Consistent deformations of dual formulations of linearized gravity: A no-go result

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The consistent, local, smooth deformations of the dual formulation of linearized gravity involving a tensor field in the exotic representation of the Lorentz group with Young symmetry type $(D-3,1)$ (one column of length $D-3$ and one column of length 1) are systematically investigated. The rigidity of the Abelian gauge algebra is first established. We next prove a no-go theorem for interactions involving at most two derivatives of the fields.

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I. INTRODUCTION

The electric-magnetic duality is one of the most fascinating symmetries of theoretical physics. Recently [1], dual formulations of linearized gravity [2] have been systematically investigated with *M*-theory motivations in mind [3,4] (see also [5]). These dual formulations involve tensor fields in "exotic" representations of the Lorentz group characterized by a mixed Young symmetry type. There exist in fact three different dual formulations of linearized gravity in generic spacetime dimension *D*. The first one is the familiar Pauli-Fierz description based on a symmetric tensor $h_{\mu\nu}$. The second one is obtained by dualizing on one index only and involves a tensor $T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}$ with

$$T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}=T_{[\lambda_1\lambda_2\cdots\lambda_{D-3}]\mu}, \tag{1.1}$$

$$T_{[\lambda_1\lambda_2\cdots\lambda_{D-3}\mu]}=0 \tag{1.2}$$

where square brackets denote antisymmetrization with strength one. Finally, the third one is obtained by dualizing on both indices and is described by a tensor $C_{\lambda_1\cdots\lambda_{D-3}\mu_1\cdots\mu_{D-3}}$ with Young symmetry type $(D-3,D-3)$ (two columns with $D-3$ boxes). Although one can write equations of motion for this theory which are equivalent to the linearized Einstein equations, these do not seem to follow (when $D>4$) from a Lorentz-invariant action principle in which the only varied field is $C_{\lambda_1\cdots\lambda_{D-3}\mu_1\cdots\mu_{D-3}}$. For this reason, we shall focus here on the dual theory based on $T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}$.

The purpose of this paper is to determine all the consistent, local, smooth interactions that this dual formulation admits. It is well known that the only consistent (local, smooth) deformation of the Pauli-Fierz theory is—under quite general and reasonable assumptions—given by the Einstein theory (see [6] and the more recent works [7,8] for systematic analyses). Because dualization is a non-local process, one does not expect the Einstein interaction vertex to have a local counterpart on the dual $T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}$-side. This does not *a priori* preclude the existence of other local interaction vertices, which would lead to exotic self-interactions of "spin-2" particles. Our main—and somewhat disappointing—result is, however, that this is not the case.

The first instance for which $T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}$ transforms in a true exotic representation of the Lorentz group occurs for $D=5$, where one has

$$T_{[\alpha\beta]\gamma} \simeq \begin{array}{|c|c|}\hline \beta & \gamma \\ \hline \alpha \\ \cline{1-1}\end{array} \; .$$

The action of this dual theory is given in [2] (see also [9–11]). We shall explicitly investigate the $T_{[\alpha\beta]\gamma}$ case in this paper and comment on general gauge fields $T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}$ at the end.

Our precise result is that the free field dual theory based on $T_{\lambda_1\lambda_2\mu}$, admits no consistent local deformation which (i) is Lorentz invariant, and (ii) contains no more than two derivatives of the field [i.e., the allowed interaction terms under consideration contains at most $\partial^2 T$ or $(\partial T)^2$]. No restriction is imposed on the polynomial degree of the interaction. Our result confirms previous unsuccessful attempts [1,2,12]. We also demonstrate the rigidity, to first order in the deformation parameter, of the algebra of the gauge symmetries without making any assumption on the number of derivatives.

Besides their occurrence in dual formulations of linearized gravity, tensor fields in exotic representations of the Lorentz group arise in the long-standing related problem of constructing consistent interactions among particles with higher spins [13–17]. A further motivation for the analysis of exotic higher spin gauge fields come from recent developments in M theory, where a matching between the $D=11$ supergravity equations [18] and the $E_{10|+10}/K(E_{10})$ coset model equations [$K(E_{10})$ being the maximal compact subgroup of the split form of $E_{10|+10}$ of E_{10}] was exhibited up to height 30 in the E_{10} roots [19] (the relevance of E_{10} in the supergravity context was indicated much earlier in [20]).

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One possibility for going beyond this height would be to introduce additional higher spin fields, most of which would be in exotic representations of the Lorentz group. Indeed, a quick argument shows that such fields might yield the exponentials associated with the higher height E_{10} roots—if they can be consistently coupled to gravity, an unsolved problem so far. The introduction of such additional massless fields would also be in line with what one expects from string theory (in the high energy limit where the string tension goes to zero [21]). The same motivations come from the covariant coset construction of [22] where $D=11$ supergravity is conjectured to provide a non-linear realization of E_{11}. The dual tensor field $T_{\lambda_1\lambda_2\cdots\lambda_8\mu}$ has actually already been identified in connection with both the E_{11} [22] and the E_{10} roots [19,23]. Note that mixed symmetry fields appear also in the models of [24,25].

