Stabilization of the electroweak Z string in the early Universe

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The standard electroweak theory admits a string solution, the Z string, in which only the electrically neutral Higgs fields are excited. This solution is unstable at zero temperature: Z strings decay by exciting charged Higgs modes. In the early Universe, however, there was a long period during which the Higgs particles were out of equilibrium but the photon field was in thermal equilibrium. We show that in this phase Z strings are stabilized by interactions of the charged Higgs modes with the photons. In a first temperature range immediately below the electroweak symmetry breaking scale, the stabilized embedded defects are symmetric in internal space (the charged scalar fields are not excited). There is a second critical temperature below which the stabilized embedded strings undergo a core phase transition and the charged scalar fields take on a nonvanishing value in the core of the strings. We show that stabilized embedded defects with an asymmetric core persist to very low temperatures. The stabilization mechanism discussed in this paper is a prototypical example of a process which will apply to a wider class of embedded defects in gauge theories.

I. INTRODUCTION

In a previous paper [1] we suggested that a wide class of embedded defects may be stabilized by plasma effects. We studied a toy model consisting of four real scalar fields ϕ_i; $i=1,\ldots,4$ with the "generalized Mexican hat" potential

$$V(\phi) = \lambda \left(\sum_{i=1}^{4} \phi_i^2 - \eta^2 \right)^2 , \qquad (1)$$

two of which (ϕ_1 and ϕ_2 to be specific) being electrically charged, the other two neutral. At zero temperature, the vacuum manifold in this model is $\mathcal{S}^3$ and hence does not admit any stable topological defects. However, there are *embedded* defects [2,3], configurations which span only a subset of the vacuum manifold. One example is the global cosmic string solution of the subspace of the theory with $\phi_1 = \phi_2 = 0$. At zero temperature this string configuration is unstable and can decay through the excitation of the charged fields. In more graphic terms, the field configuration slips off the top of the potential in the charged field directions and approaches the vacuum manifold everywhere in space. However, in the presence of a bath of photons, interactions of the photons with the charged scalar fields will lead to an effective potential which is lifted in the directions of the charged fields, thus generating a potential barrier which can stabilize the embedded string. This toy model is a theory with a global symmetry, and is realized in the sigma model description of the low energy limit of quantum chromodynamics in the limit of vanishing pion mass (see [4] which includes a detailed discussion of effects which arise when the explicit symmetry breaking caused by the nonvanishing pion mass is taken into account).

The standard electroweak theory is a good example of a theory which contains embedded strings in which only the neutral scalar field components of the multidimensional or-

der parameter are excited. Among all possible embedded strings, (see e.g. [5] for a classification of embedded strings in the electroweak theory) the electroweak Z string [6,7] is the string configuration consisting of excitations of the neutral fields only.

In the paper we demonstrate that Z strings are stabilized during the temperature interval between the electroweak phase transition temperature and the recombination. At a certain temperature during this period, a core phase transition takes place [8] below which the embedded strings will become superconducting as shown in [4], thus enhancing their stability and leading to the formation of vortons [9]. The enhanced stability of electroweak strings due to current-carrying zero modes is similar to the stabilization by neutrino zero modes considered in [10].

The mechanism discussed in this paper generalizes to a wide class of embedded defects. Since defects can play an important role in cosmology—even if they are not stable at all times—the mechanism discussed here may have important consequences for many aspects of cosmology. One possibility is to use embedded global anomalous strings such as the pion string of low energy QCD [11] to generate primordial magnetic fields [12]. Crucial aspects in this application are, in addition to the stabilization of the embedded strings, the fact that, via the anomaly, charged zero modes on the string generate coherent magnetic fields circling the string [13], and the fact that the length scale of the string network increases in comoving coordinates (see e.g. [14]), which provides the mechanism for generating the required large coherence length of the primordial magnetic fields. QCD at large baryon density also leads to the stabilization of a type of embedded strings, K strings [15].

Stabilized embedded defects (in particular embedded walls) could also play a role in defect-mediated baryogenesis (see e.g. [16] and [17,18] for the main ideas of defect mediated grand unified and electroweak baryogenesis, respec-

tively), and they may be useful in implementing the scenario of Dvali et al. [19] for alleviating the monopole problem via defect interactions (see [20,21] for studies of the basic interaction mechanism).

As with any class of topological defects, there will be severe cosmological constraints on models which admit them, resulting from the fact that the evolution of the defects in the early Universe may lead to predictions which are in conflict with observations. The vorton abundance problem (see e.g. [22]) leads to severe constraints, as does the constraint (specific to decaying defects) that the decay not lead to spectral distortions in the cosmic microwave background [23].

