The would-be Majoron in *R*-parity-violating supersymmetry

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In lepton-number-violating supersymmetric models, there is no natural choice of basis to distinguish the down-type Higgs and lepton superfields. We employ basis-independent techniques to identify the massless Majoron and associated light scalar in the case of spontaneously broken lepton number (L). When explicit L violation is added, these two scalars can acquire masses of the order of the electroweak scale and can be identified as massive sneutrinos.

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I. INTRODUCTION

Recent data that exhibit neutrino mixing phenomena imply that the lepton sector of the standard model must be extended [1]. The simplest extension involves adding right-handed neutrinos, and then tuning the neutrino masses to be less than $\mathcal{O}(1\text{ eV})$ (if neutrinos are Dirac fermions) or by invoking the seesaw mechanism (if neutrinos are Majorana fermions). In low-energy supersymmetric models, it is possible to introduce neutrino masses in a phenomenologically acceptable way without adding right-handed neutrinos. One simply allows for renormalizable terms that violate lepton number (L), while imposing baryon number (B) invariance. This can be achieved by replacing R-parity of the minimal supersymmetric model (MSSM) with a $\mathbf{Z}_3$ triality [2]. This model provides an alternative framework for neutrino masses. Eventually, one must try to understand why the L-violating parameters of the model are small enough to yield neutrino masses at the observed level [3].

In the B-conserving, L-violating alternative to the MSSM, the L-violating terms are explicit. One can also generate L violation directly in the MSSM if one of the sneutrinos acquires a vacuum expectation value [4]. In the latter case, L is spontaneously broken, which implies that a massless Goldstone boson, the Majoron, must exist in the spectrum [5]. Since the sneutrino is an electroweak doublet, one can show that the spectrum must also include a very light CP-even scalar partner to the CP-odd Majoron [6].[1] Models of this type are excluded since the decay of the Z into the Majoron and its CP-even scalar partner is not observed [7,6]. Thus, any viable L-violating supersymmetric model whose field content is identical to that of the MSSM must possess explicit L-violating terms. There are also ways to extend the model of spontaneous L-violating supersymmetry by adding additional chiral superfields (including electroweak singlets) such that the Majoron is dominantly a singlet and all other scalar masses lie above m_Z [8]. However, such models lie outside the scope of this paper.

We consider the most general L-violating low-energy supersymmetric model, with the MSSM field content. In addition to the effects of the explicit L-violating terms, one must also consider the L-violating effect that depends on the vacuum expectation values of the sneutrino fields. Of course, the latter is basis-dependent, and it is often convenient to define the Higgs field such that the orthogonal physical sneutrino fields have no vacuum expectation value. However, other choices are possible, which suggests that the model can be viewed as a model of spontaneously broken lepton number with additional explicit L-violating terms. Since models of spontaneously broken lepton number possess a massless Majoron, when explicit L-violating terms are included, the Majoron acquires a squared-mass proportional to the relevant explicit lepton-number-violating term. Two questions immediately arise: (i) How do we identify the would-be Majoron? and (ii) If explicit lepton-number violation is very small (which is needed to explain the magnitude of neutrino masses), how does one avoid a very light would-be Majoron? These questions have been previously examined in the literature [9,10]. In this paper, we revisit both these questions and demonstrate how they can be addressed in a basis-independent formalism [11,12].

II. THE SCALAR POTENTIAL AND MINIMUM CONDITIONS

In the notation of Ref. [13], the contribution of the neutral scalar fields to the scalar potential, before imposing L conservation, is

$$\begin{aligned}V_{\text{neutral}}=&(m_U^2+|\mu|^2)|h_U|^2+[(M_{\tilde{L}}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta^*]\tilde{\nu}_\alpha\tilde{\nu}_\beta^*\\&-(b_\alpha\tilde{\nu}_\alpha h_U+b_\alpha^*\tilde{\nu}_\alpha^* h_U^*)\\&+\tfrac{1}{8}(g^2+g'^2)[|h_U|^2-|\tilde{\nu}_\alpha|^2]^2,\end{aligned}\tag{1}$$

where h_U is the neutral component of the up-type scalar doublet, and we have combined the neutral component of the down-type scalar doublet, $\tilde{\nu}_0\equiv h_D$ and the three sneutrinos, $\tilde{\nu}_i$ into a generalized sneutrino field $\tilde{\nu}_\alpha$, where $\alpha=0,\ldots,n_g$ (for $n_g=3$ generations). In minimizing the full

