D}^2$, we have expanded $\Pi(k_0=0, k_\pi)_\pi$ in the limit $k_\pi \rightarrow 0$. The solution of Eq. (20) is straightforward:

$$M_{\pi,D}^2 = 4g^2 \frac{\mathcal{F}(m, T)}{1 - g^2 \tilde{\mathcal{F}}(m, T)}, \quad (21)$$

with

$$\tilde{\mathcal{F}}(m, T) = 2 \int_0^\infty \frac{dp}{(2\pi)^2} \frac{n_\psi(\omega)}{\omega}. \quad (22)$$

B. The thermal corrected sigma propagator

1. The sigma plasmon mass

In the pole of the thermal corrected sigma boson propagator at zero (three-)momentum, we have

$$k_0^2 = \Pi(k_0, k=0)_\sigma = 4g^2 \mathcal{F}(m, T) \left[1 + \frac{k_0^2 - 4m^2}{4\omega^2 - k_0^2} \right]. \quad (23)$$

Repeating the same steps as before, for $k_{0,\sigma} = M_\sigma - i\gamma_\sigma$ and $I_\sigma = \Pi(k_{0,\sigma} = M_\sigma - i\gamma_\sigma, k=0)_\sigma$, one finds

$$\begin{aligned} M_\sigma^2 &= \mathcal{P} \operatorname{Re} I_\sigma = 4g^2 \mathcal{F}(m, T), \quad (24) \\ \gamma_\sigma &= -\frac{1}{2M_\sigma} \mathcal{P} \operatorname{Im} I_\sigma = \frac{g^2}{4\pi} M_\sigma \left(1 - \frac{4m^2}{M_\sigma^2} \right)^{3/2} n_\psi(M_\sigma/2) \\ &\rightarrow \frac{g^2}{8\pi} M_\sigma, \end{aligned} \quad (25)$$

which gives the same results as for the pion case in the zero fermion mass limit.

²In QED this definition provides for an exponential decay of the screened Coulomb potential, whereas in QCD it gives a gauge-invariant result [12].

2. The sigma Debye mass

We find next the sigma Debye mass in our ‘‘plasma’’ in the same way as we did for the pion:

$$\begin{aligned} k_\sigma^2 &= -\Pi(k_0=0, k_\sigma^2 = -M_{\sigma,D}^2)_{\sigma \rightarrow M_{\sigma,D}^2} \\ &= 4g^2 \mathcal{F}(m, T) \left[1 + \frac{M_{\sigma,D}^2 - 4m^2}{4p^2} \right], \end{aligned} \quad (26)$$

which gives

$$\begin{aligned} M_{\sigma,D}^2 &= 4g^2 \frac{\mathcal{F}(m, T) - m^2 \tilde{\mathcal{F}}(m, T)}{1 - g^2 \tilde{\mathcal{F}}(m, T)} \\ &= M_{\pi,D}^2 - 4g^2 m^2 \frac{\tilde{\mathcal{F}}(m, T)}{1 - g^2 \tilde{\mathcal{F}}(m, T)}. \end{aligned} \quad (27)$$

It should be pointed out that since we are considering vanishing zero temperature boson masses, resummation techniques are required in some order of the perturbative expansion to recover its validity. In the model and at the order we are studying, a resummation of one-loop boson diagrams [which could be interpreted as the replacement of the $m_{\sigma,\pi}^2$ in Eq. (16) by effective boson thermal masses] would only shift the bosons plasmon and Debye masses, since these effective masses does not enter the (fermion) loop. Therefore, the qualitative results found here would be kept. However, Nieves and Pal [13] considered another model where the problem of the nonanalyticity of the one-loop self-energy disappeared when the calculation was carried out using improved propagators for the particles that appear in the internal lines of the loop diagrams.

C. High temperature limit for the pion and sigma plasmon and Debye masses

The high temperature limit of the functions $\tilde{\mathcal{F}}$ and \mathcal{F} are [14]

$$\begin{aligned} \tilde{\mathcal{F}}(m, T) &= \frac{1}{2\pi^2} \left(-\frac{1}{2} \ln \frac{\mu}{\pi} - \frac{1}{2} \gamma \right. \\ &\quad \left. + \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \left[\left(1 + \frac{\mu^2}{4\pi^2 n^2} \right)^{-1/2} - 1 \right] \right) \\ &= \frac{1}{2\pi^2} \left(-\frac{1}{2} \ln \frac{\mu}{\pi} - \frac{1}{2} \gamma + \frac{\zeta(3)}{16\pi} \mu^2 \right. \\ &\quad \left. - \frac{3\zeta(5)}{64\pi^2} \mu^4 + \mathcal{O}(\mu^6) \right), \end{aligned} \quad (28)$$

where $\mu = m/T$, $\gamma = 0.57721 \dots$ is Euler’s constant and the numerical values of the ζ function at important points are $\zeta(2) = \pi^2/6$, $\zeta(3) = 1.2020 \dots$, $\zeta(4) = \pi^4/90$, $\zeta(5) = 1.0369 \dots$, and so on, whereas \mathcal{F} is given by

$$\mathcal{F}(m, T) = \frac{T^2}{2\pi^2} \left(\frac{\pi^2}{12} + \frac{1}{4} \mu^2 \ln \frac{\mu}{\pi} + \frac{1}{4} \left(-\frac{1}{2} + \gamma \right) \mu^2 - \frac{\zeta(3)}{32\pi} \mu^4 + \mathcal{O}(\mu^6) \right). \quad (29)$$