A new feature which arises in the present discussion of the electroweak theory, compared to the previous studies of toy models with a global symmetry, is the fact that the underlying symmetry is a local symmetry.

II. STABILIZATION MECHANISM

Starting point is the Lagrangian for the standard electroweak theory:

$$\mathcal{L} = -\frac{1}{4} W_{\mu\nu a} W^{\mu\nu a} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu}$$
$$+ \left| \left(\partial_\kappa - \frac{1}{2} i g \tau^a W_\kappa^a - \frac{1}{2} i g' Y_\kappa \right) \Phi \right|^2 - \lambda (\Phi^\dagger \Phi - \eta^2)^2,$$

$$(2)$$

where the indices μ and ν are Lorentz indices, the index a is an SU(2) index, g and g' are the SU(2) and U(1) coupling constant, respectively, and W and Y are the SU(2) and U(1) field strength tensors, respectively.

The field Φ is a complex Higgs doublet, the upper component Φ^+ having positive charge, the lower component Φ^0 being neutral (if the expectation value of Φ is chosen such that only the lower component is nonvanishing). The analogy with (1) is clear: the lower SU(2) doublet component gives the two neutral real scalar fields ϕ_3 and ϕ_4, the upper charged component yields ϕ_1 and ϕ_2.

We are interested in describing the physics below the phase transition temperature T_c. In this case, the only gauge field which is excited is the photon field A_μ. In terms of the SU(2) gauge field W^a and the U(1) gauge field Y (we are suppressing the Lorentz index), the Z and A fields are given by

$$Z = \cos(\theta_w) W^3 - \sin(\theta_w) Y$$
$$A = \sin(\theta_w) W^3 + \cos(\theta_w) Y,$$

$$(3)$$

where θ_w is the weak mixing angle.

The electroweak Z string is obtained by setting

$$W^1 = W^2 = A = \Phi^+ = 0$$

$$\Phi^0 = \Phi_{NO} \qquad (4)$$

$$Z = A_{NO},$$

where the subscript NO stands for the Nielsen-Olesen U(1) cosmic string configuration [24], which in cylindrical coordinates r and θ (the coordinates in the plane perpendicular to the string) and in temporal gauge $A^i_{\mu=0} = 0$ has the form

$$\Phi_{NO}(r,\theta) = \eta f(r) e^{i\theta}$$

$$A_{\mu,NO}(r,\theta) = -\frac{v(r)}{\alpha r} \delta_{\mu\theta}, \qquad (5)$$

where $f(r)$ and $v(r)$ are the profile functions of the string. Note that since for the electroweak Z string the vacuum expectation value of the upper component of Φ always vanishes, the upper component of Φ will be associated with electrically charged degrees of freedom everywhere in space.

At this point the reader may object and recall that in the electroweak theory there is only one physical Higgs degree of freedom. At any point in space one can choose a gauge in which only one of the neutral Higgs fields is nonvanishing. The three scalar field degrees of freedom corresponding to rotations in the vacuum manifold are eaten by the gauge fields acquiring masses, and it is only the massive Higgs degree of freedom which remains. However, the existence of topological defects in gauge theories is precisely a consequence of the fact that there are restrictions on the ability to impose the abovementioned gauge uniformly over space. By causality, there will be regions in space in which the Higgs field is not in its vacuum manifold. This corresponds to localized energy configurations which cannot be gauged away. To describe the physics of such defects, it is advantageous to use a gauge in which the Higgs fields are not fixed, as we do below.

In the absence of any excitations in the bulk, the electroweak Z string is unstable [25]. The unstable mode corresponds to an excitation of the charged Higgs doublet:

$$\Phi(\xi,\vec{x}) = \cos(\xi) \Phi^0(\cos(\xi)\vec{x}) + \sin(\xi) \Phi^+$$

$$Z^j(\xi,\vec{x}) = \cos(\xi) Z_j^{(0)}(\cos(\xi)\vec{x}), \qquad (6)$$

where ξ is the deformation parameter (for $\xi = 0$ the configuration reduces to the Z string), $Z_j^{(0)}$ is the Nielsen-Olesen gauge field configuration, and Φ^0 is a complex Higgs doublet whose neutral component takes on the Nielsen-Olesen string configuration, and whose charged component vanishes. The field Φ^+ stands for the complex Higgs doublet in which only the charged complex scalar is excited, and the transpose of Φ^+ is given by

$$(\Phi^+)^T = \eta(1,0). \qquad (7)$$

Since by escaping into the charged scalar field directions, the string configuration can decrease its potential energy in the string core region, the energy per unit length $E(\xi)$ of the configuration (6) is smaller than the energy per unit length E_0 of the Z string. Since the loss in energy density is propor-

tional to $\xi^2 V(0)$, and is confined to the core region of the string (core radius r_c), we obtain the following estimate for the maximal energy loss:

$$E(\xi) \geq E_0 - \kappa \lambda \xi^2 \eta^4 r_c^2, \tag{8}$$

where κ is a constant of order 1. A detailed analysis [25] shows that for realistic values of the weak mixing angle, there is indeed an energy loss.