[1]This is a feature of both the nonsupersymmetric and supersymmetric doublet Majoron models.

scalar potential, we assume that only neutral scalar fields acquire vacuum expectation values: $\langle h_U\rangle\equiv(1/\sqrt{2})v_u$ and $\langle\tilde{\nu}_\alpha\rangle\equiv(1/\sqrt{2})v_\alpha$. From Eq. (1), the minimization conditions are

$$(m_U^2+|\mu|^2)v_u^*=b_\alpha v_\alpha-\tfrac{1}{8}(g^2+g'^2)(|v_u|^2-|v_d|^2)v_u^*, \tag{2}$$

$$[(M_{\tilde{L}}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta^*]v_\beta^*=b_\alpha v_u+\tfrac{1}{8}(g^2+g'^2)(|v_u|^2-|v_d|^2)v_\alpha^*, \tag{3}$$

where

$$|v_d|^2\equiv\sum_\alpha |v_\alpha|^2. \tag{4}$$

The normalization of the vacuum expectation values has been chosen such that

$$v\equiv(|v_u|^2+|v_d|^2)^{1/2}=\frac{2m_W}{g}=246\ \text{GeV}. \tag{5}$$

It is convenient to introduce two additional quantities. We define

$$M_{\alpha\beta}^2\equiv(M_{\tilde{L}}^2)_{\alpha\beta}+\mu_\alpha\mu_\beta^*-\tfrac{1}{8}(g^2+g'^2)(v_u^2-v_d^2)\delta_{\alpha\beta}. \tag{6}$$

Using this quantity, we can simplify the second minimum condition [Eq. (3)] which now reads[2]

$$M_{\alpha\beta}^2 v_\beta^*=v_u b_\alpha. \tag{7}$$

It is also useful to define the vector c_α as follows:

$$M_{\alpha\beta}^2 b_\beta=|b|^2 c_\alpha, \tag{8}$$

where $|b|^2\equiv\Sigma_\alpha b_\alpha^* b_\alpha$.

III. SPONTANEOUS LEPTON NUMBER VIOLATION IN THE MSSM

We begin by considering the possibility of spontaneous L violation in low-energy R-parity-conserving (RPC) supersymmetry consisting only of the MSSM fields. We impose L conservation on the MSSM Lagrangian, which constrains the scalar potential [Eq. (1)]. In the usual basis choice in which h_D is a Higgs field and $\tilde{\nu}_j$ $(j=1,2,3)$ are the lepton number carrying sneutrino fields, it follows that $\mu_\alpha=(\mu,0,0,0)$, $b_\alpha=(b,0,0,0)$, and $(M_{\tilde{L}}^2)_{j0}=(M_{\tilde{L}}^2)_{0j}=0$. Note that $\tilde{\nu}_0\equiv h_D$ and the $\tilde{\nu}_j$ transform the same way under the SU(3) $\times$SU(2)$\times$U(1) gauge symmetry, but are distinguished by their L quantum numbers: $\tilde{\nu}_0$ is neutral while the $\tilde{\nu}_j$ possess nonzero L. Hence if any one of the $\tilde{\nu}_j$ acquires a vacuum expectation value, L will be spontaneously broken. Noting that $M_{j0}^2=M_{0j}^2=0$ in the basis defined above, Eq. (7) implies that $M_{ij}^2 v_j=0$. Thus, if at least one of the v_j is nonzero, it follows that $\det M^2=0$. This is a necessary (basis-independent) condition for spontaneous lepton number violation.

