One loop gauge couplings in AdS_5

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We calculate the full 1-loop corrections to the low energy coupling of a bulk gauge boson in a slice of AdS_5 which are induced by generic 5-dimensional scalar, Dirac fermion, and vector fields with arbitrary $Z_2 \times Z_2'$ orbifold boundary conditions. In the supersymmetric limit, our results correctly reproduce the results obtained by an independent method based on 4-dimensional effective supergravity. This provides a nontrivial check of our results and assures the regularization scheme independence of the results.

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I. INTRODUCTION

Models with extra dimensions have provided new insight into the large scale hierarchy between the weak scale $M_W \sim 10^2$ GeV and the Planck scale $M_{Pl} \sim 10^{18}$ GeV. In this regard, the Randall-Sundrum model (RS1) is particularly interesting as it explains the weak to Planck scale ratio using the warped 5D geometry [1]:

$$ds^2 = G_{MN} dx^M dx^N = e^{-2kR|y|} g_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad (1)$$

where $-\pi \leqslant y \leqslant \pi$, k is the AdS curvature and R is the orbifold radius. In this spacetime background, a 4-dimensional (4D) graviton is localized near the UV brane at $y=0$ whose cutoff mass scale M_{UV} is of the order of the 5D Planck scale. On the other hand, in the original RS1 model, all the standard model (SM) fields are assumed to be confined on the IR brane at $y=\pi$ whose cutoff scale $M_{IR} \sim e^{-\pi kR} M_{UV}$. Then, with a moderately large value of $kR(\sim 12)$, the model can generate the large scale hierarchy $M_{Pl}/M_W \sim M_{UV}/M_{IR} \sim 10^{16}$ without any severe fine-tuning of the fundamental parameters.

An apparent drawback of the original RS1 model is that one has to abandon the attractive possibility that the SM gauge couplings g_a^2 $(a=1,2,3)$ are unified at a high energy scale through *the quantum corrections calculable within the model*. Experimental data show that g_a^2 at M_W differs from each other by order unity:

$$\frac{1}{g_a^2(M_W)} - \frac{1}{g_b^2(M_W)} = \mathcal{O}(1) \quad (a \neq b). \quad (2)$$

On the other hand, the size of quantum corrections to $1/g_a^2$ which are calculable within the RS1 model is

$$\Delta\left(\frac{1}{g_a^2}\right) = \mathcal{O}\left(\frac{1}{8\pi^2}\ln(M_{IR}^2/M_W^2)\right) = \mathcal{O}\left(\frac{1}{8\pi^2}\right), \quad (3)$$

so the RS1 model does not give any insight on why the SM gauge couplings at M_W differ from each other by order unity.

It has been noted recently [2–7] that one can achieve the gauge unification, while still solving the hierarchy problem, within the 5D effective field theory on AdS_5 if the SM gauge bosons propagate in 5D bulk spacetime. In such a case, the size of quantum corrections calculable within the model is

$$\Delta\left(\frac{1}{g_a^2}\right) = \mathcal{O}\left(\frac{1}{8\pi^2}\ln(M_{Pl}^2/M_W^2)\right) = \mathcal{O}(1), \quad (4)$$

as in the case of conventional 4D grand unified theories (GUT). This allows that the observed differences of gauge couplings are explained in terms of quantum corrections which are calculable within the model.

Calculation of the 1-loop corrections to gauge coupling in AdS_5 was first attempted in [2] for a GUT model in which all gauge-charged matter fields are confined on the UV brane. The computation involves a Pauli-Villars regulator with regulator mass $\Lambda_{PV} \leqslant k$, which could catch only the corrections at scales significantly below k. In [3], a momentum cutoff depending on the position in the 5th dimension was proposed to regulate the 1-loop corrections. Though intuitively sensible, it is difficult to isolate the regulator-independent part from the regulator-dependent total corrections in this regularization, which makes the interpretation of the results unclear. In [4,7], the 1-loop corrections have been computed for generic supersymmetric gauge theory on AdS_5 using the gauged $U(1)_R$ symmetry and chiral anomaly in 5D supergravity (SUGRA) and also the known properties of gauge couplings in 4D effective SUGRA. In this approach, one could obtain the 1-loop corrections (including those from scales between k and the 5D cutoff scale $\Lambda > k$) in obviously regulator-independent manner. In [5,6], 1-loop corrections in 5D scalar QED on AdS_5 have been computed (using dimensional regularization and also Pauli-Villars regularization) and the results are nicely interpreted in terms of AdS and conformal field theory (CFT) correspondence.

In this paper, we present the full 1-loop corrections to the low energy coupling of bulk gauge bosons in a slice of AdS_5 which are induced by generic 5D scalar, Dirac fermion and vector fields with arbitrary $Z_2 \times Z_2'$ orbifold boundary conditions. To be explicit, we adopt dimensional regularization [8], but the results should be independent of the used regularization scheme as they correspond to the scheme-independent corrections calculable within 5D effective field

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theory. When applied to the supersymmetric case [9,10], our results correctly reproduce the expressions which are obtained in a completely independent approach based on 4D effective SUGRA. This provides a nontrivial check of our results, and also assures the scheme independence of the results. We also note that the subtraction scales of log divergences at two orbifold fixed points, i.e., $y=0$ and π, differ by the warp factor $e^{-\pi kR}$. This is physically expected, and can be confirmed by comparing the results with those of Pauli-Villars regularization as well as with the results of the 4D SUGRA calculation.

The organization of this paper is as follows. In Sec. II, we set up the notations for 5D gauge theory on a slice of AdS_5 including the Kaluza-Klein (KK) analysis for generic 5D scalar, Dirac fermion, and vector fields with arbitrary $Z_2\times Z_2'$ orbifold boundary conditions. In Sec. III, we present our main results, i.e., 1-loop gauge couplings in AdS_5 induced by generic 5D fields, obtained using the background field method with dimensional regularization. In Sec. IV, we consider the supersymmetric limit in order to confirm that our results correctly reproduce the results from the 4D SUGRA calculation, and conclude in Sec. V.

II. GAUGE THEORY ON A SLICE OF AdS_5

The model we study is a 5D gauge theory defined on a slice of AdS_5 with spacetime metric (1), containing generic gauge-charged 5D scalar, fermion and vector fields with arbitrary $Z_2\times Z_2'$ boundary conditions. The Lagrangian is given by

$$\int d^4x dy\sqrt{-G}\bigg[-\frac{1}{4g_{5a}^2}F^{aMN}F^a_{MN}-\frac{1}{2}D_M\phi D^M\phi -\frac{1}{2}m_\phi^2\phi^2-i\bar{\psi}(\gamma^M D_M-m_\psi)\psi\bigg], \tag{5}$$

where D_M is the covariant derivative containing the gauge connections as well as the spin connection of AdS_5. We parametrize the masses of scalar and fermion fields as

$$m_\phi^2=ak^2+\frac{2k}{R}[b_0\delta(y)-b_\pi\delta(y-\pi)],\quad m_\psi=ck\epsilon(y), \tag{6}$$

where $\epsilon(y)=y/|y|$, b_0, and b_π are the brane mass parameters at $y=0$ and $y=\pi$, respectively, and c is the fermion kink mass parameter. The 5D fields in the model can have arbitrary $Z_2\times Z_2'$ orbifold boundary conditions,

$$\phi(-y)=Z_\phi\phi(y),\quad \phi(-y+\pi)=Z_\phi'\phi(y+\pi),$$

$$\psi(-y)=Z_\psi\gamma_5\psi(y),\quad \psi(-y+\pi)=Z_\psi'\gamma_5\psi(y+\pi),$$

$$A^a_\mu(-y)=Z_aA^a_\mu(y),\quad A^a_\mu(-y+\pi)=Z_a'A^a_\mu(y+\pi), \tag{7}$$

with $Z_\Phi=\pm1$ and $Z_\Phi'=\pm1$ for $\Phi=\{\phi,\psi,A^a_M\}$. Though we are interested in the low energy coupling of A^a_μ having $Z_a=Z_a'=1$, there can be 5D vector fields having other $Z_2\times Z_2'$ parity which are charged for the gauge fields with $Z_a=Z_a'=1$. Note that the brane mass of the scalar field at $y=0$ ($y=\pi$) is relevant only when $Z_\phi=1$ ($Z_\phi'=1$).