Another way to view the instability of the electroweak string at zero temperature [26] is as an instability to the formation of a W condensate in the core of the string, a process which was shown to lower the energy. In [27] it was shown that this instability can be gauge transformed to an instability lowering the winding number N of the pure electroweak string by 1 (at least for $N>1$). Thus, this instability is not independent of the one analyzed in [25]. Since in our numerical work we focus on the Higgs sector, we will also focus our analytical considerations mostly on the instability mode discussed above Eq. (6), although we will estimate the energy gain and loss by this process at the end of this section.

We will now show that in the presence of a background thermal bath of photons, the Z string is stabilized. For our analysis to hold, it is important that all of the scalar fields be out of thermal equilibrium, that the photon is in thermal equilibrium, and that the other gauge fields be out of equilibrium. These conditions are naturally satisfied in the electroweak theory at temperatures below the electroweak symmetry breaking scale and below the mass of the Higgs particle, but above the temperature of recombination. In this (large) temperature interval it is justified to average over the light degrees of freedom in order to study the dynamics of the order parameter.

Thus, our procedure will be to consider the Lagrangian of the electroweak theory in the presence of a thermal bath of photons. We take the thermal average of this Lagrangian and extract the terms which act as a correction to the potential for the dynamics of the order parameter. The thermal averaging consists of setting averages of A_μ to zero, and making the replacement

$$A_\mu A^\mu \rightarrow -\alpha T^2, \tag{9}$$

where α is another positive constant of order 1 (the estimate of [4] gives a value somewhat larger than 1, and the larger the number of degrees of freedom in thermal equilibrium in the plasma, the larger the value of α will be).

Thus, the starting point is the Lagrangian (2). In this Lagrangian, we set the charged gauge fields to zero, and invert the transformation (3) of the neutral fields in order to express the Lagrangian in terms of A_μ and Z_μ. If the charged scalar fields are excited as in Eq. (6), then the transformation (3) is effected. However, the effect will be a correction of order ξ^2 in a quantity which is already of order ξ^2 and can hence be neglected. Thus, we use

$$W^3 = \cos(\theta_w) Z + \sin(\theta_w) A$$

$$Y = \cos(\theta_w) A - \sin(\theta_w) Z. \tag{10}$$

Inserting these transformations into Eq. (2) allows us to extract the extra contribution to the potential energy density of the scalar fields which stems from the presence of the bath of photons. This contribution comes from the part of the covariant derivative term which is quadratic in A. Using for Φ the Z string configuration, it is easy to verify that the *effective* potential becomes

$$V_{eff} = V_0 + \frac{1}{4} \langle A_i A^i \rangle (2g \sin(\theta_w))^2 (\Phi^+)^\dagger \Phi^+$$

$$= V_0 + \frac{1}{4} \alpha T^2 (2g \sin(\theta_w))^2 (\Phi^+)^\dagger \Phi^+, \tag{11}$$

where V_0 is the bare potential [the last term on the right hand side of Eq. (2)].

From Eq. (11) we see that the interaction with the photon plasma induces a positive contribution to the mass of the charged scalar doublet. For temperatures in excess of a new critical temperature T_d, this positive contribution will be larger than the negative contribution to the mass (when expanded about $\Phi=0$) from the bare potential V_0. For temperatures smaller than T_d, the total mass term is negative. From Eqs. (11) and (2) we can immediately read off the value of T_d:

$$T_d = \eta \left(\frac{2\lambda}{\alpha} \right)^{1/2} \frac{1}{g \sin(\theta_w)}. \tag{12}$$

We thus expect the core of the embedded defect to be symmetric (no charged scalar fields excited) for $T>T_d$, whereas we expect a core phase transition [8] (charged scalar fields non-vanishing) for $T<T_d$. However, if we consider the time evolution of an embedded string which is initially set up with symmetric core (as we will do in the simulations described below), then the temperature $\tilde{T}_d$ at which the symmetric vortex undergoes a core phase transition is expected to be lower, since gradient energy is required in order to produce an asymmetric core, not just the potential energy which enters the above considerations.