We assume that $v_u\neq 0$ and $v_0\neq 0$. Without loss of generality, we may perform a rotation of the sneutrino fields among the $\tilde{\nu}_j$ such that $v_1\neq 0$ while $v_2=v_3=0$.[3] It then follows from Eqs. (2) and (3) that

$$(M_{\tilde{L}}^2)_{11}=\tfrac{1}{8}(g^2+g'^2)(v_u^2-v_0^2-v_1^2), \tag{9}$$

$$(m_U^2+(M_{\tilde{L}}^2)_{11}+|\mu|^2)v_u=bv_0, \tag{10}$$

$$(m_D^2-(M_{\tilde{L}}^2)_{11}+|\mu|^2)v_0=bv_u, \tag{11}$$

where $m_D^2\equiv(M_{\tilde{L}}^2)_{00}$. These equations have a consistent solution for nonzero v_u, v_0, and v_1 only if

$$(m_U^2+(M_{\tilde{L}}^2)_{11}+|\mu|^2)(m_D^2-(M_{\tilde{L}}^2)_{11}+|\mu|^2)=b^2. \tag{12}$$

For this very particular choice of parameters, the quantities v_u/v_0 and $v_u^2-v_0^2-v_1^2$ are fixed, but this is not enough information to determine all three vacuum expectation values uniquely at the tree level. That is, there is a flat direction in the scalar potential at tree-level. Reference [9] demonstrates that by considering the renormalization group evolution of the potential parameters, there is generically some momentum scale Q_0 for which Eq. (12) is satisfied. Then, when the one-loop effective potential is evaluated, the flat direction is lifted and the undetermined vacuum expectation value is fixed via dimensional transmutation in terms of Q_0. The parameters of the model must be tuned to get the observed Z mass, $m_Z^2=\tfrac{1}{4}(g^2+g'^2)(v_u^2+v_0^2+v_1^2)$, as well as the correct hierarchy $v_1\ll v$ needed to explain the light neutrino mass.

If lepton number is spontaneously broken, then there must be a massless Goldstone boson—the Majoron [5]. We shall exhibit this explicitly in the case above where Eq. (12) holds. For simplicity, we assume that the model is CP-conserving.[4] We can then compute CP-even and CP-odd scalar squared-mass matrices. In Ref. [12], we showed that after removing

[2]Note that one can always choose the vacuum expectation values v_u and v_α real by suitable phase re-definitions of the scalar fields. Henceforth, we assume that all vacuum expectation values are taken to be real.

[3]If $v_1=0$, then Eqs. (2) and (3) simply reduce to the usual RPC MSSM equations for v_u and $v_d=v_0$. If we had assumed that $v_u=v_0=0$, then one finds that lepton number is spontaneously broken with $v_1^2=-8(M_{\tilde{L}}^2)_{11}/(g^2+g'^2)$. In this case, there is a consistent solution if $(M_{\tilde{L}}^2)_{11}<0$. However, a model with $v_u=v_0=0$ would not generate any quark masses, so we will not consider this case any further.

[4]In a basis where the vacuum expectation values are real, it then follows that μ_α, b_α and $(M_{\tilde{L}}^2)_{\alpha\beta}$ are real.

the Goldstone boson that gives mass to the Z, the CP-odd scalar squared-mass matrix in a general L-violating model is given by

$$M^2_{\rm odd}=\begin{pmatrix} v^2(v\cdot b)/(v_u v_d^2) & v b_\beta X_{\beta i}/v_d \\ v X_{j\alpha}b_\alpha/v_d & X_{j\alpha}M^2_{\alpha\beta}X_{\beta i}\end{pmatrix}, \tag{13}$$

where $v\cdot b\equiv v_\alpha b_\alpha$ and the $X_{\beta i}$ are chosen so that the set $\{v_\beta/v_d, X_{\beta i}\}$ forms an orthonormal set of vectors in an (n_g+1)-dimensional vector space (for n_g generations). In our notation, $X_{j\alpha}\equiv X^T_{\alpha j}$, where the superscript T denotes the matrix transpose. The following relations will be useful:

$$v_\alpha X_{\alpha i}=0,\quad X_{\alpha i}X_{\alpha j}=\delta_{ij},\quad X_{\alpha i}X_{\beta i}=\delta_{\alpha\beta}-\frac{v_\alpha v_\beta}{v_d^2}. \tag{14}$$

