Domain walls as dark energy

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The possibility that the energy density of the Universe is dominated by a network of low-tension domain walls provides an alternative to the commonly discussed cosmological constant and scalar-field quintessence models of dark energy. We quantify the lower bound on the number density of the domain walls that follows from the observed near isotropy of the cosmic microwave background radiation. This bound can be satisfied by a strongly frustrated domain wall network. No fine tuning of the parameters of the underlying field theory model is required. We briefly outline the observational consequences of this model.

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I. INTRODUCTION

In the last few years there has been a growing amount of evidence that an unknown negative pressure component (''dark energy'') accounts for $70\pm10\%$ of the energy density of the Universe. Arguably the most direct evidence comes from the luminosity-redshift relation of the type Ia supernovae [1,2]. Additional evidence is provided by a combination of data [3]. Both the analysis of the formation of the large scale structure [4] and the studies of the baryon fraction in galaxy clusters [5], combined with the big bang nucleosynthesis calculations [6], suggest the matter content significantly below the critical density. On the other hand, the cosmic microwave background radiation (CMBR) power spectrum suggests a flat universe [7]. Therefore the difference must be made up by an additional nonclustering energy density component. The relationship between the pressure p and the energy density ρ of the dark energy component is usually parametrized by $p=w\rho$, where the equation of state w could be time dependent. Theoretical considerations prefer $w\geqslant-1$. A good fit to the supernova data can be obtained assuming a constant or slowly changing equation of state satisfying $w\lesssim-0.5$ [3] or $w\lesssim-0.6$ [8], depending on the details of the analysis.

The precise nature of the dark energy component has been a subject of intense theoretical speculation. The most popular dark energy candidates include the cosmological constant, or vacuum energy, and the so-called quintessence, a nearly spatially homogeneous but time-dependent scalar field (see, for example, Refs. [9–11]). Another very interesting possibility is the ''solid'' dark energy, originally suggested by Bucher and Spergel [12,13]. In this case, one postulates that the dark energy component possesses resistance to pure shear deformations, guaranteeing stability with respect to small perturbations. (The stability condition is nontrivial for substances with negative pressure. It is violated, for example, by a perfect fluid.)

A possible microphysical origin for solid dark energy is a dense network of low-tension domain walls [13,14]. This is attractive for several reasons. First, domain walls are ubiquitous in field theory, inevitably appearing in models with spontaneously broken discrete symmetries [15]. Second, domain walls, and the solid dark energy in general, have been shown to be compatible with the observations of large scale structure [13,16]. Finally, a static wall network has an equation of state $w=-2/3$ [17], consistent with all observational data. Despite these appealing features, the idea that a domain wall network could play the role of dark energy has not received much attention in the literature. In this paper we investigate certain aspects of this scenario. We show that the requirement of the isotropy of the CMBR yields a strong lower bound on the number density of the domain walls. We then consider the implications of this bound for the field-theoretic models of the domain walls. We argue that the domain wall network has to be strongly frustrated to satisfy the CMBR constraint. If this is the case, the equation of state of dark energy is expected to be close to $-2/3$.

II. CMBR CONSTRAINTS

It is well known that the Universe is spatially homogeneous and isotropic on large scales. The best evidence for this is provided by the observed near-isotropy of the CMBR. If a network of domain walls is present, the CMBR photons would acquire different gravitational redshifts (or blueshifts) depending on the direction of their arrival [18]. Let us estimate this effect.

Until $z\sim O(1)$, the contribution of the walls to the energy density was subdominant and hence they did not significantly affect the CMBR anisotropy. Most of the anisotropy due to domain walls would be built up during the relatively recent epoch, $z\lesssim1$. This allows us to simplify the problem. Consider an observer at rest in the comoving frame. (After subtracting the effects due to the peculiar motion of the Earth, the CMBR anisotropy measured in actual experiments corresponds to what would be measured by such an ob-