Let us return to the issue of W condensate formation and briefly discuss the stabilization mechanism from this point of view. The energy gain per unit string length (computed by integrating from the center of the string to radius ρ) at zero temperature obtained from the formation of a condensate in which the amplitude of the W-field at the core radius is denoted W is of the order [26]

$$E(\rho)_{\text{gain}} \sim g^2 \eta^2 W^2 \rho^2 \tag{13}$$

(this excludes the gradient energy required to form a core condensate). At finite temperature, there is an energy loss due to the interaction of the photon with the W fields. This energy loss was used in [28] to show that Z strings in strong magnetic fields can be stabilized, and the corresponding energy per unit string length (integrated from the center to radius ρ) is given by

$$E(\rho)_{\text{loss}} \sim e^2 \alpha T^2 W^2 \rho^2. \tag{14}$$

As is obvious from comparing Eqs. (13) and (14), at sufficiently high temperatures the electroweak string will be stable towards the formation of a W condensate. The critical temperature is of the order $\alpha^{-1/2}\eta$, as in Eq. (12).

III. CORE PHASE TRANSITION

We have simulated the evolution of embedded defects in the presence of a finite temperature charged plasma using a numerical code based on the one employed in [1]. Although gauge fields are not included in this case, the stability of the string configuration can be established since it essentially comes from the modification of the effective potential for the scalar fields and the gauge field should play an important role only when one string interacts with another one.

First we set up the *initial configuration* as a two-dimensional slice of an infinitely long straight global string which is formed by the neutral components of the scalar fields and whose center resides in the midpoint of the lattice. Although such a highly symmetric configuration might be too ideal, it would be appropriate to see whether the core phase transition or the complete decay of the string occurs since the conservation of the winding number and the scalar field structure at the string core can be easily checked in contrast to full three-dimensional simulations. We then add thermal energy to the configuration in the form of kinetic energy, that is, the time derivative of the scalar fields. Its amplitude is $0.1 \times T^2$, and the allocation to the four components of the scalar field is chosen at random.

Then, the four scalar fields ϕ are *evolved numerically* on a two-dimensional lattice by means of the equations of motion derived for the scalar field with the effective potential (11). Neumann boundary conditions are employed. During each simulation, the background temperature is constant and the cosmic expansion is not taken into account. However, in order to reduce the fluctuations of the fields so that it is easier to see whether the string configuration is preserved or not, an artificial damping term is introduced. Thus, the evolution equations can be written as

$$\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi + D \frac{\partial \phi}{\partial t} = -\frac{\partial V}{\partial \phi}, \quad (15)$$

and the numerical value of D is set to be 0.1 in the simulation. The inclusion of this damping term and the choice of the numerical value of D do not affect the essential results concerning the stability of the string. All dimensional quantities are rescaled by appropriate powers of the symmetry breaking scale, η and are made to be dimensionless. Although most simulations are performed in a box size of 1000^2, we have checked that the basic results are insensitive to the box size by executing 3000^2 and 6000^2 grid point simulations. The spatial resolution is $\Delta x = 0.5\eta^{-1}$, and the time steps are chosen as $\Delta t = \frac{1}{10}\Delta x$. The values of the numerical parameters are chosen as $\lambda = 2.5 \times 10^{-3}$ and $\alpha(2g \sin(\theta_w))^2 = 2$. We have calculated for various patterns of the temperature and some of the results are depicted in the figures.
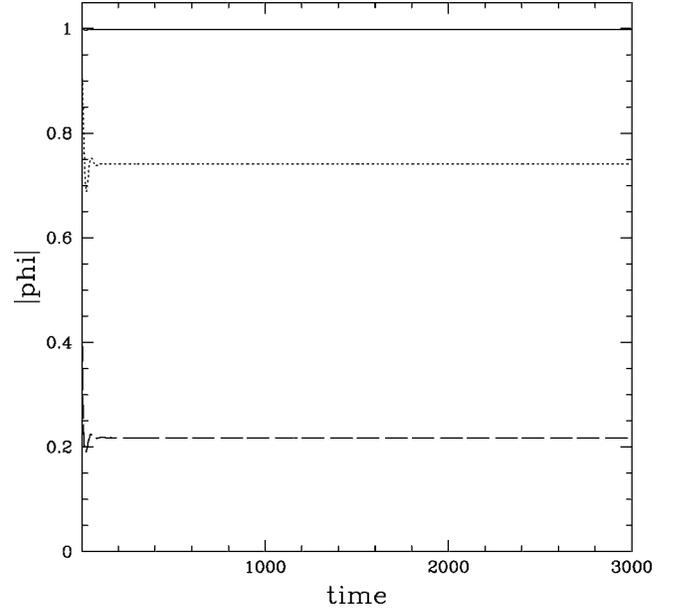
FIG. 1. The amplitude of the neutral scalar field, $\sqrt{\phi_3^2 + \phi_4^2}$, averaged over volumes of 10^2 (dashed line) and 50^2 (dotted line) grid points, and over the entire volume (solid line), as a function of time, in the high temperature simulation with $T = \eta$.