So one sees that $\tilde{\mathcal{F}}$ is less relevant at high temperature than \mathcal{F} . Thus, the pion and sigma, respectively, plasmon masses are

$$M_\pi^2 = \frac{g^2 T^2}{6} + \frac{g^2 T^2}{2\pi^2} \left(-\frac{1}{2} + \gamma + \ln \frac{\mu}{\pi} \right) \mu^2 + \mathcal{O}(\mu^4) \xrightarrow{H.T.} \frac{g^2 T^2}{6}, \quad (30)$$

$$M_\sigma^2 = \frac{g^2 T^2}{6} + \frac{g^2 T^2}{2\pi^2} \left(-\frac{1}{2} + \gamma + \ln \frac{\mu}{\pi} \right) \mu^2 + \mathcal{O}(\mu^4) \xrightarrow{H.T.} \frac{g^2 T^2}{6}, \quad (31)$$

where *H.T.* denotes the dominant term in the high temperature limit (or, equivalently, the zero fermion mass limit), while the Debye masses are written as

$$\begin{aligned} M_{\pi,D}^2 &= 4g^2 \mathcal{F}(m, T) [1 + g^2 \tilde{\mathcal{F}}(m, T) + (g^2 \tilde{\mathcal{F}}(m, T))^2 \\ &\quad + \mathcal{O}(g^6 \tilde{\mathcal{F}}(m, T)^3)] \\ &= \frac{g^2 T^2}{6} \left[1 - \frac{g^2}{(2\pi)^2} \left(\ln \frac{\mu}{\pi} + \gamma \right) + \left(\frac{g^2}{(2\pi)^2} \right)^2 \right. \\ &\quad \left. \times \left(\ln \frac{\mu}{\pi} + \gamma \right)^2 + \dots \right], \quad (32) \end{aligned}$$

$$\begin{aligned} M_{\sigma,D}^2 &= M_{\pi,D}^2 - 4g^2 m^2 \tilde{\mathcal{F}}(m, T) [1 - g^2 \tilde{\mathcal{F}}(m, T)]^{-1} \\ &= M_{\pi,D}^2 + \frac{g^2 m^2}{\pi^2} \left(\ln \frac{\mu}{\pi} + \gamma \right) \left[1 - \frac{g^2}{(2\pi)^2} \left(\ln \frac{\mu}{\pi} + \gamma \right) \right. \\ &\quad \left. + \left(\frac{g^2}{(2\pi)^2} \right)^2 \left(\ln \frac{\mu}{\pi} + \gamma \right)^2 + \dots \right]. \quad (33) \end{aligned}$$

These results clearly show that, despite the pion self-energy being analytic at the origin in the momentum space, its physical plasmon and Debye masses are different, as they should be, thanks to the consistency of the calculations at the pole of the corrected propagator.

IV. THE DISPERSION RELATION

Usually, the noncommuting limits have been traced back to the cut structure of the one-loop self-energy through the dispersion relation [2,15]

$$\text{Re } \Pi(k_0, k) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} du \frac{\text{Im } \Pi(u, k)}{u - k_0}. \quad (34)$$

However this relation is not general. If $\Pi(k_0, k) \sim k_0^n$ ($n \geq 0$) for $k_0 \rightarrow \infty$, as is the case of the pion self-energy in the

approximation used here, a more general relation should be used [16–18]. Particularly for the pion case we have

$$\text{Re } \Pi(k_0, k) = \text{Re } \Pi(0, 0) + \frac{1}{\pi} (k_0^2 - k^2) \mathcal{P} \int_{-\infty}^{\infty} du \frac{\text{Im } \Pi(u, k)}{u^2 (u - k_0)}, \quad (35)$$

with $\text{Im } \Pi(k_0, k)$ given by

$$\begin{aligned} \text{Im } \Pi(k_0, k) &= -\frac{1}{2} \int \frac{d^3 \vec{p}}{(2\pi)^2} \frac{1}{2\omega\Omega} \{ (\Omega + \omega)^2 [\delta(k_0 + \Omega + \omega) \\ &\quad - \delta(k_0 - \Omega - \omega)] + (\Omega - \omega)^2 [\delta(k_0 + \Omega - \omega) \\ &\quad - \delta(k_0 - \Omega + \omega)] \} \tanh\left(\frac{\beta\omega}{2}\right), \quad (36) \end{aligned}$$

where $\Omega \equiv \sqrt{(p-k)^2 + m^2}$.

One can check that Eq. (35) can also be used to show that the analytic behavior of $\text{Re } \Pi(k_0, k)$ is due to the kinetic term multiplying the integral and that in both limits ($k_0 = 0, k \rightarrow 0$ and $k = 0, k_0 \rightarrow 0$) this contribution vanishes leaving the first term as the sole contribution.

V. CONCLUSIONS

In this paper we have studied the analytic properties of thermal corrected boson propagators through their interaction with fermions. A particular model was chosen based on the fact that its two kinds of bosons couple differently with the fermions, which leads to distinct (unexpected) behavior of their self-energy. We have shown that the pion self-energy is analytic at the origin in the frequency-momentum space at finite temperature, whereas the sigma self-energy is nonanalytic. In spite of this, we have shown that the analytic behavior found for the pion self-energy does not spoil the difference between the plasmon and Debye pion masses. We have also shown that the two physical masses arise from the consistent calculation at the pole of the corrected propagator. Then, we have derived the plasmon and Debye masses for both the pion and sigma bosons. We note here that the answer to the question of which mass of a certain field is manifested in a plasma in a given temperature T depends strictly on the situation encountered (or assumed) by its four momentum. Besides, the specific dependence on the external momenta of the pion self-energy graph implies a modification in the usual dispersion relation which allows us to trace back the origin of the analyticity. Similar behavior seems also to be found in derivative coupling models [19]. This has been shown to be a criteria other than the existence of distinct masses running in the upper and lower internal lines of a diagram [2], which could be used to predict the analytic behaviors of bubble diagrams as the external momentum $K_\mu \rightarrow 0$.

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