Could thermal fluctuations seed cosmic structure?

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We examine the possibility that thermal, rather than quantum, fluctuations are responsible for seeding the structure of our universe. We find that while the thermalization condition leads to nearly Gaussian statistics, a Harrisson-Zeldovich spectrum for the primordial fluctuations can only be achieved in very special circumstances. These depend on whether the universe gets hotter or colder in time, while the modes are leaving the horizon. In the latter case we find a no-go theorem which can only be avoided if the fundamental degrees of freedom are not particlelike, such as in string gases near the Hagedorn phase transition. The former case is less forbidding, and we suggest two potentially successful ''warming universe'' scenarios. One makes use of the Phoenix universe, the other of ''phantom'' matter.

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I. INTRODUCTION

A major success of the big bang theory is its ability to account for the detailed structure of the Universe, such as galaxy clustering, or the temperature fluctuations in the cosmic microwave background. Paramount to this picture is the phenomenon of gravitational instability, by means of which primordial small departures from homogeneity can grow into the observed structures. Within the big bang theory the required primordial fluctuations are treated merely as initial conditions. Vacuum *quantum* fluctuations in inflationary scenarios have been shown to lead to the required initial conditions, fitting current large scale structure data. It is, however, important to know just how unavoidable is the conclusion that cosmic structures have a quantum origin. Could these primordial fluctuations have a *thermal* origin instead?

The possibility that we are descendent from thermal fluctuations was advanced by Peebles, in his book *Principles of Physical Cosmology*, pp. 371–373 [1]. Peebles pointed out that if the Universe was in thermal equilibrium on the comoving scale of 10 Mpc when its temperature was $T=10^{11}$ GeV, then the observed value of σ_{10} could be explained. This fascinating remark leaves several questions unanswered. Such a scenario may explain the observed value of σ_{10}, but what about fluctuations on other scales? Furthermore, thermal fluctuations are not strictly Gaussian—does this scenario conflict with observations?

Motivated by these unsolved issues, in this paper we go further and examine under which conditions primordial thermal fluctuations lead to a Harrison-Zeldovich (HZ) [2] spectrum of approximately Gaussian fluctuations. Such a scenario would fit *all* existing data in the same way that the usual inflationary quantum fluctuation scenario does. Indeed, the only potentially distinguishing feature would be a different signature (or absence thereof) of gravitational waves (tensor modes) in the thermal scenario.

A matter not addressed by Peebles is how to establish thermalization on the relevant scales. In pure big bang cosmology all observed fluctuations spanned causally disconnected regions when the universe was at the required high temperatures to induce the appropriate level of fluctuations. In addition, simply *postulating* exact thermalization over all scales, say at Planck time, leads to grossly inappropriate results (thermal fluctuations are white-noise rather than scale invariant). Hence, we should assume that exact thermalization only applies to sub-horizon modes, and that there is a mechanism for pushing sub-horizon thermal modes outside the horizon, where they freeze and become non-thermal. Since different modes freeze at different temperatures, it may be that the final super-horizon spectrum is indeed of HZ type.

We shall consider three types of mechanisms for pushing modes outside the horizon: accelerated expansion, varying speed of light (VSL) and a contracting universe. In the first case we consider inflation models driven by a dominant component of thermal radiation, as in [3], where deformed dispersion relations affect common radiation at high temperatures. This is not to be confused with inflationary models in which there is a finite radiation component during inflation [4,5]. There the dominant component is always the inflaton field (even though the thermal bath drives inflaton fluctuations). The second possibility is a varying speed of light (VSL), either in the form of a space-time field $(c(x^\mu))$ [6–9], or as an energy-dependent effect $(c(E))$ [11]. There are VSL models [8] in which quantum vacuum fluctuations can produce the HZ spectrum [10]. However, in a large class of VSL scenarios the universe is never vacuum dominated. Hence subhorizon scale thermalization is the reason for the apparent homogeneity of the Universe—and likewise thermal fluctuations are responsible for the primordial fluctuations. In the third case, we consider the possibility of thermal fluctuations seeding structures within Lemaitre's Phoenix universe [12].

The paper is organized as follows. In Sec. II we examine the statistics of thermal fluctuations. In Sec. III we derive the necessary conditions for thermal fluctuations to have a scale-invariant spectrum. In Sec. IV we show that these conditions cannot be fulfilled in universes dominated by radiation comprised of conventional particles and in which the temperature decreases with time. At the end of Sec. IV and in Sec. V we propose a set of models that may bypass this no-go theorem. We summarize our results in Sec. VI.

II. THE STATISTICS OF THERMAL FLUCTUATIONS

Most of the currently available data is consistent with the hypothesis that primordial fluctuations are Gaussian distributed [13]. A possible exception is the report of non-vanishing "inter-scale" components of the cosmic microwave background (CMB) bispectrum calculated from the four year Cosmic Background Explorer (COBE) Differential Microwave Radiomatry (DMR) data [14]. New CMB measurements, in particular by the Microwave Anisotropy Probe (MAP) satellite, are expected to provide tighter constraints on the amount of cosmological non-Gaussianity.