server.) Let R denote the comoving distance corresponding to $z=z_0<1$, and consider the sphere $\mathcal{S}$ of radius R centered on the observer. We assume that the CMBR photons arriving at the surface of this sphere are isotropic. The anisotropy induced when the photons are travelling from the surface of $\mathcal{S}$ to the observer can be simply estimated, up to corrections suppressed by powers of z_0, using Newtonian theory. Indeed, the results of the general theory of relativity on subhorizon scales with receding velocities $v \ll c$ must be equally well described using Newtonian gravity. Recall that in the Newtonian picture, the observer is taken to be at "the center of the Universe." Homogeneous matter around him creates a gravitational field which grows with distance as

$$\mathbf{g}(r) = -G\frac{4\pi}{3}(\rho+3p)r\hat{\mathbf{r}}. \tag{1}$$

Here we are following the general principle that the source of Newtonian gravity is the combination $(\rho+3p)$ and not the density itself (see, for example, [19]). The redshifts of distant objects in this picture result from the Doppler shift due to the expansion of the Universe, and the effect of the gravitational field in Eq. (1). In the Appendix we discuss the calculation of these effects in a homogeneous Universe, and show that the Newtonian calculation is accurate up to and including the second order terms in the relative velocity of the emitter and the observer.

If domain walls are present, an additional, anisotropic gravitational field will be induced on top of Eq. (1), leading to anisotropies in the CMBR temperature. Denoting the Newtonian potential of the wall network by $\Phi(\mathbf{r})$, we can write the observed temperature difference between points **1** and **2** on the surface of the sphere $\mathcal{S}$ as

$$\frac{T(\hat{\mathbf{r}}_\mathbf{1})-T(\hat{\mathbf{r}}_\mathbf{2})}{\langle T\rangle} = \Phi(\mathbf{r}_\mathbf{1})-\Phi(\mathbf{r}_\mathbf{2}), \tag{2}$$

where $\mathbf{r}_{\mathbf{1,2}}$ are the vectors pointing from the center of the sphere to points **1** and **2** and $\hat{\mathbf{r}}_{\mathbf{1,2}}=\mathbf{r}_{\mathbf{1,2}}/R$.

The Newtonian potential at point $\mathbf{x}$ from a single planar wall is given by [20]

$$\phi(\mathbf{x};\hat{\mathbf{n}},\mathbf{x_0}) = -2\pi G_N\sigma\hat{\mathbf{n}}\cdot(\mathbf{x}-\mathbf{x_0}), \tag{3}$$

where σ is the wall tension, $\mathbf{x_0}$ is a point on the wall and $\hat{\mathbf{n}}$ is a unit vector normal to the wall. The force on a matter particle from the wall is repulsive. The total potential due to an arbitrary network of planar walls is the sum $\Phi(\mathbf{x}) = \Sigma_i \phi(\mathbf{x};\ \hat{\mathbf{n}}_i, \mathbf{x}_i)$. Because the potential (3) grows linearly at large distances, the walls outside the sphere cannot be neglected in the calculation of Φ. On the other hand, it is clear that the contribution of the walls lying outside the event horizon has to vanish. To model this effect, we introduce a regulator which is spherically symmetric with respect to the observer and cuts off the effects of the walls beyond certain radius $\tilde{R}>R$. (Notice that such a regulator is also necessary in the usual Newtonian treatment of a matter dominated FRW cosmology.) The dependence on the regulator will disappear in the final results.

To see how the CMBR anisotropy generated by the walls depends on their distribution in space, let us consider a "one-dimensional" toy model in which all the walls are planar and perpendicular to the x axis. The anisotropy, defined in this model as the temperature difference for the photons arriving from the $\pm x$ directions and measured by an observer at $x=0$, is

$$\Delta\Phi = \Phi(R)-\Phi(-R). \tag{4}$$

Let N denote the number of walls between $x=0$ and $x=R$. Then, we obtain

$$\Phi(R) = FR + 4\pi N G_N\sigma\left(R-\frac{1}{N}\sum_{i=1}^{N} x_i\right), \tag{5}$$

where

$$F = 2\pi G_N\left(-\sum_{i=1}^{\infty}\sigma f(x_i)+\sum_{i=1}^{\infty}\sigma f(x_{-i})\right) \tag{6}$$