In order to investigate the consistent, local, smooth deformations of the theory, we shall follow the cohomological approach of [26], based on the antifield formalism [27–29]. An alternative, Hamiltonian based deformation point of view has been developed in [30]. One advantage of the cohomological approach, besides its systematic aspect, is that it minimizes the work that must be done because most of the necessary computations are either already in the literature [31] or are direct extensions of existing developments carried out for 1-forms [32,33], p-forms [34] or gravity [8,35] (see also [36,37] for recent developments on the 1-form–p-form case). To a large extent, our no-go theorem is obtained by putting together, in a standard fashion, various cohomological computations which have an interest in their own right and which have been already published or can be obtained through by-now routine techniques.

II. THE FREE THEORY

A. Lagrangian, gauge symmetries

As stated above, we first restrict the explicit analysis to the case of a tensor T with 3 indices, $T=T_{\alpha\beta\mu}$, which is dual to linearized gravity in $D=5$ (but we shall carry the analysis without specifying D, taken only to be stricly greater than 4, $D>4$, so that the theory carries local degrees of freedom). The symmetry properties read

$$T_{\alpha\beta\gamma}=T_{[\alpha\beta]\gamma}, \quad T_{[\alpha\beta]\gamma}+T_{[\beta\gamma]\alpha}+T_{[\gamma\alpha]\beta}=0. \tag{2.1}$$

As shown in [38,39], the appropriate algebro-differential language for discussing gauge theories involving exotic representations of the Lorentz group is that of multiforms, or more accurately, that of hyperforms [38,41,43]. Multiforms were discussed recently in [40] as an auxiliary tool for investigating questions concerning N complexes associated with higher spin gauge theories. It turns out that hyperforms have been introduced much earlier in the mathematical literature by Olver in the analysis of higher order Pfaffian systems with integrability criteria (Olver, unpublished work [41]). We shall not use here the language of multiforms or hyperforms, however, because the relevant tensors involve only a few indices.

The Lagrangian for the gauge tensor field $T_{\lambda_1\lambda_2\mu}$ reads

$$\mathcal{L}=-\frac{1}{12}\left(F_{[\alpha\beta\gamma]\delta}F^{[\alpha\beta\gamma]\delta}-3F^{\xi}_{[\alpha\beta\xi]}F^{[\alpha\beta\lambda]}_{\lambda}\right), \tag{2.2}$$

where F is the tensor

$$F_{[\alpha\beta\gamma]\delta}=\partial_\alpha T_{[\beta\gamma]\delta}+\partial_\beta T_{[\gamma\alpha]\delta}+\partial_\gamma T_{[\alpha\beta]\delta}\equiv 3\partial_{[\alpha}T_{\beta\gamma]\delta}. \tag{2.3}$$

The gauge invariances are

$$\delta_{\sigma,\alpha}T_{[\alpha\beta]\gamma}=2(\partial_{[\alpha}\sigma_{\beta]\gamma}+\partial_{[\alpha}\alpha_{\beta]\gamma}-\partial_\gamma\alpha_{\alpha\beta}), \tag{2.4}$$

where $\sigma_{\alpha\beta}$ and $\alpha_{\alpha\beta}$ are arbitrary symmetric and antisymmetric tensor fields. The tensor F is invariant under the σ-gauge symmetries, but not under the α-ones. To get a completely gauge-invariant object, one must take one additional derivative. The tensor

$$E_{[\alpha\beta\delta][\varepsilon\gamma]}\equiv\frac{1}{2}(\partial_\varepsilon F_{[\alpha\beta\delta]\gamma}-\partial_\gamma F_{[\alpha\beta\delta]\varepsilon}) \tag{2.5}$$

is easily verified to be gauge invariant. Moreover its vanishing implies that $T_{[\alpha\beta]\gamma}$ is pure gauge [38]. The most general gauge invariant object depends on the field $T_{\alpha\beta\mu}$ and its derivatives only through the "curvature" $E_{[\alpha\beta\delta][\varepsilon\gamma]}$ and its derivatives. It is convenient to define the Ricci-like tensor $E_{[\alpha\beta]\gamma}$ and its trace:

$$E_{[\alpha\beta]\gamma}=\eta^{\varepsilon\delta}E_{[\alpha\beta\delta][\varepsilon\gamma]}, \qquad E_\alpha=\eta^{\beta\gamma}E_{[\alpha\beta]\gamma}. \tag{2.6}$$

The equations of motion are then

$$\frac{\delta\mathcal{L}}{\delta T_{[\alpha\beta]\gamma}}=3[E^{[\alpha\beta]\gamma}+\eta^{\gamma[\alpha}E^{\beta]}]=0. \tag{2.7}$$