Figures 1 and 2 show the resulting scalar fields as a function of time and averaged over various sizes of square boxes centered at the midpoint of the simulation box, that is, the string core of the initial configuration, for a high temperature of $T = \eta$. Figure 1 depicts the amplitude of the neutral scalar field averaged over boxes of 10^2 and 50^2 grid points as well as averaged over the entire volume (1000^2 grid points). Figure 2 shows the average of the charged scalar field over the
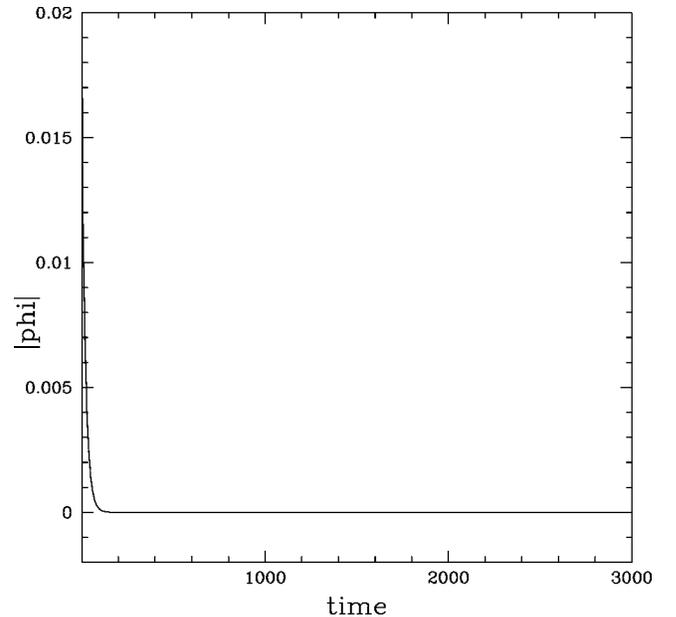
FIG. 2. The amplitude of the charged scalar field, $\sqrt{\phi_1^2 + \phi_2^2}$, averaged over the entire volume, as a function of time, in the high temperature simulation with $T = \eta$.
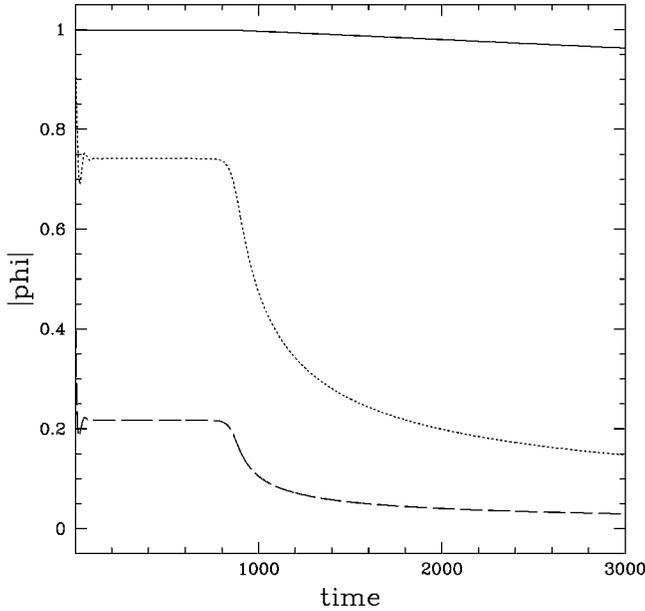
FIG. 3. The amplitude of the neutral scalar field averaged over volumes of 10^2 (dashed line) and 50^2 (dotted line) grid points, and over the entire volume (solid line), as a function of time, in the low temperature simulation with $T = 10^{-3}\eta$. The core phase transition occurs at a time $t \sim 800$ and is marked by a sharp decrease in the average value of field in the string core and surrounding regions.

FIG. 4. The amplitude of the charged scalar field averaged over volumes of 10^2 (dashed line) and 50^2 (dotted line) grid points, and over the entire volume(solid line), as a function of time, in the low temperature simulation with $T = 10^{-3}\eta$. At the time of the core phase transition $t \sim 800$, the charged scalar field takes on an average value over the string core region and the neighboring volume of 50^2 grid points which is of the order η.

entire volume (the average values over the smaller volumes are almost identical).