Based on current observational evidence, one has to require that at least on very large scales, primordial fluctuations must be sufficiently well described by a Gaussian distribution. Here we encounter the first obstacle, since strictly speaking thermal fluctuations are not Gaussian. In what follows we shall show that these fluctuations are Gaussian to a very good approximation under the same set of conditions which assure thermalization.

Fluctuations in a thermal (canonical) ensemble can be determined from the partition function

$$Z=\sum_r e^{-\beta E_r}, \tag{1}$$

where $\beta=T^{-1}$. This is true even if deformed dispersion relations are introduced [11]. Hence the total energy U inside a volume V is given by

$$U=\langle E\rangle=\frac{\sum_r E_r e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}=-\frac{d\log Z}{d\beta}. \tag{2}$$

In general this integral needs not be proportional to T^4, and indeed under deformed dispersion relations $U\propto T^{\gamma}$, with $1<\gamma<4$. However, because energy is an extensive quantity, we always have that $U=\rho V$, that is U is proportional to the volume. If $\gamma\neq 4$ there is a preferred length scale l_T, and we can choose units so that

$$\rho\approx T^4(l_T T)^{\gamma-4} \tag{3}$$

(specifically we use units such that $G=\hbar=c_0=k_B=1$, where c_0 is current value of c, and neglect factors of order 1).

The energy variance is given by

$$\sigma^2(E)=\langle E^2\rangle-\langle E\rangle^2=\frac{d^2\log Z}{d\beta^2}=-\frac{dU}{d\beta} \tag{4}$$

and so the relative variance is

$$\sigma_U^2=\frac{\sigma^2(E)}{U^2}=\frac{T^2\rho'}{\rho^2}\frac{1}{V} \tag{5}$$

where prime denotes differentiation with respect to temperature. We see that whereas the amplitude of these fluctuations is model dependent, their spectrum (i.e. the fact that $\sigma_U^2\propto 1/V$) is not. The white noise nature of thermal fluctuations follows from the fact that energy is an extensive quantity (i.e. proportional to the volume).

The fact that $\sigma_U^2\propto 1/V$ leads to an interesting heuristic interpretation of thermal fluctuations. It seems to imply that thermal fluctuations may be seen as a Poisson process involving a set of regions with coherence length λ dependent only on the temperature (and not the sample volume). In a volume V there are $n=V/\lambda^3$ such regions, so that a Poisson process results in variance

$$\sigma_U^2=\frac{\sigma^2(n)}{\bar{n}^2}=\frac{1}{\bar{n}}=\frac{\lambda^3}{V} \tag{6}$$

implying that any white noise spectrum of fluctuations may be seen as a Poisson process. The dependence $\lambda(T)$ can be inferred from Eq. (5) and in general takes the form

$$\lambda^3=\frac{T^2\rho'}{\rho^2} \tag{7}$$

translating, for Eq. (3), into

$$\lambda^3\approx\frac{\gamma}{T^3}\frac{1}{(l_T T)^{\gamma-4}}. \tag{8}$$

We see that $\lambda\propto T^{-1}$ only for $\gamma=4$. For $\gamma=1$ (realized in non-commutative geometry [11]) λ is temperature independent and equals the length scale of non-commutativity. In general the thermal coherence length decreases with increasing temperature, *except* if $\gamma<1$. In the exceptional case $\gamma<1$ the coherence length increases with the temperature and the relative energy fluctuations anomalously increase with the temperature; in Sec. IV we shall rule out this exceptional case.

The non-Gaussianity of thermal fluctuations may now be studied in terms of the cumulants of the distribution. The third order centered cumulant is given by [15]

$$\kappa_3=\langle E^3\rangle-3\langle E^2\rangle\langle E\rangle+2\langle E\rangle^3=-\frac{d^3\log Z}{d\beta^3}=\frac{d^2U}{d\beta^2} \tag{9}$$

so that the relative skewness is

$$s_3=\frac{\kappa_3}{\sigma^3}\approx\frac{\gamma(\gamma+1)}{\gamma^{3/2}}\frac{(l_T T)^{2-\gamma/2}}{(VT^3)^{1/2}}. \tag{10}$$

Hence for large volumes $s_3\ll 1$. Likewise for higher cumulants

$$\kappa_n=(-1)^n\frac{d^n\log Z}{d\beta^n}=(-1)^{n-1}\frac{d^{n-1}U}{d\beta^{n-1}} \tag{11}$$

and so

$$s_n=\frac{\kappa_n}{\sigma^n}\approx c_n(\gamma)\frac{(l_T T)^{(n/2-1)(4-\gamma)}}{(VT^3)^{n/2-1}} \tag{12}$$

with the proportionality constant $c_n(\gamma)=\gamma(\gamma+1)\cdots(\gamma+n-2)/\gamma^{n/2}$. Setting $V=L^3$ we find the unsurprising result that this can be written as

$$s_n\approx\frac{c_n(\gamma)}{\gamma^{n/2-1}}\left(\frac{\lambda}{L}\right)^{3(n/2-1)} \tag{13}$$

and so if the volume under study is much larger than the thermal coherence length as defined above we have indeed that $s_n\ll 1$.