Because the action is gauge-invariant, the equations of motion satisfy the "Bianchi identities"

$$\partial_\alpha(E^{[\alpha\beta]\gamma}+\eta^{\gamma[\alpha}E^{\beta]})\equiv 0. \tag{2.8}$$

One easy way to check these identities is to observe that one has

$$\frac{\delta\mathcal{L}}{\delta T_{[\mu\nu]\rho}}\equiv\partial_\lambda G^{\lambda\mu\nu\rho} \tag{2.9}$$

where the tensor $G^{\lambda\mu\nu\rho}$ is completely antisymmetric in its first three indices, $G^{\lambda\mu\nu\rho}=G^{[\lambda\mu\nu]\rho}$. Explicitly,

$$G^{\lambda\mu\nu\rho}=\frac{3}{2}\left(\partial^{[\lambda}T^{\mu\nu]\rho}-\eta^{\rho\lambda}\partial^{[\mu}T^{\nu\alpha]}_{\alpha}-\eta^{\rho\mu}\partial^{[\nu}T^{\lambda\alpha]}_{\alpha}-\eta^{\rho\nu}\partial^{[\lambda}T^{\mu\alpha]}_{\alpha}\right). \tag{2.10}$$

The gauge symmetries (2.4) are reducible. Indeed,

$$\delta_{\tilde\sigma,\tilde\alpha}T_{[\alpha\beta]\gamma}\equiv 0 \tag{2.11}$$

when

$$\tilde\sigma_{\alpha\beta}=6\partial_{(\alpha}\gamma_{\beta)}, \qquad \tilde\alpha_{\alpha\beta}=2\partial_{[\alpha}\gamma_{\beta]} \tag{2.12}$$

where γ_α are *arbitrary fields*. There is no further local reducibility identity.

The problem of introducing (smooth) consistent interactions is that of smoothly deforming the Lagrangian (2.2),

$$\mathcal{L}\rightarrow\mathcal{L}+g\mathcal{L}_1+g^2\mathcal{L}_2+\cdots, \tag{2.13}$$

the gauge transformations (2.4),

$$\delta_{\sigma,\alpha}T_{[\alpha\beta]\gamma}=(2.4)+g\ \delta^{(1)}_{\sigma,\alpha}T_{[\alpha\beta]\gamma}+g^2\delta^{(2)}_{\sigma,\alpha}T_{[\alpha\beta]\gamma}+\cdots \tag{2.14}$$

and the reducibility relations (2.12) in such a way that (i) the new action is invariant under the new gauge symmetries; and (ii) the new gauges symmetries reduce to zero on-shell when the gauge parameters fulfill the new reducibility relations. By developing these requirements order by order in the deformation parameter g, one gets an infinite number of consistency conditions, one at each order.

We shall impose the further requirement that the first order vertex $\mathcal{L}_1$ be Lorentz-invariant. Under this sole condition (together with consistency), we show that one can always redefine the fields and the gauge parameters in such a way that the gauge structure is unaffected by the deformation (to first order in g). That is, the gauge transformations remain abelian and the reducibility relations remain unchanged (‘‘rigidity of the gauge algebra’’). We next restrict the deformations to contain at most two derivatives of the fields, as the original free Lagrangian. This still leave a priori an infinite number of possibilities, of the schematic form $T^k(\partial T)^2$ where k is arbitrary [a term $T^l\partial^2T$ is of course equivalent to $T^{l-1}(\partial T)^2$ upon integration by parts]. We show, however, that within this infinite class, there is no non-trivial deformation. Any deformation can be redefined away by a local change of field variables.

B. BRST differential

As shown in [26], the first-order consistent local interactions correspond to elements of the cohomology $H^{D,0}(s|d)$ of the Becchi-Rouet-Stora-Tyutin (BRST) differential s modulo the spacetime exterior derivative d, in maximum form degree D and in ghost number 0. That is, one must compute the general solution of the cocycle condition

$$sa+db=0, \tag{2.15}$$

where a is a D-form of ghost number zero and b a $(D-1)$-form of ghost number one, with the understanding that two solutions a and a' of Eq. (2.15) that differ by a trivial solution

$$a'=a+sm+dn \tag{2.16}$$

should be identified as they define the same interactions up to field redefinitions. The cochains a, b, etc. that appear depend polynomially on the field variables (including ghosts and antifields) and their derivatives up to some finite order (‘‘local polynomials). Given a non trivial cocycle a of $H^{D,0}(s|d)$, the corresponding first-order interaction vertex $\mathcal{L}_1$ is obtained by setting the ghosts equal to zero.