is the gravitational force exerted by the walls on a unit mass placed at the origin, and x_i denote the positions of the walls to the right $(i>0)$ or to the left $(i<0)$ of the observer. The regulator function f in Eq. (6) is even, $f(x)=f(-x)$, and smoothly cuts off the force due to the walls beyond the observer's event horizon: $f(x)=1$ for $|x|<\tilde{R}$, $f(x)\to 0$ for $|x|>\tilde{R}$. Using Eq. (5) and the analogous expression for $\Phi(-R)$, we can obtain simple analytic estimates of the observed anisotropy. In our estimates, we will always assume $N\gg 1$; clearly, this condition has to be satisfied for the Universe to be even approximately homogeneous on large scales. In fact, the CMBR bound derived below will *require* N to be large, so our analysis is self-consistent.

If the walls form a perfectly regular lattice structure with period $L\ll R$, the leading dependence on R in Eq. (4) cancels out and one finds [18]

$$\Delta\Phi \sim 2\pi G_N\sigma L. \tag{7}$$

This is exactly what one expects on physical grounds. Indeed, on scales much larger than L, a regular wall structure behaves like uniform dark energy, with its Newtonian potential approaching a symmetric parabola; all deviations from the parabola occur because of the granularity of the structure on scales $\sim L$. However, Eq. (7) only holds if the wall structure is perfectly regular on all scales up to the present size of the horizon.[1] Both causality considerations and the inherent randomness of the system make it extremely hard to believe that such a regular structure can be realized physically.

[1]Although the final result is regulator independent, the cancellation of the R-dependent terms requires a conspiracy between the "nearby" walls at $x<R$ and the "distant" walls at $x\sim\tilde{R}$.

In realistic models, the main contribution to the anisotropy comes not from the granularity of the wall network, but from long-wavelength fluctuations of the effective average energy density of the walls due to deviations from perfect regularity. As an illustration, consider an example in which the walls are displaced from their lattice positions by random amounts $|\delta x_i| \sim L/2$. In this case, we find

$$\Delta\Phi \sim 2\pi G_N \sigma (LR)^{1/2}, \tag{8}$$

a much stronger anisotropy than indicated by Eq. (7). Even this model, however, is hardly realistic, since it requires that the number of walls to the left and to the right of the observer be identical. Removing even a single wall on one side of the observer leads, on average, to an anisotropy

$$\Delta\Phi \sim 2\pi G_N \sigma R \sim 2\pi G_N \sigma N L. \tag{9}$$

In the absence of long-distance correlations between wall positions, the average difference in the number of walls to the left and to the right of the observer is of order $\sqrt{N}$, leading to

$$\Delta\Phi \sim 2\pi G_N \sigma R^{3/2} L^{-1/2} \sim 2\pi G_N \sigma N^{3/2} L. \tag{10}$$

The preceding discussion can be summarized by writing the anisotropy in the form

$$\Delta\Phi \sim 2\pi G_N \sigma N^{\nu} L, \tag{11}$$

where the value of the exponent ν depends on the details of the wall configuration. What values of ν correspond to realistic domain wall networks? To answer this question, let us consider a field theory with N_v distinct vacua. Domain walls form during a cosmological phase transition due to the fact that the field may choose different vacua in causally disconnected regions of space. Immediately after the phase transition, each causally connected region contains either no walls, with probability $p \sim 1/N_v$, or a single wall with probability $1-p$. First, consider the case when the wall network does not evolve (apart from conformal Hubble stretching) after the transition. In this case, the CMBR anisotropy induced by the walls today can be estimated as

$$\Delta\Phi \sim 2\pi G_N \sigma N^{3/2} L \sqrt{p(1-p)}. \tag{12}$$

Thus, for moderate p, a non-evolving domain wall network induces the anisotropy of the size indicated by Eq. (10). The same estimate applies if the network does evolve, but the evolution does not make the structure more regular. On the other hand, if more regular wall configurations are favored dynamically and emerge as the network is evolving, the induced anisotropy could be weaker. While we are not aware of any numerical simulations that conclusively demonstrate such behavior in a given model, this remains a logical possibility. A reasonable lower bound on the induced anisotropy is provided by Eq. (9), since even a single ''defect'' in the regular wall structure inside the present Hubble volume would induce an anisotropy of that size. Below, we will derive the bounds on the parameters of the model using Eq. (12) (the ''non-evolving network'' case, $\nu = 3/2$) and Eq. (9) (the ''regular network'' case, $\nu = 1$). The constraints in any realistic model are expected to lie between these two limiting cases.