We conclude that thermal fluctuations are Gaussian to a very good approximation if, when they leave the horizon, the thermal coherence length is much smaller than the horizon size. The departures from Gaussianity always lead to positive cumulants and these decay with the order n as in Eq. (13). Since the condition for thermalization is that the coherence length is much smaller than the scales under study, we may conclude that Gaussianity is part and parcel of the self-consistency conditions for studying thermal fluctuations in thermal equilibrium.

III. THE POWER SPECTRUM

Gaussian fluctuations are fully described by their two-point function. This can be encoded in σ_U^2, but is more often expressed in terms of the power spectrum $P(k)=\langle|\delta_k|^2\rangle$, where δ_k are the Fourier modes of the density contrast $\delta\rho/\rho$. The two can be related via the integral [16]:

$$\sigma_U^2=\frac{1}{2\pi^2}\int_0^\infty P(k)W_F^2(kL)k^2dk, \tag{14}$$

where $W_F(kL)$ is a filter function and $L\sim V^{1/3}$ is the smoothing scale (so that $U=\rho V$). Assuming a power law dependence for the power spectrum, $P(k)=A^2k^n$, and, for instance, a Gaussian filter function, $W_F(kL)=e^{-k^2L^2/2}$, integration of Eq. (14) gives

$$\sigma_U^2=\frac{\tilde{A}^2}{L^{3+n}}=\frac{\tilde{A}^2}{V^{1+n/3}}, \tag{15}$$

where $\tilde{A}^2=A^2\Gamma[(n+3)/2]/(4\pi^2)$, that is, $\tilde{A}\approx A$. By comparing Eqs. (5) and (15) one can see that thermal density fluctuations have a white noise spectrum $(n=0)$ with amplitude,

$$P(k)=\langle|\delta_k|^2\rangle\approx\frac{T^2\rho'}{\rho^2}k^0 \tag{16}$$

where we have ignored factors of order 1. This result only applies to modes which are in causal contact, i.e. sub-horizon modes. More precisely it only applies when self-gravity is negligible, and so to modes smaller than the Jeans length.

Suppose we have a model in which a certain range of Fourier modes of thermal density fluctuations are forced outside the horizon and are allowed to re-enter it at a later time.[1] Then, at first horizon crossing we have

$$k_h\sim\frac{|\dot{a}|}{c} \tag{17}$$

where k are comoving wave numbers, a is the scale factor and c is the speed of light.[2] The modulus sign in Eq. (17) accounts for the possibility of such a crossing happening during a contracting phase in a bouncing universe—a model discussed in Sec. V B. We parametrize the temperature dependence of the first horizon crossing with

$$k_h=T^\mu\lambda_1^{\mu-1}, \tag{18}$$

where λ_1 is some characteristic length scale. We will consider two types of scenarios: those in which the temperature of the radiation is decreasing with the evolution, and those in which the temperature is increasing.

If the universe cools as it expands then solving the horizon problem and the existence of a first crossing require that $\mu<0$. More generally, one needs $dk_h/dT<0$ or, from Eq. (18),

$$\frac{\ddot{a}}{\dot{a}}-\frac{\dot{c}}{c}>0. \tag{19}$$

Hence, while modes are being forced outside the horizon one must have either accelerated expansion or a decreasing speed of light, or a combination of both.

If the universe heats up as it evolves in time, then $\mu>0$ (or $dk_h/dT>0$). We consider this case in some detail in Sec. V.

In both types of scenarios we will be interested in identifying conditions for a HZ spectrum of density fluctuations to be left outside the horizon. If there is no significant evolution of the gravitational potential ϕ outside the horizon while the modes are being pushed out (this is usually enforced by requiring that the equation of state remains more or less constant), then density fluctuations have a HZ spectrum when the equal-time power spectrum of ϕ has a form

$$k^3P_\phi=B^2, \tag{20}$$

with $B\approx 10^{-5}$. If in addition at horizon crossing we have $\phi_h\approx\delta_h$, then we need[3]

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[1]This condition is more restrictive than requiring that the horizon problem be solved—for instance the Milne universe (with $a\propto t$) does not have horizons, and yet sub-horizon modes are not pushed outside the horizon. The reader is also referred to Ref. [17] for a simple condition for solving the horizon and flatness problems.

[2]c is the speed of light at the time of the horizon crossing, which could be different from c_0.

[3]This may be seen as an independent assumption, or justified using Einstein's equations. The perturbed Friedmann equations in the comoving longitudinal gauge imply that $k^2\phi\approx a^2\rho\delta$ and, using the Friedmann equation, $\dot{a}^2\approx\rho a^2$, and Eq. (17), we obtain $\phi\approx(k_h/k)^2\delta$. Hence $\phi_h\approx\delta_h$.

$$k_h^3\delta_h^2 \approx B^2 \approx 10^{-10}, \tag{21}$$

which is sometimes used as the definition of the HZ spectrum. Since $\phi_h \approx \delta_h$, this expression is true in any gauge because the δ_h defined in the various gauges are all proportional to each other.