According to the general rules, the spectrum of fields and antifields is given by the fields $T_{[\alpha\beta]\gamma}$, with ghost number zero and antifield number zero; the ghosts $S_{(\alpha\beta)}$ and $A_{[\alpha\beta]}$ with ghost number one and antifield number zero; the ghosts of ghosts C_α with ghost number two and antifield number zero, which appear because of the reducibility relations; the antifields $T^{*[\alpha\beta]\gamma}$, with ghost number minus one and antifield number one; the antifields $S^{*(\alpha\beta)}$ and $A^{*[\alpha\beta]}$: ghost number minus two and antifield number two; the antifields $C^{*\alpha}$ with ghost number three and antighost number three.

The antifield number is also called ‘‘antighost number.’’ Since the theory at hand is a free theory, the BRST differential takes the simple form

$$s=\delta+\gamma \tag{2.17}$$

The decomposition of s into δ plus γ is dictated by the antifield number : δ decreases the antifield number by one unit, while γ leaves it unchanged. Combining this property with $s^2=0$, one concludes that

$$\delta^2=0,\ \delta\gamma+\gamma\delta=0,\ \gamma^2=0. \tag{2.18}$$

A grading is associated to each of these differentials: γ increases by one unit the ‘‘pure ghost number’’ denoted *puregh* while δ increases the ‘‘antighost number’’ *antigh* by one unit. The ghost number *gh* is defined by

$$gh=puregh-antigh. \tag{2.19}$$

The action of the differentials γ and δ on all the fields of the formalism is displayed in the following array which indicates also the pureghost number, antighost number and Grassmannian parity of the various fields:

Z	$\gamma(Z)$	$\delta(Z)$	$puregh(Z)$	$antigh(Z)$	parity
$T_{[\alpha\beta]\gamma}$	$2(\partial_{[\alpha}S_{\beta]\gamma}+\partial_{[\alpha}A_{\beta]\gamma}-\partial_\gamma A_{\alpha\beta})$	0	0	0	0
$S_{(\alpha\beta)}$	$6\partial_{(\alpha}C_{\beta)}$	0	1	0	1
$A_{[\alpha\beta]}$	$2\partial_{[\alpha}C_{\beta]}$	0	1	0	1
C_α	0	0	2	0	0
$T^{*[\alpha\beta]\gamma}$	0	$3[E^{[\alpha\beta]\gamma}+\eta^{\gamma[\alpha}E^{\beta]}$	0	1	1
$S^{*\alpha\beta}$	0	$-\partial_\gamma(T^{*[\gamma\alpha]\beta}+T^{*[\gamma\beta]\alpha}$	0	2	0
$A^{*\alpha\beta}$	0	$-3\partial_\gamma(T^{*[\gamma\alpha]\beta}-T^{*[\gamma\beta]\alpha})$	0	2	0
$C^{*\alpha}$	0	$6\partial_\mu S^{*\mu\alpha}+2\partial_\mu A^{*\mu\alpha}$	0	3	1

It is convenient to perform a change of variables in the *antigh*=2 sector in order for the Koszul-Tate differential to take a simpler expression when applied on all the antifields of *antigh*$\geqslant 2$. We define

$$C^{*\alpha\beta}=3S^{*\alpha\beta}+A^{*\alpha\beta}. \tag{2.20}$$

It leads to the following simple expressions

$$\delta C^{*\alpha\beta}=-6\partial_\gamma T^{*[\gamma\alpha]\beta}, \tag{2.21}$$

$$\delta C^{*\mu}=2\partial_\nu C^{*\nu\mu}. \tag{2.22}$$

C. Strategy

To compute $H^{D,0}(s|d)$, one proceeds as in [32,33]: one expands the cocycle condition $sa+db=0$ according to the antifield number. To analyze this resulting equations, one needs to know the cohomological groups $H(\gamma)$, $H(\gamma|d)$ in strictly positive antighost number, $H(\delta|d)$ and $H^{inv}(\delta|d)$.

III. STANDARD RESULTS

Of the cohomologies just listed, some are already known while some can be computed straightforwardly.

A. Cohomology of γ

The cohomology of γ (space of solutions of $\gamma a=0$ modulo trivial coboundaries of the form γb) has been explicitly worked out in [31] and turns out to be generated by the following variables: the antifields and all their derivatives, denoted by $[\Phi^*]$, the undifferentiated ghosts of ghosts C_μ, the following "field strength" components of the ghosts $A_{[\alpha\beta]}$: $H^A_{[\alpha\beta\gamma]}\equiv\partial_{[\alpha}A_{\beta\gamma]}$ (but not their derivatives, which are exact), the *T*-field strength components defined in Eq. (2.5) and all their derivatives denoted by $[E_{[\alpha\beta\gamma][\delta\varepsilon]}]$.

Therefore, the cohomology of γ is isomorphic to the algebra

$$\{f([E_{[\alpha\beta\gamma][\delta\varepsilon]}],[\Phi^*],C_\mu,H^A_{[\alpha\beta\gamma]})\} \tag{3.1}$$

of functions of the generators. The ghost-independent polynomials $\alpha([E_{[\alpha\beta\gamma][\delta\varepsilon]}],[\Phi^*])$ are called "invariant polynomials."