The KK spectrum of bulk fields on a slice of AdS_5 has been discussed in detail in [10]. It is rather straightforward to generalize the analysis of [10] to the field with arbitrary $Z_2\times Z_2'$ parity. A generic 5D field Φ can be decomposed as

$$\Phi(x,y)=\sum\Phi_n(x)f_n(y),$$

where the KK wave function f_n satisfies

$$[-e^{skR|y|}\partial_y(e^{-skR|y|}\partial_y)+R^2k^2\hat{M}_\Phi^2]f_n=R^2e^{2kR|y|}m_n^2f_n \tag{8}$$

for the KK mass eigenvalue m_n. Here $s=\{2,4,1\}$ and the bulk mass parameters

$$\hat{M}_\Phi^2=\{0,a,c(c\pm1)\}\quad\text{for}\quad \Phi=\{A_\mu,\phi,e^{-2kR|y|}\psi_{L,R}\}. \tag{9}$$

This determines f_n to be

$$f_n(y)=e^{skR|y|/2}\bigg[J_\alpha\bigg(\frac{m_n}{k}e^{kR|y|}\bigg)+b_\alpha(m_n)Y_\alpha\bigg(\frac{m_n}{k}e^{kR|y|}\bigg)\bigg], \tag{10}$$

where

$$\alpha=\sqrt{(s/2)^2+\hat{M}_\Phi^2}. \tag{11}$$

To determine the corresponding KK mass spectrum, one needs to impose the orbifold boundary condition. Parity-even conditions under the reflection at $y=0$ or π leads to

$$\frac{df_n}{dy}=rkRf_n\quad\text{at}\quad y=0\quad\text{or}\quad\pi, \tag{12}$$

for the brane mass parameter

$$r=\{0,b_0\ \text{or}\ b_\pi,\mp c\}\quad\text{for}\quad\Phi=\{A_\mu,\phi,e^{-2kR|y|}\psi_{L,R}\}. \tag{13}$$

Then using Eqs. (10) and (12), one finds

$$b_\alpha(m_n)=-\frac{\left(\frac{s}{2}-r\right)J_\alpha\left(\frac{m_n}{k}e^{kR\tilde{y}}\right)+\frac{m_n}{k}e^{kR\tilde{y}}J_\alpha'\left(\frac{m_n}{k}e^{kR\tilde{y}}\right)}{\left(\frac{s}{2}-r\right)Y_\alpha\left(\frac{m_n}{k}e^{kR\tilde{y}}\right)+\frac{m_n}{k}e^{kR\tilde{y}}Y_\alpha'\left(\frac{m_n}{k}e^{kR\tilde{y}}\right)}, \tag{14}$$

where $\tilde{y}=0$ or π. Parity-odd conditions under the reflection at $y=0$ or π leads to

$$f_n=0\quad\text{at}\quad y=0\quad\text{or}\quad\pi, \tag{15}$$

yielding

$$b_\alpha(m_n) = -\frac{J_\alpha\left(\frac{m_n}{k}e^{kR\tilde{y}}\right)}{Y_\alpha\left(\frac{m_n}{k}e^{kR\tilde{y}}\right)}. \tag{16}$$

With the above results, the KK spectrum of the 5D field Φ can be determined by the so-called N function $N(q) = N(-q)$ which has simple zeros at $q = \pm m_n \neq 0$:

$$N(m_n) = 0. \tag{17}$$

If there exists a massless mode, N has a double zero at $q = 0$. For later use, here we summarize the N functions for all $Z_2 \times Z_2'$ boundary conditions of the corresponding 5D field. Let r_0 denote the brane mass parameter at $y=0$ and r_π the brane mass parameter at $y=\pi$. The N function for $(Z_\Phi, Z_\Phi') = (+,+)$ is given by

$$\begin{aligned}N_{++}(q) = &\left\{\left(\frac{s}{2}-r_0\right)J_\alpha\left(\frac{q}{k}\right)+\frac{q}{k}J_\alpha'\left(\frac{q}{k}\right)\right\}\left\{\left(\frac{s}{2}-r_\pi\right)Y_\alpha\left(\frac{q}{T}\right)\right.\\ &\left.+\frac{q}{T}Y_\alpha'\left(\frac{q}{T}\right)\right\}-\left\{\left(\frac{s}{2}-r_\pi\right)J_\alpha\left(\frac{q}{T}\right)\right.\\ &\left.+\frac{q}{T}J_\alpha'\left(\frac{q}{T}\right)\right\}\left\{\left(\frac{s}{2}-r_0\right)Y_\alpha\left(\frac{q}{k}\right)+\frac{q}{k}Y_\alpha'\left(\frac{q}{k}\right)\right\}\end{aligned} \tag{18}$$

where $T = ke^{-\pi kR}$. As for the fields with other boundary conditions, i.e., $(Z_\Phi, Z_\Phi') = (+,-),(-,+),(-,-)$, we find

$$\begin{aligned}N_{+-}(q) &= Y_\alpha\left(\frac{q}{T}\right)\left[\left(\frac{s}{2}-r_0\right)J_\alpha\left(\frac{q}{k}\right)+\frac{q}{k}J_\alpha'\left(\frac{q}{k}\right)\right]\\ &\quad -J_\alpha\left(\frac{q}{T}\right)\left[\left(\frac{s}{2}-r_0\right)Y_\alpha\left(\frac{q}{k}\right)+\frac{q}{k}Y_\alpha'\left(\frac{q}{k}\right)\right],\\ N_{-+}(q) &= J_\alpha\left(\frac{q}{k}\right)\left[\left(\frac{s}{2}-r_\pi\right)Y_\alpha\left(\frac{q}{T}\right)+\frac{q}{T}Y_\alpha'\left(\frac{q}{T}\right)\right]\\ &\quad -Y_\alpha\left(\frac{q}{k}\right)\left[\left(\frac{s}{2}-r_\pi\right)J_\alpha\left(\frac{q}{T}\right)+\frac{q}{T}J_\alpha'\left(\frac{q}{T}\right)\right],\\ N_{--}(q) &= J_\alpha\left(\frac{q}{k}\right)Y_\alpha\left(\frac{q}{T}\right)-J_\alpha\left(\frac{q}{T}\right)Y_\alpha\left(\frac{q}{k}\right).\end{aligned} \tag{19}$$

As we will see in the next section, one can choose an appropriate gauge fixing to make sure that the KK spectrum of A_5 is determined by the N function of 5D scalar field ϕ with a specific mass:

$$N_{A_5} = N_\phi \quad \text{for} \quad m_\phi^2 = -4k^2 + \frac{4k}{R}(\delta(y) - \delta(y-\pi)). \tag{20}$$

In fact, one needs to know the asymtotic bahaviors of these N functions at $|q| \to \infty$ to regulate the UV divergence and also the behaviors at $|q| \to 0$ to find the 1-loop couplings in the IR limit. Some properties of the N functions including those asymptotic behaviors are summarized in Appendix A.

III. ONE LOOP EFFECTIVE COUPLINGS

In this section, we calculate the 1-loop effective coupling of gauge field zero mode in AdS$_5$ using the background field method [11] with dimensional regularization [8]. Let us first describe the calculation scheme. We split the gauge field as

$$A_M^a = \bar{A}_M^a + \tilde{A}_M^a, \tag{21}$$

where $\bar{A}_M^a$ denotes the background gauge field in the gauge $\bar{A}_5^a = 0$ and $\tilde{A}_M^a$ is the quantum fluctuation. We choose the gauge fixing term

$$-\frac{1}{2g_{5a}^2}\int d^5x\sqrt{-G}\left[e^{2kR|y|}g^{\mu\nu}D_\mu\tilde{A}_\nu^a + \frac{e^{2kR|y|}}{R^2}\partial_y(e^{-2kR|y|}\tilde{A}_5)\right]^2 \tag{22}$$

where D_μ is defined by the background gauge field $\bar{A}_\mu^a$. The corresponding ghost action is given by

$$\int d^5x\sqrt{-G}\left[e^{2kR|y|}\bar{\xi}^a D^2\xi^a + \frac{e^{2kR|y|}}{R^2}\bar{\xi}^a\partial_y(e^{-2kR|y|}\partial_y\xi^a)\right], \tag{23}$$

where $D^2 = g^{\mu\nu}D_\mu D_\nu$. It is then straightforward to find the following gauge-fixed actions which are quadratic in $\tilde{A}_\mu^a, \tilde{A}_5^a$ and ξ^a:

$$\begin{aligned}\int d^5x\Bigg[&-\frac{1}{4g_{5a}^2}\Bigg(-2R\tilde{A}_\mu^a D^2\tilde{A}^{a\mu} + 4Rf_{abc}\bar{F}_{\mu\nu}^a\tilde{A}^{b\mu}\tilde{A}^{c\nu}\\ &-\frac{2}{R}\tilde{A}_\mu^a\partial_y(e^{-2kR|y|}\partial_y)\tilde{A}^{a\mu} - \frac{2}{R}e^{-2kR|y|}\tilde{A}_5^a D^2\tilde{A}_5^a\\ &-\frac{2}{R^3}e^{-2kR|y|}\tilde{A}_5^a\partial_y^2(e^{-2kR|y|}\tilde{A}_5^a)\Bigg)\\ &+e^{-2kR|y|}R\left\{\bar{\xi}^a D^2\xi^a - \frac{1}{R^2}\bar{\xi}^a\partial_y(e^{-2kR|y|}\partial_y\xi^a)\right\}\Bigg].\end{aligned} \tag{24}$$

