Testable anthropic predictions for dark energy

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In the context of models where the dark energy density ρ_D is a random variable, anthropic selection effects may explain both the "old" cosmological constant problem and the "time coincidence." We argue that this type of solution to both cosmological constant problems entails a number of definite predictions, which can be checked against upcoming observations. In particular, the anthropic approach predicts that the dark energy equation of state is $p_D = -\rho_D$ with a very high accuracy, and that the dark energy density is greater than the currently favored value $\Omega_D \approx 0.7$. Another prediction, which may be testable with an improved understanding of galactic properties, is that the conditions for civilizations to emerge arise mostly in galaxies completing their formation at low redshift, $z \approx 1$. Finally, there is a prediction which is not likely to be tested observationally: our part of the universe is going to recollapse eventually, but it will take more than a trillion years of accelerated expansion before this happens.

I. INTRODUCTION

The "old" cosmological constant problem—why don't we see the large vacuum energy density ρ_Λ which is expected from particle physics?—and the "time coincidence" problem—why do we live at the epoch when the dark energy component ρ_D starts dominating?—may find a natural explanation in models where ρ_D is a random variable. The idea is to introduce a dynamical dark energy component X whose contribution ρ_X varies from place to place, due to processes which occurred in the early universe. Then

$$\rho_D = \rho_\Lambda + \rho_X$$

will also vary from place to place, and the old cosmological constant problem takes a different form. The question is not why ρ_Λ is much smaller than η^4, where η is some high energy physics mass scale, such as the supersymmetry breaking scale $\eta \sim$ TeV, but why we happen to live in a place where ρ_Λ is almost exactly canceled by ρ_X. This line of enquiry is rather quantitative, since we can ask what is the probability for us to observe certain values of ρ_D $\sim 10^{-11} (\text{eV})^4$, or what is the probability for the time coincidence.

Explicit particle physics models for a variable ρ_X have been reviewed in [1]. Two examples which have been thoroughly discussed in the literature are a four-form field strength, which can vary through nucleation of membranes [2,3], and a scalar field with a very low mass [4,3]. Assuming one such mechanism, and using a theory of initial conditions such as inflation, one can calculate the "*a priori*" probability distribution $\mathcal{P}_*(\rho_D) d\rho_D$. This is defined as the fraction of comoving volume which at some fiducial initial time (which we conventionally take to be the time of recombination) had the value of the dark energy density in the interval $d\rho_D$. Inflation is also responsible for smoothing out the value of ρ_D over comoving distances much larger than the size of our presently observable universe.

By itself, $\mathcal{P}_*$ is not sufficient to calculate probabilities for our observations. Selection effects which bias the measurement of ρ_D must be included, and the most important one in this case is anthropic [5–9].[1] While $|\rho_D|$ may be very large in most places, there is nobody there to observe such extreme values. If $\rho_D > 0$, galaxy formation stops once the dark energy becomes dominant over the matter density. Some galaxies are seen at redshifts of order $z \sim 5$, but not much higher, indicating [9] that galaxies will not form in regions where $\rho_D \gtrsim (1 + z_{EG})^3 \rho_0$. Here, ρ_0 is the matter density at the present time t_0, and $z_{EG} \approx 5$ is the redshift at the time $t_{EG} \sim (1 + z_{EG})^{-3/2} t_0$ when the earliest galaxies formed. Also, for a negative ρ_D the universe recollapses on a time scale $t_D \sim |G\rho_D|^{-1/2}$, where G is Newton's constant. This time should be larger than the earliest time t_{EI} which is required for intelligence to develop [7,13]. Thus, observers will only exist within a tiny "anthropic range":

$$-(Gt_{EI}^2)^{-1} \lesssim \rho_D \lesssim (Gt_{EG}^2)^{-1}. \tag{1}$$

It should be noted that, aside from the above minimal requirements, anthropic selection includes all other ways in which ρ_D disfavors the existence of observers. For instance, in regions where $\rho_D < 0$, the matter density is larger than $|\rho_D|$ throughout the cosmic evolution. If $|\rho_D|$ is too large, all galaxies formed in that region will be very dense, and as a result, very inhospitable. This occurs also for a large $\rho_D > 0$, since galaxies must form before ρ_D starts dominating. We shall come back to this issue in Secs. III and V.

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[1] Anthropic selection effects associated with the possible variation of the amplitude of density fluctuations [10,11] and of the baryon to photon ratio [10,12] have also been discussed in the literature.

The selection effect can be implemented quantitatively by assuming the mediocrity principle, according to which our civilization is typical in the ensemble of all civilizations in the universe. The probability to find ourselves in a region with given values of ρ_D is thus given by [14]

$$d\mathcal{P}(\rho_D) \propto \mathcal{P}_*(\rho_D) n_{civ}(\rho_D) d\rho_D. \qquad (2)$$

Here, $n_{civ}(\rho_D)$ refers to the number of civilizations which will ever form per unit comoving volume in regions where the dark energy density was equal to ρ_D at the time of recombination.[2]

Needless to say, the determination of both factors in the right-hand side (RHS) of Eq. (2) leaves room for some uncertainties. However, we shall argue that there are reasons to be optimistic. If the distribution (2) is to explain both cosmological constant problems, then a number of rather generic predictions can be made, rendering these ideas very testable.

In the next section, we review the calculation of the prior probability distribution $\mathcal{P}_*(\rho_D)$. The anthropic factor $n_{civ}(\rho_D)$ is discussed in Sec. III. In the same section, we argue that the anthropic approach can succeed only if the conditions for civilizations to evolve arise mostly in galaxies formed at low redshifts, $z \sim 1$. The reason is quite simple. If most civilizations could form much earlier, then the cosmological constant could in fact be much larger than observed. In Sec. IV we discuss the equation of state of dark energy. In models where both cosmological constant problems are solved anthropically, the time variation of the vacuum energy ρ_D is generally slow on the Hubble scale. We argue that this condition is likely to be satisfied by excess, rather than marginally. This leads to the prediction that the dark energy equation of state is $p_D = -\rho_D$ to very good accuracy. In Sec. V we discuss the predictions for the dark energy density ρ_D, and for the Hubble parameter h. These follow from a quantitative determination of $\mathcal{P}(\rho_D)$, based on the standard Λ cold dark matter picture for structure formation. A key input in this picture is the amplitude of primordial density fluctuations, which is inferred from cosmic microwave background (CMB) measurements. This inference depends on the value of the Hubble parameter, and therefore our predictions have some dependence on h. A common feature of anthropic models is that the universe is bound to a big crunch once negative values of ρ_D are achieved. We elaborate on this prediction in Sec. VI. Finally, our conclusions are briefly summarized and discussed in Sec. VII.