To show that there is a Majoron in the case of spontaneously broken L, we exhibit the eigenvector of $M^2_{\rm odd}$ with zero eigenvalue. Consider the eigenvector:[5]

$$J_\beta\equiv\begin{pmatrix} \dfrac{-v_u}{v} \\ \dfrac{v_d(v\cdot b)b_\rho X_{\rho i}}{|v\times b|^2}\end{pmatrix}. \tag{15}$$

A simple calculation shows that $(M^2_{\rm odd})_{\alpha\beta}J_\beta=0$ [after applying Eq. (7)], if

$$(v\cdot b)M^2_{\alpha\beta}b_\beta-v_u b^2 b_\alpha=0. \tag{16}$$

It is easy to check that Eq. (16) is satisfied under the assumption of L conservation of the MSSM Lagrangian.[6] It is interesting to note that Eq. (16) can be written more simply as $b_\alpha=(v\cdot b/v_u)c_\alpha$, where c_α is defined in Eq. (8). It then follows that $|b\times c|^2=0$, and we conclude that the necessary and sufficient basis independent condition for spontaneously broken lepton number is $|b\times c|^2=0$, with $|v\times b|^2\neq 0$.[7]

We now turn to the CP-even scalar that is associated with the CP-odd Majoron. Again following Ref. [12], the CP-even scalar squared-masses of the model can be determined by computing the eigenvalues of the following squared-mass matrix:

$$M^2_{\rm even}=\begin{pmatrix} m_Z^2\cos^2 2\beta & -m_Z^2\sin 2\beta\cos 2\beta & 0 \\ -m_Z^2\sin 2\beta\cos 2\beta & m_Z^2\sin^2 2\beta+v^2(v\cdot b)/(v_u v_d^2) & -v b_\beta X_{\beta i}/v_d \\ 0 & -v X_{j\alpha}b_\alpha/v_d & X_{j\alpha}M^2_{\alpha\beta}X_{\beta i}\end{pmatrix}, \tag{17}$$

where $\tan\beta\equiv v_u/v_d$, with v_d given by Eq. (4). First, we note that if $\cos 2\beta=0$, then there is a massless scalar state at the tree level in all circumstances (i.e., conserved L, spontaneously broken L or explicitly broken L). In the case of spontaneously broken L, we can identify this state as the massless scalar state associated with the Majoron. Henceforth, we shall assume that $\cos 2\beta\neq 0$. Then, one can easily verify that the eigenvector

$$\rho_\beta\equiv\begin{pmatrix} \dfrac{v_u\sin 2\beta}{v\cos 2\beta} \\ \dfrac{v_u}{v} \\ \dfrac{v_d(v\cdot b)b_\rho X_{\rho i}}{|v\times b|^2}\end{pmatrix}, \tag{18}$$

satisfies $(M^2_{\rm even})_{\alpha\beta}\rho_\beta=0$ provided that Eq. (16) holds. That is, there exists a massless CP-even scalar at the tree level, ρ, associated with the massless Majoron, J. When radiative corrections are incorporated, the mass of ρ is not protected (it is not a Goldstone boson). Thus, ρ gains a small mass of $\mathcal{O}(v_1)$. Nevertheless, the experimental absence of the decay $Z\to J\rho$ implies that the model of spontaneously broken R-parity described above is ruled out.

IV. EXPLICIT *L* VIOLATION AND THE WOULD-BE MAJORON

We now consider the introduction of explicit L-violating terms. Clearly, the Majoron eigenstate identified in Eq. (15) is no longer an eigenstate of the CP-odd squared-mass matrix. But, to the extent that explicit L-violation is small, the Majoron identified above is an approximate eigenstate, but with a nonzero mass. We denote this state as the would-be Majoron. It is a simple matter to use first-order perturbation theory to compute its mass.