Comments

Let $\{\omega^I(C_\mu,H^A_{[\alpha\beta\gamma]})\}$ be a basis of the algebra of polynomials in the variables C_μ and $H^A_{[\alpha\beta\gamma]}$. Any element of $H(\gamma)$ can be decomposed in this basis, hence for any γ-cocycle α

$$\gamma\alpha=0\Leftrightarrow\alpha=\alpha_I([E_{[\alpha\beta\gamma][\delta\varepsilon]}],[\Phi^*])\omega^I(C_\mu,H^A_{[\alpha\beta\gamma]})+\gamma\beta \tag{3.2}$$

where the α_I are invariant polynomials. Furthermore, $\alpha_I\omega^I$ is γ exact if and only if all the coefficients α_I are zero:

$$\alpha_I\omega^I=\gamma\beta,\Leftrightarrow\alpha_I=0,\quad\text{for all } I. \tag{3.3}$$

Another useful property of the ω^I is that their derivatives are γ exact and thus, in particular,

$$d\omega^I=\gamma\hat{\omega}^I \tag{3.4}$$

for some $\hat{\omega}^I$.

B. General properties of $H(\gamma|d)$

The cohomological space $H(\gamma|d)$ is the space of equivalence classes of forms a such that $\gamma a+db=0$, identified by the relation $a\sim a'\Leftrightarrow a'=a+\gamma c+df$. We shall need properties of $H(\gamma|d)$ in strictly positive antighost (= antifield) number. To that end, we first recall the following theorem on invariant polynomials (pure ghost number $=0$):

Theorem III.1. In form degree less than n and in antifield number strictly greater than 0, the cohomology of d is trivial in the space of invariant polynomials.

The argument runs as in [32,33], to which we refer for the details.

Theorem III.1, which deals with d-closed invariant polynomials that involve no ghosts (one considers only invariant polynomials), has the following useful consequence on general γ-mod-d-cocycles with *antigh*>0.

Consequence of Theorem III.1

If a has strictly positive antifield number (and involves possibly the ghosts), the equation $\gamma a+db=0$ is equivalent, up to trivial redefinitions, to $\gamma a=0$. That is,

$$\left.\begin{array}{r}\gamma a+db=0,\\ antigh(a)>0\end{array}\right\}\Leftrightarrow\left\{\begin{array}{l}\gamma a'=0,\\ a'=a+dc.\end{array}\right. \tag{3.5}$$

Thus, in antighost number >0, one can always choose representatives of $H(\gamma|d)$ that are strictly annihilated by γ. Again, see [32,33].

C. Characteristic cohomology $H(\delta|d)$

We now turn to the groups $H(\delta|d)$, i.e., to the solutions of the condition $\delta a+db=0$ modulo trivial solutions of the form $\delta m+dn$. As shown in [32], these groups are isomorphic to the groups $H(d|\delta)$ of the characteristic cohomology, describing ordinary and higher order conservation laws (i.e., n-forms built out of the fields and their derivatives that are closed on-shell). Without loss of generality, one can assume that the solution a of $\delta a+db=0$ does not involve the ghosts, since any solution that vanishes when the ghosts are set equal to zero is trivial [42]. By applications of the results and methods of [32], one can establish the following theorems (in $H^D_q(\delta|d)$, D is the form degree and q the antighost (=antifield) number):

Theorem III.1. The cohomology groups $H^D_q(\delta|d)$ vanish in antifield number q strictly greater than 3,

$$H^D_q(\delta|d)=0\quad\text{for}\quad q>3. \tag{3.6}$$

Theorem III.2. A complete set of representatives of $H^D_3(\delta|d)$ is given by the antifields $C^{*\mu}$ conjugate to the ghost of ghosts, i.e.,

$$\delta a_3^D + d a_2^{D-1} = 0 \Rightarrow a_3^D = \lambda_\mu C^{*\mu} dx^0 \wedge dx^1 \wedge \ldots \wedge dx^{D-1} \quad (3.7)$$

where the λ_μ are constants and modulo trivial terms.

Theorem III.3. In antighost number 2, the general solution of

$$\delta a_2^D + d a_1^{D-1} = 0 \quad (3.8)$$

reads, modulo trivial terms,

$$a_2^D = C^{*\mu\nu} t_{\mu\nu\rho} x^\rho dx^0 \wedge dx^1 \wedge \ldots \wedge dx^{D-1} \quad (3.9)$$

where $t_{\mu\nu\rho}$ is an arbitrary, completely antisymmetric, constant tensor, $t_{\mu\nu\rho} = t_{[\mu\nu\rho]}$. If one considers cochains a that have no explicit x dependence (as it is necessary for constructing Poincaré-invariant Lagrangians), one thus find that the cohomological group $H_2^D(\delta|d)$ vanishes.