The width of the field configuration of a Nielsen-Olesen string is about $\lambda^{-1/2}$. Thus, the averaging in the smallest of our three volumes (the 10^2 grid point volume) is probing mainly the core region of the embedded string, and thus the amplitude of the neutral field component is small compared to unity, which is equal to the symmetry breaking scale, η, in our normalization scheme, whereas it is already of order unity on the scale of the 50^2 grid point volume. The fact that the neutral scalar field averaged over the smallest box (enclosing the initial electroweak string core region) remains small indicates that the electroweak string does not decay. The fact that the charged scalar field averaged over the core region remains vanishingly small indicates that no core phase transition takes place: the defect is a symmetric embedded defect.

Figures 3 and 4 show the corresponding curves in the case of a much lower temperature $T = 10^{-3}\eta$. Note that this temperature is lower than the critical temperature T_d when the effective barrier stabilizing $\phi = 0$ in the effective potential V_{eff} disappears. The field configuration begins in the same state as in the high temperature case. However, after a short time a core phase transition sets in during which the charged scalar field components take on a nonvanishing value which is of the order of η. The fact that the neutral scalar field remains small when averaged over the smallest of the three boxes demonstrates that the embedded string remains stable.

In fact, the absolute value of the neutral scalar field components is observed to decrease slightly during the core phase transition and, in contrast, the amplitude of the charged scalar fields averaged over the whole simulation box

is increasing. This represents the gradual increase in the core size, an increase which will stop once the core size R is of the order T^{-1}. This can be seen as follows: If the core phase transition occurs via the excitation of a single of the two charged scalar fields (no winding number in the charged scalar field sector generated), then the energy is lowered by eliminating the angular gradient energy of the neutral scalar fields. The energy per unit length thus gained will be of the order

$$E(R)_{\text{grad}} \sim \eta^2 \ln\left(\frac{R}{w}\right), \tag{16}$$

where w is the core width before the core phase transition. However, there is an energy cost associated with the generation of a nonvanishing value for the charged field, and this energy cost (per unit string length) is proportional to

$$E(R)_{\text{cost}} \sim \eta^2 T^2 R^2. \tag{17}$$

By balancing the energy gain and the energy loss, one obtains an optimal core width which is of the order of $R \sim T^{-1}$. Based on this consideration, we predict that the increase of the average value of the charged field over the entire box will eventually come to a halt, on a time scale which is proportional to T^{-1}. We have confirmed these predictions concerning the final value of R and the time scale in our numerical work.

The time at which the core phase transition takes place depends sensitively on the value of the damping term added to the equation of motion; the smaller the damping effect is,
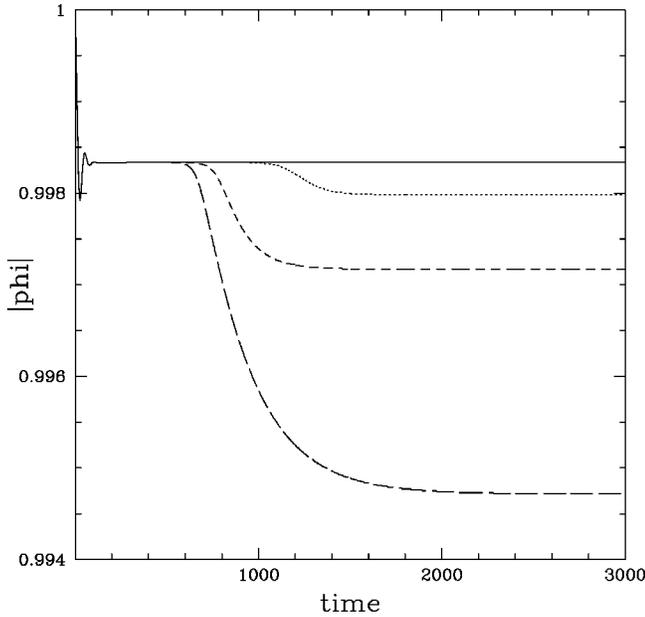
FIG. 5. The amplitude of the neutral scalar field averaged over the entire volume, as a function of time, for four different temperatures near the critical temperature $\tilde{T}_d$, $T=0.02\eta$ (long dashed line), $T=0.03\eta$ (short dashed line), $T=0.04\eta$ (dotted line) and $T=0.05\eta$ (solid line).