[5]Although the cross product technically exists only in three dimensions, the dot product of two cross products can be expressed in terms of dot products and thus exists in any number of dimensions. For example, $|v\times b|^2=v_d^2b^2-(v\cdot b)^2$. Note that by assumption in this calculation, $b_\alpha=(b,0,0,0)$ and $v_\alpha=(v_0,v_1,0,0)$ with $v_1\neq 0$. Hence $|v\times b|^2\neq 0$.

[6]L conservation implies that one can choose a basis in which $M^2_{0j}=M^2_{j0}=b_j=0$. Equation 7 then implies that $M^2_{00}=v_u b/v_0$.

[7]Note that from Eq. (7), $|v\times b|^2=0$ implies that $|b\times c|^2=0$, but the converse is true only if $M^2_{\alpha\beta}$ is an invertible matrix. But, we noted previously that $\det M^2=0$ is a necessary condition for spontaneously broken lepton number.

Suppose we write: $M^2_{\rm odd}=M^{(0)2}_{\rm odd}+M^{(1)2}_{\rm odd}$, where J_β [Eq. (15)] is the eigenvector of $M^{(0)2}_{\rm odd}$ with zero eigenvalue. Using first order perturbation theory, the squared-mass is computed by evaluating the expectation value of $M^{(1)2}_{\rm odd}$ with respect to the unperturbed normalized eigenvalue (i.e., J_β normalized to unit length). Since the unperturbed Majoron is massless, this is equivalent to computing the expectation value of the full squared-mass matrix $M^2_{\rm odd}$. Thus, the squared-mass of the would-be Majoron, J, is

$$m_J^2=\frac{(M^2_{\rm odd})_{\alpha\beta}J_\alpha J_\beta}{N_o^2}, \tag{19}$$

where $N_o^2\equiv\Sigma_\alpha J_\alpha J_\alpha$. After much algebraic simplification, the end result is

$$m_J^2=\frac{v_d^2v^2(v\cdot b)[(v\cdot b)M^2_{\alpha\beta}b_\alpha b_\beta-v_ub^4]}{|v\times b|^2[v_u^2|v\times b|^2+v^2(v\cdot b)^2]}, \tag{20}$$

where $b^2\equiv\Sigma_\alpha b_\alpha b_\alpha$. It is useful to define the basis-independent quantity:

$$v_L^2\equiv\frac{|v\times b|^2}{b^2}=v_d^2-\frac{(v\cdot b)^2}{b^2}. \tag{21}$$

Note that in a basis where $b_j=0$, one obtains $v_L^2\equiv v_d^2-v_0^2=\Sigma_i v_i^2$. That is, $v_L\ll v_d$, assuming that L-violating effects are small. Hence, we can drop the first term relative to the second in the denominator of Eq. (20). In addition, using the definition of c_α [Eq. (8)], the above result can be further simplified. We then obtain

$$m_J^2=\frac{v_d^2(v\times b)\cdot(b\times c)}{(v\cdot b)v_L^2}. \tag{22}$$

Note that if we go to the spontaneous L-violating limit in which $|b\times c|=0$ [with $v_L\neq 0$], one finds a massless Majoron as expected. Further, in the case of explicit L violation, it is easy to check that $v_L\neq 0$.[8] One notable feature of Eq. (22) is that it provides a basis-independent expression for the mass of the would-be Majoron.

Finally, we can address the puzzle of how the would-be Majoron mass can be of $\mathcal{O}(m_Z)$ even if the explicit L violation is small [10]. It is convenient to choose a basis in which $b_i=0$. Using the minimum condition [Eq. (7)] and Eq. (20), and assuming that $v_L\ll v$, we end up with

$$m_J^2=\frac{\sum_{ij} M^2_{ij}v_iv_j}{\sum_i v_i^2}\left[1+\mathcal{O}\left(\frac{v_L^2}{v^2}\right)\right]. \tag{23}$$