The action of scalar and fermion fields can be written as

$$\begin{aligned}\int d^5x\Bigg[&e^{-2kR|y|}R\frac{1}{2}\phi\left(D^2 + \frac{1}{R^2}e^{2kR|y|}\partial_y e^{-4kR|y|}\partial_y\right.\\ &\left.-e^{-2kR|y|}m_\phi^2\right)\phi - e^{-3kR|y|}R(\bar{\psi}_L i\gamma^\mu D_\mu\psi_L\\ &+\bar{\psi}_R i\gamma^\mu D_\mu\psi_R) - e^{-4kR|y|}(\bar{\psi}_L i\gamma^5\partial_y\psi_R + \bar{\psi}_R i\gamma^5\partial_y\psi_L)\\ &-iRe^{-4kR|y|}m_\psi(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)\Bigg].\end{aligned} \tag{25}$$

Note that the quadratic action of $\tilde{A}_5^a$ has the same form as the action of 5D real scalar ϕ with $m_\phi^2=-4k^2+4kR^{-1}[\delta(y)-\delta(y-\pi)]$, justifying the relation (20).

One-loop effective action of the gauge field zero mode can be obtained by integrating out all quantum fluctuation fields at 1-loop order. This procedure yields

$$S_{eff}=\int d^4x\left(-\frac{\pi R}{4g_{5a}^2}F^{a\mu\nu}F^a_{\mu\nu}\right)+\Gamma_\phi[A_\mu]+\Gamma_\psi[A_\mu]+\Gamma_A[A_\mu], \tag{26}$$

where the first term is obviously the tree level action, and Γ_ϕ, Γ_ψ, and Γ_A represent the 1-loop corrections due to the loops of ϕ, ψ, and A_M^a (and also the ghost fields $\xi^a,\overline{\xi^a}$), respectively:

$$i\Gamma_\phi=-\frac{1}{2}\mathrm{Tr}_\phi\ln(-D^2+M^2(\phi)),$$

$$i\Gamma_\psi=\frac{1}{2}\mathrm{Tr}_\psi\ln(-D^2+M^2(\psi)+F_{\mu\nu}J^{\mu\nu}_{1/2}),$$

$$i\Gamma_A=-\frac{1}{2}\mathrm{Tr}_{A^\mu}\ln(-D^2+M^2(A_\mu)+F_{\mu\nu}J^{\mu\nu}_1),$$
$$-\frac{1}{2}\mathrm{Tr}_{A_5}\ln(-D^2+M^2(A_5))+\mathrm{Tr}_{\xi,\bar{\xi}}\ln(-D^2+M^2(\xi)). \tag{27}$$

Here we replace the background gauge field $\overline{A^a_\mu}$ by unbarred A^a_μ, and $M^2(\Phi)$ is the mass-square operator whose eigenvalues m_n^2 are determined by the zeros of the corresponding N function. $J_j^{\mu\nu}$ is the 4D Lorentz spin generator normalized as $\mathrm{tr}(J_j^{\mu\nu}J_j^{\rho\sigma})=C(j)(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})$ where $C(j)=(0,1,2)$ for $(j=0,1/2,1)$.

The above 1-loop effective action is divergent, so it needs to be regulated. As in the case of a flat 5D orbifold, the UV divergence structure of 5D gauge theory on AdS_5 is given by

$$-\int d^5x\sqrt{-G}\left[\frac{\gamma_a}{32\pi^3}\Lambda F^a_{MN}F^{aMN}+\frac{\ln\Lambda}{32\pi^2}\left(\lambda_0\frac{\delta(y)}{\sqrt{G_{55}}}+\lambda_\pi\frac{\delta(y-\pi)}{\sqrt{G_{55}}}\right)F^a_{\mu\nu}F^{a\mu\nu}\right] \tag{28}$$

where the coefficient of linear divergence (γ_a) is highly sensitive to the used regularization scheme, while those of log divergences at fixed points ($\lambda_{0,\pi}$) are scheme independent. In dimensional regularization, $\gamma_a=0$, however this does not have any special physical meaning. As for the coefficients of log divergences, it is straightforward to find [12]

$$\lambda_0=\frac{1}{24}[T_a(\phi_{++})+T_a(\phi_{+-})-T_a(\phi_{-+})-T_a(\phi_{--})]-\frac{23}{24}[T_a(A_{++})+T_a(A_{+-})-T_a(A_{-+})-T_a(A_{--})],$$

$$\lambda_\pi=\frac{1}{24}[T_a(\phi_{++})-T_a(\phi_{+-})+T_a(\phi_{-+})-T_a(\phi_{--})]-\frac{23}{24}[T_a(A_{++})-T_a(A_{+-})+T_a(A_{-+})-T_a(A_{--})], \tag{29}$$

where $T_a(\Phi)=\mathrm{Tr}(T_a^2)$ for the gauge group representation given by Φ, $\phi_{zz'}$ $(z,z'=\pm)$ is a 5D real scalar field with $Z_2\times Z_2'$ parity (z,z'), and $A_{zz'}$ is a 5D real vector field.

With the UV divergences given by Eq. (28), the low energy effective gauge coupling can be written as

$$\frac{1}{g_a^2(p,k,R)}=\left[\frac{1}{g_{5a}^2(\Lambda)}+\frac{\gamma_a}{8\pi^3}\right]\pi R+\left[\frac{1}{g_{0a}^2(\Lambda)}+\frac{1}{g_{\pi a}^2(\Lambda)}+\frac{\lambda_0+\lambda_\pi}{8\pi^2}\ln\Lambda\right]+\frac{1}{8\pi^2}\tilde{\Delta}_a(p,k,R)+\mathcal{O}(1/\Lambda) \tag{30}$$

where p is the 4D momentum of the external gauge boson zero mode, $g_{0a}^2(\Lambda)$ and $g_{\pi a}^2(\Lambda)$ denote the bare brane gauge couplings at the orbifold fixed points $y=0$ and $y=\pi$, respectively, and $\mathcal{O}(1/\Lambda)$ stands for the part suppressed by $1/\Lambda$. Here the conventional momentum running and also the finite KK threshold corrections are encoded in $\tilde{\Delta}_a$. The bare brane couplings $g_{0a}^2(\Lambda)$ and $g_{\pi a}^2(\Lambda)$ can be interpreted as the Wilsonian brane couplings at Λ in the metric frame of G_{MN} [see Eq. (1)]. However, when measured in the metric frame of 4D massless graviton $g_{\mu\nu}=e^{2kR|y|}G_{\mu\nu}$, they should be interpreted as the Wilsonian couplings at different scales: $g_{0a}^2(\Lambda)$ at the scale Λ and $g_{\pi a}^2(\Lambda)$ at the rescaled scale $e^{-\pi kR}\Lambda$. One can then assume that $g_{0a}^2(\Lambda)$ and $g_{\pi a}^2(\Lambda)$ are of order $8\pi^2$, under which

$$\frac{1}{g_a^2(p,k,R)}=\frac{\pi R}{\hat{g}_{5a}^2}+\frac{1}{8\pi^2}\Delta_a(p,k,R,\ln(\Lambda))+\mathcal{O}\left(\frac{1}{8\pi^2}\right), \tag{31}$$

where $1/\hat{g}_{5a}^2=1/g_{5a}^2+\gamma_a\Lambda/8\pi^3$ and $\Delta_a=\tilde{\Delta}_a+(\lambda_0+\lambda_\pi)\ln\Lambda$. Note that $1/\hat{g}_{5a}^2$ represents the 5D bare coupling which is *not* calculable within 5D effective field theory. (But it would be determined by the UV dynamics at scales above Λ.) On the other hand, Δ_a represents the corrections from scales below Λ which are unambiguously calculable within 5D effective field theory. In the following, we compute Δ_a induced by generic 5D scalar, Dirac fermion and vector fields with arbitrary $Z_2\times Z_2'$ boundary conditions.