II. THE PRIOR DISTRIBUTION

The first task in determining Eq. (2) is to estimate $\mathcal{P}_*$. The vacuum energy density is of order $\rho_\Lambda \sim \eta^4 \gtrsim (\text{TeV})^4$, and therefore ρ_D must have a natural range of variation of

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[2]As we shall argue, in models where both cosmological constant problems can be solved anthropically, ρ_D has not varied appreciably since the time of recombination, and therefore it can be treated as constant in time.

order η^4 or larger. Weinberg noted [9] that a function $\mathcal{P}_*(\rho_D)$ that varies smoothly on scales $\rho_D \sim \eta^4$, should behave as a constant in the utterly narrower interval (1)— unless of course, the function would happen to have a zero or a pole in that interval (which would be an utter coincidence). This led him to conjecture that for values of ρ_D in the anthropic range the prior probability would be constant,

$$\mathcal{P}_*(\rho_D) \approx \text{const.} \qquad (3)$$

Outside of this range the form of $\mathcal{P}_*$ is irrelevant, because the factor n_{civ} vanishes. Weinberg's conjecture is subject to verification. As mentioned in the Introduction, $\mathcal{P}_*$ is calculable, provided that the dynamics of ρ_X is known, and assuming an inflationary model which would determine its spatial distribution at the time of recombination. Analysis of explicit models shows that Eq. (3) is not automatically guaranteed [4], but it does seem to be satisfied in generic models.

There are basically two reasons [4,1] why a nonflat $\mathcal{P}_*$ may result from the process of randomization of ρ_D which occurs during inflation (this randomization is due to quantum diffusion in the case where X is a scalar field, or to nucleation of membranes in the case when X is a four-form). The first reason is the differential expansion induced by the dark energy component. During inflation, the expansion rate is determined by $H^2 = (8\pi G/3)(V_{inf} + \rho_D)$. Although ρ_D is very small compared with the inflationary potential V_{inf}, its effect may build up over time, in such a way that more thermalized volume is generated with high values of ρ_D. In this way, $\mathcal{P}_*(\rho_D)$ could be biased towards large values. Let us denote by $\tau(X,H)$ the characteristic time needed for the dynamics of X to sample (at a fixed point in space) all values of ρ_D within the anthropic range $(\Delta\rho_D)_{anth}$. The differential expansion is characterized by the parameter

$$q = (\Delta H)\tau = (4\pi G/3)H^{-1}(\Delta\rho_D)_{anth}\tau(X,H). \qquad (4)$$

If $q \gg 1$, then $\mathcal{P}_*$ is exponentially steep in the range of interest. This case is ruled out by observations, because it predicts a very large ρ_D, even after selection effects have been factored in. If $q \sim 1$, the distribution $\mathcal{P}_*$ may have a moderate dependence on ρ_D within the anthropic range. This dependence affects the position of the peak of the distribution for the observed values of ρ_D, Eq. (2), and hence it affects our predictions. While models of this sort are not ruled out, they require a very unnatural adjustment of parameters, since q is determined by a combination of rather different pieces of dynamics. Hence, we shall disregard this marginal possibility as nongeneric. Finally, there is a wide class of models where $q \ll 1$ is satisfied without any fine tuning [4,1], and hence we shall take this to be the generic case. Numerical simulations confirm that in this case the bias effect due to differential expansion is insignificant [15].

The second reason why $\mathcal{P}_*$ may be nonflat is the following. Even if the differential expansion is negligible, and the prior distribution for X is flat, this does not automatically guarantee that the prior for ρ_D will be flat, unless the relation between X and ρ_D is linear in the range of interest. Through this effect, it is possible to have a moderate variation of

$\mathcal{P}_*(\rho_D)$ within the anthropic range. But again, this would require a contrived adjustment of parameters and we shall dismiss this case as nongeneric (see also [16] for a discussion of this issue).

As an example, let us consider the case where $\rho_X = V(\phi)$ is the potential energy density of a scalar field ϕ,

$$\rho_D = \rho_\Lambda + V(\phi). \tag{5}$$

The field must change very slowly on a cosmological time scale, so that its potential energy behaves as an effective cosmological constant. This requires the slow-roll conditions [4]

$$|V'| \ll 10\rho_D/m_p, \quad |V''| \ll 10^2\rho_D/m_p^2 \tag{6}$$

to be satisfied up to the present time (when $\rho_D \sim \rho_0$, with ρ_0 the present matter density). The constraint $q \ll 1$ on the differential expansion yields [4]

$$V'^2 H^4/GV^3 \gg 1. \tag{7}$$

During inflation, the scalar field is randomized by quantum fluctuations, and at recombination it is distributed according to the "length" in field space,

$$\mathcal{P}_*(\phi)d\phi \propto d\phi. \tag{8}$$

Therefore,[3]

$$\mathcal{P}_*(\rho_D)d\rho_D \propto \frac{d\rho_D}{|V'(\phi)|}. \tag{9}$$

Thus, the flatness of the prior depends on how much V' changes in the anthropic range. As we shall see, variations in this range may occur, but they do not bias the probability distribution for ρ_D in any significant way, unless we adjust some parameters specifically for this purpose.

Consider a potential of the form

$$V(\phi) = \frac{1}{2}\mu^2\phi^2, \tag{10}$$

where $\mu^2 \rho_\Lambda < 0$, so that it is possible to have $|\rho_D|$ very small even if $|\rho_\Lambda|$ is large. Eqs. (6) lead to the condition [4]

$$|\mu| \ll 10^{-120}m_p^3|\rho_\Lambda|^{-1/2}. \tag{11}$$

Such a small mass parameter may seem unrealistic, but it can naturally arise, for instance, in a low energy effective theory with a suitable discrete symmetry [3] (for other proposals, see [1,16,17] and references therein). Note that Eq. (11) does not correspond to a fine tuning, but just to a strong suppression. The condition (7) translates into

$$|\mu| \gg H_0^3 H^{-2} \sim 10^{-169}m_p, \tag{12}$$

where H_0 is the present Hubble rate, and in the last step we have used $H \sim 10^{-7}m_p$, corresponding to a grand unified theory (GUT) scale of inflation. The conditions (11) and (12) leave very many orders of magnitude available for the parameter μ, and so fine tuning is not necessary. From Eq. (5),

$$\rho_D = \kappa(\phi - \phi_0) + \frac{\mu^2}{2}(\phi - \phi_0)^2, \tag{13}$$

where $\phi_0^2 = -2\rho_\Lambda/\mu^2$ and $\kappa = \mu^2\phi_0$. We are interested in the vicinity of $\rho_D = 0$, where it is easy to show from Eq. (9) that [4]

$$\mathcal{P}_*(\rho_D)d\rho_D \propto [1 + \mathcal{O}(\rho_D/\rho_\Lambda)]d\rho_D \approx d\rho_D. \tag{14}$$

Since $\rho_D \ll \rho_\Lambda$ in the anthropic range, the distribution is indeed flat to a very good accuracy.