While we have used a one-dimensional toy model to derive Eqs. (7)–(12), it is possible to show that the same estimates hold for three-dimensional domain wall networks with the corresponding regularity properties. Thus, the temperature anisotropy created by domain walls inside a sphere of radius R for the non-evolving network case can be written as

$$\frac{\delta T}{\langle T \rangle} = 2\pi a G_N \sigma N^{3/2} L \sqrt{p(1-p)}, \tag{13}$$

where a is a numerical coefficient of order unity that depends on the detailed properties of the network, L is the average separation between the walls, and $N = R/L$ is the average number of walls crossed by a CMBR photon traveling from the surface of the sphere to the observer. For a regular network, we obtain

$$\frac{\delta T}{\langle T \rangle} = 2\pi a G_N \sigma N L. \tag{14}$$

This anisotropy is generated by the fluctuations of the wall number density at large (Hubble) distance scales. Predicting the power spectrum of the wall-induced anisotropy, as well as possible deviations from Gaussianity, would require detailed knowledge of the network geometry.

To be a viable dark energy candidate, the network of domain walls must have the average energy density $\rho = 3\sigma/L = \Omega_w \rho_{\rm crit}$, where $\rho_{\rm crit} = 3H_0^2/8\pi G_N$ is the current critical density, and $0.6 \lesssim \Omega_w \lesssim 0.8$. (We assume that the walls are the only form of dark energy present: for example, the cosmological constant vanishes.) To avoid conflict with the precise measurements of the CMBR anisotropy [21], the wall contribution should be at most at the level $\delta T/T \sim 10^{-6}$. Using Eq. (11), we find

$$N \gtrsim (2\times 10^5 a b^2)^{1/(2-\nu)}, \tag{15}$$

where b is the radius of the sphere in Hubble units, $b = RH_0 \approx z_0$. In the case of a non-evolving network, we obtain

$$N \gtrsim 3\times 10^{10} a^2 b^4 p(1-p), \tag{16}$$

$$\sigma \lesssim \frac{2\times 10^{-16}}{a^2 b^3 p(1-p)}\ \ {\rm GeV}^3, \tag{17}$$

$$L \lesssim \frac{0.15}{a^2 b^3 p(1-p)}\ \ {\rm pc}, \tag{18}$$

while in the case of a regular network

$$N \gtrsim 2\times 10^5 a b^2, \tag{19}$$

$$\sigma \lesssim \frac{1}{ab} 4\times 10^{-11}\ \ {\rm GeV}^3, \tag{20}$$

$$L \lesssim \frac{1}{ab} 3 \times 10^4 \text{ pc.} \quad (21)$$

Since the wall energy density becomes comparable to that of matter at $z \simeq 0.5$, we will use the value $b = 0.5$ for our numerical estimates. The other two parameters entering the above bounds, a and p, depend on many factors, such as the geometry of the wall network, the properties of the underlying field theory model, etc. We will keep the dependence on these parameters explicit throughout the discussion.

Our calculation has only included the anisotropy due to the walls at low redshifts, where Newtonian approximation is applicable. The effects of walls at higher redshifts could be included using the full general relativistic solution for the metric perturbation created by the walls in the Friedmann-Robertson-Walker (FRW) universe [22]. In this formalism, the redshift is computed using the well known Sachs-Wolfe formula [23]

$$\left(\frac{\Delta T}{T} \right)_{SW} = \Phi|_e^o - \mathbf{v} \cdot \mathbf{e}|_e^o - \frac{1}{2} \int_e^o h_{\rho\sigma,0} \dot{x}^{(0)\rho} \dot{x}^{(0)\sigma} d\xi, \quad (22)$$