Comment

The cycle $C^{*\mu}$ is associated to the conservation law $d{*G} \approx 0$ for the $(D-3)$-form $*G$ dual to $G^{[\lambda\mu\nu]\rho}$ (the equations of motion read $\partial_\lambda G^{\lambda\mu\nu\rho} \approx 0$). The cycle $C^{*\mu\nu} t_{\mu\nu\rho} x^\rho$ is associated to the conservation law $\partial_\lambda I^{\lambda\sigma\mu\nu\rho} \approx 0$ where $I^{\lambda\sigma\mu\nu\rho}$ is equal to the tensor $G^{\lambda\sigma\mu\nu} x^\rho + 3\eta^{\lambda\mu} T^{\nu\rho\sigma} - 3\eta^{\lambda\mu}\eta^{\sigma\nu} T_\alpha^{\alpha\rho}$ completely antisymmetrized in the three indices μ, ν, ρ and in the pair λ, σ. The above theorems provide a complete description of $H_k^D(\delta|n)$ for $k>1$ and show that these groups are finite-dimensional. In contrast, the group $H_1^D(\delta|d)$, which is related to ordinary conserved currents, is infinite-dimensional since the theory is free. It is not computed here.

D. Invariant characteristic cohomology: $H^{inv}(\delta|d)$

The crucial result that underlies all consistent interactions deals not with the general cohomology of δ modulo d but rather with the *invariant* cohomology of δ modulo d. The group $H^{inv}(\delta|d)$ is important because it controls the obstructions to removing the antifields from a s-cocycle modulo d, as we shall see explicitly below.

The central theorem that gives $H^{inv}(\delta|d)$ in antighost number $\geqslant 2$ is

Theorem III.1. Assume that the invariant polynomial a_k^p ($p =$ form-degree, $k =$ antifield number) is δ-trivial modulo d,

$$a_k^p = \delta \mu_{k+1}^p + d\mu_k^{p-1} \quad (k \geqslant 2). \quad (3.10)$$

Then, one can always choose μ_{k+1}^p and μ_k^{p-1} to be invariant.

Hence, we have $H_k^{n,inv}(\delta|d) = 0$ for $k>3$ while $H_3^{n,inv}(\delta|d)$ is given by theorem III.2 and $H_2^{n,inv}(\delta|d)$ vanishes (in the space of translation-invariant cochains), by theorem III.3.

The proof of this theorem proceeds exactly as the proofs of similar theorems established for vector fields [33], p-forms [34] or gravity [8]. We shall therefore skip it.

IV. RIGIDITY OF THE GAUGE ALGEBRA

We can now proceed with the derivation of the cohomology of s modulo d in form degree D and in ghost number zero. A cocycle of $H^{0,D}(s|d)$ must obey

$$sa + db = 0 \quad (4.1)$$

(besides being of form degree D and of ghost number 0). To analyze Eq. (4.1), we expand a and b according to the antifield number, $a = a_0 + a_1 + \cdots + a_k$, $b = b_0 + b_1 + \cdots + b_k$, where, the expansion stops at some finite antifield number [33]. We recall [26] (i) that the antifield-independent piece a_0 is the deformation of the Lagrangian; (ii) that a_1, which is linear in the antifields $T^{*[\alpha\beta]\gamma}$ contains the information about the deformation of the gauge transformations of the fields, given by the coefficients of $T^{*[\alpha\beta]\gamma}$; (iii) that a_2 contains the information about the deformation of the gauge algebra (the term $C_A^* f^A_{BC} C^B C^C$ with $C_A^* \equiv S^{*\alpha\beta}, A^{*\alpha\beta}$ and $C^A \equiv S_{\alpha\beta}, A_{\alpha\beta}$ gives the deformation of the structure functions appearing in the commutator of two gauge transformations, while the term T^*T^*CC gives the on-shell terms) and about the deformation of the reducibility functions (terms containing the ghosts of ghosts and the antifields conjugate to the ghosts); and (iv) that the a_k ($k>3$) give the information about the deformation of the higher order structure functions, which appear only when the algebra does not close off-shell. Thus, if one can show that the most general solution a of Eq. (4.1) stops at a_1, the gauge algebra is rigid: it does not get deformed to first order.

Writing s as the sum of γ and δ, the equation $sa + db = 0$ is equivalent to the system of equations $\delta a_i + \gamma a_{i-1} + db_{i-1} = 0$ for $i = 1, \ldots, k$, and $\gamma a_k + db_k = 0$.