For contrast, we may consider the "washboard" potential

$$\rho_D = \rho_\Lambda + \kappa\phi + M^4\sin(\phi/\eta), \tag{15}$$

where κ was given above and M and η are different mass scales. Let us assume that[4]

$$M^4 \ll H_0^2 m_p \eta \sim (\eta/m_p)\rho_0. \tag{16}$$

Then the field will typically be found away from the local minima, with a probability distribution

$$\mathcal{P}_*(\rho_D)d\rho_D = \frac{d\rho_D}{|\kappa + (M^4/\eta)\cos(\phi/\eta)|^{-1}}. \tag{17}$$

Both κ and M^4/η should be much smaller than $H_0^2 m_p$ in order to satisfy the slow roll condition. In the case $\kappa \gg M^4/\eta$, the distribution (17) is still flat, as in Eq. (3). In the opposite case, where $M^4/\eta \gg \kappa$, the *a priori* distribution can have a sizable variation within the anthropically allowed range. If $\eta \ll m_p$, this range is very wide in the field space, $\delta\phi \gtrsim \rho_0/\kappa \gg m_p$. This means that the oscillations in $\mathcal{P}_*$ will average out on scales much smaller than the anthropic range, and effectively we recover Eq. (3). Clearly, the only way to avoid this averaging effect is if $\eta \gtrsim m_p$, and

$$M^4 \sim (\Delta\rho_D)_{anth}. \tag{18}$$

The last equation is to ensure that a significant range of values of ϕ/η is sampled in the anthropic range $(\Delta\rho_D)_{anth} \lesssim 10^3\rho_0$, so that changes in the slope of the potential are appreciable. Otherwise the distribution for $\mathcal{P}_*$ will be almost flat. Thus, aside from the fact that the washboard potential is already a somewhat contrived example [16], Eq. (18) implies an otherwise unnecessary adjustment of the parameter M.

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[3]Note that near the points where $V'(\phi) = 0$, we have $\rho_D \approx A + B\phi^2$ and $V'(\phi) \sim \phi \sim (\rho_D - A)^{1/2}$, which is integrable. Hence, the zeroes of $V'(\phi)$ are not a concern.

[4]If $M^4 \gg H_0^2 m_p \eta$, the slow roll condition is not satisfied today and the field ϕ will be in any one of the local minima of the washboard. With some generic requirements on the inflationary parameters, the minima will have equal *a priori* probability within the anthropic range [1].

In what follows, we shall only consider models where there is no such *ad hoc* adjustment. In this sense, our predictions may not be completely inescapable, but they can be considered generic. The situation can be compared with the predictions of inflation that the density parameter is $\Omega = 1$ and the spectrum of density perturbations is nearly flat. It is certainly possible, in the context of inflation, to have an open universe with $\Omega < 1$, or to have a markedly non-flat spectrum of density perturbations. But to achieve this, additional parameters must be introduced and adjusted to the desired outcome.

III. THE ANTHROPIC FACTOR

We now consider the effect of the anthropic factor n_{civ} in Eq. (2). The physical situation is rather different for positive and negative ρ_D, so we consider these two cases separately.

For positive ρ_D, the main change introduced by n_{civ} is that the time of earliest galaxy formation t_{EG} in the anthropic range (1) is effectively replaced by the time at which the bulk of galaxy formation occurs. This is because a few early birds will not make a difference once we apply the principle of mediocrity. More precisely, we should take into consideration that the morphology of some galaxies could make them less suitable for the development of civilizations, and therefore

$$n_{civ}(\rho_D) = \int \, d\alpha \, \, n(\alpha, \rho_D) N_{civ}(\alpha). \qquad (19)$$

Here, α denotes the set of parameters characterizing the type of galaxy (e.g. its size, density, etc.), $n(\alpha, \rho_D)$ is the number density of such galaxies that form per comoving volume in regions characterized by ρ_D, and $N_{civ}(\alpha)$ is the number of civilizations per galaxy of type α. Suppose that the above integral receives a dominant contribution from galaxies of type α_G. Then

$$n_{civ}(\rho_D) \propto n(\alpha_G, \rho_D), \qquad (20)$$

and the relevant time for anthropic considerations is the time at which this type of galaxies form, which we shall denote by t_G. With the assumption of a flat prior $\mathcal{P}_*$, it was shown in [11,18] that the most probable value for a positive ρ_D is the one characterized by

$$t_D \sim t_G. \qquad (21)$$

This fact was used in order to explain the observed time coincidence

$$t_D \sim t_0. \qquad (22)$$

The last relation follows from Eq. (21), assuming that stars and civilizations develop on a timescale not much greater than t_G, and therefore t_G is comparable to t_0, defined as the time when most civilizations make their first determination of ρ_D.

Connected with the above discussion, there is a prediction of the anthropic approach, which can be checked by a combination of observations and theoretical analysis. In a not so distant future, our understanding of galactic evolution and morphology may improve to the point where we can tell with some confidence which galaxies are suitable for sustaining planetary systems similar to our own, where civilizations can develop. The anthropic approach to the cosmological constant problems (CCPs) predicts that the conditions for civilizations to emerge will be found mostly in galaxies that formed (or completed their formation) at a low redshift, $z \sim 1$.

In the standard cold dark matter cosmology, galaxy formation is a hierarchical process, with smaller objects merging to form more and more massive ones. We know from observations that some galaxies existed already at $z = 5$, and the theory predicts that some dwarf galaxies and dense central parts of giant galaxies could form as early as $z = 10$ or even 20. The fraction of matter bound in giant galaxies ($M \sim 10^{12} M_\odot$) at $z = 1$ ($\sim 20\%$) is somewhat less than that in objects of mass $\sim 10^9 M_\odot$ at $z = 3$, or in objects of mass $\sim 10^7 M_\odot$ at $z = 5$ [19]. If civilizations were as likely to form in early galaxies as in late ones, then Eq. (21) would indicate that, for a typical observer, the cosmological constant should start dominating at a redshift $z_G \gtrsim 5$. The corresponding dark energy density,

$$\rho_D \sim (1 + z_G)^3 \rho_0, \qquad (23)$$

would be far greater than observed. Clearly, the agreement becomes much better if we assume that the conditions for civilizations to emerge arise mainly in the types of galaxies which form at lower redshifts, $z_G \sim 1$.

We now point to some directions along which the choice of $z_G \sim 1$ may be justified. One problem with dwarf galaxies is that if the mass of a galaxy is too small, then it cannot retain the heavy elements dispersed in supernova explosions. Numerical simulations suggest that the fraction of heavy elements retained is $\sim 30\%$ for a $10^9 M_\odot$ galaxy and is negligible for much smaller galaxies [20]. The heavy elements are necessary for the formation of planets and of observers, and thus one has to require that the structure formation hierarchy should evolve up to mass scales $\sim 10^9 M_\odot$ or higher prior to the dark energy domination. This gives the condition $z_G \lesssim 3$, but falls short of explaining $z_G \sim 1$.