If we identify these fluctuations with thermal fluctuations about to leave the horizon, then, using Eq. (16), this is equivalent to

$$\frac{T^2}{\rho^2}\frac{d\rho}{dT}k_h^3(T) \approx B^2 \tag{22}$$

(where again we have neglected factors of order 1). Equation (3) then leads to

$$\frac{\gamma}{l_T^{\gamma-4}}k_h^3 T^{1-\gamma} \approx B^2. \tag{23}$$

Taking the time derivative of the above expression leads to

$$3\frac{\dot{k}_h}{k_h} + (1-\gamma)\frac{\dot{T}}{T} = 0. \tag{24}$$

Since $\dot{k}_h > 0$ is the condition for modes to be leaving the horizon, it follows from Eq. (24) that

$$(1-\gamma)\frac{\dot{T}}{T} < 0. \tag{25}$$

Hence scale invariance requires that $\gamma < 1$ or $\gamma > 1$ depending on whether the Universe is getting colder or hotter. We recall that $\gamma < 1$ is equivalent to saying that the thermal coherence length λ *increases* with the temperature [see Eq. (8)], or that the fractional energy fluctuations increase with the temperature. This is quite anomalous and we shall rule it out explicitly in the next section. Hence we are left with warming universes as a possibility for HZ fluctuations of thermal origin.

These results may be expressed more quantitatively by noting that Eqs. (18) and (23) lead to the condition for scale invariance:

$$\gamma = 1 + 3\mu, \tag{26}$$

or more generally the expression for the tilt

$$n = 4 + \frac{1-\gamma}{\mu}. \tag{27}$$

In our argument so far we have abstained from using Einstein's equations. Instead, we have treated Eq. (21) as an independent assumption and also *assumed* that the gravitational potential would freeze outside the horizon. However we can go further if we are prepared to use Friedmann equations:

$$\frac{\dot{a}}{a} \propto \sqrt{\rho} \quad \text{and} \quad \rho \propto \frac{1}{a^{3(1+w)}} \tag{28}$$

(where w is the equation of state), and if we parametrize the dependence of the speed of light on the temperature as

$$c \propto T^{\alpha}. \tag{29}$$

Then from Eqs. (17), (18), (28) and (29) the condition for scale invariance reduces to

$$\frac{\gamma}{1-3\alpha} = 2\frac{1+w}{1-w} \tag{30}$$

or, alternatively, to

$$\alpha = \frac{(2-\gamma)+(2+\gamma)w}{6(1+w)}. \tag{31}$$

IV. A NO-GO THEOREM FOR COOLING UNIVERSES

If at all times the universe has been cooling ($\dot{T} < 0$), then we arrive at a forbidding set of conditions for a scale-invariant spectrum. In this case k_h must decrease with the temperature ($\mu < 0$). As already noted, from Eq. (25) it then follows that this requires

$$\gamma < 1. \tag{32}$$

This condition is a strict inequality: $\gamma = 1$ leads to either $n = 4$ (if the modes are indeed being pushed out of the horizon), or to $n = 0$ (if the Hubble length stagnates).

If $\gamma < 1$ the fractional amplitude of thermal fluctuations anomalously *increases* with the temperature. As Eq. (8) shows, this also implies that the thermal correlation length increases with the temperature. Below we present a no-go theorem which shows that it is unlikely that this condition is satisfied assuming that the weak energy condition is satisfied and that the fundamental degrees of freedom are particlelike.

Consider radiation in thermal equilibrium with a certain dispersion relation $p(E)$ which becomes the usual $p^2 = E^2$ at sufficiently low energies. The energy density is proportional to the integral:

$$\rho(T) \propto I(T) = \int dE \frac{Ep^2(E)}{e^{E/T}-1}\left|\frac{dp}{dE}\right|, \tag{33}$$

where the integration is over all allowed values of energy. For convenience, let us define $F(E) \equiv Ep^2(E)|dp/dE|$ and re-write $I(T)$ as

$$I(T) = T\int \frac{dE}{T}\frac{F(E)}{e^{E/T}-1} \equiv Tf(T). \tag{34}$$

In order to have $\gamma < 1$, $f(T)$ must be a decreasing function of temperature at sufficiently high values of T, i.e. $f'(T) < 0$. Let us evaluate $f'(T)$:

$$f'(T) = \int \frac{dE}{T^2}\frac{F(E)}{e^{E/T}-1}\left[\frac{E}{T}\frac{e^{E/T}}{e^{E/T}-1} - 1\right]. \tag{35}$$

If we only consider non-negative energies, then the factor under the integral appearing in front of the square brackets is non-negative, while the expression inside the brackets is a non-negative, monotonically increasing function of E/T. Hence, $f'(T)>0$ for all T. Thus, the inequality (32) cannot be satisfied.

The above proof is quite general and is valid for all models that would aim to achieve $\gamma<1$ by modifying the dispersion relations without altering the statistical properties of the gas. In particular, this proof implies that the modified dispersion relations considered in Refs. [18,11,3] could not result in a HZ spectrum.

The no-go argument assumed that the thermalized radiation was made of particles obeying Bose-Einstein statistics. A radically different statistics is likely to be needed in order to obtain the desired exponent in the Stefan-Boltzmann relation. Thus the only way we see of bypassing this no-go theorem is to allow for non-particle-like degrees of freedom. We conclude this section by suggesting a scenario which may make use of this possibility.