where $h_{\mu\nu}$ is the metric perturbation due to the walls,[2] $\Phi = h_{00}/2$ is the "conformal" Newtonian potential, and $x^{(0)} = (\text{const} + \xi, \xi \mathbf{e})$ is the unperturbed photon path. The subscripts e and o refer to the "emitter" (i.e., a point on the surface of last scattering) and the observer, respectively. Note that in an expanding Universe, even static domain walls induce a time-dependent metric perturbation, and the third term in Eq. (22) does not vanish. It was shown in [22] that in a matter-dominated Universe, the anisotropy due to a wall at a redshift z_w scales as $(1+z_w)^{-3/2}$ (the redshift of a wall is defined as the redshift corresponding to the point on the wall closest to the observer). Moreover, a constant average number density of walls in comoving coordinates implies that $dN/dz_w \propto (1+z_w)^{-3/2}$. Both these effects lead to a severe suppression of the effects of the walls at high redshifts. We have checked that the calculation of Ref. [22] leads to the same order-of-magnitude estimates (9) and (13) for the anisotropy created by the walls as the Newtonian treatment of our paper, justifying the latter.

III. IMPLICATIONS

Let us discuss the implications of the CMBR bounds [Eqs. (16)–(21)] for the field-theoretic models responsible for the walls. Domain walls necessarily appear in theories with spontaneously broken discrete symmetries. If all the dimensionless parameters of the model are of order one, the wall tension is determined by the symmetry breaking scale v. The bounds (17) and (20) then imply

$$v \lesssim \frac{10}{a^{2/3} p^{1/3} (1-p)^{1/3}} \text{ keV} \quad (23)$$

and

$$v \lesssim \frac{4 \times 10^2}{a^{1/3}} \text{ keV} \quad (24)$$

for the non-evolving and the regular network cases, respectively. Note that the dependence on a and p is rather mild. In supersymmetric models, such low energy scales can be generated naturally and be radiatively stable, provided that the supersymmetry (SUSY) breaking is communicated to the field(s) responsible for domain walls only by gravitational interactions. In this case, the natural value of the discrete symmetry breaking scale is given by $v \sim F/M_{Pl}$, where $\sqrt{F}$ is the scale at which SUSY is broken. The constraints (23) and (24) are satisfied if $\sqrt{F} \lesssim 10^4 - 10^5$ TeV, which is allowed phenomenologically if the breaking is mediated to the visible sector (standard model fields and their superpartners) by gauge interactions [24]. In this respect the domain wall models of dark energy are much more attractive than the scalar-field quintessence models which contain a superlight scalar field with a mass of order 10^{-33} eV. In the latter case, it is difficult to understand how such a low energy scale can arise from particle physics and not be destabilized by radiative corrections. Moreover, the superlight scalar will in general mediate a phenomenologically problematic new long-range force [25].

The simplest field theory model in which domain walls arise contains a single real scalar field with a Z_n-invariant potential. (A well-known example is an axion of Peccei-Quinn models [26].) Numerical simulations [27,28] and analytic calculations [29] show that the wall network of this model enters the so-called *scaling regime* shortly after its formation. In this regime, there is only one wall (or at most a few walls) per horizon volume at any given time. This is in gross contradiction with the bound (16) and, more generally, with the whole concept of the isotropic FRW cosmology. The problem could be avoided by introducing a very large number ($\sim 10^8$) of independent scalar fields, or considering models with a very large number of vacua. (In the context of the axion, the latter possibility would correspond to a humongous value of the color anomaly of the Peccei-Quinn symmetry, $N_{PQ} \sim 10^8$.) Needless to say, both possibilities are extremely unattractive.