Another point to note is that smaller galaxies, formed at earlier times, have a higher density of matter. This may increase the danger of nearby supernova explosions and the rate of near encounters with stars, large molecular clouds, or dark matter clumps. Gravitational perturbations of planetary systems in such encounters could send a rain of comets from the Oort-type cloud towards the inner planets, causing mass extinctions.[5]

Our own Galaxy has definitely passed the test for the evolution of intelligence, and the principle of mediocrity suggests that most observers may live in galaxies of this

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[5]The cross section for disruption of planetary orbits is much smaller, and it would take a rather substantial increase of the density for this process to become statistically important. A.V. is grateful to David Spergel for a discussion of this issue.

type. Our Milky Way is a giant spiral galaxy. The dense central parts of such galaxies were formed at a high redshift $z \gtrsim 5$, but their discs were assembled at $z \sim 1$ or later [21]. Our Sun is located in the disc, at a distance ~ 8.5 kpc from the galactic center.[6] If this situation is typical, then the relevant epoch to use in Eq. (23) is the epoch $z_G \sim 1$ associated with the formation of discs of giant galaxies.

The above remarks may or may not be on the right track, but we emphasize once again that if CCPs have an anthropic resolution, then, for one reason or another, the evolution of intelligent life should require conditions which are found mainly in giant galaxies, which completed their formation at $z_G \sim 1$.

In order to estimate $n(\alpha_G, \rho_D)$ in Eq. (20), we shall need a simple quantitative criterion to specify the relevant type of galaxies. The most important parameter characterizing a galaxy is its mass M. For the Milky Way it is $M_{MW} \sim 10^{12} M_\odot$ [23], and the above discussion suggests that we identify the relevant galaxies with gravitationally bound halos of this mass. (Note that this is also the typical mass of L_* galaxies, which contain most of the luminous stars in the Universe.) It should be recognized, however, that the choice of this characteristic mass scale is somewhat uncertain, so we shall illustrate how our results are affected by choosing a larger or a smaller mass.

Our Galaxy is a member of the local group cluster, whose mass has been estimated as [24] $M_{LG} \sim 4 \times 10^{12} M_\odot$. It is conceivable that the gas captured in this cluster is later accreted onto the member galaxies and thus affects the properties of their disks. There seems to be no justification to consider larger mass objects, and we shall regard M_{LG} as an upper bound on the potentially relevant mass scales. On the lower mass end, we shall use $M \sim 10^{11} M_\odot$, which is roughly the mass of the bright part of our Galaxy, up to ~ 10 kpc from the center. (We note that M_{MW} is probably a more reasonable choice, because the properties of the disk depend on the total mass of the halo [25].)

We now consider negative ρ_D. The scale factor of a universe filled with nonrelativistic matter and dark energy with $\rho_D < 0$ is given by

$$a(t) = \sin^{2/3}\left(\frac{t}{t_D}\right), \qquad (24)$$

where $t_D \equiv (1/6\pi G |\rho_D|)^{1/2}$. The matter density ρ_M initially decreases while the universe expands, but at $t = \pi t_D/2$, when it reaches the value $\rho_M = -\rho_D$, the universe stops its expansion and starts recontraction. The matter density grows in the contracting phase, and thus $\rho_M \gtrsim |\rho_D|$ throughout the evolution. The structure formation in a universe with a negative ρ_D proceeds as usual until $t \sim t_D$, but then the growth of density perturbations accelerates during the contraction, so that all overdensities collapse to form bound objects prior to the big crunch. For $t_D \gtrsim t_0$, giant galaxies will form at about the same time as they did in our part of the universe and will have similar properties (with a possible caveat indicated below). However, for $t_D \ll t_0$ halos of the galactic size will be forced to collapse at a much earlier time $t \sim t_D$, and their density will therefore be much higher than that of our Galaxy. This would probably make such halos unsuitable for life.

These considerations suggest that the anthropic factor effectively constrains t_D to be in the range

$$t_D \gtrsim t_0 \qquad (25)$$

for both positive and negative ρ_D. There is, however, an additional factor that could make negative ρ_D less probable. For $\rho_D > 0$, structure formation effectively stops at $t > t_D$, and the existing structures evolve more or less in isolation. This may account for the fact that disks of giant galaxies take their grand-design spiral form only relatively late, at $z \sim 0.3$. The disks are already in place at $z \sim 1$, but they have a very unsettled, irregular appearance [21]. On the other hand, for $\rho_D < 0$ the clustering hierarchy only speeds up at $t > t_D$, and quiescent disks which may be necessary for the evolution of fragile creatures like ourselves may never be formed.

Another factor to consider is the characteristic time t_I needed for intelligence to develop. For positive ρ_D, this factor is unimportant, since the time after the dark energy domination is practically unlimited, but for negative ρ_D the available time is bounded by $t < \pi t_D$, and the effect of t_I requires a closer examination.

We first note that $t_I \ll t_0$ is unlikely, since then it is not clear why it took so long for intelligence to develop on Earth. (The total time of biological evolution, from the origin of life on Earth till present, is estimated at $\sim 3.5 \times 10^9$ yr.) For $t_I \gtrsim t_0$, we note that the main sequence lifetime of stars believed to be suitable to harbor life is $t_\star \sim (5-20) \times 10^9$ yr $\sim t_0$ (see [11] for a discussion of this point). If $t_I \gg t_0 \sim t_\star$, most of these stars will explode as red giants before intelligence has a chance to develop. Carter [6] has argued that this is the most likely scenario.[7] In this case, the number N_{civ} is suppressed by a factor $\sim \min\{t_\star, t_D\}/t_I \sim t_0/t_I$, where we have used Eq. (25) in the last step. For positive ρ_D, the suppression is by a factor $\sim t_\star/t_I$, which is of the same order of magnitude.

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[6]It has been noted [22] that this distance is close to the corotation radius, where the orbital velocity of the stars coincides with the rotational velocity of the spiral pattern. In other words, the motion of the Sun relative to the spiral arms is rather slow, and as a result, the periods between spiral arm crossings are rather long ($\sim 10^8$ yr). Spiral arms are the primary sites of supernova explosions. They are also rich in giant molecular clouds, and are therefore very hazardous to life. It has been argued in [22] that spiral arm crossings are responsible for the major mass extinctions observed in the fossil record. Then one expects that habitable planetary systems are to be found mainly in the vicinity of the corotation radius, since mass extinctions at a rate much greater than once in 10^8 yr may be too frequent for intelligent life to evolve. (Note that it took us 6.5 $\times 10^7$ yr to evolve since the last great extinction.)

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[7]The coincidence $t_I \sim t_\star$ is unlikely, since the evolution of life and evolution of stars are governed by completely different processes.

We conclude that the precise value of t_I has little effect on the relative probability of positive and negative ρ_D. If the accelerated clustering hierarchy is detrimental for life, then the probability for negative ρ_D is suppressed; otherwise the two signs of ρ_D are equally likely. In either case, we should not be surprised that ρ_D is positive in our part of the universe. In the following sections we shall focus on the positive values of ρ_D.