FIG. 6. The amplitude of the charged scalar field averaged over the entire volume, as a function of time, for four different temperatures near the critical temperature $\tilde{T}_d$, $T=0.02\eta$ (long dashed line), $T=0.03\eta$ (short dashed line), $T=0.04\eta$ (dotted line) and $T=0.05\eta$ (solid line).

the earlier the transition occurs. However, such a difference is not cosmologically significant when we consider the stability of the electroweak string since the time it takes for the core phase transition to occur is much smaller than the Hubble time in both the cases of $T=10^{-3}\eta$ and $T=\eta$. The asymptotic amplitudes of the fields averaged over the larger two of the three volumes do not depend on the presence or absence of the damping term, although the amplitude of the fields in the smallest volume does. This is due to the fact that in the absence of damping, the thermal fluctuations will cause the string core to move by more than a core radius.

Note that in both the high and the low temperature simulations, the winding number (in the neutral scalar field sector) is conserved not only for the entire simulation box but also when it is calculated around the string core, the 10^2 grid points square region. This is a further test of the stability of the embedded string.

We have also investigated numerically at which temperature $\tilde{T}_d$ the core phase transition takes place. We expect [1] the phase transition to happen at the temperature T_d when the potential barrier at $\phi=0$ due to the plasma terms disappears. This occurs when the positive quadratic contribution to V_{eff} due to the plasma equals the negative quadratic contribution due to the bare potential [see Eq. (12)]. For the values of λ and $\alpha(2g\sin(\theta_w))^2$ chosen the value of T_d is 0.1η. However, an initially symmetric core can only undergo a core phase transition if there is sufficient energy released from potential energy to create the required gradient energy. Our numerical simulations show that this happens at a temperature $\tilde{T}_d$ which lies in the interval between 0.04 and 0.05 in units of η (see Figs. 5 and 6). Note that the numerical value of $\tilde{T}_d$ for
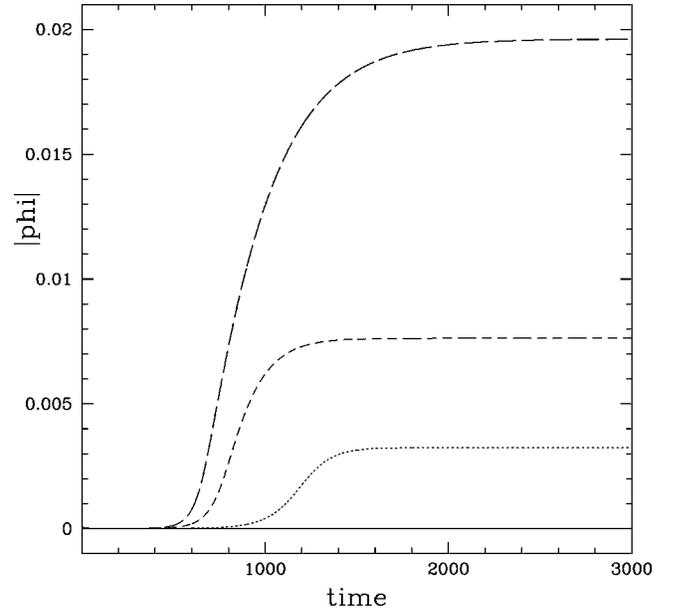
different model parameters such as λ or α can be obtained by simple scaling using Eq. (12).

Moreover, we have investigated the stability of the embedded electroweak string at very low temperatures, $T=10^{-6}\eta$. By tracking the winding number in the neutral field sector, as well as by tracking the ratio of the neutral to the charged scalar fields in the string core region, we find no evidence for a decay of the string. The time evolution of the fields is almost as shown in Figs. 3 and 4, except for the fact that the core phase transition sets in later since the thermal fluctuations are weaker. Obviously, our approximate analysis is only valid at temperatures for which the photon is in equilibrium, and thus breaks down before the cosmic temperature reaches that of last scattering.

IV. CONCLUSIONS AND DISCUSSION

We have studied the stability of the embedded Z string of the standard electroweak theory in the presence of an electromagnetic plasma in an approximate treatment in which we only follow the dynamics of the Higgs fields and treat the gauge fields as either vanishing (the W and Z fields) or else (the photon) as being in a thermal bath. We find that the electroweak string is stabilized at all temperatures above which the photon is in thermal equilibrium. In the temperature range $T_d<T<T_c$ the embedded string is symmetric in the sense that the charged scalar fields are not excited in the core, for lower temperatures the charged scalar fields are non-vanishing in the string core, but the string itself is preserved in the sense that the winding number in the neutral scalar field sector is conserved.

The basic mechanism which stabilizes the electroweak Z

string is the plasma mass for the charged Higgs doublet induced via the interactions with the plasma. This lifts the vacuum manifold in the direction of the charged Higgs doublet, leaving an effective vacuum manifold $\mathcal{M} = S^1$ which admits stable cosmic string solutions.[1]

Our numerical work is based on simulations in which only the scalar fields are evolved. From a gauge theory perspective this is an inconsistent approach. However, we feel that since the stability of the embedded defects is determined by the scalar field effective potential, neglecting the gauge fields should not adversely affect our results. However, it would be interesting to perform full local field theory numerical simulations to verify our conclusions.