The scaling evolution of a domain wall network relies on the fact that the network can disentangle as fast as allowed by causality. In models with more complicated vacuum structure, the disentanglement process could be slowed down, since domains of the same vacuum would generally be well separated from each other in space. While no convincing simulations of domain wall networks exhibiting such behavior exist at present, a similar phenomenon has indeed been observed in the simulations of cosmic string networks [30]. Alternatively, the walls could experience strong friction forces, for example due to their interaction with the particles of dark matter [31]. In both cases, one expects substantial deviations from the scaling law. The networks of this kind are referred to as *frustrated*. For frustrated networks, the number of walls in the present horizon volume depends on

[2]Explicitly, $ds^2 = a^2(\eta)(g^{(0)}_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu$, where η is conformal time.

their evolution, and predicting it would require detailed numerical simulations. On the other hand, one can obtain a simple analytical upper bound on this number as a function of the wall formation temperature T_f. Assuming that the walls form during the radiation dominated epoch, and that the fields of the hidden sector responsible for the walls are in thermal equilibrium at approximately the same temperature as the visible sector fields,[3] we obtain

$$N_h \lesssim z^{-1}(T_f) \frac{T_f^2}{M_{Pl} H_0}, \tag{25}$$

where $z(T)$ is the redshift corresponding to the given temperature. (This bound is saturated by a non-evolving network, which in the present context can also be termed *maximally frustrated*.) In a model with no unnatural dimensionless parameters, $T_f \sim v$, and Eq. (23) leads to $N_h \lesssim 10^5 a^{-2/3} p^{-1/3} (1-p)^{-1/3}$ for the non-evolving case. This value does not contradict the CMBR constraint (16) for reasonable values of a and p (for example, $a \sim p \sim 0.1$.) For the case of a regular network, Eqs. (24) and (25) imply that $N_h \lesssim 6 \times 10^6 a^{-1/3}$, although the true bound is probably somewhat lower since some of the walls are likely to be destroyed during the evolution leading to a regular structure. In any case, the bound seems to be compatible with the CMBR constraint (19). Thus, we conclude that in the presence of frustration, it should be possible to build realistic models of domain wall dark energy without any fine tuning (apart from the tuning required to cancel the cosmological constant.) It would be very interesting to find explicit field theory models leading to a frustrated wall network.

The upper bounds on N derived in the previous paragraph and the lower bounds in Eqs. (16) and (19) have to be nearly saturated to be compatible with each other for reasonable values of a and p. This observation leads to two interesting predictions. First, the wall network has to be close to maximal frustration, or, equivalently, be nearly static. Second, the CMBR anisotropy induced by the walls should be close to the current bounds, and therefore improved measurements of the anisotropy have a good chance of detecting the wall contribution if this model is realized.

IV. EQUATION OF STATE

It is well known that the equation of state of a maximally frustrated (static) network of planar domain walls is $w = -2/3$. Above, we have argued that a domain wall network has to be close to the maximally frustrated regime to play the role of dark energy and be consistent with the CMBR anisotropy limits. One may worry, however, that even small deviations from this regime may cause a large change in the value of w, making it an unacceptable dark energy candidate. This turns out not to be the case. Let us demonstrate that the wall equation of state lies in the allowed range $-2/3 \leqslant w \leqslant -1/2$, regardless of the evolution.

Energy conservation for domain walls reads $d(\rho V) = -p dV - \delta E$, where δE is the energy lost by the network in the process of evolution. (The energy can be radiated away in form of gravitons, elementary excitations of the fields responsible for the walls, etc.) Let us tentatively set $\delta E = 0$. The wall equation of state can then be found from the dependence of the energy density of the wall network ρ_w on the scale factor of the Universe a,

$$w = -1 - \frac{1}{3} \frac{d \ln \rho_w}{d \ln a}. \tag{26}$$

The number of walls in a fixed comoving volume scales as $\eta^{-\alpha}$, where η is conformal time, $d\eta = dt/a(t)$. When the wall network is static, $\alpha = 0$. On the other hand, by causality there must be at least one wall per horizon volume at any given time, implying $\alpha \leqslant 1$. The physical density of the wall rest energy therefore scales as $\propto a^{-1} \eta^{-\alpha}$. Neglecting the kinetic energy of the walls, we obtain

$$w = -\frac{2}{3} + \frac{\alpha}{3} \frac{d \ln \eta}{d \ln a}. \tag{27}$$

During the matter-dominated epoch the total pressure of the Universe $p_{\rm tot} = 0$, while after the wall energy takes over $p_{\rm tot}$ becomes negative. It can then be shown that in an expanding Universe $0 < d \ln \eta / d \ln a \leqslant 1/2$, and therefore