IV. PREDICTION FOR THE EQUATION OF STATE

A generic prediction of models where both CCP's are solved anthropically is that the equation of state of dark energy is given by $p_D = w\rho_D$, with

$$w = -1 \pm 10^{-5}. \tag{26}$$

The error bars correspond to the precision to which the observable universe can be approximated by a homogeneous and isotropic model. In models where ρ_X is the energy density of a four-form field, this equation of state is guaranteed by the fact that the four-form energy density is a constant and can only change by the nucleation of branes (other than that, it behaves exactly like an additional cosmological constant). If ρ_X is a generic scalar field potential, the slow roll conditions (6) are likely to be satisfied by excess, by many orders of magnitude, rather than marginally. For instance, for the quadratic potential (10), these conditions imply the constraint (11). It would be contrived to arrange for the condition to be satisfied marginally, since the whole point of the present approach is to have ρ_Λ canceled regardless of its precise value (which is not known to us even by order of magnitude). If the slow roll conditions are satisfied by excess by just more than three orders of magnitude, then the kinetic energy of the scalar field will be less than its potential energy by more than six orders of magnitude, and Eq. (26) follows.

There are certainly models for dark energy, some of them with anthropic input, were Eq. (26) is not satisfied. For instance, Kallosh and Linde [13] recently considered a supergravity model where the time coincidence problem is solved anthropically, and where Eq. (26) does not hold. However, their model does not solve the old CCP, since it is assumed that the cosmological constant vanishes in the observable matter sector due to some unspecified mechanism. Likewise, Eq. (26) does not hold in the usual quintessence models [26], which have no anthropic input at all, but which do not address the CCP's [1,16], or in models of k essence [27], where only the time coincidence is partially addressed.

A possibility worth discussing is the case of models where the slow roll parameters are themselves random fields. Consider, for instance, the following model:

$$\rho_D = \rho_\Lambda + \mu^2(\psi)\phi^2. \tag{27}$$

If the probability distribution for the new scalar field ψ were such that all values of μ^2 are equiprobable, then one might imagine that the order of magnitude of μ^2 would be such that the slow roll conditions would be marginally satisfied. However, this new field must also be a light field and hence its distribution is calculable. It is then easy to show that the marginal values will not be preferred generically. We can actually consider a more general form of the potential for ψ and ϕ,

$$\rho_D = \rho_\Lambda + V(\psi, \phi). \tag{28}$$

Around any point (ψ_0, ϕ_0) on the curve γ defined by $V(\psi, \phi) = -\rho_\Lambda$, the potential can be approximated by a linear function of the fields. Moreover, we can always rotate coordinates in field space so that ψ is directed along γ, and ϕ is orthogonal to it,

$$\rho_D \approx V_\phi(\psi_0, \phi_0)(\phi - \phi_0).$$

Here V_ϕ is the gradient of the potential at that point. During inflation, both fields ψ and ϕ are randomized by quantum fluctuations. Hence, the prior probability distribution is given by the area in field space $\mathcal{P}_* d\psi d\phi \propto d\psi d\phi$, which leads to

$$\mathcal{P}_*(\rho_D) d\rho_D = \left[\int_\gamma \frac{d\psi}{|V_\phi|} \right] d\rho_D. \tag{29}$$

Along the curve γ, the values of V_ϕ that will carry more weight are those for which the slope is smaller, since for equal intervals of ρ_D they correspond to larger portions of field space. Thus, given a model where the slope of the potential is variable, smaller values of the slope are preferred *a priori*, and there is no reason to expect that the slow roll conditions should be satisfied only marginally.

V. PREDICTIONS FOR Ω_D AND H

Currently favored values for the dark energy density and for the Hubble parameter are $\Omega_D \approx .7$ and $h \approx .7$ [28,29], both with error bars of the order of 10%. While observations are not very accurate, we would like to challenge the status quo and boldly use the anthropic approach to the CCP's to make predictions for these two parameters. As we shall see, this approach predicts that Ω_D is likely to be somewhat higher, and that h is likely to be smaller than those currently favored values.

The basic reason why we expect Ω_D to be larger is the following [14,30]. The growth of density fluctuations in a universe with a positive cosmological constant effectively stops at the redshift z_D when the cosmological constant starts dominating. This is given by $(1 + z_D) \sim (\Omega_D/\Omega_M)^{1/3}$, where $\Omega_M = 1 - \Omega_D$ is the matter density parameter. According to Eq. (21), we expect $z_D \sim z_G$, where z_G is the epoch when the relevant galaxies were formed. With $z_G \sim 1$, this corresponds to $(\Omega_D/\Omega_M) \sim 8$, which in turn implies $\Omega_D \sim .9$. (For $z_G > 1$, we would obtain an even higher value for ρ_D.) This prediction can be made more quantitative [31,32] by using the distribution (2). As we shall see, the precise predictions depend not only on Ω_D but also on h.

Throughout this section, we shall assume that $\rho_D > 0$ as part of our prior. In a universe filled with pressureless matter and with a dark energy component $\rho_D > 0$, the scale factor behaves as

$$a(t) = \sinh^{2/3}\left(\frac{t}{t_D}\right),$$

where $t_D \equiv (1/6\pi G\rho_D)^{1/2}$. A primordial overdensity will eventually collapse, provided that its value at the time of recombination is larger than a certain value δ_c^{rec}. In the spherical collapse model, this is estimated as $\delta_c^{rec}(\rho_D) = 1.13x_{rec}^{1/3}$, where $x_{rec} = x(t_{rec})$ [33]. Here, we have introduced the variable

$$x(t) \equiv \frac{\Omega_D(t)}{\Omega_M(t)} = \sinh^2\left(\frac{t}{t_D}\right). \quad (30)$$

The number of galaxies $n(M,\rho_D)$ of mass M that will form per unit comoving volume in a region characterized by the value ρ_D of the dark energy density, is proportional to the fraction of matter that eventually clusters into this type of galaxies. In the Press-Schechter approximation [33,34], this is given by

$$n_{civ}(\rho_D) \propto n(M,\rho_D) \propto \mathrm{erfc}\left(\frac{\delta_c^{rec}(\rho_D)}{\sqrt{2}\sigma_{rec}(M)}\right). \quad (31)$$

Here, erfc is the complementary error function, and $\sigma_{rec}(M)$ is the dispersion in the density contrast at the time of recombination t_{rec}. As argued in the preceding section, we shall assume that most civilizations are formed in galaxies characterized by a mass $M \sim M_{MW} \approx 10^{12} M_\odot$ (although we shall also consider slightly larger and smaller masses).

The factor n_{civ} depends on the parameter σ_{rec}, which in turn depends on the amplitude of density perturbations generated during inflation. The value of σ_{rec} can be inferred from the normalization of CMB anisotropies, but for this task, both the present value of Ω_D and the value of the Hubble parameter h would be needed. Since these are the parameters we wish to make predictions about, it would be somewhat contrived to use them at this point to make an inference about σ_{rec}.