To understand the physical implication of this result, let us choose the direction of v_i to point along the kth direction. Then $m_J^2=M^2_{kk}$. But, in the limit of small explicit L violation, M^2_{kk} is the squared-mass of the kth sneutrino (in the RPC limit). Thus we have identified the would-be Majoron as one of the sneutrinos. Since the model parameters can easily be chosen such that $M^2_{kk}\sim\mathcal{O}(v^2)$, we see that there is no contradiction in having the would-be Majoron mass of $\mathcal{O}(v)$, even in the limit of arbitrarily small explicit L breaking [10]. Nevertheless, the limit of vanishing explicit lepton number violation is smooth. In particular, note that for $b_i=0$, Eq. (7) implies that $M^2_{i0}v_0=-M^2_{ij}v_j$. In the limit of an L-conserving Lagrangian in which L is spontaneously broken, $M_{i0}=0$ while one of the v_i is nonzero. This implies that $M^2_{ij}v_iv_j=0$ and the massless Majoron is regained.

These results can also be understood in a basis-independent language using the results of Eq. (22). The squared-mass of the would-be Majoron is proportional to the dimensionless ratio of two small parameters, $(v\times b)\cdot(b\times c)/[(v\cdot b)v_L^2]$. The numerator is a consequence of explicit L breaking and the denominator is proportional to the square of the sneutrino vacuum expectation value in the case of spontaneous L breaking. Nevertheless, the ratio of these two small quantities can be $\mathcal{O}(1)$, in which case m_J is of order the electroweak scale.

To see that this last result does not contradict our usual intuition about explicit symmetry breaking, consider for simplicity the one generation case. Then, we can write $m_J^2=M^2_{11}=-M^2_{10}v_0/v_1$. We then see explicitly that m_J^2 is *linear* in the explicit L-violating parameter M^2_{10}. Nevertheless, in the limit of small M^2_{10}, because M^2_{10}/v_1 can be of the same order as v_0, it follows that m_J^2 can be of $\mathcal{O}(v^2)$ without an unnatural tuning of the parameters. A simple exercise shows that this is in accord with the expectations of Dashen's formula [14]. For example, consider the linear O(4) sigma model [15] consisting of σ and $\vec{\pi}$, with the usual Mexican hat potential and corresponding vacuum expectation value v. If we now break the O(4) symmetry with $\mathcal{L}_{\rm break}=a\sigma$, then the Goldstone boson (π) acquires a mass that is linear in a and is given by Dashen's formula:

$$v^2m_\pi^2=\langle 0|[Q,[Q,\mathcal{L}_{\rm break}]]|0\rangle=av, \tag{24}$$

where Q is the Noether symmetry charge and $v=\langle 0|\sigma|0\rangle$ is the vacuum expectation value in the absence of explicit symmetry breaking. Thus, $m_\pi^2=a/v$, which has the same behavior as $m_J^2\propto M^2_{10}/v_1$. Of course, in QCD the relevant chiral symmetry breaking parameters are such that $m_\pi\ll\Lambda\sim 4\pi v$ [16]. In contrast, one must choose $M^2_{10}\sim\mathcal{O}(v_0v_1)$ in order to ensure that the sneutrino mass is of order the electroweak scale (light sneutrinos are ruled out by the absence of Z decay into sneutrino pairs).

[8]In a basis where $b_i=0$, $v_L=0$ implies that $v_i=0$. Then from Eq. (7) one obtains $M^2_{i0}=0$. In this case, barring the unlikely cancellation $M^2_{i0}=(M^2_{\tilde L})_{i0}+\mu_i\mu_0=0$ for nonvanishing $(M^2_{\tilde L})_{i0}$ and μ_i, it follows that the scalar potential is L conserving in contradiction to our assumption.

For completeness, we evaluate the mass of the CP-even scalar ρ associated with the Majoron when explicit L violation is introduced. Following the method of computation of m_J^2, we again use first-order perturbation theory. Writing $M^2_{\rm even}=M^{(0)2}_{\rm even}+M^{(1)2}_{\rm even}$, and using the fact that ρ_β [Eq. (18)] is an eigenvector of $M^{(0)2}_{\rm even}$ with zero eigenvalue, it follows that

$$m_\rho^2=\frac{(M^2_{\rm even})_{\alpha\beta}\rho_\alpha\rho_\beta}{N_e^2}, \tag{25}$$

where $N_e^2\equiv\Sigma_\alpha\rho_\alpha\rho_\alpha$. The end result is

$$m_\rho^2=\frac{v_d^2v^2(v\cdot b)(v\times b)[(v\cdot b)M^2_{\alpha\beta}b_\alpha b_\beta-v_u b^4]\cos^2 2\beta}{|v\times b|^2[v_u^2|v\times b|^2+v^2(v\cdot b)^2\cos^2 2\beta]}. \tag{26}$$