Regularizing a field theory on compact space involves the regularization of the KK summation. It is then convenient to

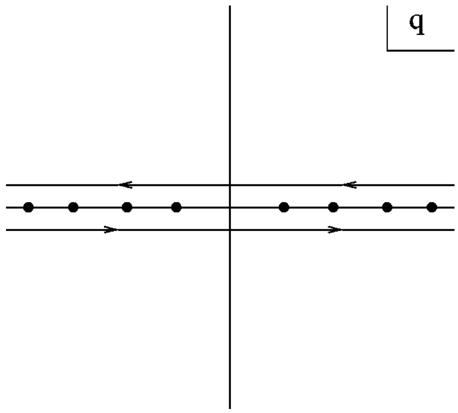

FIG. 1. Contour $\leftrightharpoons$ in the complex q plane. Bold dots represent the mass poles.

convert the KK summation into an integral by introducing a pole function $P(q)$ [8] having the following properties: (i) $P(q)$ has poles at $q=m_n$, (ii) each pole has the residue 1; (iii) there exists $\delta>0$ such that $P\to B$ for $|\mathrm{Re}(q)|\to\infty$ and $\mathrm{Im}(q)>\delta$, while $P\to -B$ for $|\mathrm{Re}(q)|\to\infty$ and $\mathrm{Im}(q)<-\delta$, where B is an *imaginary* constant. These conditions uniquely determine the pole function. In our case, it is given by

$$P(q)=\frac{N'(q)}{2N(q)}, \tag{32}$$

for which

$$\sum_{m_n}\int d^4p f(p,m_n)=\int_{\leftrightharpoons}\frac{dq}{2\pi i}\int d^4p P(q)f(p,q), \tag{33}$$

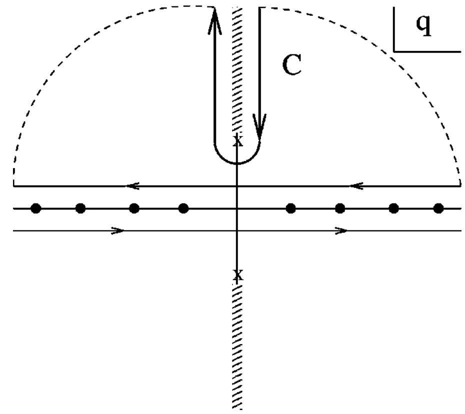

FIG. 2. For the contribution from $\tilde{P}(q)$, the contour $\leftharpoondown$ can be deformed to the contour C represented by the bold line since the contribution vanishes on the dotted infinite half circle. Hatched lines on the imaginary axis are logarithmic branch cuts. After integrating by parts, the point **x** where the branch cut starts becomes a simple pole. Then the integral along C is given by the values of the integrand at the boundary of C at infinity and the residue value at the point **x**. The integral along $\rightharpoondown$ can be similarly treated in the lower half plane.

where $\leftrightharpoons$ denotes the contour depicted in Fig. 1.

To obtain the 1-loop effective action of gauge field zero mode, one needs to compute

$$\mathrm{Tr}\ln(-D^2+M^2(\Phi)+F_{\mu\nu}J_j^{\mu\nu}) \tag{34}$$

which contains the following two-point amplitude:

$$\int_{\leftrightharpoons}\frac{dq}{2\pi i}P(q)\int\frac{d^4p}{(2\pi)^4}A_\mu^a(-p)A_\nu^a(p)T_a(\Phi)\Bigg[d(j)\int\frac{d^4k}{(2\pi)^4}\frac{g^{\mu\nu}((p+k)^2+q^2)-\frac{1}{2}(p+2k)^\mu(p+2k)^\nu}{(k^2+q^2)((p+k)^2+q^2)}$$
$$-2C(j)(p^2g^{\mu\nu}-p^\mu p^\nu)\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2+q^2)((p+k)^2+q^2)}\Bigg]\equiv i\int\frac{d^4p}{(2\pi)^4}\mathcal{G}_a(p)A_\mu^a(-p)(p^2g^{\mu\nu}-p^\mu p^\nu)A_\nu^a(p), \tag{35}$$

where $d(j)=(1,4,4)$ and $C(j)=(0,1,2)$ for $j=(0,1/2,1)$. For the computation of the above integral, it is convenient to split the pole function into two parts:

$$P(q)=\tilde{P}(q)+P_\infty(q), \tag{36}$$

where $\tilde{P}\to\mathcal{O}(q^{-2})$ at $|q|\to\infty$. Then P_∞ can be written as

$$P_\infty(q)=-\frac{A}{q}-B\epsilon(\mathrm{Im}(q)), \tag{37}$$

where $\epsilon(x)=x/|x|$ and A and iB are some real constants, which gives

$$\tilde{P}(q)=\frac{N'(q)}{2N(q)}+\frac{A}{q}+B\epsilon(\mathrm{Im}(q)). \tag{38}$$

With the decomposition (36), all UV divergences appear in the contribution from P_∞ in a manner allowing simple dimensional regularization, while the contribution from $\tilde{P}$ is *finite*.

The 4D momentum integral d^4p in Eq. (35) exhibits a branch cut on the imaginary axis of q. For the contribution from $\tilde{P}$, one can change the contour as in Fig. 2 since the contribution from the infinite half-circle vanishes. After integrating by parts, we find that the part of $\mathcal{G}_a$ from $\tilde{P}$ is given by

$$\Delta\mathcal{G}_a=\frac{T_a(\Phi)}{8\pi^2}\left(\frac{1}{6}d(j)-2C(j)\right)\mathcal{F}(q)|_{q\to i\infty}$$
$$-\frac{1}{8\pi^2}\int_0^1 dx\left(\frac{1}{2}d(j)(1-2x)^2-2C(j)\right)$$
$$\times\mathcal{F}(q)|_{q=i\sqrt{x(1-x)p^2}}, \tag{39}$$

where

$$\mathcal{F}(q)=\frac{1}{2}\ln N+A\ln q+Bq.$$

The contribution from P_∞ includes the log divergence from the pole term $1/q$. This can be regulated by the standard dimensional regularization of 4D momentum integral, $d^4p\to d^Dp$, yielding a $1/(D-4)$ pole. On the other hand, the step-function contribution from $\epsilon(\mathrm{Im}(q))$ involves a 5D momentum integral which is linearly divergent, but it simply gives a finite result in dimensional regularization. Adding the divergent contribution from P_∞ to the finite part from $\tilde{P}$, we obtain

$$\mathcal{G}_a=\frac{T_a(\Phi)}{8\pi^2}\Bigg[\left(\frac{1}{6}d(j)-2C(j)\right)\mathcal{F}(q)|_{q\to i\infty}$$
$$+\int_0^1 dx\left(-\frac{1}{2}d(j)(1-2x)^2+2C(j)\right)$$
$$\times\left(\frac{1}{2}\ln N\right)\Bigg|_{q=i\sqrt{x(1-x)p^2}}+A\int_0^1 dx\Bigg(-\frac{1}{2}d(j)(1-2x)^2$$
$$+2C(j)\Bigg)\left(\frac{1}{D-4}\right)\Bigg]. \tag{40}$$

In fact, the values of A and $\mathcal{F}(q)$ at $q\to i\infty$ depend only on the $Z_2\times Z_2'$ parity of the corresponding 5D field, *not* on the spin of the field. We then find

$$A=(-1/2,0,0,1/2)$$

for $Z_2\times Z_2'$ parity $(Z_\Phi,Z_\Phi')=(++,+-,-+,--)$ and

$$\mathcal{F}|_{q\to i\infty}=\Bigg(\frac{1}{4}\pi kR-\frac{1}{2}\ln k,\ -\frac{1}{4}\pi kR,\ \frac{1}{4}\pi kR,$$
$$-\frac{1}{4}\pi kR+\frac{1}{2}\ln k\Bigg)$$

for the same $Z_2\times Z_2'$ parity.