Another factor to consider is that σ_{rec} may be different in distant regions of the universe (where, as a consequence, galaxies would form earlier or later). In models where the inflaton field has only one component, the value of σ_{rec} is the same in all regions of the universe. However, if the inflaton field has more than one component, the amplitude of density perturbations depends on the path followed by the inflaton on its way to the minimum of the potential. In such models, it is possible for σ_{rec} to vary over distances much larger than the presently observable universe.

To make our discussion sufficiently general, we shall consider that σ_{rec} is itself a random variable with unspecified prior. This prior may be determined by processes occurring during inflation, or it may just reflect our ignorance of the actual value of the fixed parameter σ_{rec}. Then, Eq. (2) is generalized to

$$d\mathcal{P}(\rho_D,\sigma_{rec}) \approx n_{civ}\mathcal{P}_*(\rho_D,\sigma_{rec})d\rho_D d\sigma_{rec}. \quad (32)$$
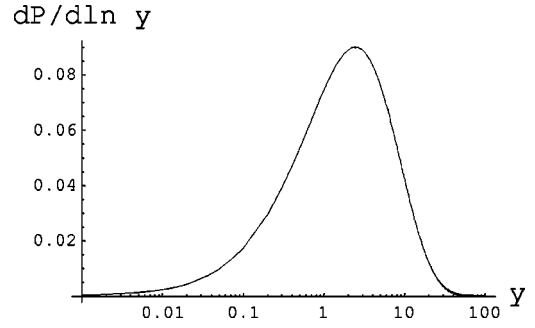

FIG. 1. The distribution (34).

In this context, the generic expectation that the prior does not depend on ρ_D in the anthropic range [see Eq. (3)], translates into

$$\mathcal{P}_*(\rho_D,\sigma_{rec}) \approx \mathcal{P}_*(\sigma_{rec}).$$

Substituting Eq. (31) into Eq. (32), we have

$$d\mathcal{P}(\rho_D,\sigma_{rec}) \approx \frac{1}{\sqrt{\pi}}\left(\frac{.80}{\sigma_{rec}}\right)^3 \mathrm{erfc}\left(\frac{.80x_{rec}^{1/3}}{\sigma_{rec}}\right)$$
$$\times \mathcal{P}_*(\sigma_{rec})dx_{rec}d\sigma_{rec}, \quad (33)$$

where we have used that $\Omega_M(t_{rec}) \approx 1$ in all regions of interest, so that $d\rho_D \propto dx_{rec}$. Introducing $y = x_{rec}\sigma_{rec}^{-3}$, the change of variables $(x_{rec},\sigma_{rec}) \to (y,\sigma_{rec})$ produces a Jacobian proportional to σ_{rec}^3, and we have $d\mathcal{P}(y,\sigma_{rec}) \approx f(y)P_*(\sigma_{rec})dy d\sigma_{rec}$, where $f(y)$ does not depend on σ_{rec}. Integrating over σ_{rec} leads to the normalized distribution

$$d\mathcal{P}(y) = (.80)^3\pi^{-1/2}\mathrm{erfc}(.80y^{1/3})y\ d\ln y, \quad (34)$$

which is uncorrelated with σ_{rec}.

The variable y can be expressed in terms of observable quantities, as we shall see below, and from Eq. (34) we should expect $y \sim 1$ by order of magnitude (see Fig. 1). More precisely, we expect $y > .79$ with probability

$$P(y > .79) = .68 \quad (1\sigma\ \mathrm{C.L.}), \quad (35)$$

and $y > .07$ with probability

$$P(y > .07) = .95 \quad (2\sigma\ \mathrm{C.L.}). \quad (36)$$

We shall denote these two equations as the 1σ and 2σ confidence level predictions for y. Let us now show how these translate into confidence level curves for the expected values of the parameters Ω_D and h. Here, and in what follows, Ω_D will denote the *present* value of the dark energy density parameter in our observable universe.

Let us first express the "observed" value of y, which we shall denote as y_0, in terms of Ω_D and h. The density contrast at present is given by $\sigma_0 = G(x_0,x_{rec})\sigma_{rec}$, where, assuming $z_{rec} \gg 1$, the growth factor is given by [32] $G(x_0,x_{rec}) = x_{rec}^{-1/3}F(\Omega_D)$, with

$$F(\Omega_D) = \frac{5}{6}\Omega_D^{-1/2} \int_0^{\Omega_D/(1-\Omega_D)} \frac{dw}{w^{1/6}(1+w)^{3/2}}. \quad (37)$$

Therefore,

$$y_0 = \left[\frac{F(\Omega_D)}{\sigma_0}\right]^3. \quad (38)$$

The linearized density contrast at present σ_0 can be inferred from measurements of CMB temperature anisotropies, as described e.g. in [32,35]. Since the spectrum is expressed as a function of wavelength, the mass scale has to be converted into a length scale. A halo of mass M corresponds to a comoving radius $R(M) = (3M/4\pi\rho_0)^{1/3}$. The mean matter density of the universe is given by $\rho_0 = 1.88 \times 10^{-29}\Omega_M h^2$ g/cm^3, which leads to

$$R(M) = .98h^{-1}\Omega_M^{-1/3}\left(\frac{M}{10^{12}M_\odot}\right)^{1/3} \text{Mpc}.$$

Assuming an adiabatic primordial spectrum of scalar density perturbations, characterized by a spectral index n, we have

$$\sigma_0(R) = (c_{100}\Gamma)^{(n+3)/2}\delta_H K^{1/2}(R). \quad (39)$$

Here, $c_{100} = 2.9979$ is the speed of light in units of 100 km s^{-1} and

$$\Gamma = \Omega_M h \exp[-\Omega_b(1+\sqrt{2h}\Omega_M^{-1})]$$

is the so-called shape parameter, with Ω_b the density parameter in baryons. For numerical estimates, we shall take $\Omega_b h^2 \approx .02$. The dimensionless amplitude of cosmological perturbations inferred from the COBE DMR experiment is given by [35,36]

$$\delta_H = 1.91 \times 10^{-5}\frac{\exp[1.01(1-n)]}{\sqrt{1+r(.75-.13\Omega_D^2)}}\Omega_M^{-.80-.05\ln\Omega_M}$$

$$\times[1-.18(1-n)\Omega_D - .03r\Omega_D]. \quad (40)$$

The parameter r denotes the ratio of tensor to scalar amplitudes. Note that the effect of tensors is to make δ_H a bit smaller (although not very significantly). Finally,

$$K(R) = \int_0^\infty q^{(n+2)}T^2(q)W^2(q \ \Gamma \ hR \ Mpc^{-1})dq,$$

where the transfer function is given by $T(q) = (2.34q)^{-1}\ln(1+2.34q)[1+3.89q+(16.1q)^2 +(5.46q)^3+(6.71q)^4]^{-1/4}$ and the window function is given by $W(u) = 3u^{-3}(\sin u - u\cos u)$.