Saturating temperature

An example of a system in which it is possible to have $\gamma<1$ is a gas of strings at temperatures close to the so-called Hagedorn temperature, T_H [19]. In a gas of strings, the number of degenerate states increases exponentially with energy [20] and the canonical partition function diverges for all $T>T_H$. This does not necessarily mean that temperature higher than T_H are unphysical. In fact, all physical quantities, such as energy density and specific heat are actually finite at $T\geqslant T_H$ [21]. In [22] it was suggested that T_H corresponds to a phase transition, somewhat analogous to the deconfining transition in QCD. At temperatures close to T_H, the canonical ensemble description of string gases becomes invalid due to increasingly large energy fluctuations [23]. One must use the microcanonical ensemble instead, which is well defined only if all spatial dimensions were compactified [21].[4]

At least within the canonical ensemble formalism, T_H can be interpreted as the limiting temperature of the gas—as energy is increased, the temperature remains constant. In the language of Eq. (3) this corresponds to $\gamma\to 0$, in agreement with the constraint (32). A straightforward examination of Eq. (30) with $\gamma=0$ shows that scale invariance can be satisfied if either $w=-1$, as in inflation, or if $\alpha=1/3$, as in VSL models.

String-driven inflationary models, making use of the existence of a limiting temperature, have been considered in the late 1980s [24,25]. More recently, in Ref. [26], it was proposed that winding modes of open strings on D-branes above the Hagedorn phase transition can provide the negative pressure necessary to drive inflation. In particular, it was suggested that one could achieve a period of exponential inflation (with $w=-1$) if not all transverse dimensions supported winding modes [26].

It would be very interesting to investigate if thermal fluctuations could indeed be viable candidates for structure formation in these models and the degree of fine-tuning it would involve.

The other possibility, that of a VSL theory with $\alpha=1/3$, is currently lacking a specific model realization.

V. WARMING UNIVERSES

But it could be that at the early stage when the modes are being pushed out of the horizon the universe is getting hotter. Such is the case of thermal radiation with $\gamma(1+w)<0$. If $\gamma>0$, denser radiation means hotter radiation; however if $w<-1$ the universe gets denser (and so hotter) as it expands. Alternatively we could have $w>-1$, so that the universe gets less dense as it expands; but then if $\gamma<0$ this translates into a higher temperature.

Another possibility is a stage of radiation injection, either from particle or antiparticle annihilation, from false vacuum decay, or from a cosmological constant discharge.

Yet another possibility is the Phoenix universe of Lemaitre [12], where modes would be pushed outside the horizon with temperature increasing during the contracting phase.

If the universe gets hotter in time, we need k_h to increase with time, and with temperature, so that $\mu>0$. A necessary condition for scale invariance is then

$$\gamma>1, \tag{36}$$

bypassig the no-go theorem in the previous section. Again, from Eqs. (17), (18), (28) and (29) we obtain

$$\mu=\gamma\left[\frac{1}{2}-\frac{1}{3(1+w)}\right]-\alpha>0 \tag{37}$$

or equivalently

$$\frac{\alpha}{\gamma}<\frac{1+3w}{6(1+w)}. \tag{38}$$

Conditions (30) and (31) still apply.

We now consider particular solutions to these conditions.

A. A phantom phase

''Phantom'' matter [27] exhibits an equation of state with $w<-1$, and it may constitute the dark energy of the universe. It has also been conjectured that normal radiation at high temperatures could behave like phantom matter [3]. For these models there is a critical density, ρ_c, such that $w>-1$ for $\rho<\rho_c$, while for $\rho>\rho_c$ one has $w<-1$. If the Universe starts off with $\rho>\rho_c$ and expanding then $a\propto(-t)^{2/3(1+w)}$. As the universe expands it gets denser and

[4]In Ref. [21] it is further suggested that in this picture one needs a mechanism which would later make three of the spatial dimensions sufficiently large for us to live in.

hotter. Eventually a phase transition brings it to the sub-Planckian regime.

In such a scenario there is hyper-inflation, so the modes are pushed out of the horizon without a VSL. However, in order for density fluctuations to have a scale invariant spectrum, the condition in Eq. (30) must be satisfied. Since $\gamma > 1$ and $w < -1$, we find that

$$\alpha > \frac{1}{2} - \frac{1}{3(\omega+1)} > \frac{1}{2}. \tag{39}$$

Hence, in order to obtain scale-invariance, one needs a VSL.

Regarding the spectrum's amplitude, from Eq. (23) one can obtain the requirement

$$\frac{\gamma}{l_T^{\gamma-4}} \lambda_1^{(1-\mu)(1-\gamma)/\mu} \approx B^2 \approx 10^{-10}, \tag{40}$$

or, using Eq. (26) to express μ in terms of λ,

$$\gamma \left(\frac{\lambda_1}{l_T} \right)^{\gamma-4} \approx B^2. \tag{41}$$

Thus, given a value of γ, Eq. (41) constrains the ratio of the two characteristic length scales l_T and λ_1. Within any specific model providing a relation between the scale factor a and the temperature T it should be possible to express λ_1 in terms of l_T. Consequently, these two length scales should not be considered as independent.