$$-\frac{2}{3} \leqslant w \leqslant -\frac{1}{2}. \tag{28}$$

The lower limit is achieved when the domain wall network is static, while the upper limit corresponds to the scaling regime. Notice that even in the scaling regime, while the Universe is matter dominated, the wall energy density varies as $\propto a^{-3/2}$. Hence, regardless of the details of the evolution, the wall network eventually comes to dominate the Universe and its equation of state always lies within the range allowed by the analysis of [3]. Of course, as already mentioned, the network must be nearly maximally frustrated to satisfy the isotropy constraint, so we generally expect $w \simeq -2/3$. In this case, the more restrictive bound obtained in [8] is also satisfied.[4]

In the above derivation, we have neglected the energy loss by the wall network δE and the kinetic energy of the walls $E_{\rm kin}$. Obviously, both approximations are valid for static networks. Moreover, numerical simulations [27,28] show that even in the scaling regime $\delta E \simeq 0$ and $E_{\rm kin} \ll E_{\rm rest}$ throughout the evolution. This justifies our assumptions. [Notice also that the condition $\delta E = 0$ can be relaxed; the effect of finite $\delta E / \delta a$ is to shift w closer to $-2/3$, and Eq. (28) still holds.] On the other hand, the condition $p_{\rm tot} \leqslant 0$ is essential for de-

[3] An upper bound on the temperature of the hidden sector fields is provided by big bang nucleosynthesis. Unless the hidden sector possesses a large number of degrees of freedom, this bound can be satisfied with $T_{\rm hid}$ somewhat lower than, but of the same order as, $T_{\rm vis}$.

[4] Recent studies disfavor $w = -2/3$ at 95% confidence level [34].

riving Eq. (28). For example, in the radiation-dominated Universe the wall equation of state is $w=-1/3$ in the scaling regime.

V. CONCLUSIONS

We have considered the constraints on domain wall models of dark energy from the observed near-isotropy of the CMBR. We have shown that these constraints can be satisfied by a strongly frustrated domain wall network. The scale of spontaneous symmetry breaking responsible for the walls is expected to lie in the 10–100 keV range, and can arise naturally in supersymmetric theories. This makes these models quite attractive from the particle physics point of view.

Domain wall models of dark energy have important observational predictions. The dark energy equation of state is predicted to be close to $-2/3$. This value can be clearly distinguished from the case of the cosmological constant by the SNAP experiment [32]. The CMBR anisotropy induced by the walls is likely to be close to the current bounds, and could be observable in the near future. In general, inhomogeneities of the wall distribution are also expected to induce peculiar velocity flows on large scales. However, this effect is small: a network satisfying the CMBR constraints will produce velocities of order 300 m/s, which is not in conflict with current observations.

The most important outstanding issue in making domain wall dark energy models fully realistic is finding explicit field theories which lead to highly frustrated wall networks. Frustration could arise as a result of the complex dynamics of the system. This possibility can only be addressed by numerical simulations. Such simulations are also necessary to make more detailed predictions of the phenomenological signatures of the walls, such as the power spectrum of the wall-induced CMBR anisotropies. At this time, we are aware of only one numerical study of frustrated networks [33], whose usefulness is severely limited by its insufficient dynamical range. Clearly, further work in this direction is necessary. An interesting alternative possibility is to consider domain wall networks whose evolution is slowed down by their interaction with dark matter. This idea was introduced in [31]. In the specific model of Ref. [31], the average wall velocity is determined by the ratio of wall and dark matter energy densities. In our case, this ratio is of order one, and the mechanism does not work. However, this class of ideas certainly deserves further investigation.

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APPENDIX: NEWTONIAN TREATMENT OF THE PHOTON REDSHIFT

The rigorous way of computing the temperature anisotropy acquired by the photons on the way from the surface of last scattering to the observer is by using the techiques of general relativity. On the other hand, the anisotropy induced when the photons are travelling inside a sphere of a radius corresponding to $z_0 \lesssim 1$ and centered on the observer can be simply estimated using Newtonian theory. (The estimate is expected to be accurate up to corrections suppressed by powers of z_0.) To illustrate how the Newtonian theory can be applied to give the correct redshifts, accurate to the second order in v/c, let us explicitly compare the redshifts of a photon emitted at some distance R computed by the two methods in the simplest case of exactly homogeneous, uniformly expanding Universe.