Substituting Eq. (39) in Eq. (38), and using $\Omega_D + \Omega_M = 1$, we obtain the function $y_0 = y_0(\Omega_D, h)$. Contour lines of this function, corresponding to the 1σ and 2σ predictions represented by Eqs. (35), (36), are plotted in Fig. 2, assuming that the dominant contribution to n_{civ} is in galaxies of mass $M = M_{MW} = 10^{12}M_\odot$ (thick solid lines). We also consider the predictions for different choices of the mass, as discussed in
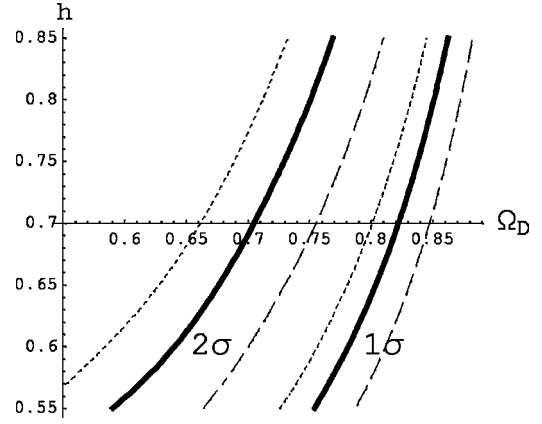

FIG. 2. Contours of the function $y_0(\Omega_D, h)$ given in Eq. (38), corresponding to the 1σ (lower curves) and 2σ (upper curves) predictions represented by Eqs. (35), (36). The excluded region lies to the left of the curves. The thick solid lines assume that the dominant contribution to n_{civ} is in galaxies of mass $M = M_{MW} = 10^{12}M_\odot$. For comparison, we show the predictions for different choices of the mass. The short dashed curves correspond to the mass of the local group $M_{LG} = 4\times10^{12}M_\odot$, and the long dashed curves correspond to the mass of the bright inner part of our galaxy $M = 10^{11}M_\odot$. A scale invariant spectrum of density perturbations is assumed.

Sec. III. The short dashed curves correspond to the mass of the local group $M_{LG} = 4\times10^{12}M_\odot$, and the long dashed curves correspond to the mass of the bright inner part of our galaxy $M = 10^{11}M_\odot$. The effect of a tilt in the spectral index is plotted in Fig. 3. Both of these figures ignore the effect of tensor modes in the normalization (40). Tensor modes tend to lower the value of δ_H, and hence they tend to make the bounds somewhat less stringent. The effect, however, is not dramatic. Even for r as large as .5, the effect on the curves is comparable to the effect of lowering the spectral index by .05.

Expressions similar to Eqs. (34)–(38) were already contained in the exhaustive analysis of the problem given by
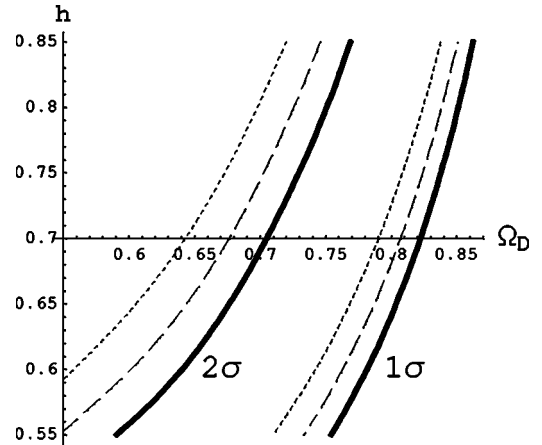

FIG. 3. Effect of a tilt in the spectral index of density perturbations. As in Fig. 2, the thick solid lines correspond to a scale invariant spectrum $n = 1$, and a mass $M = M_{MW} = 10^{12}M_\odot$. The long dashed line and the short dashed lines correspond to tilted spectra, with $n = .95$ and $n = .9$ respectively.

Martel, Shapiro and Weinberg (MSW) in [32], where σ_{rec} was treated as a fixed parameter. However, our use of these expressions is somewhat different. MSW noted that the existing observations indicate a value of $\Omega_D \sim 0.6$–0.7 and used Eq. (33), with $h=0.7$ to show that this range corresponds to probabilities from 2% to 12%, depending on the values chosen for the galactic scale M and the spectral index of perturbations n. They concluded that "anthropic considerations do fairly well as an explanation of a cosmological constant with $[\Omega_D]$ in the range 0.6–0.7." However, one cannot help but feel disappointed by the somewhat low values of the probabilities.

Our approach here is that anthropic models should be used as any other models—to make testable predictions. Thus, the goal is not so much to explain the value of Ω_D after it is determined by observations, but to predict that value at a specified confidence level. The contour lines in Figs. 2,3 indicate the 1σ and 2σ predictions of the model. If $M \sim M_{MW}$ proves to be the relevant mass giving the dominant contribution to n_{civ}, then the currently favored model with $\Omega_D \approx .7$ and $h \approx .7$ is virtually excluded by the anthropic approach at the 2σ level. Instead, this approach favors lower values of h and higher values of Ω_D.

These predictions can be turned around. If the values $\Omega_D \lesssim 0.7$, $h \gtrsim 0.7$ are confirmed by future measurements, then our model will be ruled out at a 95% confidence level, again assuming $M \sim M_{MW}$ and a scale invariant spectrum. For a tilted spectrum, slightly lower values of Ω_D are allowed at the same confidence level. The observational situation at the time of this writing is far from being clear. CMB and supernovae measurements yield [28,37] $\Omega_D \approx 0.7$, while the observations of galaxy clustering give [38] $\Omega_M = 0.18 \pm 0.8$, and thus $\Omega_D \approx 0.8$.

VI. THE FUTURE OF THE UNIVERSE

We finally discuss the anthropic prediction which is not likely to be tested any time soon. In all anthropic models, ρ_D can take both positive and negative values, so the observed positive dark energy will eventually start decreasing and will turn negative, and our part of the universe will recollapse to a big crunch.

To be specific, we shall consider a scalar field model with a very flat potential. In the anthropic range (1), the potential can be approximated as a linear function,

$$V(\phi) \approx -V_0' \phi, \tag{41}$$

where V_0' is a constant and we have set $\phi=0$ at $V=0$. Once the dark energy dominates, the evolution is described by the usual slow roll equations

$$3H\dot{\phi} = V_0', \tag{42}$$

$$H^2 = \frac{8\pi}{3m_p^2} V_0' \phi, \tag{43}$$

where $H=\dot{a}/a$ and $a(t)$ is the scale factor. The solution of Eqs. (42), (43) is

$$\phi(t) = -\phi_*[1-(t/t_*)]^{2/3}, \tag{44}$$

$$a(t) = \exp[4\pi m_p^{-2}(\phi_*^2 - \phi^2(t))], \tag{45}$$

where $-\phi_*$ is the present value of ϕ and

$$t_* = 8\pi t_D(\phi_*/m_p)^2 \tag{46}$$

is the time from the present to the beginning of recollapse.