In order to get a physical result from Eq. (40), we still need to subtract the $1/(D-4)$ pole. When written in the position space of 5th dimension, $1/(D-4)$ term in Eq. (40) eventually leads to a term $\propto[\lambda_0\delta(y)+\lambda_\pi\delta(y-\pi)]F^a_{\mu\nu}F^{a\mu\nu}/(D-4)$ in the 1-loop effective action. [See Eqs. (28) and (29) for the definition of λ_0 and λ_π.] Then the subtraction procedure should take into account that the cutoff scales at $y=0$ and π differ by the warp factor $e^{-\pi kR}$. The correct subtraction scheme is to add a counterterm

$$\int d^4x dy\sqrt{G}\frac{1}{32\pi^2}\Bigg[\lambda_0\left(\frac{1}{(D-4)}-\ln(\Lambda)\right)\frac{\delta(y)}{\sqrt{G_{55}}}$$
$$+\lambda_\pi\left(\frac{1}{(D-4)}-\ln(\Lambda e^{-\pi kR})\right)\frac{\delta(y-\pi)}{\sqrt{G_{55}}}\Bigg]F^a_{\mu\nu}F^{a\mu\nu}, \tag{41}$$

which gives an extra R-dependent contribution $\propto\lambda_\pi\pi kR$ to the low energy gauge coupling. This can be considered in principle as a different choice of the bare IR brane coupling $g^2_{\pi a}(\Lambda)$. However if the 5D orbifold field theory is regulated in an R-independent manner, which is the most natural choice in view of the fact that R is a dynamical field in 5D theory, this extra piece should be considered as a part of a calculable correction. Also the strong coupling assumption on the bare brane couplings [13], $g^2_{0a}(\Lambda)\approx g^2_{\pi a}(\Lambda)=\mathcal{O}(8\pi^2)$, applies for the R-independent part. As we will see in the next section, our subtraction scheme correctly reproduces the results in the supersymmetric case which can be obtained by a completely independent method based on 4D effective SUGRA whose regulator mass is R independent. We also explicitly show in Appendix B that our subtraction scheme gives precisely the same result as the R-independent Pauli-Villars regularization for the case of 5D scalar QED.

With the prescription to compute the regularized one-loop gauge coupling which has been discussed so far, it is now straightforward to compute Δ_a induced by generic 5D fields with arbitrary $Z_2\times Z_2'$ boundary condition. The correction due to 5D scalar fields is given by

$$\Delta_a(\phi)=\frac{1}{12}\Bigg[T_a(\phi_{++})\Bigg\{\ln\left(\frac{\Lambda}{k}\right)$$
$$-3\int_0^1 du\,F(u)\ln N_{\phi_{++}}\left(\frac{iu}{2}\sqrt{p^2}\right)\Bigg\}$$
$$-3T_a(\phi_{+-})\int_0^1 du\,F(u)\ln N_{\phi_{+-}}\left(\frac{iu}{2}\sqrt{p^2}\right)$$
$$-3T_a(\phi_{-+})\int_0^1 du\,F(u)\ln N_{\phi_{-+}}\left(\frac{iu}{2}\sqrt{p^2}\right)$$
$$-T_a(\phi_{--})\Bigg\{\ln\left(\frac{\Lambda}{k}\right)$$
$$+3\int_0^1 du\,F(u)\ln N_{\phi_{--}}\left(\frac{iu}{2}\sqrt{p^2}\right)\Bigg\}\Bigg] \tag{42}$$

where the part with coefficient $T_a(\phi_{zz'})$ represents the contribution from the loops of 5D scalar field $\phi_{zz'}$ and

$$F(u)=u(1-u^2)^{1/2}.$$

Here $N_{\phi_{zz'}}$ $(z=\pm,z'=\pm)$ are the N functions of Eqs. (18) and (19) for

$$(Z_\Phi,Z'_\Phi,s,r_0,r_\pi,\alpha)=(z,z',2,b_0,b_\pi,\sqrt{4+a}).$$

The 1-loop corrections due to 5D fermion and vector fields are similarly obtained to be

$$\begin{aligned}\Delta_a(\psi)=\frac{1}{3}\Bigg[&T_a(\psi_{++})\left\{2\ln\left(\frac{k}{p}\right)-\pi kR\right.\\&\left.+3\int_0^1 du\ G(u)\ln N_{\psi_{++}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}\\&+T_a(\psi_{+-})\left\{-\pi kR\right.\\&\left.+3\int_0^1 du\, G(u)\ln N_{\psi_{+-}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}+T_a(\psi_{-+})\\&\times\left\{\pi kR+3\int_0^1 du\, G(u)\ln N_{\psi_{-+}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}\\&+T_a(\psi_{--})\left\{2\ln\left(\frac{k}{p}\right)-\pi kR\right.\\&\left.+3\int_0^1 du\, G(u)\ln N_{\psi_{--}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}\Bigg],\end{aligned}\tag{43}$$

$$\begin{aligned}\Delta_a(A)=\frac{1}{12}\Bigg[&T_a(A_{++})\left\{23\ln\left(\frac{p}{\Lambda}\right)+21\ln\left(\frac{p}{k}\right)+22\pi kR\right.\\&\left.+\int_0^1 du\, K(u)\ln N_{A_{++}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}+T_a(A_{+-})\\&\times\left\{-\pi kR+\int_0^1 du\, K(u)\ln N_{A_{+-}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}\\&+T_a(A_{-+})\left\{\pi kR+\int_0^1 du\, K(u)\ln N_{A_{-+}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}\\&+T_a(A_{--})\left\{23\ln\left(\frac{\Lambda}{k}\right)+2\ln\left(\frac{k}{p}\right)-\pi kR\right.\\&\left.+\int_0^1 du\, K(u)\ln N_{A_{--}}\left(\frac{iu}{2}\sqrt{p^2}\right)\right\}\Bigg],\end{aligned}$$

where

$$G(u)=u(1-u^2)^{1/2}-u(1-u^2)^{-1/2},$$

$$K(u)=-9u(1-u^2)^{1/2}+24u(1-u^2)^{-1/2}.$$

Here $N_{\psi_{++}}$, $N_{\psi_{+-}}$, $N_{\psi_{-+}}$ and $N_{\psi_{--}}$ are the N functions of Eqs. (18) and (19) for

$$\begin{aligned}&(Z_\Phi,Z'_\Phi,s,r,\alpha)\\&\quad=(-,-,1,c,|c-1/2|),(+,-,1,-c,|c+1/2|),\\&\quad(-,+,1,-c,|c+1/2|),(-,-,1,-c,|c+1/2|),\end{aligned}$$

where $r=r_0=r_\pi$, and $N_{A_{++}},N_{A_{+-}},N_{A_{-+}}$ and $N_{A_{--}}$ are the N functions for

$$\begin{aligned}(Z_\Phi,Z'_\Phi,s,r,\alpha)=(-,-,4,2,0),(+,-,2,0,1),\\(-,+,2,0,1),(-,-,2,0,1).\end{aligned}$$

Note that $N_{\psi_{++}}$ and $N_{A_{++}}$ are given by N_{--} in Eq. (19), not N_{++} in Eq. (18).

For a practical application of the above results, one may consider the low momentum limit $p\ll m_1$ where m_1 denotes the *lowest* mass eigenvalue determined by the corresponding N function. The results of Δ_a in such a limit are summarized in Table I. We also provide in Table II the expressions of Δ_a induced by a scalar field with particular values of bulk and brane mass parameters, i.e., $b_0=b_\pi$ and $\alpha=|2-b_0|$, which corresponds to the scalar field in supersymmetric theory.

IV. 4D SUPERGRAVITY CALCULATION

In [4,7], 1-loop low energy gauge couplings in AdS_5 have been obtained in the supersymmetric case using the gauged $U(1)_R$ symmetry and chiral anomaly [14] in 5D SUGRA in AdS_5 [9,10] and also the known properties of gauge couplings in 4D effective SUGRA [15]. In this section, we confirm that the results of the previous section correctly reproduce the SUGRA results when applied in the supersymmetric case.

To proceed, let us briefly discuss supersymmetric 5D theory on AdS_5. The theory contains two types of 5D supermultiplets other than the SUGRA multiplet: one is the hypermultiplet H containing two 5D complex scalar fields h^i $(i=1,2)$ and a Dirac fermion ψ, and the other is the vector multiplet V containing a 5D vector A_M, real scalar Σ and a symplectic Majorana fermion λ^i. In the supersymmetric model, all scalar fields have $b_0=b_\pi=b$ and $\alpha=|2-b|$ [see Eqs. (6) and (11) for the definitions of $b_{0,\pi}$ and α] and their superpartner fermion has a kink mass parameter $c=\pm(3-2b)/2$. Also the $U(1)_R$ symmetry is gauged with the graviphoton B_M in the following way:

$$D_M h^i=\partial_M h^i-i\left(\frac{3}{2}(\sigma_3)^i_j-c\,\delta^i_j\right)k\epsilon(y)B_M h^j+\cdots$$

$$D_M\psi=\partial_M\psi+ick\epsilon(y)B_M\psi+\cdots$$

TABLE I. One loop corrections for $p\ll m_1$ where m_1 is the lowest nonzero KK mass.