We may expect one of the models based upon the formalism presented in [11] to satisfy these conditions, placing further restrictions upon deformations of dispersion relations. The spectrum's amplitude in this case will be fixed by the ratio between the non-commutative length scale (related to l_T) and the Planck length (related to λ_1).

B. A bouncing universe

In the bouncing model a closed universe goes through a series of cycles starting with a big bang and expansion, followed by re-contraction and a big crunch. Every time the universe approaches the crunch it bounces into a new bang, and if entropy increases at the bounce the new cycle lasts longer (i.e. expands to a larger and colder turnaround point) in each new cycle. Although the idea goes back to Lemaitre [12], it has often been forgotten and revived a few times, e.g. more recently in [28].

It is questionable that this model dispenses with fine-tuning, specially when dealing with the homogeneity and entropy problems. It may be that just too much junk (entropy) is left over from previous cycles. This can usually be avoided combining the bouncing universe with inflation, such as in its ekpyrotic incarnation [29].

Another possibility is that the speed of light decreases at the bounce, so that all the relevant scales we see today were sub-horizon modes at some point deep in the radiation epoch in the contracting phase. Hence homogeneity (and absence of pervading coalescing black holes) could have been established on all the relevant scales for the current cycle. But more importantly in this scenario thermal fluctuations could become the Harrison-Zeldovich spectrum of initial conditions which we observed in our current cycle. We now illustrate how this could happen.

As long as $w > -1$ (and $\gamma > 0$) the universe heats up in time during the contracting phase. Hence a necessary condition for scale invariance is that $\gamma > 1$, evading the no-go theorem presented in Sec. IV. Modes must leave the horizon as the universe contracts, and as Eq. (37) shows this can be achieved without a VSL if $w > -1/3$, i.e. the universe undergoes the necessary accelerated contraction when the strong energy condition *is* satisfied. A general condition for scale invariance is then Eq. (30). We find the notable result that standard radiation ($w = 1/3$, $\gamma = 4$, $\alpha = 0$) satisfies the condition for scale invariance. Hence, as long as the scales we can observe today were sub-horizon deep in the radiation epoch during the contracting phase of the previous cycle, thermal fluctuations in standard radiation lead to a Harrison-Zeldovich spectrum of initial fluctuations in the new phase.[5]

A problem arises with the amplitude, which may be remedied if there is a drop in the speed of light at the bounce. For $\gamma = 4$ the scale l_T drops out of the problem [cf. Eq. (3)]. Also for $w = 1/3$ and $\alpha = 0$ we have $k_h \propto T$, so $\mu = 1$ and the scale λ_1 disappears from Eq. (18). Hence the ratio of these parameters disappears from the expression for the amplitude, as can be seen directly from Eq. (41); the amplitude becomes then a combination of numerical factors of order 1 clearly in contradiction with observations. This can also be illustrated using the 10 Mpc comoving scale as the normalization point, as in Peebles' example [1]. Rephrasing Peebles argument in terms of this model, we know that in the case of a perfectly symmetric bounce the right normalization σ_{10} can be obtained if the 10 Mpc comoving scale leaves the horizon in the contracting phase when the universe was at 10^{11} GeV. However, if the relation between temperature and time on either side of the bounce is symmetric, this comoving scale would leave the horizon much earlier in the contracting phase, when the universe was much colder and therefore the fractional fluctuations were much larger.

The problem with the normalization mentioned above, could be resolved if the bounce was asymmetrical, allowing for the 10 Mpc comoving scale to leave the horizon when the universe was indeed at 10^{11} GeV. One possible scenario is when the speed of light is constant during the previous ($c = c_-$) and current ($c = c_+$) cycles, but $c_- \gg c_+$, i.e. the speed of light decreases at the bounce. One can make additional simplifying assumptions just to illustrate the point. Let us assume that the fundamental constants G and $\hbar$ also change at the bounce in such a way so as to preserve the relation between the temperature and time (this can be achieved if $\hbar^3 c^5/G$ is the same before and after the bounce). Hence the relation between time and temperature is sym-

[5]This assumes certain mode matching at the bounce. In general, the picture will be much more complicated. This issue is discussed in Sec. V B 1.