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[1]An interesting question is what effect quantum vacuum fluctuations might have on the possible stabilization of embedded defects. We are grateful to Leandros Perivolaropoulos and Bill Unruh for raising this question.

The plasma stabilization mechanism discussed in this work obviously extends to a wide class of gauge theories which might be relevant in the early Universe. A new class of stabilized topological defects will thus create a new and interesting avenue to explore the interface of particle physics and cosmology.

[1] M. Nagasawa and R.H. Brandenberger, Phys. Lett. B **467**, 205 (1999).

[2] M. Barriola, T. Vachaspati, and M. Bucher, Phys. Rev. D **50**, 2819 (1994).

[3] A. Achucarro and T. Vachaspati, Phys. Rep. **327**, 347 (2000); **327**, 427 (2000).

[4] B. Carter, R.H. Brandenberger, and A.C. Davis, Phys. Rev. D **65**, 103520 (2002).

[5] T. Vachaspati, hep-ph/9301201.

[6] Y. Nambu, Nucl. Phys. **B130**, 505 (1977); K. Huang and R. Tipton, Phys. Rev. D **23**, 3050 (1981); N.S. Manton, *ibid.* **28**, 2019 (1983).

[7] T. Vachaspati, Phys. Rev. Lett. **68**, 1977 (1992); **69**, 216(E) (1992); Nucl. Phys. **B397**, 648 (1993).

[8] M. Axenides and L. Perivolaropoulos, Phys. Rev. D **56**, 1973 (1997); M. Axenides, L. Perivolaropoulos, and M. Trodden, *ibid.* **58**, 083505 (1998); M. Axenides, L. Perivolaropoulos, and T.N. Tomaras, *ibid.* **58**, 103512 (1998).

[9] R.L. Davis and E.P. Shellard, Nucl. Phys. **B323**, 209 (1989).

[10] G. Starkman, D. Stojkovic, and T. Vachaspati, Phys. Rev. D **65**, 065003 (2002); D. Stojkovic, Int. J. Mod. Phys. A **16S1C**, 1034 (2001).

[11] X. Zhang, T. Huang, and R.H. Brandenberger, Phys. Rev. D **58**, 027702 (1998).

[12] R.H. Brandenberger and X.M. Zhang, Phys. Rev. D **59**, 081301 (1999).

[13] D.B. Kaplan and A. Manohar, Nucl. Phys. **B302**, 280 (1988).

[14] R.H. Brandenberger, Int. J. Mod. Phys. A **9**, 2117 (1994).

[15] K.B. Buckley and A.R. Zhitnitsky, J. High Energy Phys. **08**, 013 (2002).

[16] R.H. Brandenberger, A.C. Davis, and M. Hindmarsh, Phys. Lett. B **263**, 239 (1991).

[17] R.H. Brandenberger and A.C. Davis, Phys. Lett. B **308**, 79 (1993).

[18] R.H. Brandenberger, A.C. Davis, and M. Trodden, Phys. Lett. B **335**, 123 (1994); R.H. Brandenberger, A.C. Davis, T. Prokopec, and M. Trodden, Phys. Rev. D **53**, 4257 (1996).

[19] G.R. Dvali, H. Liu, and T. Vachaspati, Phys. Rev. Lett. **80**, 2281 (1998).

[20] S. Alexander, R.H. Brandenberger, R. Easther, and A. Sornborger, hep-ph/9903254.

[21] L. Pogosian and T. Vachaspati, Phys. Rev. D **62**, 105005 (2000).

[22] R.H. Brandenberger, B. Carter, A.C. Davis, and M. Trodden, Phys. Rev. D **54**, 6059 (1996).

[23] R.H. Brandenberger, B. Carter, and A.C. Davis, Phys. Lett. B **534**, 1 (2002).

[24] H.B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).

[25] M. James, L. Perivolaropoulos, and T. Vachaspati, Nucl. Phys. **B395**, 534 (1993); Phys. Rev. D **46**, 5232 (1992); M. Goodband and M. Hindmarsh, Phys. Lett. B **363**, 58 (1995).

[26] W.B. Perkins, Phys. Rev. D **47**, 5224 (1993).

[27] A. Achucarro, R. Gregory, J.A. Harvey, and K. Kuijken, Phys. Rev. Lett. **72**, 3646 (1994).

[28] J. Garriga and X. Montes, Phys. Rev. Lett. **75**, 2268 (1995).