Type	(zz')	$\Delta_a(p,k,R,\ln\Lambda)$
Real scalar ϕ	$(++)$	if $Q_{++}\neq 0$. $\frac{1}{12}T_a(\phi_{++})[\ln\Lambda-\ln k-\ln Q_{++}]$ if $Q_{++}=0$, $\frac{1}{12}T_a(\phi_{++})[\ln\Lambda+\ln k-2\ln p-\ln R_{++}]$
	$(+-)$	if $Q_{+-}\neq 0$, $-\frac{1}{12}T_a(\phi_{+-})\ln Q_{+-}$ if $Q_{+-}=0$, $\frac{1}{12}T_a(\phi_{+-})[-2\ln p+2\ln k-\ln R_{+-}]$
	$(-+)$	if $Q_{-+}\neq 0$, $-\frac{1}{12}T_a(\phi_{-+})\ln Q_{-+}$ if $Q_{-+}=0$, $\frac{1}{12}T_a(\phi_{-+})[-2\ln p+2\ln k-\ln R_{-+}]$
	$(--)$	$\frac{1}{12}T_a(\phi_{--})\left[-\ln\Lambda+\ln k-\ln\left(\frac{e^{\alpha\pi kR}-e^{-\alpha\pi kR}}{\pi\alpha}\right)\right]$
Spinor ψ	$(++)$	$\frac{1}{3}T_a(\psi_{++})\left[-\pi kR+2\ln k-2\ln p-2\ln\times\left\{\frac{e^{(c-1/2)\pi kR}-e^{-(c-1/2)\pi kR}}{\pi(c-1/2)}\right\}\right]$
	$(+-)$	$\frac{2}{3}T_a(\psi_{+-})c\pi kR$
	$(-+)$	$-\frac{2}{3}T_a(\psi_{-+})c\pi kR$
	$(--)$	$\frac{1}{3}T_a(\psi_{--})\left[-\pi kR+2\ln k-2\ln p-2\ln\times\left\{\frac{e^{(c+1/2)\pi kR}-e^{-(c+1/2)\pi kR}}{\pi(c+1/2)}\right\}\right]$
Vector A	$(++)$	$\frac{1}{12}T_a(A_{++})[-23\ln\Lambda+22\pi kR-21\ln k+44\ln p+21\ln 2kR]$
	$(+-)$	$-\frac{11}{6}T_a(A_{+-})\pi kR$
	$(-+)$	$\frac{11}{6}T_a(A_{-+})\pi kR$
	$(--)$	$\frac{1}{12}T_a(A_{--})\left[23\ln\Lambda-\pi kR-21\ln k-2\ln p+21\ln\left(\frac{e^{\pi kR}-e^{-\pi kR}}{\pi}\right)\right]$

$$D_M\lambda^i=\partial_M\lambda^i-i\frac{3}{2}(\sigma_3)^i_jk\epsilon(y)B_M\lambda^j+\cdots, \tag{44}$$

where ψ has a kink mass $ck\epsilon(y)$ and the ellipses stand for the couplings with other gauge fields. Taking into account the $Z_2\times Z_2'$ parity, the supermultiplet structure is given by

$$H_{zz'}(c)=\left(h^1_{zz'}\left(b=\frac{3}{2}-c\right),h^2_{\tilde{z}\tilde{z}'}\left(b=\frac{3}{2}+c\right),\psi_{zz'}(c)\right),$$

$$V_{zz'}=\left(A^\mu_{zz'},A^5_{\tilde{z}\tilde{z}'}(b=2),\lambda^i_{zz'}\left(c=\frac{1}{2}\right),\Sigma_{\tilde{z}\tilde{z}'}(b=2)\right), \tag{45}$$

TABLE II. 5D scalar contribution for $p\ll m_1$ when $b_0=b_\pi$ and $\alpha=|2-b_0|$.

$(++)$	$\frac{1}{12}T_a(\phi_{++})\left[\ln\Lambda+\ln k-2\ln p-\pi kR-\ln\left\{\frac{e^{(1-b)\pi kR}-e^{-(1-b)\pi kR}}{\pi(1-b)}\right\}\right]$
$(+-)$	$\frac{1}{12}T_a(\phi_{+-})(2-b)\pi kR$
$(-+)$	$-\frac{1}{12}T_a(\phi_{-+})(2-b)\pi kR$
$(--)$	$\frac{1}{12}T_a(\phi_{--})\left[-\ln\Lambda+\ln k-\ln\left\{\frac{e^{(2-b)\pi kR}-e^{-(2-b)\pi kR}}{\pi(2-b)}\right\}\right]$

where the subscripts z,z' denote the $Z_2\times Z_2'$ parity, $\tilde{z}=-z$, $\tilde{z}'=-z'$, b is the brane mass parameter and c is the kink mass parameter.

Let us assume that our 5D theory is compactified in a manner preserving $D=4$ $N=1$ supersymmetry. This allows the low energy physics to be described by 4D effective SUGRA whose action can be written as

$$S_{4D}=\int d^4x\left[\int d^4\theta\left\{-3\exp\left(-\frac{K}{3}\right)\right\}+\left(\int d^2\theta\frac{1}{4}f_aW^{a\alpha}W^a_\alpha+\text{H.c.}\right)\right], \tag{46}$$

where W^a_α is the chiral spinor superfield for the 4D gauge multiplet and we set the 4D gravity multiplet by their vacuum values. The Kähler potential K can be expanded in powers of generic gauge-charged chiral superfield Q:

$$K=K_0(\mathcal{T},\mathcal{T}^*)+Z_Q(\mathcal{T},\mathcal{T}^*)Q^*e^{-V}Q+\cdots, \tag{47}$$

where $\mathcal{T}$ denotes the radion superfield whose scalar component is given by

$$\mathcal{T}=R+iB_5,$$

and the gauge kinetic function f_a is a *holomorphic* function of $\mathcal{T}$. Then the 1-loop gauge couplings in effective 4D SUGRA can be determined by f_a containing the 1-loop threshold correction from massive KK modes and also the tree-level Kähler potential K [15]:

$$\frac{1}{g_a^2(p)}=\text{Re}(f_a)+\frac{b_a}{16\pi^2}\ln\left(\frac{M_{Pl}^2}{e^{-K_0/3}p^2}\right)-\sum_Q\frac{T_a(Q)}{8\pi^2}\ln(e^{-K_0/3}Z_Q)+\frac{T_a(\text{Adj})}{8\pi^2}\ln(\text{Re}(f_a)), \tag{48}$$

where $b_a=\sum T_a(Q)-3T_a(\text{Adj})$ is the 1-loop beta function coefficient and M_{Pl} is the Planck scale of $g_{\mu\nu}$ which defines $p^2=-g^{\mu\nu}\partial_\mu\partial_\nu$.

Let us consider the 4D effective SUGRA of a 5D theory which contains $H_{++},H_{+-},H_{-+},H_{--}$ as well as $V_{++},V_{+-},V_{-+},V_{--}$. The 5D vector multiplet V_{++} gives a massless 4D gauge multiplet containing A^{μ}_{++} whose low energy couplings are of interest to us, while V_{--} gives a massless 4D chiral multiplet containing $\Sigma_{++}+iA^5_{++}$. H_{++} and H_{--} also give massless 4D chiral multiplets containing h^1_{++} and h^2_{++}, respectively, whose tree level Kähler metrics are required to compute the 1-loop gauge coupling (48). Other 5D multiplets, i.e., V_{+-},V_{-+},H_{+-} and H_{-+} do not give any massless 4D mode. Let $Y_Q=e^{-K_0/3}Z_Q$ where $Z_Q(Q=H_{++},H_{--},V_{--})$ denote the Kähler metric of the 4D massless chiral superfields coming from the 5D multiplets H_{++},H_{--} and V_{--}, respectively. Following Refs. [7,16], it is straightforward to find the *tree level* $Z_{H_{++}},Z_{H_{--}}$ and also f_a containing *the 1-loop threshold corrections* from massive KK modes:

$$M_{Pl}^2=e^{-K_0/3}M_5^2=\frac{M_5^3}{k}(1-e^{-k\pi(\mathcal{T}+\mathcal{T}^*)}),$$

$$Y_{H_{++}}=\frac{M_5}{\left(\frac{1}{2}-c_{++}\right)k}(e^{(1/2-c_{++})\pi k(\mathcal{T}+\mathcal{T}^*)}-1),$$

$$Y_{H_{--}}=\frac{M_5}{\left(\frac{1}{2}+c_{--}\right)k}(e^{(1/2+c_{--})\pi k(\mathcal{T}+\mathcal{T}^*)}-1),$$

$$Y_{V_{--}}=\frac{k}{M_5}\frac{1}{e^{\pi k(\mathcal{T}+\mathcal{T}^*)}-1},$$

$$f_a=\frac{\pi\mathcal{T}}{\hat{g}_{5a}^2}+\frac{z'}{8\pi^2}\left(\frac{3}{2}\sum_{V_{zz'}}T_a(V_{zz'})-\sum_{H_{zz'}}c_{zz'}T_a(H_{zz'})\right)k\pi\mathcal{T}, \tag{49}$$

where M_5 is the 5D Planck scale, and $c_{zz'}$ is the kink mass of $H_{zz'}$. As was noted in [7], the KK threshold correction to f_a can be entirely determined by the chiral anomaly with respect to the following B_5-dependent phase transformation:

$$\lambda^{ai}\to e^{3ik|y|B_5/2}\lambda^{ai},\quad \psi\to e^{-ick|y|B_5}\psi. \tag{50}$$