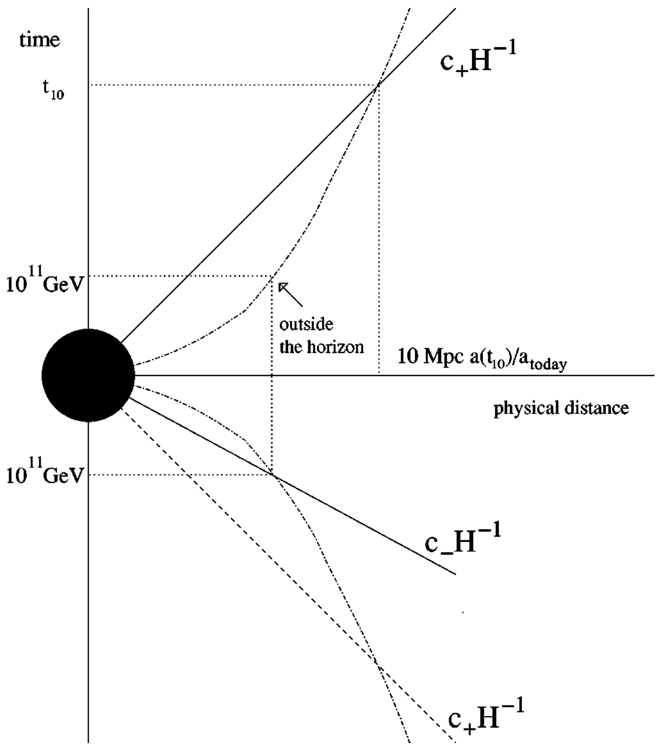

FIG. 1. The normalization problem in bouncing universes and a way to solve it. In the case of a perfectly symmetrical bounce, density perturbations on 10 Mpc scale exit the horizon during the contracting phase at temperatures much lower than 10^{11} GeV. The problem could be fixed if the speed of light in the contracting phase (c_-) had a much bigger constant value than it has today (i.e. $c_- \gg c_+$).

metrical around the bounce, but for $c_-/c_+ \ll 1$ the 10 Mpc comoving scale would leave the horizon in the contracting phase much later and at a higher temperature, as illustrated in Fig. 1.

Using $t/(1\ \mathrm{sec}) \approx (T/(1\ \mathrm{MeV}))^{-2}$, we find that $t_\star = 10^{-28}$ sec before the bounce the temperature is 10^{11} GeV. The comoving horizon at this time is 10 Mpc across if

$$c_- t_\star = 10\ \mathrm{Mpc}\frac{a_\star}{a_0} \tag{42}$$

Since $a_\star/a_0 = T_{CMB}/10^{11}$ GeV (where $T_{CMB} \approx 2.7$ K) we get

$$\frac{c_-}{c_+} \approx 10^{21}. \tag{43}$$

Note that unlike other VSL arguments, which lead to lower bounds on c_-/c_+, this argument leads to an identity: the spectrum amplitude results directly from a given value of c_-/c_+. It may be possible to obtain a similar effect by another way of making the bounce asymmetrical. For instance if G and $\hbar$ vary differently at the bounce a different constraint is obtained.

Certainly, the validity of the above discussion depends upon the matching of growing and decaying modes during the bounce dynamics—something which is unknown in the absence of a specific model for the bounce. We conclude with a discussion of the two main uncertainties faced by this model.

1. Mode matching at the bounce

Bounce dynamics will in general mix the modes in either phase [34,35], a matter which may upset our previous arguments. In the contracting phase we have

$$\phi = A_- \phi_0 + \frac{B_-}{|t|} \tag{44}$$

and in the expanding phase

$$\phi = A_+ \phi_0 + \frac{B_+}{t} \tag{45}$$

where ϕ is the gravitational potential and $t=0$ at the bounce. Matching these modes depends on the bounce dynamics, and everything we have said previously assumed that B_- mode was not excited and that the A_- mode was matched onto the A_+ mode.

If the B_- mode is excited, but then matches on to the B_+ mode, then the earlier discussion still applies. However if the B_- mode matches onto the A_+ mode then the conclusions in this paper have to be revised: in addition to the HZ spectrum predicted here, there would be a very red component ($n = -3$). Another pathological case is when A_- and B_+ are excited, while $B_- = A_+ = 0$.

We defer to a future publication a more careful study of this situation. It certainly depends on how the bounce is actually produced. Mode matching in pre-big-bang [36] and ekpyrotic [29] cosmologies has been discussed in considerable detail in Refs. [34,35].

2. Entropy production at the bounce

Another matter which may change the spectrum and normalization of the fluctuations is entropy production at the bounce (which almost certainly occurs). Indeed for normal radiation, i.e. with $\gamma=4$, the amplitude of thermal density fluctuations on a given scale L can be expressed in terms of the entropy contained in a sphere of radius L [Eq. (15.26) of [1]]:

$$\delta_L^2 = \frac{16}{3S}. \tag{46}$$

Thus, if entropy is produced at different rates on different scales the spectrum of fluctuations could be modified. In addition this issue is bound to interfere with any normalization condition for the amplitude.

VI. CONCLUSIONS

We have studied the possibility of thermal fluctuations providing seeds for currently observed cosmic structures. Assuming thermal equilibrium, we have identified the necessary conditions under which these fluctuations are Gaussian and scale invariant. We have shown that Gaussianity constraints are automatically satisfied in thermal equilibrium. The situation with the scale invariance of the power spectrum is more problematic. In order to have a HZ spectrum in a universe that cools in time one needs significant modifications to the Stefan-Boltzmann law, namely, $\rho \propto T^{\gamma}$ with $\gamma < 1$. We have shown that this condition is unlikely to be satisfied for radiation comprised of Bose-Einstein particles. If we are prepared to assume that the radiation is not made of particles, then naturally a number of possibilities could open up. If over a certain period the Universe was warming up with time, then the condition for scale invariance becomes $\gamma > 1$ and can, in principle, be achieved with a gas of Bose-Einstein particles with appropriate dispersion relations.

An approach different from ours was taken in Ref. [30]. There, authors have considered thermal fluctuations in a system of particles with an attractive short range potential and a repulsive $1/r^2$ potential at large scales. They have shown that the resulting power spectrum of density fluctuations is scale invariant on sufficiently large scales (small ks). We find this direction of thought interesting and deserving further development.