From the point of view of general relativity, the ratio of the emitted and observed frequencies, ω_1/ω_2 is simply the relative change of the scale factor of the Universe a during the time between emission and absorption:

$$\omega_1/\omega_2 = a_2/a_1. \tag{A1}$$

If the Universe is filled with a component at critial density with the equation of state w, the scale factor has a power law dependence on time, $a \sim t^n$, where $n=2/[3(1+w)]$. The velocity with which the emitter is receding from the observer at the time of emission t_1 is

$$v = R\frac{\dot{a}(t_1)}{a(t_1)} = R\frac{n}{t_1}. \tag{A2}$$

The time it takes for the light to travel to the observer, $\Delta t \equiv t_2 - t_1$, can be found from the equation

$$\int_{t_1}^{t_2} \frac{dt}{a(t)} = \frac{R}{a(t_1)}. \tag{A3}$$

Upon integration, we find

$$R = (t_2^{1-n} t_1^n - t_1)\frac{1}{1-n}, \tag{A4}$$

so that

$$t_2 = \left(\frac{(1-n)R + t_1}{t_1^n}\right)^{1/(1-n)}. \tag{A5}$$

Using this result, we find for the ratio of the scale factors

$$\frac{a(t_2)}{a(t_1)} = \left(\frac{t_2}{t_1}\right)^n = \left((1-n)\frac{R}{t_1} + 1\right)^{n/(1-n)}, \tag{A6}$$

or, in terms of the recess velocity $v = nR/t_1$,

$$\frac{a(t_2)}{a(t_1)} = \left(1 + v\frac{(1-n)}{n}\right)^{n/(1-n)}. \tag{A7}$$

Finally, the ratio of the emitted to absorbed frequency is

$$\frac{\omega(t_2)}{\omega(t_1)}=\left(1-v\frac{(n-1)}{n}\right)^{n/(n-1)}\simeq 1-v+\frac{1}{2n}v^2+\cdots$$

$$\simeq 1-v+\frac{3(1+w)}{4}v^2+\cdots. \tag{A8}$$

Now let us compute the frequency redshift using Newtonian gravity. For that we take the observer to be at ''the center of the Universe,'' with the surrounding matter inducing a gravitational field

$$\mathbf{g}(r)=-G\frac{4\pi}{3}(\rho+3p)r\hat{\mathbf{r}}. \tag{A9}$$

The frequency shift has two components, the kinematic Doppler shift and the blueshift because the photon falls into the potential well. The Doppler shift is simply given by

$$\omega_2^{\text{Doppler}}=\frac{\omega_1(1-v)}{\sqrt{1-v^2}}\simeq\omega_1(1-v+v^2/2+\cdots). \tag{A10}$$

The gravitational blueshift is computed as follows:

$$\frac{(\omega_2-\omega_1)^{\text{Grav}}}{\omega_1}=\Delta\phi=-\int_0^R g(r)dr=G\frac{4\pi}{3}(\rho+3p)\frac{R^2}{2}. \tag{A11}$$

Since in the flat Universe the recession velocity obeys $v(R)^2/2=GM(R)/R=4\pi G\rho R^2/3$, Eq. (A11) could be rewritten as

$$\frac{(\omega_2-\omega_1)^{\text{Grav}}}{\omega_1}=(1+3w)\frac{v^2}{4}. \tag{A12}$$

We observe that, by combining Eqs. (A10) and (A12), we recover precisely the result of Eq. (A8). Thus, in a homogeneous Universe, the Newtonian calculation of the photon redshifts due to the expansion of the Universe is correct up to and including the second order terms in v. In Sec. II we use the Newtonian picture to estimate the anisotropies due to the presense of domain walls. Since the walls only become dominant for $z\sim 1$, this approximation should be sufficiently good for an order-of-magnitude estimate. This intuition is confirmed by studying the full general relativistic calculation of Ref. [22].

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