Using the above results, we finally find

$$\begin{aligned}\Delta_a={}&T_a(H_{++})\left[\ln\left(\frac{k}{p}\right)-c_{++}\pi kR-\ln\left(\frac{e^{(1-2c_{++})\pi kR}-1}{\pi(1-2c_{++})}\right)\right]-T_a(V_{++})\left[3\ln\left(\frac{M_5}{p}\right)-\frac{3}{2}\pi kR-\ln(M_5R)\right]\\&+c_{+-}T_a(H_{+-})\pi kR-\frac{3}{2}T_a(V_{+-})\pi kR-c_{-+}T_a(H_{-+})\pi kR+\frac{3}{2}T_a(V_{+-})\pi kR+T_a(H_{--})\left[\ln\left(\frac{k}{p}\right)+c_{--}\pi kR\right.\\&\left.-\ln\left(\frac{e^{(1+2c_{--})\pi kR}-1}{\pi(1+2c_{--})}\right)\right]+T_a(V_{--})\left[\ln\frac{M_5}{p}+\ln\frac{M_5}{k}+\frac{1}{2}k\pi R+\ln(1-e^{-2\pi kR})\right]\end{aligned} \tag{51}$$

for $p\ll m_1$ where m_1 is the lowest nonzero KK mass. Note that $m_1\sim ke^{-\pi kR}$ for the bulk fields other than H_{+-} or H_{-+}, while H_{+-} has $m_1\sim ke^{-(1/2+c_{+-})\pi kR}$ for $c_{+-}\geqslant 1/2$ and $m_1\sim ke^{-\pi kR}$ for $c_{+-}\leqslant 1/2$, and H_{-+} has $m_1\sim ke^{(-1/2+c_{-+})\pi kR}$ for $c_{-+}\leqslant -1/2$ and $m_1\sim ke^{-\pi kR}$ for $c_{-+}\geqslant -1/2$. The above result obtained by 4D SUGRA analysis perfectly agrees with the result that one would obtain using the results of Tables I and II when M_5 is replaced by Λ. This provides a nontrivial check for the results obtained in the previous section and assures us that our results are truly scheme independent.

V. CONCLUSION

In this paper, we have calculated the full 1-loop corrections to the low energy coupling of bulk gauge bosons in AdS_5 induced by generic 5D scalar, fermion and vector fields with arbitrary $Z_2\times Z_2'$ orbifold boundary conditions. The used calculation scheme is the background field method with dimensional regularization. We noted that the subtraction scale for the log divergence at the IR brane ($y=\pi$) should be taken to be $\Lambda e^{-\pi kR}$ where Λ is the subtraction scale for the UV brane ($y=0$). We also considered the supersymmetric case to assure us that our results correctly reproduce the results obtained by a completely independent method based on 4D effective supergravity analysis.

Note added. While this work was in completion, we received [17,18] discussing the 1-loop gauge coupling renormalization due to 5D scalar loops in AdS_5 background and its interpretation in the context of AdS-CFT correspondence and also [19] discussing the 1-loop renormalization in the context of deconstructed AdS_5.

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APPENDIX A: SOME PROPERTIES OF THE N FUNCTIONS

In this appendix, we present some properties of the N functions, $N_{zz'}$ $(z,z'=\pm)$, given in Eqs. (18) and (19). Using $Y_\alpha(x)=[\cos\alpha\pi J_\alpha(x)-J_{-\alpha}(x)]/\sin\alpha\pi$ and also the fact that $N_{zz'}$ are antisymmetric under the exchange of J_α and Y_α, one can rewrite the N functions as

$$\begin{aligned}N_{++}(q)=&-\frac{1}{\sin\alpha\pi}\left[\left\{\left(\frac{s}{2}-r_0\right)J_\alpha\left(\frac{q}{k}\right)+\frac{q}{k}J'_\alpha\left(\frac{q}{k}\right)\right\}\left\{\left(\frac{s}{2}-r_\pi\right)J_{-\alpha}\left(\frac{q}{T}\right)+\frac{q}{T}J'_{-\alpha}\left(\frac{q}{T}\right)\right\}-\left\{\left(\frac{s}{2}-r_\pi\right)J_\alpha\left(\frac{q}{T}\right)\right.\right.\\&\left.\left.+\frac{q}{T}J'_\alpha\left(\frac{q}{T}\right)\right\}\left\{\left(\frac{s}{2}-r_0\right)J_{-\alpha}\left(\frac{q}{k}\right)+\frac{q}{k}J'_{-\alpha}\left(\frac{q}{k}\right)\right\}\right],\\
N_{+-}(q)=&-\frac{1}{\sin\alpha\pi}\left[\left\{\left(\frac{s}{2}-r_0\right)J_\alpha\left(\frac{q}{k}\right)+\frac{q}{k}J'_\alpha\left(\frac{q}{k}\right)\right\}J_{-\alpha}\left(\frac{q}{T}\right)-J_\alpha\left(\frac{q}{T}\right)\left\{\left(\frac{s}{2}-r_0\right)J_{-\alpha}\left(\frac{q}{k}\right)+\frac{q}{k}J'_{-\alpha}\left(\frac{q}{k}\right)\right\}\right],\\
N_{-+}(q)=&\frac{1}{\sin\alpha\pi}\left[\left\{\left(\frac{s}{2}-r_\pi\right)J_\alpha\left(\frac{q}{T}\right)+\frac{q}{T}J'_\alpha\left(\frac{q}{T}\right)\right\}J_{-\alpha}\left(\frac{q}{k}\right)-J_\alpha\left(\frac{q}{k}\right)\left\{\left(\frac{s}{2}-r_\pi\right)J_{-\alpha}\left(\frac{q}{T}\right)+\frac{q}{T}J'_{-\alpha}\left(\frac{q}{T}\right)\right\}\right],\\
N_{--}(q)=&-\frac{1}{\sin\alpha\pi}\left[J_\alpha\left(\frac{q}{k}\right)J_{-\alpha}\left(\frac{q}{T}\right)-J_\alpha\left(\frac{q}{T}\right)J_{-\alpha}\left(\frac{q}{k}\right)\right],\end{aligned}\tag{A1}$$

where $T=ke^{-\pi kR}$. Then using $J_\alpha(x)=x^\alpha f(x^2)$, one can easily see that all N functions are even functions:

$$N_{zz'}(q)=N_{zz'}(-q).$$

We already know $N_{zz'}(q)$ is analytic near $q=0$, allowing an expansion around $q=0$:

$$N_{zz'}(q)=Q_{zz'}+\frac{q^2}{k^2}R_{zz'}+\mathcal{O}(q^4),\tag{A2}$$

where

$$\begin{aligned}Q_{++}=&\frac{1}{\pi\alpha}\left[\left(\alpha+r_\pi-\frac{s}{2}\right)\left(\alpha-r_0+\frac{s}{2}\right)e^{-\alpha\pi kR}-\left(\alpha+r_0-\frac{s}{2}\right)\left(\alpha-r_\pi+\frac{s}{2}\right)e^{\alpha\pi kR}\right],\\
Q_{+-}=&-\frac{1}{\pi\alpha}\left[\left(\alpha-r_0+\frac{s}{2}\right)e^{-\alpha\pi kR}+\left(\alpha+r_0-\frac{s}{2}\right)e^{\alpha\pi kR}\right],\\
Q_{-+}=&\frac{1}{\pi\alpha}\left[\left(\alpha-r_\pi+\frac{s}{2}\right)e^{\alpha\pi kR}+\left(\alpha+r_\pi-\frac{s}{2}\right)e^{-\alpha\pi kR}\right],\\
Q_{--}=&\frac{1}{\pi\alpha}[e^{\alpha\pi kR}-e^{-\alpha\pi kR}],\\
R_{++}=&\frac{1}{4\pi}\left[\frac{1}{\alpha(\alpha-1)}\left\{\left(2-\alpha-r_0+\frac{s}{2}\right)\left(\alpha-r_\pi+\frac{s}{2}\right)e^{\alpha\pi kR}+\left(-2+\alpha+r_\pi-\frac{s}{2}\right)\left(\alpha-r_0+\frac{s}{2}\right)e^{(2-\alpha)\pi kR}\right\}\right.\\&\left.+\frac{1}{\alpha(\alpha+1)}\left\{\left(-\alpha-r_\pi+\frac{s}{2}\right)\left(2+\alpha-r_0+\frac{s}{2}\right)e^{-\alpha\pi kR}+\left(\alpha+r_0-\frac{s}{2}\right)\left(2+\alpha-r_\pi+\frac{s}{2}\right)e^{(\alpha+2)\pi kR}\right\}\right],\\
R_{+-}=&\frac{1}{4\pi}\left[-\frac{1}{\alpha(\alpha-1)}\left\{\left(-2+\alpha+r_0-\frac{s}{2}\right)e^{\alpha\pi kR}+\left(\alpha-r_0+\frac{s}{2}\right)e^{(2-\alpha)\pi kR}\right\}+\frac{1}{\alpha(\alpha+1)}\left\{\left(2+\alpha-r_0+\frac{s}{2}\right)e^{-\alpha\pi kR}+\left(\alpha+r_0-\frac{s}{2}\right)e^{(2+\alpha)\pi kR}\right\}\right],\\
R_{-+}=&-\frac{1}{4\pi}\left[\frac{1}{\alpha(1-\alpha)}\left\{\left(-2+\alpha+r_\pi-\frac{s}{2}\right)e^{(2-\alpha)\pi kR}+\left(\alpha-r_\pi+\frac{s}{2}\right)e^{\alpha\pi kR}\right\}+\frac{1}{\alpha(1+\alpha)}\left\{\left(2+\alpha-r_\pi+\frac{s}{2}\right)e^{(2+\alpha)\pi kR}+\left(\alpha+r_\pi-\frac{s}{2}\right)e^{-\alpha\pi kR}\right\}\right],\\
R_{--}=&\frac{1}{4\pi}\left[-\frac{1}{\alpha(\alpha-1)}\{e^{-(\alpha-2)\pi kR}-e^{\alpha\pi kR}\}+\frac{1}{\alpha(\alpha+1)}\{e^{-\alpha\pi kR}-e^{(\alpha+2)\pi kR}\}\right].\end{aligned}\tag{A3}$$

The KK mass eigenvalue m_n is determined by the zeros of N function: $N(m_n)=0$. Obviously a 5D field has a massless 4D mode iff $Q_{zz'}=0$. Generically, a nonzero KK mass eigenvalue starts to appear from $m_n=\mathcal{O}(T)$. However, in some special cases, there can be nonzero mass eigenvalues much smaller than $T=ke^{-\pi kR}$. For instance, if $\alpha=s/2-r_0$ and α has a large value, $Q_{+-}\sim e^{-\alpha\pi kR}$ and $R_{+-}\sim e^{\alpha\pi kR}$, giving a very light state of Φ_{+-} with $m_n\sim ke^{-\alpha\pi kR}$. Similarly, if $\alpha=r_\pi-s/2$, Φ_{-+} can also have a very small m_n. However, Φ_{--} does have neither a massless state nor a very light state with $m_n\ll ke^{-\pi kR}$.

The asymptotic behavior of an N function at $|q|\to\infty$ is essential for regularizing the 1-loop gauge coupling. Using the asymptotic formulas of Bessel functions:

$$J_\alpha(x)\to\sqrt{\frac{2}{\pi x}}\cos\left[x-\left(\alpha+\frac{1}{2}\right)\right],$$

$$Y_\alpha(x)\to\sqrt{\frac{2}{\pi x}}\sin\left[x-\left(\alpha+\frac{1}{2}\right)\right],$$

we find

$$N_{++}(q)\to-\frac{2qe^{\pi kR/2}}{\pi k}\sin\left(\frac{(1-e^{\pi kR})q}{k}\right),$$

$$N_{+-}(q)\to-\frac{2}{\pi}e^{-\pi kR/2}\cos\left(\frac{(1-e^{\pi kR})q}{k}\right),$$

$$N_{-+}(q)\to\frac{2}{\pi}e^{\pi kR/2}\cos\left(\frac{(1-e^{\pi kR})q}{k}\right),$$

$$N_{--}(q)\to-\frac{2k}{\pi q}e^{-\pi kR/2}\sin\left(\frac{(1-e^{\pi kR})q}{k}\right).$$

APPENDIX B: COMPARISON WITH PAULI-VILLARS REGULARIZATION

The natural regularization in 5D theory is to cut off 5D momentum in the 5D metric frame of G_{MN}: $-G^{MN}\partial_M\partial_N<\Lambda^2$. In AdS background, this would correspond to an effective y-dependent cutoff of 4D momentum in the 4D metric frame of $g_{\mu\nu}$: $p^2=-g^{\mu\nu}\partial_\mu\partial_\nu<e^{-2kR|y|}\Lambda^2$. In dimensional regularization, such a feature is not manifest, but can be taken into account by choosing the subtraction scale $\sim\Lambda e^{-kR\tilde{y}}$ where $\tilde{y}=0$ or π is the location of log divergence. On the other hand, such feature is rather manifest in Pauli-Villars (PV) regularization in which Λ corresponds to a 5D regulator mass. In this appendix, we compare our result using dimensional regularization with the subtraction scheme (41) to the PV result for scalar QED. For simplicity, we consider the massless scalar QED with $Z_2\times Z_2'$ parity $(++)$.

In the PV scheme, the UV divergence is regulated by a PV regulator with 5D mass Λ which has the same $Z_2\times Z_2'$ boundary condition as ϕ but opposite statistics:

$$\sum_n\int\frac{d^4p}{(2\pi)^4}f(p,m_n)\to\sum_n\left\{\int\frac{d^4p}{(2\pi)^4}f(p,m_n)-\int\frac{d^4p}{(2\pi)^4}f(p,M_n)\right\},\quad\text{(B1)}$$

where M_n is the KK spectrum for the PV regulator. We convert the summation into an integral using the pole functions:

$$P_\phi=\frac{N'_\phi}{2N_\phi},\qquad P_{\rm PV}=\frac{N'_{\rm PV}}{2N_{\rm PV}},$$

and then the regulated amplitude is given by

$$\int_{\leftrightarrows}\frac{dq}{2\pi i}P_{\rm reg}(q)\int\frac{d^4p}{(2\pi)^4}f(p,q),\quad\text{(B2)}$$

where $P_{\rm reg}(q)\equiv P_\phi(q)-P_{\rm PV}(q)$. Since N_ϕ and $N_{\rm PV}$ are the same limiting behavior at $|q|\to\infty$, $P_{\rm reg}(q)$ vanishes at infinity. After a partial integration along q, we find

$$\Delta_{\rm PV}=8\pi^2\int_C\frac{dq}{2\pi i}\left(\frac{1}{2}\ln N_\phi-\frac{1}{2}\ln N_{\rm PV}\right)\frac{d}{dq}\left\{\frac{1}{2}\int dx(1-2x)^2\frac{1}{(4\pi)^2}\ln(x(1-x)p^2+q^2)\right\}$$
$$=-\frac{1}{4}\int dx(1-2x)^2(\ln N_\phi-\ln N_{\rm PV})|_{q=i\sqrt{x(1-x)p^2}},\quad\text{(B3)}$$

where C is the contour line described in Fig. 2. For $q\ll ke^{-\pi kR}$,

$$N_\phi\approx\frac{q^2}{k^2}e^{\pi kR}\left(\frac{e^{\pi kR}-e^{-\pi kR}}{\pi}\right),\quad\text{(B4)}$$

$$N_{\rm PV}\approx\frac{(\alpha-2)(\alpha+2)}{\pi\alpha}(e^{-\alpha\pi kR}-e^{\alpha\pi kR}).\quad\text{(B5)}$$

For $\Lambda\gg k$, $\alpha\equiv\sqrt{4+\Lambda^2/k^2}\approx\Lambda/k$, and so

$$\ln N_{\rm PV}\approx\pi\Lambda R+\ln\Lambda-\ln k.\quad\text{(B6)}$$

We then find

$$\Delta_{\rm PV}=\frac{1}{12}\left[\pi\Lambda R+\ln\Lambda+\ln k-2\ln p-\pi kR-\ln\left(\frac{e^{\pi kR}-e^{-\pi kR}}{\pi}\right)\right],\quad\text{(B7)}$$

which is precisely the same as the result in Table II for a massless real ϕ_{++} obtained using dimensional regularization with the subtraction scheme (41). In scalar QED, the charged scalar field should be complex, so that it gives a loop correction twice that of the above result.

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