As noted previously [see discussion below Eq. (17)], if $\cos 2\beta=0$, then $m_\rho=0$ is an exact tree-level result, even in the presence of L-violating terms. Assuming that $|\cos 2\beta| \gg v_L/v$ and that L-violating effects are small, we may again drop the first term relative to the second in the denominator of Eq. (26). As before, we obtain

$$m_\rho^2=\frac{v_d^2(v\times b)\cdot(b\times c)}{(v\cdot b)v_L^2}. \tag{27}$$

That is,

$$m_\rho^2=m_J^2\left[1+\mathcal{O}\left(\frac{v_L^2}{v^2}\right)\right]. \tag{28}$$

Following the discussion below Eq. (23), we identify ρ as a sneutrino (in the RPC limit). Moreover, since ρ and J are degenerate in the RPC limit, these two real scalars can be combined to make a (complex) sneutrino state of definite lepton number [17].

At the tree level, the squared-mass splitting, $\Delta m^2\equiv m_\rho^2-m_J^2$ is nonzero when explicit L-violation is present. The analysis above seems to imply that $\Delta m^2\sim\mathcal{O}(v_L^2/v^2)$. However, an explicit expression for Δm^2 to first order in v_L^2/v^2 would require a second-order perturbation theory computation of m_J^2 and m_ρ^2. In the presence of explicit L violation, if m_J, $m_\rho\sim\mathcal{O}(v)$ then we may use the results of Ref. [12] to obtain a basis-independent expression for Δm^2. This case corresponds to sneutrino masses of order the electroweak scale, and we indeed verify that $\Delta m^2\sim\mathcal{O}(v_L^2/v^2)$. On the other hand, if m_J, $m_\rho\ll v$, then the results of Ref. [12] do not directly apply, since there is an independent small parameter which must be treated consistently in the expansion around the L-conserving limit. In this case, the tree-level value of Δm^2 can be significantly smaller than $\mathcal{O}(v_L^2/v^2)$. Consequently, one must not neglect the radiative corrections that could end up as the dominant contribution to the squared-mass difference.

V. CONCLUSIONS

In models of R-parity-violating supersymmetry, there is no longer a distinction between the hypercharge $Y=-1$ Higgs superfield and the lepton superfields. In computing physical quantities involving the scalar Higgs and slepton sectors, one can either choose a basis in the generalized Higgs-lepton flavor space or employ basis-independent techniques. For example, one could choose to define the Higgs field direction so that the neutral slepton vacuum expectation values vanish. However, in this case, the distinction between spontaneous lepton number violation (typically associated with non-zero sneutrino vacuum expectation values) and explicit lepton number violation is unclear. By employing basis-independent methods, we are able to provide an unambiguous condition for the existence of spontaneous lepton number violation.

In the latter case, the spectrum contains a massless Goldstone boson—the CP-odd Majoron. The simplest models of this type also predict the existence of a very light CP-even scalar partner. Such models are ruled out by precision Z decay data. Thus, any realistic L-violating model (based solely on the superfields of the MSSM) must contain some explicit L breaking. The would-be Majoron acquires a squared-mass that depends linearly on the explicit L-breaking squared-mass parameter. We demonstrate how to compute the mass of the would-be Majoron using basis-independent techniques, and identify this CP-odd scalar and its CP-even scalar partner as approximate sneutrino states. Finally, we have shown how it is possible for the mass of the would-be Majoron and its CP-even scalar partner to be of $\mathcal{O}(v)$ despite the fact that the explicit L violation must be small enough to account for neutrino masses less than of $\mathcal{O}$(eV).

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