A. Terms a_k, $k>3$

To begin with, let us assume $k>3$. Then, using the consequence of theorem III.1 , one may redefine a_k and b_k so that $b_k = 0$, i.e., $\gamma a_k = 0$. Then, $a_k = \alpha_J \omega^J$ (up to trivial terms), where the α_J are invariant polynomials and where the $\{\omega^J\}$ form a basis of the algebra of polynomials in the variables C_μ and $H^A_{[\alpha\beta\gamma]}$. Acting with γ on the second to last equation and using $\gamma^2 = 0$, $\gamma a_k = 0$, we get $d\gamma b_{k-1} = 0$ i.e., $\gamma b_{k-1} + dm_{k-1} = 0$; and then, thanks again to the consequence of theorem III.1 , b_{k-1} can also be assumed to be invariant, $b_{k-1} = \beta_J \omega^J$. Substituting these expressions for a_k and b_{k-1} in the second to last equation, we get

$$\delta[\alpha_J \omega^J] + d[\beta_J \omega^J] = \gamma(\cdots). \quad (4.2)$$

This equation implies

$$[\delta\alpha_J + d\beta_J]\omega^J = \gamma(\cdots) \quad (4.3)$$

because the exterior derivative of a ω^J is equivalent to zero in $H(\gamma)$. Then, as dicussed at the end of Sec. III A, this leads to

$$\delta\alpha_J + d\beta_J = 0, \quad \forall\ J. \quad (4.4)$$

If the antifield number of α_J is strictly greater than 3, the solution is trivial, thanks to our results on the cohomology of δ modulo d : $\alpha_J = \delta\mu_J + d\nu_J$. Furthermore, theorem III.1 tells us that μ_J and ν_J can be chosen invariants. This is the crucial place where we need theorem III.1 . Thus $a_k = (\delta\mu_J + d\nu_J)\omega^J = s(\mu_J\omega^J) + d(\nu_J\omega^J) \pm \nu_J d\omega^J$. The last term $\nu_J d\omega^J$ is equal to $\nu_J s\hat{\omega}^J$ and differs from the s-exact term $s(\pm\nu_J\hat{\omega}^J)$ by the term $\pm\delta\nu_J\hat{\omega}^J$, which is of lowest antifield number. Trivial redefinitions enable one to set a_k to zero. Once this is done, β_J must satisfy $d\beta_J = 0$ and is then exact in the space of invariant polynomials, $\beta_J = d\rho_I$, and b_{k-1} can be removed by appropriate trivial redefinitions. One can next repeat the argument for antifield number $k-1$, etc, until one reaches antifield number 3. This case deserves more attention, but what we can stress already now is that *we can assume that the expansion of a in $sa + db = 0$ stops at antifield number 3 and takes the form $a = a_0 + a_1 + a_2 + a_3$ with $b = b_0 + b_1 + b_2$*. Note that this result is independent of any condition on the number of derivatives or of Lorentz invariance. These requirements have not been used so far. The crucial ingredient of the proof is that the cohomological groups $H_k^{inv}(\delta|d)$, which control the obstructions to remove a_k from a, vanish for $k > 3$.

B. Computation of a_3

We have now the following descent:

$$\delta a_1 + \gamma a_0 + db_0 = 0, \tag{4.5}$$

$$\delta a_2 + \gamma a_1 + db_1 = 0, \tag{4.6}$$

$$\delta a_3 + \gamma a_2 + db_2 = 0, \tag{4.7}$$

$$\gamma a_3 = 0. \tag{4.8}$$

We write $a_3 = \alpha_I\omega^I$ and $b_2 = \beta_I\omega^I$. Proceeding as before we find that a necessary (but not sufficient) condition for a_3 to be a non-trivial solution of Eq. (4.7), so that a_2 exists, is that α_I be a non-trivial element of $H_3^n(\delta|d)$. The theorem III.2 imposes then $\alpha_I \sim C^{*\mu}$. We then have to complete this α_I with an ω^I of ghost number 3 in order to build a candidate $\alpha_I\omega^I$ for a_3. There are *a priori* a lot of possibilities to achieve this, but if one demands Lorentz invariance, only two possibilities emerge

$$a_3 = C^{*\mu} H_{\mu\alpha\beta} H^{\alpha\nu\rho} H^{\beta}{}_{\nu\rho}, \tag{4.9}$$

$$a_3 = C^{*\mu} \varepsilon_{\mu\nu\rho\lambda\sigma} H^{\nu\rho\lambda} C^{\sigma}, \tag{4.10}$$

where we recall that $H_{\mu\alpha\beta} \equiv \partial_{[\mu}A_{\alpha\beta]} \in H(\gamma)$ at ghost number one, and $C^\sigma \in H(\gamma)$ at ghost number two. Since $C^{*\mu}$ has antighost number three [i.e. ghost number $(-)3$], we indeed have two ghost-number-zero a_3 candidates. The first is quartic in the fields and, if consistent, would lead to a quartic interaction vertex. The second is cubic and, if consistent, would lift to an a_0 which breaks PT invariance.

However, neither of these candidates can be lifted all the way to a_0. Both get obstructed at antighost number one: a_2 exists, but there is no a_1 that solves Eq. (4.6) (given a_3 and the corresponding a_2). The computation is direct but not very illuminating and so will not be reproduced here.