To conclude, we have discussed an alternative, "thermal," way of obtaining scale-invariant initial spectrum of density fluctuations. Other ways to match the observations without inflationary quantum fluctuations have been previously discussed, e.g. active seed models [31] with [32] or without VSL [33]. One may argue that some of these models are less "natural" than inflation. However, they make the point that the recent observational victories in modern cosmology are a success of the Harrison-Zeldovich spectrum plus gravitational instability rather than a "proof" of the inflationary origin of the initial fluctuations.

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[1] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, NJ, 1993).
[2] E.R. Harrison, Phys. Rev. D **1**, 2726 (1970); Ya.B. Zeldovich, Mon. Not. R. Astron. Soc. **160**, 1 (1972).
[3] S. Alexander, R. Brandenberger, and J. Magueijo, hep-th/0108190.
[4] J. Yokoyama and K. Maeda, Phys. Lett. B **207**, 31 (1988); J. Yokoyama and A. Linde, Phys. Rev. D **60**, 083509 (1999).
[5] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995).
[6] J. Moffat, Int. J. Mod. Phys. D **2**, 351 (1993).
[7] A. Albrecht and J. Magueijo, Phys. Rev. D **59**, 043516 (1999); J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002).
[8] M.A. Clayton and J.W. Moffat, Phys. Lett. B **460**, 263 (1999); **477**, 269 (2000); Int. J. Mod. Phys. D **11**, 187 (2002); I. Drummond, gr-qc/9908058.
[9] J. Magueijo, Phys. Rev. D **62**, 103521 (2000).
[10] J.W. Moffat, hep-th/0208122.
[11] S. Alexander and J. Magueijo, hep-th/0104093.
[12] G. Lemaitre, Ann. Soc. Sci. Bruxelles A **53**, 51 (1933).
[13] A.F. Heavens, Mon. Not. R. Astron. Soc. **299**, 805 (1998); A. Banday, S. Zaroubi, and K.M. Górski, Astrophys. J. **533**, 575 (2000); M. Kunz *et al.*, Astrophys. J. Lett. **563**, L99 (2001); M.G. Santos *et al.*, Phys. Rev. Lett. **88**, 241302 (2002).
[14] J. Magueijo, Astrophys. J. Lett. **528**, L57 (2000).
[15] A. Stuart and K. Ord, *Kendall's Advanced Theory of Statistics* (Wiley, London, 1994).
[16] P. Coles and F. Luccin, *Cosmology: The Origin and Evolution of Cosmic Structure* (Wiley, London, 1995).
[17] P.P. Avelino and C.J.A.P. Martins, Phys. Rev. D **67**, 027302 (2003).
[18] G. Amelino-Camelia and S. Majid, Int. J. Mod. Phys. A **15**, 4301 (2000); G. Amelino-Camelia, Int. J. Mod. Phys. D **11**, 35 (2002).
[19] R. Hagedorn, Nuovo Cimento, Suppl. **3**, 147 (1965).
[20] K. Huang and S. Weinberg, Phys. Rev. Lett. **25**, 895 (1970); S. Fubini and G. Veneziano, Nuovo Cimento A **64**, 1640 (1969).
[21] R. Brandenberger and C. Vafa, Nucl. Phys. **B316**, 391 (1989).
[22] J.J. Atick and E. Witten, Nucl. Phys. **B310**, 291 (1988).
[23] D. Mitchell and N. Turok, Phys. Rev. Lett. **58**, 1577 (1987).
[24] Y. Aharonov, F. Englert, and J. Orloff, Phys. Lett. B **199**, 366 (1987).
[25] N. Turok, Phys. Rev. Lett. **60**, 549 (1988).
[26] S. Abel, K. Freese, and I. Kogan, J. High Energy Phys. **01**, 039K (2001).
[27] R.R. Caldwell, Phys. Lett. B **545**, 23 (2002).
[28] R. Durrer and J. Laukenmann, Class. Quantum Grav. **13**, 1069 (1996).
[29] J. Khoury, B.A. Ovrut, P.J. Steinhardt, and N. Turok, Phys. Rev. D **64**, 123522 (2001); P.J. Steinhardt and N. Turok, *ibid.* **65**, 126003 (2002).
[30] A. Gabrielli *et al.*, Phys. Rev. D **67**, 043506 (2003).
[31] A. Albrecht *et al.*, Phys. Rev. Lett. **76**, 1413 (1996); J. Magueijo *et al.*, *ibid.* **76**, 2617 (1996); J. Magueijo *et al.*, Phys. Rev. D **54**, 3727 (1996).
[32] P. Avelino and C. Martins, Phys. Rev. Lett. **85**, 1370 (2000).
[33] N. Turok, Phys. Rev. Lett. **77**, 4138 (1996).
[34] R. Durrer and F. Vernizzi, Phys. Rev. D **66**, 083503 (2002).
[35] S. Tsujikawa, R. Brandenberger, and F. Finelli, Phys. Rev. D **66**, 083513 (2002).
[36] G. Veneziano, Phys. Lett. B **265**, 287 (1991); M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993).