The slow roll condition (6) implies that $\phi_* \gtrsim m_p$. As we discussed in Sec. IV, we do not expect this condition to be only marginally satisfied, and thus $\phi_* \gg m_p$. Then it follows from Eqs. (46) and (45) that $t_* \gg 8\pi t_D$ and therefore we should expect our region of the universe to undergo accelerated expansion for at least another trillion years before recollapse.[8]

The slow roll approximation breaks down at $\phi \sim -m_p$, so the above equations cannot be used to describe the evolution at $\phi>0$, where the potential becomes negative. A general analysis of models with negative potentials has been given in [39], where it is shown that at $\phi \gg m_p$ the dynamics becomes dominated by the kinetic energy of the field, $\dot{\phi}^2 \gg |V(\phi)|$. The corresponding evolution is described by

$$\phi(t) = \frac{m_p}{\sqrt{6\pi}}\ln(t_c-t) + \text{const}, \tag{47}$$

$$a(t) \propto (t_c-t)^{1/3}, \tag{48}$$

where t_c is the time of the big crunch. The linear approximation (41) for the potential breaks down at sufficiently large ϕ, but in this regime the form of the potential is unimportant and Eqs. (47), (48) still apply.

During the dark energy dominated expansion, the ordinary nonrelativistic matter is diluted by the exponential factor (45). When the contraction starts, the density of matter begins to grow as $\rho_M \propto (t_c-t)^{-1}$. However, the kinetic energy of the field ϕ grows much faster, $\dot{\phi}^2 \propto (t_c-t)^{-2}$, and thus ordinary matter forever remains a subdominant component of the universe.

VII. CONCLUSIONS

We now summarize the predictions that follow from the anthropic approach to the CCP's.

(1) The dark energy equation of state is predicted to be that of the vacuum,

$$p_D = w\rho_D, \tag{49}$$

where $w=-1$ with a very high accuracy. This distinguishes the anthropic models we discussed here from other approaches, such as quintessence [26] or k essence [27].

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[8]This is in contrast with the model of Kallosh and Linde [13] discussed in Sec. IV, where the universe is expected to recollapse within 10–20 billion years.

(2) The anthropic predictions for the dark energy density Ω_D and for the Hubble parameter h are given in Figs. 2 and 3 of Sec. V.[9] We show the areas in the $\Omega_D - h$ plane that are excluded at 1σ and 2σ confidence levels. The excluded areas depend on the assumed galactic mass M and on the spectral index n of the density fluctuations. For $M = M_{MW} = 10^{12} M_\odot$ the currently popular values $\Omega_D = 0.7$, $h = 0.7$ are marginally excluded at 2σ confidence level for a scale invariant spectrum $n = 1$. Lowering the spectral index relaxes the bounds somewhat. For $h > 0.65$ and $n > .95$, the 1σ prediction is $\Omega_D > 0.79$. These anthropic constraints get weaker when the relevant mass scale M is increased. For example, with $M = 4 \times 10^{12} M_\odot$ a value as low as $\Omega_D = 0.63$ is still allowed at the 2σ level for a scale invariant spectrum. The 1σ prediction in this case is $\Omega_D > 0.78$ (for $h = 0.65$).

(3) Conditions for intelligent life to evolve are expected to arise mainly in giant galaxies that form (or complete their formation) at low redshifts, $z_G \lesssim 1$.

(4) The accelerated expansion will eventually stop and our part of the universe will recollapse, but it will take more than a trillion years for this to happen. Of course, this prediction is not likely to be tested anytime soon.

The above predictions apply to models where both CCP's are solved anthropically. For comparison, we may consider other models. For instance, it is conceivable that a small value of the cosmological constant will eventually be explained within the fundamental theory. (We note the interesting recent proposal by Dvali, Gabadadze and Shifman [40] in this regard.) Even then, the coincidence problem will still have to be addressed. One possibility is that ρ_D is truly a constant, while the amplitude of the density fluctuations σ_{rec} is a stochastic variable. With some mild assumptions about the prior probability distribution $\mathcal{P}_*(\sigma_{rec})$, it can be shown [1] that most galaxies are then formed at about the time of vacuum domination. In this class of models, predictions (1) and (3) still hold, while the other two predictions no longer apply.

Another possibility has been recently discussed by Kallosh and Linde [13]. They assumed an M-theory inspired potential

$$V(\phi) = \Lambda(2 - \cosh \sqrt{2}\phi) \qquad (50)$$

with a stochastic variable Λ. An interesting property of this potential is that its curvature is correlated with its height (at $\phi = 0$). As a result, the universe tends to recollapse within a few Hubble times after the dark energy comes to dominate. Assuming that other contributions to the vacuum energy are somehow cancelled (that is, that the old CCP is solved by some unspecified mechanism), Kallosh and Linde argue that the coincidence $t_D \sim t_I$ is to be expected, where t_I is the time it takes intelligent life to evolve (they assume it to be $\sim 10^{10}$ yr). Predictions (1)–(3) are not applicable to this

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[9]These predictions are implicit in the earlier analysis by Martel, Shapiro and Weinberg [32].

model. The model does predict recollapse of the universe, but the corresponding timescale ($\sim 10^{10}$ yr) is much shorter than the anthropic prediction (4).

Here we have made predictions which apply to all presently known mechanisms for generating a range of values of Λ, and which allow for a solution of both CCP's. In principle, the predictions are vulnerable to the discovery of new mechanisms. For instance, the prior distribution for Λ might not be flat for some particular new mechanisms. However, from the general arguments given by Weinberg [9], this possibility seems quite unlikely. Likewise, our prediction for the equation of state of dark energy may also be vulnerable. Here we have restricted attention to the case where the random values are generated by four-forms or light fields which fluctuate during inflation. In the context of inflationary models, it seems hard to imagine anything different: either the dark energy takes discrete values, in which case it has the equation of state of a cosmological constant, or it behaves as a light field (otherwise its value would change too fast). Under these hypotheses, our prediction for the equation of state follows. It is conceivable that the theory of initial conditions which randomizes Λ is not inflation. In this case, one is free to speculate that a different equation of state might be possible. For example, in a model with several light fields, as in Eq. (28), the equation of state could be different from $p_D = -\rho_D$ if the prior distribution did not favor small slopes of the potential (as it does in the case of inflation).

Anthropic arguments are sometimes perceived as hand-waving, unpredictive and unfalsifiable lore, of questionable scientific validity. In our view, the results presented in this paper should dispel this notion. Here, we have used the anthropic approach to make several quantitative predictions, some of which may soon be checked against observations. It should also be emphasized that, for the particular case of dark energy, there are at present no alternative theories explaining both CCP's, or making generic predictions of comparable accuracy.

The present bound on the equation of state parameter w from the CMB and supernovae measurements is [41] $w < -0.7$, which is consistent with the anthropic prediction of $w = -1$. The value of $w = -1$ is usually associated with a plain cosmological constant. However, if in addition to this equation of state, observations confirm some of the other predictions presented above, this may be taken as an indication that the dark energy is dynamical. Thus, a better understanding of structure formation and galactic evolution may in fact reveal a crucial property of dark energy, with important implications for particle physics.

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