C. Computation of a_2

Continuing with our analysis, we set $a_3 = 0$ and get the system

$$\delta a_1 + \gamma a_0 + db_0 = 0, \tag{4.11}$$

$$\delta a_2 + \gamma a_1 + db_1 = 0, \tag{4.12}$$

$$\gamma a_2 = 0. \tag{4.13}$$

Now a_2 has to be found in $H_2^n(\delta|d)$, but this latter group vanishes (in the space of translation-invariant deformations), as shown in theorem III.3. We can thus conclude that there is no possibility of deforming the free theory to obtain an interacting theory whose gauge algebra is non-Abelian. To obtain this no-go result we just asked for locality, Lorentz invariance and the assumption that the deformed theory reduces smoothly to the free one as the deformation parameter goes to zero.

V. NO-GO THEOREM

With $a_i = 0$ for $i > 1$, the cocycle condition (4.1) reduces to

$$\delta a_1 + \gamma a_0 + db_0 = 0, \tag{5.1}$$

$$\gamma a_1 = 0. \tag{5.2}$$

The last equation forces a_1 to take the schematic form $a_1 = T^* Hp(E, \partial E, \dots)$ where the constant term in p is zero because $T^{*[\alpha\beta]\gamma}H_{\alpha\beta\gamma}$ (the only E-independent possibility allowed by Lorentz invariance) is identically zero due to opposite symmetries for $T^{*[\alpha\beta]\gamma}$ and $H_{\alpha\beta\gamma}$. But all these candidates a_1 involve at least four derivatives (two in E, one in H, and one in T^*—which counts for one because δT^* contains two derivatives, see e.g. [33]), so we reject this possibility on the assumption that the interaction terms in the Lagrangian should not have more than two derivatives. Thus, $a_1 = 0$ and the deformations not only do not modify the gauge algebra, but actually also leave unchanged the gauge transformations of the field $T_{[\alpha\beta]\gamma}$.

We are then reduced to look for a_0 solutions of $\gamma a_0 + db_0 = 0$, i.e., for deformations of the Lagrangian which must be gauge invariant up to a total derivative. Because these deformations are gauge invariant up to a total derivative, their Euler-Lagrange derivatives are strictly gauge invariant. These Euler-Lagrange derivatives contains two derivatives of the fields and satisfies Bianchi identities of the type (2.8) (because of the gauge invariance of $\int a_0$). It is easy to see that the only gauge-invariant object satisfying these conditions are the Euler-Lagrange derivatives of the original Lagrangian itself, so we conclude that $a_0 \sim \mathcal{L}$: the deformation only changes the coefficient of the free Lagrangian and is not essential. In fact, allowing for an a_0 containing three derivatives would not change the conclusions. Indeed, starting from the first derivative of the curvature, ∂E, there is no way to contract the indices in order to form a candidate $X^{[\alpha\beta]\gamma}$ with the symmetries of the field $T^{[\alpha\beta]\gamma}$. Hence, acceptable a_0 (other than the original Lagrangian) should involve at least four derivatives. This completes the proof of the rigidity of the free theory.

VI. COMMENTS AND CONCLUSIONS

We can summarize our results as follows: under the hypothesis of locality, and Lorentz invariance there is no smooth deformation of the free theory which modifies the gauge algebra. If one further excludes deformations involving four derivatives or more in the Lagrangian, then there is just no smooth deformation of the free theory at all.

Without this extra condition on the derivative order, one can introduce Born-Infeld-like interactions that involve powers of the gauge-invariant curvatures $E_{[\alpha\beta\gamma][\lambda\mu]}$. Such deformations modify neither the gauge algebra nor the gauge transformations.

The same no-go result can easily be extended to a collection of fields $T_{[\alpha\beta]\gamma}$, as in [8], or to a system of one $T_{[\alpha\beta]\gamma}$ and one Pauli-Fierz field $h_{\mu\nu}$.

We have considered here explicitly the dual formulation of gravity in $D=5$ dimensions, with a 3-index tensor $T_{\alpha\beta\gamma}$. The general case of exotic representations $T_{\lambda_1\lambda_2\cdots\lambda_{D-3}\mu}$ with more indices in the first row, which is relevant in D spacetime dimensions, is dealt with similarly, provided one takes into account the additional reducibility identities that appear then. There are more candidates analogous to a_3 (but now in higher antighost number) but we have checked that all these candidates are eliminated if one restricts the derivative order to two (in fact, the higher derivative terms are in any case probably obstructed, but we have not verified this explicitly, the derivative argument being sufficient to rule them out). In the end (going though all a_i's), one finds again that there is no deformation with no more than two derivatives.

Finally, we note that the same techniques can be used to analyze consistent deformations of more general exotic gauge fields. We plan to return to this question in the future.

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